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A new model and a reformulation for the crude distillation unit charging problem with oil blends and sequence-dependent changeover costs

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ABSTRACT

In this paper, we address the problem of planning the crude distillation unit charging process with oil blend. It is well known that blending and splitting operations can lead together to both non-linearities and concavities in mathematical programming models. As result, many proposed models for this problem use simplifying assumptions to keep the formulation computationally tractable. However, we show the existence of splitting operations that can lead to inconsistencies in the solutions obtained by the previous MILP models from the literature. Then, we propose a way to address this issue through an aggregated inventory capacity combined with a disaggregation algorithm. Furthermore, we develop a mathematical reformulation that improves the solving efficiency of the method. Then, we report experiments that show that the reformulated MILP model presents significant gains concerning linear relaxation gaps and run times, and the disaggregation algorithm leads to feasible solutions for all the tested instances.

1. Introduction

The crude oil supply process is a crucial part of the oil & gas supply chain, since it supports the connection between the upstream and the downstream stages, i.e. the link between the exploration and production of crude oil and the refining and distribution of its derivatives.

Due to its complexity, this process is usually managed through the hierarchical logistic planning, which is composed by three sequenced and interdependent levels: strategical, tactical and operational (Miller, 2002; Rocha et al., 2009). The decisions made in a previous level directly affect the next one, since they are used as inputs to the next decision-making process.

In the first level, the entire supply chain is regarded in a monthly granularity and a long-term horizon, in order to decide the required amount of produced and imported crude oil streams to attend the refineries portfolios and expected processing yields, which are defined in the petroleum derivatives production planning in order to meet the demand of each market place to be supplied. In a daily basis and a medium-term horizon, the next level splits the aggregated monthly amounts into transportation lots between platforms and refineries. Finally, the operational level unfolds the previous decisions in a short-term and more detailed schedule, defining the crude oil tankers which are going to perform the transportation of the settled lots from platforms to waterway terminals, and the schedule of the pumping lots through the pipelines network from the storage tanks of the waterway terminals to the charging tanks of the refineries, meeting the demand of crude oil blends which continually supply the crude distillation units (CDU) as the planned processing campaign.

In this paper, we study an operational optimization model for the CDU supply process. The research problem consists of scheduling the pumping lots since the tanker arrivals at the waterway terminal berths until the crude oil blend supply for the crude distillation units at the refineries. The crude oil lots are pumped from the vessels to the terminal storage tanks, and then to the refinery charging tanks. The percentage of each different crude oil type that composes the total volume in the tanks at the terminals and refineries are controlled, aiming to assure the quality demanded for the crude oil blend that charges continually the CDUs along the time. The blend composition settles the yields of the derivatives obtained from the distillation process.

The optimization objective is to find out the minimum cost schedule, regarding the costs of waiting times for berthing, unloading, tank inventories (at the terminals and refineries), and charging tank changeovers. The constraints are summarized in the following groups: tanker arrivals and departure rules at the waterway

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terminals; balance and operational limits concerning the inventory in the tankers, terminal storage tanks and refinery charging tanks, including controlled components; demand meeting for the crude oil blend; and operational rules for the CDU charging process.

This problem has already been addressed in Lee et al. (1996) and Yüzgeç et al. (2010), both being considered as a basis to the model proposed in the present paper. Both references regard the following assumptions: the time is discretized; the amount of crude oil remaining in the pipeline is neglected; changeover times are neglected since they are small in comparison with the scheduling horizon; there is perfect mixing in both the storage and the charging tanks, and the additional mixing time is neglected; and concerning the components control, nonlinear mixing equations are reformulated into linear ones, which is possible since this scheduling system involves only mixing operation without splitting operations (Quesada and Grossmann, 1995).

There are two main contributions in this article. First, it presents a new model to the problem that drives a discussion towards the statement in Lee et al. (1996) and Yüzgeç et al. (2010) that this scheduling system involves only mixing operations without splitting operations, consequently allowing the reformulation of nonlinear mixing equations into linear ones. We show that the obtained solutions may indeed contain splitting operations in two dimensions: stages and times. Because of that, the component flows may violate the assumption of perfect mixing in the tanks in an arbitrary way, i.e., the compositions sent from the same source to different branches may be different from each other, each composition being decided by the solver under the linear constraints that are imposed by the model. This behavior, which we call *false split*, does not necessarily correspond to any non-homogenous splitting cause by particularities of the chemical process. This fact occurs because the linear constraints are not sufficient to perfectly represent the set of feasible solutions determined by the perfect mixing assumption. To fix this problem, a different way to handle the flow splitting issue has been developed which avoids the non-linearity through the aggregation of the storage tanks in the Mixed Integer Linear Programming (MILP) formulation. Then, after the MILP model is solved, a disaggregation algorithm is executed to obtain the detailed pumping schedule.

