1 鎹熷け鍑芥暟浣滀笟

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(a) 鐢诲嚭鍑芥暟鍥惧儚濡備笅:

鍙闃伕滃c鐨勫鍔狅紝鍑芥暟鍥惧儚闀愭笎鐢卞绉板彉寰椾笉瀵圭О銆

們(b) $E(L(\theta,\delta)|\mathbf{x}) = \mathrm{e}^{c\delta}E(\mathrm{e}^{-c\theta}|\mathbf{x}) - c(\delta - E(\theta|\mathbf{x})) - 1$

姹傚鍙緱$c\mathrm{e}^{c\delta}E(\mathrm{e}^{-c\theta}|\mathbf{x}) - c = 0 \Rightarrow \delta = -\frac{1}{c}\log E(\mathrm{e}^{-c\theta}|\mathbf{x})$

鑰$\mathrm{ce}^{c\delta}E(\mathrm{e}^{-c\theta}|\mathbf{x}) - c$闅$\delta$鐨勫崟璋冮拱锛屾瘮鏈佸间负鏈佸皬鍊煎¤涙瑕佹

瑕锛屾墍浠$\delta^{\pi}(x) = -\frac{1}{c}\log E(\mathrm{e}^{-c\theta}|\mathbf{x})$

(c) 杩欓噷鐨$\pi(\theta)$浠 h 〃鍏ㄥ疄杞寸殑鍧囧寑鍒嗗竷鍑芥暟锛屽彲鍏堝

$\pi(\theta) = \frac{1}{D}$鏈鍚庝护$D \to \infty$(鍏跺疄鍚庨潰鐨勮绠椾笉D鏃犲叧)

$f(\mathbf{x},\theta) = C\mathrm{e}^{-\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{2\sigma^2}}\mathrm{e}^{-\frac{n(\bar{x}-\theta)^2}{2\sigma^2}} = p(\mathbf{x})\mathrm{e}^{-\frac{n(\bar{x}-\theta)^2}{2\sigma^2}}$

$m(\mathbf{x}) = \int f(\mathbf{x}|\theta)\pi(\theta)\mathrm{d}\theta = p(\mathbf{x})\int \mathrm{e}^{-\frac{n(\bar{x}-\theta)^2}{2\sigma^2}}\mathrm{d}\theta$

$\pi(\theta|\mathbf{x}) = \dfrac{f(\mathbf{x},\theta)}{m(\mathbf{x})} = \dfrac{\mathrm{e}^{-\frac{n(\bar{x}-\theta)^2}{2\sigma^2}}}{\int \mathrm{e}^{-\frac{n(\bar{x}-\theta)^2}{2\sigma^2}}\mathrm{d}\theta}$

鍗$\pi(\theta|\mathbf{x}) \sim \mathrm{n}(\bar{x}, \frac{\sigma^2}{n})$ 鍙互寰楀埌

$E(\mathrm{e}^{-c\theta}|\mathbf{x}) = M_{\theta|\mathbf{x}}(-c) = \mathrm{e}^{E\bar{x}+\frac{\sigma^2c^2}{2n}}$

$\delta^B(\bar{X}) = \bar{X} - \dfrac{c\sigma^2}{2n}$

(d) $E(\theta|\mathbf{x}) = \bar{x}$

$E(L(\theta,\delta^B)|\mathbf{x}) = \mathrm{e}^{c\delta}E(\mathrm{e}^{-c\theta}|\mathbf{x}) - c(\delta - E(\theta|\mathbf{x})) - 1 = \dfrac{c^2\sigma^2}{2n}$

$E(L(\theta,\bar{x})|\mathbf{x}) = \mathrm{e}^{c\bar{x}}E(\mathrm{e}^{-c\theta}|\mathbf{x}) - c(\bar{x} - E(\theta|\mathbf{x})) - 1 = \mathrm{e}^{\frac{\sigma^2c^2}{2n}} - 1$

(e) $L_2(\theta, a) = |\theta - a|^2$

$E(L_2(\theta, \delta^B)|\mathbf{x}) = E\left[(\bar{x}-\theta)^2 + (\frac{c\sigma^2}{2n})^2 - 2(\bar{x}-\theta)\frac{c\sigma^2}{2n}\Big|x\right] = \frac{\sigma^2}{n} + \left(\frac{c\sigma^2}{2n}\right)^2$

$E(L_2(\theta, \bar{x})|\mathbf{x}) = E\left[(\bar{x}-\theta)^2\Big|x\right] = \frac{\sigma^2}{n}$

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(a) $\because\ n\bar{X} \sim \mathrm{B}(n, \theta)\mathrm{Var}(n\bar{X}) = E(n\bar{X})^2 - (En\bar{X})^2 \neq \theta^2$

