Ultrasonic backscattering from human tissue: a realistic model

1977 Phys. Med. Biol. 22 317

(http://iopscience.iop.org/0031-9155/22/2/011)

PHYS. MED. BIOL., 1977, VOL. 22, NO. 2, 317–326.

Ultrasonic Backscattering from Human Tissue: A Realistic Model

J. C. GORE and S. LEEMAN

Department of Medical Physics, Royal Postgraduate Medical School, Hammersmith Hospital, London W12 0HS

Received 5 *August* 1976

ABSTRACT. The propagation of ultrasound pulses in inhomogeneous media is described, and it is shown that they are scattered by fluctuations in density and compressibility. It is proposed that some of the echoes recorded by diagnostic pulse echo equipment are produced in this way. The precise form of the acoustic field backscattered from tissues is calculated using realistic approximations about the nature of tissue inhomogeneities and the form of the pulses used. It is shown that only limited information about tissue structure is contained in these signals, and the restrictions imposed by the use of typical pulses are indicated. The implications of this analysis for methods of tissue characterization and clinical imaging are discussed.

1. Introduction

The sequences of echoes produced when tissue is irradiated with ultrasound pulses contain not only the relatively isolated and distinct echoes, which are generated at abrupt changes in the physical composition of the tissues, but also the numerous low-level signals which are generally held to carry information about the internal structure of the media investigated. All echoes derive from the same basic physical interactions in tissue, but the former class are generated approximately specularly by structures large compared to the dominant wavelength in the pulse, λ, while the latter class come from the small scale ($\lesssim \lambda$) structures which scatter the incident radiation into other directions. Interest in analysing the scattered component as a means of tissue characterization has been renewed recently by a number of papers on the subject, showing a variety of approaches to the problem: the backscattered signal from a small region of tissue has been analysed for its angular dependence (Nicholas and Hill 1975) and spectral content (Chivers and Hill 1975, Lele and Senapati 1975), the frequency dependence of the scattering into a fixed angle has been demonstrated to contain structural information (Waag and Lerner 1973) while the amplitudes of the sequence of echoes from a fixed direction through tissue have been utilized to provide information of diagnostic significance with varying degrees of success (Mountford and Wells 1972, Ossoinig 1974). Waag and Lerner (1973) interpreted the scattering as being generated from velocity variations, but little has otherwise been published on the quantitation of realistic physical mechanisms for the ultrasound–tissue interaction. Moreover, experiments are often described in terms of continuous wave fields, which gives rise to conceptual difficulties when considering the transient nature of the ultrasound excitations used in practice (Chivers and Hill 1975) although it must be pointed out that the

time-independent (continuous wave) and time-dependent (pulse–echo) formulations of scattering theory are known to be equivalent (Goldberger and Watson 1964).

For a true understanding of the ultrasound–tissue interaction, a precise specification of the scattering elements in tissue (a tissue model) is important and a theoretical treatment such as that presented here can tell whether a postulated tissue model may be characterized by, or even reconstructed from, the scattering data. It also indicates what experiments are, in fact, necessary, for unique tissue characterization but the relevance of the model to human tissue (or rather, to the true scattering elements in that medium) can be deduced only from direct observations. In this paper, a tissue model is chosen on the basis of simple but realistic assumptions and the scattering of a typical diagnostic pulse is calculated. The information which may in principle be derived from a simple backscattering experiment is seen, and the light this sheds on the work of the authors mentioned above is reviewed. Modifications introduced by scanner parameters such as emission pulse shape are explicitly demonstrated, so that a meaningful comparison between different workers may be made.

Ideally, the scattering from a particular region should be specified in a way which allows the effect of the overlying tissue to be easily quantified; in particular, the effect of frequency dependent attenuation should be included. For simplicity, however, such absorption effects are not considered here, but the introduction of simple exponential absorption leads to only minor changes in the theory. Furthermore, for scattering data to provide meaningful diagnostic information, a demonstration of the uniqueness of the solution of the inverse scattering problem is essential, to ensure that the echo field is unambiguously characteristic of the tissue structure. No attempt is made to prove this here, but uniqueness for a similar class of interactions has been established (Leeman 1970) so that it may be tentatively assumed that no fundamental problems will arise in this context.