In other front, the MILP formulations found in the literature present more emphasis on the problem representation perspective than on the computational cost and solving efficiency. It can be noticed by the large linear relaxation gaps and non-negligible run times on solving the proposed instances, which are significantly small comparing to realistic ones. It is worth mentioning that the proposed approach previously described can be applied using the same MILP formulation found in the previous works. For that, it suffices to replace the storage tanks by a single aggregated one, and post-process the obtained solution with the proposed disaggregation algorithm. So, as the second main contribution, we develop a reformulation concerning the performance of the MILP solvers when using this formulation. To improve this performance, we propose a reformulation of the changeover variables, based on the approach introduced by Pochet and Wolsey (2006) for the pigment sequencing problem, in order to provide a better quality for both the linear relaxation gaps and the obtained feasible solutions. To evaluate the new formulation, we create 48 instances based on the case study presented in Leiras (2010). Then, we report experiments that show a significant evolution with respect to the original model.

As an example concerning the comparison between the size of instances from the literature and tested instances in the present work, the largest instances contained in Lee et al. (1996), Jia and Ierapetritou (2004), Karuppiah et al. (2008), Yüzgeç et al. (2010), and Chen et al. (2012) present 12 time periods, 3 vessels, 3 tanks on each stage (terminal and refinery), and 2 controlled components. Although, the largest instance solved to optimality by the developed reformulation presents 20 time periods, 6 vessels, 6 storage tanks on each stage, and 3 controlled components.

1.1. Literature review

The problem of scheduling and planning in petroleum companies has appeared since the introduction of linear programming (Symonds, 1955; Manne, 1956; Saharidis et al., 2009). However, the MILP formulation developed by Lee et al. (1996) has been chosen among the main references as a starting point for the reformulation proposed in this article.

Based on Pochet and Wolsey (2006), this problem can be classified as production planning, featuring characteristics of multi-stage discrete lot sizing with multiple items, inventory management with Wagner–Whiting costs (Pochet and Wolsey, 2010), blending, and scheduling with sequence-dependent changeovers. The Wagner–Whiting costs are non-speculative, i.e. constant unit production costs and non-negative unit holding costs (the production and inventory costs at time t are equal or greater than the production cost at time $t+1$, so there is no incentive based on these costs to produce and storage items in order to meet future demand).

Besides the deterministic MILP models with discrete time representation, focus of the present paper, several other approaches have been developed for the problem, e.g. mixed integer non-linear programming (MINLP) models (Karuppiah et al., 2008; Mouret et al., 2011), heuristics and simulation methods (Chryssolouris et al., 2005; Leiras, 2010), continuous representation of time (Reddy et al., 2003, 2004; Jia and Ierapetritou, 2004; Rejowski and Pinto, 2008; Chen et al., 2012), stochastic models in chance constrained or robust optimization (Wang and Rong, 2009; Cao et al., 2009, 2010), as well as hybrid approaches combining some of these issues (Pan et al., 2009).

The previously cited works were not selected for a more detailed comparison against the developed model based on the following reasons: different scope of the problem (Rejowski and Pinto, 2008), since it is focused on detailing the representation of the pipeline operation, including pumping costs and pipeline segments; absence of modeling the changeover costs (Jia and Ierapetritou, 2004; Karuppiah et al., 2008; Mouret et al., 2011), since they are the most relevant costs in the practical instances; significant inferior performance of solvers (Reddy et al., 2003, 2004; Chen et al., 2012; Pan et al., 2009), since they present results only for small instances, most of them with only one level of storage tanks; usage of techniques other than mono-objective (Chryssolouris et al., 2005; Leiras, 2010), respectively simulation and multi-objective optimization; and usage of techniques that are focused on modeling uncertainty (Wang and Rong, 2009; Cao et al., 2009, 2010). To the best of our knowledge, this is the first model proposed in the literature that handles both blending and changeover costs, using MILP formulation without the false split issue. Our choice for the linear formulations (as Lee et al., 1996; Yüzgeç et al., 2010) is related to the solver performance since there is a broader collection of formulations, decomposition techniques and solution algorithms found in the literature (e.g. Pochet and Wolsey, 2006) that allows to solve larger problems. Thus, in addition to the results presented in this paper, we believe that future works will be able to improve even more the solution times.

