基科 32 曾柯又 2013012266

5.42

(a) $P(X_{(n)} \le 1-\epsilon) = P(X_i \le 1-\epsilon) = \big(P(X_i \le 1-\epsilon)\big)^n$

$\because P(X_i \le \epsilon) = \int_0^{1-\epsilon} \frac{1}{B(1,\beta)}(1-x)^{\beta-1}\mathrm{d}x = \int_\epsilon^1 \beta x^{\beta-1}\mathrm{d}x = 1-\epsilon^\beta$

$\therefore P(X_{(n)} \le 1-\epsilon) = (1-\epsilon^\beta)^n$

令$\epsilon = \frac{t}{n^{1/\beta}}$

有$P(1-X_{(n)} \ge \frac{t}{n^{1/\beta}}) = (1-\frac{t^\beta}{n})^n$

$\lim\limits_{n\to\infty} P(n^{1/\beta}(1-X_{(n)}) \ge t) = e^{-t^\beta}$

即 $\lim\limits_{n\to\infty} P(n^{1/\beta}(1-X_{(n)}) \le t) = 1-e^{-t^\beta}$

(b) $P(X_{(n)} \le \epsilon) = \big(P(X_i \le \epsilon)\big)^n$

而$P(X_i \le \epsilon) = \int_0^\epsilon e^{-x}\mathrm{d}x = 1-e^{-\epsilon}$

取$\epsilon = \ln(\frac{n}{t})$

$P(X_{(n)} \le \ln(\frac{n}{t})) = (1-\frac{t}{n})^n$

$\lim\limits_{n\to\infty} P(X_{(n)} - \ln n \le \ln(\frac{1}{t})) = e^{-t}$

$\lim\limits_{n\to\infty} P(X_{(n)} - \ln n \le t) = \exp(-e^{-t})$

5.43

(a) $P(|Y_n-\mu| \ge \epsilon) = P(|\sqrt{n}(Y_n-\mu)| \ge \sqrt{n}\epsilon)$

$\forall\delta,\ \exists M,$ 使得$\big(1-\int_{-M}^{M}\frac{1}{\sqrt{2\pi}\sigma}e^{-x^2/(2\sigma^2)}\mathrm{d}x\big) < \delta$

而$\forall M$, $\exists N$ 使得$\forall n > N$ $\sqrt{n}\epsilon > M$

可得 $\lim\limits_{n\to\infty} P(|\sqrt{n}(Y_n - \mu)| \geq \sqrt{n}\epsilon) \leq \lim\limits_{n\to\infty} P(|\sqrt{n}(Y_n - \mu)| \geq M) \leq \delta$

由δ的任意性 $\lim\limits_{n\to\infty} P(|(Y_n - \mu)| \geq \epsilon) = 0$

(b) 由 slutsky 定理 $\sqrt{n}g'(\theta)(Y_n - \theta) \xrightarrow{\mathcal{D}} g'(\theta)X$, 其中$X \sim \mathrm{n}(0,1)$,对式子$\sqrt{n}(g(Y_n) - g(\theta)) = \sqrt{n}g'(\theta)(Y_n - \theta)$, 可以得到$\sqrt{n}(g(Y_n) - g(\theta)) \xrightarrow{\mathcal{D}} g'(\theta)X \sim \mathrm{n}(0, \sigma^2[g'(\theta)]^2)$,