(b) $\therefore E\bar{X}^2 = \frac{1}{n^2}E(n\bar{X})^2 = \frac{1}{n^2}\left[n\theta(1-\theta) + n^2\theta^2\right] = \theta^2 + \frac{\theta(1-\theta)}{n}$

$T_n = \bar{X}^2$

$T_n^{(i)} = \left(\frac{\sum_{k=1}^n X_k - X_i}{n-1}\right)^2 = \left(\frac{\sum_{k=1}^n X_k}{n-1}\right)^2 + \left(\frac{X_i}{n-1}\right)^2 - \frac{2X_i(\sum_{k=1}^n X_k)}{(n-1)^2}$

$\sum_{i=1}^n T_n^{(i)} = \frac{(n-2)(\sum_{k=1}^n X_k)^2 + \sum_{i=1}^n X_i^2}{(n-1)^2} = \frac{(n-2)n^2T_n + \sum_{i=1}^n X_i^2}{(n-1)^2}$

$$\begin{aligned} JK(T_n)1 &= nT_n - \frac{n-1}{n}\sum_{i=1}^n T_n^{(i)} \\ &= nT_n - \frac{n(n-2)T_n}{n-1} - \frac{\sum_{i=1}^n X_i^2}{n(n-1)} \\ &= \frac{n}{n-1}T_n - \frac{\sum_{i=1}^n X_i^2}{n(n-1)} \end{aligned}$$

鑰屼0-1鍒嗗竷$X_i = X_i^2$,鏁$JK(T_n) = \frac{n}{n-1}T_n - \frac{\bar{X}}{(n-1)}$

(c) $EJK(T_n) = \frac{n}{n-1}(\theta^2 + \frac{\theta(1-\theta)}{n}) - \frac{\theta}{n-1} = \theta$

(d) 浼姬鍒╋垎甯冨畾睘浜座寨鏁板垎甯冩槑铏庣岃槗鍏涓θ鐨勮畲鍊∠厐鍒
唠粺璁⊄噺銆

鑰$JK(T_n) = \frac{n}{n-1}\bar{X}^2 + \frac{\bar{X}}{n-1}$鏄鐨勮嚧鏁帮紝鍥犳$JK(T_n)$鏄$\theta^2$鐨勬渶
浣虫棤鍋忎及璁

(d)

閿忔湇璇嗘獓楠岃瘉 鍒╋敤R鐢熸垚闅忔満鏁帮紝閫夊彇$\theta = 0.3, 0.5, 0.7$锛屽垎鍒敓鎴0涓狦鏈烘暟(n 涓嶈兘閫夊彇澶 ぇ 锛屽惁鍒$\theta(1-\theta)_{\frac{}{n}}$瓒嬭繎0鑰屽緱涓嶅埌鍙嶅簲)锛岃绠椾及璁 ¢ 嗗$T_n$, $JK(T_n)$锛屽苟閲嶅10000娆★紝鐢熸垚鍗曚釜10000娆＄殑骞冲潎鍊肩粨鏋 互楠岃瘉涓婅堪a),(c)涓ゆ潯閫

們傛偋傈傳偋傰

傪偋備傈	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.7$
θ^2	偦偙偦傪	偦偙偟偲	偦偙偡傪
T_n	偦偙傑傑傑偤傪	偦偙偟偤偲偣偤	偦偙偲傑偦偟偤
JKT_n	偦偙偦傪偦偤偟	偦偙偟偲偦偣偣	偦偙偡偛傪偟偛

T_n鐨勫钩鍧囧兼槑鏄炬瘮θ^2瑕佸ぇ 锛岃JKT$_n$鐨勫钩鍧囧间笌θ^2寰堟帴杩戙

2

們閿忔湇妯℃嫙锛氬瘡〣曞鐝扮殑浼拌

1 姹傚嚭C鐨勫间负$C = 1.0001397$, f鐨勫嚱鏁板泦鍍忓涓嬶細

2 灏嗗瘡〣曞颁笌鐩存帴鍒嗘瀽鐨勫尯闂村悓涓婅堪鍏嬪埄鍏厛缁撴灉姣旇緝锛

瑰＄畻鍑000娆℃ā鍓熸垚鍥寰殑RAISE鍊煎緱鍧囧煎拰鏍瑰樊涓哄叾涓嬶

〃鎵绀

RAISE	mean	sd
n = 100	0.01691	0.00301
n = 1000	0.00750	0.00129

鑰岃饮鍙互鍙戠幇RAISE鐨勫垎甯冨熀鏈湁浠庡鎬佸垎甯璇佹槑鏄惧綏鏈変簨鍥伴毦)銆傜涔嶇瑰泦濡備笅

鍙互鐪嬪嚭鍙杦犻鏁伴噺n瓒婂ぇ锛屼竴鑸瘜銆嶉鏁扮殑浼拌瓒婂ソ銆傝纜浠庢垜浠瑙備笉涔熷鏌撶悊瑙ｃ俗犻鏈秺澶氳紝