The recent advent and success of grey-scale imaging systems has underlined the importance of understanding the scattering which generates the patterns of echoes from inside tissues that are widely recognized as an aid to interpreting scans and diagnosis (Kossoff 1972). To date, such images have not been successfully analysed quantitatively, although their potential use as indicators of the changes produced in tissue by disease has been discussed (Rettenmaier 1974, Taylor 1974). It is hoped that the fuller understanding of the dominant scattering processes presented here may make it possible to extract diagnostically useful parameters from a direct analysis of the B-scan pattern itself.

2. The wave equation for ultrasound propagation through tissue

Sound waves in homogeneous media are described by a homogeneous wave equation which predicts that plane waves of sound propagate without scattering if the acoustic properties are uniform. The density and compressibility of small tissue samples fluctuate from place to place about their mean values so that in any region the local acoustic properties differ from the average. Sound waves

will be scattered in such an inhomogeneous medium and the echoes from inside tissue can be attributed to density-compressibility fluctuations, which may be distributed randomly or regularly throughout the tissue. Such a model includes as special cases the approaches of Waag and Lerner (1973) and Atkinson and Berry (1974), and is consistent with the observations of Fields and Dunn (1973). No assumptions are necessary as to the random nature, or otherwise, of the variables of interest, but the weakness of the scattering observed in practice implies that the magnitude of fluctuations may be considered to be small.

It is convenient to consider formally the scattering region V to be embedded in some non-dispersive medium with constant density, ρ_0, and compressibility $\kappa_0 = (\rho_0 c_0^2)^{-1}$, with c_0 the acoustic velocity in the embedding medium. The values of these parameters are chosen to be the mean values they assume inside V. Inside the region of inhomogeneity the equation for acoustic propagation may be written (Morse and Ingard 1968)

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}\gamma(\boldsymbol{r}) + \nabla\cdot(\mu(\boldsymbol{r})\nabla p) \tag{1}$$

where $p(\boldsymbol{r}, t)$ is the acoustic pressure at position $\boldsymbol{r}$ at time t,

$$\gamma(\boldsymbol{r}) \equiv \frac{\kappa - \kappa_0}{\kappa_0}$$

$$\mu(\boldsymbol{r}) \equiv \frac{\rho - \rho_0}{\rho}$$

with $\kappa(\boldsymbol{r})$ and $\rho(\boldsymbol{r})$ denoting the compressibility and density in V. Outside V both γ and μ vanish. The wave equation (1) may be shown to be the same as that considered by Chernov (1960).

Expressing eqn (1) as

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -f(\boldsymbol{r}, t) \tag{2}$$

emphasizes that the source term for the scattering, $f(\boldsymbol{r}, t)$, vanishes for uniform media ($\mu = \gamma = 0$). It is possible to rewrite eqn (2) using a time-dependent Green's function to formulate the scattering solution as

$$p(\boldsymbol{r}, t) = \int \mathrm{d}^3\boldsymbol{r}' \int \mathrm{d}t' f(\boldsymbol{r}', t')\, g(\boldsymbol{r}, t;\, \boldsymbol{r}', t') \tag{3}$$

with the Green's function

$$g(\boldsymbol{r}, t;\, \boldsymbol{r}', t') = \frac{\delta[t - t' - (1/c_0)|\boldsymbol{r} - \boldsymbol{r}'|]}{4\pi|\boldsymbol{r} - \boldsymbol{r}'|}.$$