In the context of MILP models, we highlight the work of Saharidis et al. (2009) that developed an optimization model using an event-based time representation. This reference addresses a different modeling approach which regards only one stage for the storage tanks and there are blending operations without charging tanks (flows sent from the tanks directly to CDUs via manifold). Furthermore, Yüzgeç et al. (2010) changed the formulation of Lee et al. (1996) in order to capture additional problem characteristics and fixed some inconsistencies of the instances. It presented for the research problem a model predictive control (MPC) strategy to determine the optimal control decisions for adjustments along the time. The improved formulation of Yüzgeç et al. (2010) is the chosen basis for the proposed changeover reformulation in this paper.

1.2. Paper organization

The remainder of the paper is organized as follows. Section 2 presents the problem definitions and details the original formulation of Yüzgeç et al. (2010). Section 3 presents the false split issue identified in the literature models and proposes a new approach to deal with this problem. Section 4 details the new proposed model and the developed reformulation. Section 5 describes the proposed instances and presents a evaluation of the new approach and a comparison between the original formulation and the proposed reformulation. At last, Section 6 contains the conclusions of this work.

2. Problem definition

The basic representation of the crude distillation unit charging problem with oil blends and sequence-dependent changeover costs can be verified in Fig. 1.

The optimization problem defined in Yüzgeç et al. (2010), used as initial reference to the present paper is the following. The time horizon is discretized in equal time units indexed by $t \in \{1, \ldots, SCH\}$. We are given NV vessels, each vessel v arriving at the time $TArr_v$ on the unique berth of the waterway terminal to unload one or more crude oil types to the storage tanks. The crude oil types are kept segregated in the vessels but mix when stored in tanks. For each vessel v, let VV_v^0 be the initial volume of each vessel, $CWait_v$ the cost incurred from the waiting time between $TArr_v$ and the beginning of the unloading operation, and $CUnl_v$ the unloading operational cost.

The waterway terminal contains storage tanks $i \in \{1, \ldots, NST\}$, and the components $k \in \{1, \ldots, NCE\}$ are controlled to compose the refinery blend demand. For each charging tank $j \in \{1, \ldots, NBT\}$, DM_j is the related demand which must be sent to one of the NCDU crude distillation units. Let VS_i^0, $VS_i^{\min}$ and $VS_i^{\max}$ be, respectively, the initial, minimum and maximum inventory level of the storage tank i. Analogously, VB_j^0, $VB_j^{\min}$ and $VB_j^{\max}$ are the inventory parameters for the charging tank j. There are storage and charging tank inventory costs along the time horizon, being respectively $CInvST_i$ and $CInvBT_j$.

The crude oil lots are pumped from the vessel v to the storage tank i in a flow range between $FVS_{v,i}^{\min}$ and $FVS_{v,i}^{\max}$, and from the storage tank i to the charging tank j between $FSB_{i,j}^{\min}$ and $FSB_{i,j}^{\max}$. The charging tanks must keep the crude distillation units $l \in \{1, \ldots, NCDU\}$ in continuous supply. When supplying the distillation unit l, the charging tank j must maintain the flow on the range between $FBD_{j,l}^{\min}$ and $FBD_{j,l}^{\max}$. A changeover occurs when some CDU l charged by a tank j changes to another tank j', incurring in a setup cost $CSetup_{j,j',l}$. Neither storage tanks nor charging tanks can send and receive crude oil at the same time t. If one given tank receive a lot of crude oil at time t, we assume the mixture inside this tank as already perfectly homogenous at time $t+1$.

Concerning the controlled components, let $yV_{v,k}$ be the percentage of the component k in the vessel v. The parameters $yS_{i,k}^0$, $yS_{i,k}^{\min}$, $yS_{i,k}^{\max}$, $yB_{j,k}^0$, $yB_{j,k}^{\min}$, and $yB_{j,k}^{\max}$ are the initial, minimum and maximum percentage of the component k in the storage tank i, and the initial, minimum and maximum percentage of the same component in the charging tank j, respectively.

2.1. Original formulation

The decision variables for the original formulation proposed by Yüzgeç et al. (2010) can be defined in five groups: arrival and unloading of the vessels, flows among stages, sending and receiving lots, inventory levels, and changeover occurrences.

2.1.1. Decision variables

Arrival and unloading of the vessels:

- $XF_{v,t}$, $XL_{v,t}$: Binary variables which denote, respectively, the beginning and the end of the unloading operation performed by the vessel v at time t.
- $XW_{v,t}$: Binary variables which denote the occurrence of unloading operation of the vessel v at time t.
- TF_v, TL_v: Integer variables which denote, respectively, the initial and completion time of the unloading operation performed by the vessel v.