For weak scattering the sound field within V may be written as the sum of the incident field p_i, and a weak scattered field p_s

$$p = p_i + p_s \quad \text{with} \quad \frac{|p_s|}{|p_i|} \ll 1.$$

To first approximation (the Born approximation) the scattered field (measured

at some location and time for which there is no contribution from the incident field) is given by

$$p_s(\boldsymbol{r},t) = -\int \mathrm{d}^3\boldsymbol{r}' \int \mathrm{d}t' \left[\frac{1}{c_0^2}\frac{\partial^2 p_i(\boldsymbol{r}',t')}{\partial t^2}\gamma(\boldsymbol{r}') + \nabla\cdot(\mu(\boldsymbol{r}')\nabla p_i(\boldsymbol{r}',t'))\right] g(\boldsymbol{r},t;\boldsymbol{r}',t) \quad (4)$$

where the vector differentiation operator ∇ is understood to be with respect to $\boldsymbol{r}'$.

Multiple scattering effects have been neglected, which is consistent with the assumption of weak scattering, and the possibility of a strongly reflecting interface within the region of interest V is clearly excluded. The latter circumstance may, however, be handled by regarding the interface as a boundary between two smaller regions of weak scattering, provided that the coupling between the two sub-regions is adequately described. Ultrasound propagation by modes other than purely longitudinal is neglected, not only for reasons of simplicity, but also because their significance is not well documented or understood for scattering from tissue.

3. The backscattering of pulses

The incident field, p_i, of eqn (4) is a solution of the wave equation in the embedding medium ($f(\boldsymbol{r},t) = 0$) and can be chosen to represent an ultrasound pulse in realistic conformance with those generated by practical pulse–echo scanners. One such representation (Atkinson and Berry 1974), for a pulse propagating with fixed shape and group velocity, v, in the z-direction, and located at the origin at $t = 0$ is

$$p_i(\boldsymbol{r},t) = a(z-vt)\,b(h)\exp[\mathrm{i}k_0(z-vt)] \quad (5)$$

where $a(z)$ describes the axial pulse shape, $b(h)$ the transverse beam profile which is assumed axially symmetric and k_0 is the magnitude of the wave vector corresponding to the carrier frequency of the pulse. In practice, eqn (5) approximates closely to the emission from typical pulse–echo equipment (Gore 1976, to be published), with an axial pulse shape:

$$a(z) = a_0 z\exp(-\alpha z) \quad (a_0, \alpha, \text{constants}; z \geqslant 0)$$

and beam profile

$$b(h) = \exp(-\beta^2 h^2) \quad (\beta \text{ constant}).$$

An equivalent representation, and a more convenient one for calculation, is to consider a superposition of travelling plane waves, spread about the carrier wave vector $\boldsymbol{k}_0$:

$$p_i(\boldsymbol{r},t) = \int \mathrm{d}^3\boldsymbol{k}' G(\boldsymbol{k}'-\boldsymbol{k}_0)\exp[\mathrm{i}(\boldsymbol{k}'\cdot\boldsymbol{r}-\omega' t)] \quad (6)$$

with

$$\omega' \equiv c_0|\boldsymbol{k}'|.$$

Expressing the vectors in cylindrical polar coordinates, with the z-axis chosen along the direction of $\boldsymbol{k}_0$,

$$\boldsymbol{k}' \equiv (\eta,\phi,k), \quad \boldsymbol{k}_0 \equiv (0,0,k_0), \quad \boldsymbol{r}' \equiv (h,\theta,z)$$

and assuming axial symmetry, with a separable spectrum

$$G(\boldsymbol{k}' - \boldsymbol{k}_0) = A(k - k_0)\, B(\eta)$$

and supposing that G is sharply peaked in the direction of $\boldsymbol{k}_0$, so that the only $\boldsymbol{k}'$ vectors of interest satisfy

$$\frac{|\eta|}{|k|} \ll 1 \tag{7}$$

then the representation in eqn (6) reduces to (5) with the correspondence

$$v = c_0$$

$$a(z) = \int_{-\infty}^{\infty} A(s) \exp(\mathrm{i}sz)\, \mathrm{d}s$$

$$b(h) = 2\pi \int_0^{\infty} B(\eta) J_0(\eta h)\, \eta \mathrm{d}\eta.$$

J_0 is the zeroth order Bessel function.

Both (5) and (6) are readily shown to be solutions of the wave equation in the embedding medium; a and A are Fourier Transform pairs, whilst b and B are Hankel Transform pairs. The two pulse representations are equivalent, and a knowledge of one is sufficient, in principle, to calculate the other. The limited spectrum condition (7) is valid for typical ultrasound B-scanner pulses (Gore 1976, to be published) but is not a bandwidth restriction on the axial pulse shape.

The representation (6) may be substituted into the scattering integral (3), and the backscattered acoustic pressure at the observation point, $\boldsymbol{R}$, calculated (see fig. 1).

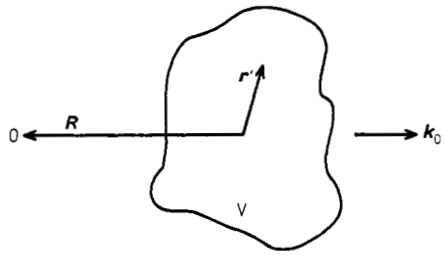

Fig. 1. Definition of coordinates. The sound backscattered from V is calculated at 0, the position of the transducer. The z direction is chosen to lie along k_0.

For simplicity, it is assumed that the observation point $\boldsymbol{R}$ is quite distant from the scattering region (Fraunhofer approximation):

$$\frac{r'}{R} \ll 1 \quad \text{for all } r' \text{ in V}$$

so that, to first order

$$|\boldsymbol{R} - \boldsymbol{r}'| \approx R + z$$

and

$$|\boldsymbol{R}-\boldsymbol{r}'|^{-1}\approx R^{-1}.$$

In a typical pulse-echo backscattering experiment, $\boldsymbol{R}$ denotes both the observation point and the site of generation of the emission pulse. It is more in accord with the usual description to measure the time t from the emission instant so that at $t = 0$ the incident pulse is located at $\boldsymbol{R}$; the scattering integral (3) may then be evaluated, neglecting all second order terms, to give the back-scattered echo train at $\boldsymbol{R}$:

$$p_s(\boldsymbol{R},t) = \frac{1}{4\pi R}\int_0^\infty \eta\,\mathrm{d}\eta \int_0^{2\pi}\mathrm{d}\phi \int_{-\infty}^{\infty}\mathrm{d}k \int_0^\infty h\,\mathrm{d}h \int_0^{2\pi}\mathrm{d}\theta \int_{-\infty}^{\infty}\mathrm{d}z\, A(k-k_0)\,B(\eta)\,I(h,\theta,z)$$
$$\exp\left[\mathrm{i}\eta h\cos(\phi-\theta)\right]\exp\left[2\mathrm{i}k(R+z)-\mathrm{i}c_0 kt\right]$$

with

$$I(h,\theta,z)\equiv k^2[\mu(h,\theta,z)+\gamma(h,\theta,z)]-\mathrm{i}\left[\eta\cos(\phi-\theta)\frac{\partial\mu}{\partial h}+\frac{\eta}{h}\sin(\phi-\theta)\frac{\partial\mu}{\partial\theta}+k\frac{\partial\mu}{\partial z}\right].$$

After some algebraic manipulation, and without introducing any further approximations, the above integral reduces to

$$p_s(\boldsymbol{R},t) = \frac{1}{4\pi R}\int k^2\,\mathrm{d}k\int h\mathrm{d}h\int\mathrm{d}z\int\mathrm{d}\theta\, A(k-k_0)\,b(h)\,[\gamma(h,\theta,z)-\mu(h,\theta,z)]$$
$$\exp\left[\mathrm{i}k(2z+2R-c_0 t)\right] \qquad (8)$$

providing that μ is differentiable inside V. On introducing the following notation

$$\bar{f}(h,z) = \frac{1}{2\pi}\int_0^{2\pi} f(h,\theta,z)\,\mathrm{d}\theta$$

$$f_b(\theta,z)\equiv\int_0^\infty h f(h,\theta,z)\,b(h)\,\mathrm{d}h$$

eqn (8) reduces to

$$p_s(\boldsymbol{R},t) = \frac{1}{2R}\int_{-\infty}^{\infty}\mathrm{d}k\,A(k-k_0)\,k^2[\Gamma_\gamma(2k)-\Gamma_\mu(2k)]\exp[\mathrm{i}k(2R-c_0 t)] \qquad (9)$$

where

$$\Gamma_\gamma(2k)\equiv\int_{-\infty}^{\infty}\mathrm{d}z\;\bar{\gamma}_b(z)\exp(2\mathrm{i}kz),\quad\text{and similarly for }\Gamma_\mu.$$

The Fourier transform of the scattered pressure wave is

$$P_s(\boldsymbol{R},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p_s(\boldsymbol{R},t)\exp(\mathrm{i}\omega t)\,\mathrm{d}t$$
$$= \frac{1}{2Rc_0}\left(\frac{\omega}{c_0}\right)^2 A\left(\frac{\omega}{c_0}-k_0\right)\Gamma\left(\frac{2\omega}{c_0}\right)\exp(2\mathrm{i}\omega R/c_0)$$

so that

$$|P_s(\boldsymbol{R},\omega)|^2 \propto \omega^4 \left| A\left(\frac{\omega}{c_0} - k_0\right)\right|^2 \left|\Gamma\left(\frac{2\omega}{c_0}\right)\right|^2 \tag{10}$$

with

$$\Gamma = \Gamma_\gamma - \Gamma_\mu.$$

If the assertion of Fields and Dunn (1973) is generally valid, viz. that the ultrasound scattering in soft tissue is generated mainly by the connective tissues of the organ scanned, then Γ will be approximately the spatial frequency spectrum of the connective tissue matrix only.

4. Discussion and implications of results

The backscattered train of echoes has been shown in the previous section to depend not only on the intrinsic properties of the tissue itself (γ, μ) but also on the interrogating pulse shape. These relationships are made explicit in eqn (9) above. Of course, in practice, there is a further modification by the spectral transfer properties of the receiver and display units, as well as by the frequency response and sampling characteristics of the transducer in reception (Chivers and Hill 1975, Gore and Leeman 1975) but these may, in principle, be accurately determined independently of the backscattering experiment.

The influence of pulse shape is not trivial; it is apparent that, as a consequence of the angle θ and transverse profile h averaging implied in eqn (9), a knowledge of the time course of the back-scattered echo amplitudes at a fixed observation point is not sufficient, in general, to determine uniquely the elasticity and density variations in the tissue being probed, even in the case of weak scattering considered here. Translated into pulse-echo terminology, this implies that the A-scan display of the backscattered echoes does not reflect directly the scattering from tissue 'microstructure', but only from a pulse-smoothed version of that structure. Thus, there is no guarantee, except in the simplest of cases, that scanning with another transducer, producing a different beam profile, even at the same carrier frequency, will generate the same backscattered field, since the smoothing operation will be different in the two cases. Two experiments may produce, in extreme cases, apparently contradictory results for back-scattering from the same organ, a fact to be borne in mind when assessing grey-scale images.

The tissue property that is measured by backscattering analysis is neither γ nor μ, but their smoothed out difference, Γ. Moreover, only those spatial frequency components of Γ that are twice those present in the axial pulse shape contribute to the scattering, and are, in principle at least, measurable. For typical pulses in the 'diagnostic' carrier frequency range $(1 \rightarrow 5\,\text{MHz})$ structure information in the spatial frequency range $\sim 2 \rightarrow 50$ cycles mm^{-1} only is seen. Conclusions about tissue characteristics made on the basis of A- (and B-) scans relate, therefore, primarily to the ordering and distribution of aggregates of cells, which may or may not be too large to be diagnostically significant,

depending on the circumstances. The same considerations are seen, from eqn (10), to apply to observations in which the power spectrum of the echo signals is analysed.

The usefulness of any particular method of identifying or characterizing tissue is seen to be dependent on the nature of the structure of that tissue itself. If the distribution of scattering elements is isotropic and homogeneous, then the backscattered echo sequences from all directions through that medium will be equivalent, although, as we have shown above, two different tissues (different γ, μ but same Γ) may, in principle, give the same results to first order. For a less restricted class of scattering elements, results would not be rotationally invariant. The studies of Chivers and Hill (1975) and Nicholas and Hill (1975) indicate that angle effects are important for many tissues, including liver, in contrast to the findings of Mountford and Wells (1972) which imply directional invariance for that organ. However, even the measurement of the angular variation of the backscattering produced by rotation of the tissue sample about a single axis (Nicholas and Hill 1975) cannot be sufficient to characterize completely a general three-dimensional structure, and is itself useful for a restricted class of structures only, e.g. where a preferred direction, such as a symmetry axis, through the tissue of interest exists, so that the rotation axis may be uniquely prescribed with respect to it.

In general, three independent experiments are necessary to measure a function of three independent variables, such as γ or μ. For tissue, a further series of experiments would be required to unravel the individual γ and μ functions. The above analysis suggests that this is possible, either by measuring more subtle second order effects, or by examining the angular components of the scattered radiation other than the backscattered one (e.g. Waag and Lerner 1973) since the γ and μ source terms individually generate a different angular dependence of the scattered field. But a backscattering technique is obviously attractive technically, and in many cases the only practicable one for *in vivo* investigations, so it seems reasonable to look to methods of characterizing the echo sequence by single-variable analysis (e.g. power spectrum, angle-dependence, autocorrelation, amplitude statistics) of backscattered echo data in the hope that one or more of these may prove sufficiently reliable for diagnostic purposes. The limitations of each approach would have to be established by experiment, until such time as a more general multivariable analysis becomes feasible, if necessary, for *in vivo* assessment.

A true appreciation of the differential diagnostic capability ascribed to grey-scale B-scan images requires not only a knowledge of the scattering processes within tissue, but also quantitative image analysis as well as perception and pattern recognition studies. At least one attempt has been made to quantitate a clinically useful, but subjective, interpretation of grey-scale images (Taylor and Milan 1976) but for such analytic procedures to be universally applicable the influence of scanner characteristics must be clearly outlined and allowed for. This includes specification not only of instrumental effects such as scanner gain settings, but also, as is indicated above, of the scanner–transducer transfer properties and emission pulse shapes.

The tissue model discussed here may be extended to include effects such as absorption, and further study of this is under way. However, it is already clear from the present analysis that the apparently different tissue characterization techniques of several workers, including the autocorrelation studies reported by us (British Medical Ultrasonics Group Annual Meeting, Bristol, December 1975), may be described by the same formalism, which also links such investigations to the quantitative treatment of grey-scale images.

We gratefully acknowledge the guidance and enthusiasm of the late Professor R. Oliver who encouraged this work. We thank Dr. E. W. Emery for his useful discussions and comments. S. L. thanks the Wellcome Trust for financial support.

Résumé

Rétrodiffusion des ondes ultra-sonores des tissus humains: un modèle réaliste

La propagation des impulsions ultra-sonores dans un milieu non homogène est décrite dans exposé qui montre que ces impulsions sont dispersées par fluctuation de densité et de compressibilité. On suggère de produire de cette façon certains des échos enregistrés par du matériel de diagnose par échos pulsométriques. On calcule la forme exacte du champ acoustique rétrodiffusé des tissus en se servant d'approximations réalistes sur la nature des non homogénéités du tissu et la forme des impulsions utilisées. On montre que ces signaux ne contiennent que des informations limitées sur la structure des tissus et l'on indique les restrictions imposées par l'emploi d'impulsions typiques. Les implications de cette analyse pour les méthodes de caractérisation des tissus et la représentation clinique sont exposées.

Zusammenfassung

Überschallrückstreuung von menschlichem Gewebe: ein realistisches Modell

Es wird die Ausbreitung von Ultraschallpulsen in inhomogenen Medien beschrieben und bewiesen, dass sie durch Fluktuationen in Dichte und Kompressibilität gestreut werden. Die von diagnostischen Pulsechogeräten registrierten Echos werden teilweise auf diesen Effekt zurückgeführt. Mit Hilfe realistischer Schätzungen der Form von Gewebeinhomogenität und Form der verwandten Pulse wird die präzise Gestalt des vom Gewebe rückgestreuten akustischen Feldes berechnet. Es wird bewiesen, dass diese Signale nur begrenzte Information zur Gewebestruktur enthalten. Die unter Verwendung typischer Pulse bedingten Restriktionen werden aufgezeigt, die Implikationen dieser Analyse für Methoden der Gewebecharakterisierung und klinischen Abbildung erörtert.

Резюме

Ультразвуковое обратное рассеяние человеческой тканью: реальная модель

Статья описывает распространение ультразвуковых импульсов в неоднородной среде, показано, что они рассеиваются из-за флуктуаций плотности и сжимаемости. Предполагается, что часть эха, регистрируемого диагностическим оборудованием, использующим принцип импульсных эхо-сигналов, возникает по этой причине. Дается расчет точной конфигурации звукового поля, вызываемого обратным рассеянием от ткани, используя реальные приближения для природы неоднородностей ткани и формы используемых импульсов. Показано, что эти сигналы содержат только ограниченные сведения о структуре ткани, также обращается внимание на ограничения, налагаемые использованием типичных импульсов. В статье обсуждаются последствия этого анализа для методов характеристики тканей и клинического воспроизведения изображения.

References

Atkinson, P., and Berry, M. V., 1974, *J. Phys. A: Math. Nucl. Gen.*, **7**, 1293.
Chernov, L. A., 1960, *Wave Propagation in Random Media* (New York: McGraw-Hill).

CHIVERS, R. C., and HILL, C. R., 1975, *Phys. Med. Biol.*, **20**, 799.
FIELDS, S., and DUNN, F., 1973, *J. Acoust. Soc. Am.*, **54**, 809.
GOLDBERGER, M. L., and WATSON, K. M., 1964, *Collision Theory* (New York: John Wiley).
GORE, J. C., and LEEMAN, S., 1975, in *Abstracts, Second European Congress on Ultrasonics in Medicine, Munich.*
KOSSOFF, G., 1972, *Ultrasonics*, **10**, 221.
LEEMAN, S., 1970, *Proc. R. Soc., Lond.* A, **315**, 49.
LELE, P. P., and SENAPATI, N., 1975, in *Abstracts, Second European Congress on Ultrasonics in Medicine, Munich.*
MORSE, P. M., and INGARD, K. N., 1968, *Theoretical Acoustics* (New York: McGraw-Hill).
MOUNTFORD, R. A., and WELLS, P. N. T., 1972, *Phys. Med. Biol.*, **17**, 14.
NICHOLAS, D., and HILL, C. R., 1975, *Nature, Lond.*, **257**, 305.
OSSOINIG, K. C., 1974, in *Proc. Second World Congress on Ultrasonics in Medicine*, Ed. de Vlieger *et al.* (Amsterdam: Excerpta Medica), p. 49.
RETTENMAIER, G., 1974, in *Proc. Second World Congress on Ultrasonics in Medicine*, Ed. de Vlieger *et al.* (Amsterdam: Excerpta Medica) p. 199.
TAYLOR, K. J. W., 1974, *J. Clin. Ultrasound*, **2**, 74.
TAYLOR, K. J. W., and MILAN, J., 1976, *Br. J. Radiol.*, **49**, 519.
WAAG, R. C., and LERNER, R. M., 1973, in *Ultrasonics Symposium Proceedings*, IEEE Cat 73, CHO 807–8SU.