Flows among stages:

- $FVS_{v,i,t}$, $fVS_{v,i,k,t}$: Continuous variables which denote, respectively, the total flow and the component k flow between the vessel v and the storage tank i at time t.
- $FSB_{i,j,t}$, $fSB_{i,j,k,t}$: Continuous variables which denote, respectively, the total flow and the component k flow between the storage tank i and the charging tank j at time t.
- $FBD_{j,l,t}$, $fBD_{j,l,k,t}$: Continuous variables which denote, respectively, the total flow and the component k flow between the charging tank j and the crude distillation unit l at time t.

Sending and receiving lots:

- $DSB_{i,j,t}$: Binary variables which denote the occurrence of crude oil transfer between storage tank i and charging tank j at time t.

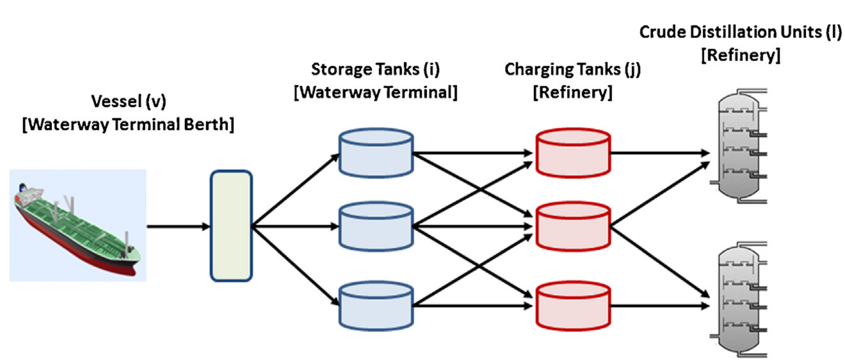

Fig. 1. Representation of the problem.

- $DBD_{j,l,t}$: Binary variables which denote the occurrence of crude oil transfer between charging tank j and crude distillation unit l at time t.

Inventory levels:

- $VV_{v,t}$: Continuous variable which denote the total volume contained in the vessel v at time t.
- $VS_{i,t}$, $vS_{i,k,t}$: Continuous variables which denote, respectively, the total volume and the component k volume contained in the storage tank i at time t.
- $VB_{j,t}$, $vB_{j,k,t}$: Continuous variables which denote, respectively, the total volume and the component k volume contained in the charging tank j at time t.

Changeover occurrences:

- $Z_{j,j',l,t}$: Binary variables which denote the occurrence of a changeover from the charging tank j to the charging tank j' on the supplying for the crude distillation unit l at time t.

Furthermore, the constraints can be defined in four groups: vessels operations, material balance, flow control and charging operations.

2.1.2. Constraints

Vessels operations:

$$\sum_{t=1}^{SCH} XF_{v,t} = 1, \quad \sum_{t=1}^{SCH} XL_{v,t} = 1 \quad \forall v \tag{1}$$

$$TF_v = \sum_{t=1}^{SCH} tXF_{v,t}, \quad TL_v = \sum_{t=1}^{SCH} tXL_{v,t} \quad \forall v \tag{2}$$

$$TF_v \geq TArr_v, \quad TF_{v+1} \geq TL_v + 1, \quad TL_v - TF_v \geq 1 \quad \forall v \tag{3}$$

$$XW_{v,t} \leq \sum_{u=1}^{t} XF_{v,u}, \quad XW_{v,t} \leq \sum_{u=t}^{SCH} XL_{v,u} \quad \forall v, t \tag{4}$$

The constraints (1) assure the uniqueness of the start and completion times of the vessel unloading operation, (2) guarantee the consistency between the $XF_{v,t}$ and the TF_v variables and between the $XL_{v,t}$ and the TL_v variables, (3) establishes the sequence of vessel arrivals and the minimum time for the unloading operations, and (4) defines the unloading occurrence for each time t within the operation time range, i.e. between the vessel arrival and departure.

Material balance:

$$VV_{v,t} = VV_v^0 - \sum_{i=1}^{NST} \sum_{u=1}^{t} FVS_{v,i,u} \quad \forall v, t \tag{5}$$

$$VS_{i,t} = VS_i^0 + \sum_{v=1}^{NV} \sum_{u=1}^{t} FVS_{v,i,u} - \sum_{j=1}^{NBT} \sum_{u=1}^{t} FSB_{i,j,u} \quad \forall i, t \tag{6}$$

$$VB_{j,t} = VB_j^0 + \sum_{i=1}^{NST} \sum_{u=1}^{t} FSB_{i,j,u} - \sum_{l=1}^{NCDU} \sum_{u=1}^{t} FBD_{j,l,u} \quad \forall j, t \tag{7}$$

$$vS_{i,k,t} = vS_{i,k}^0 + \sum_{v=1}^{NV} \sum_{u=1}^{t} fVS_{v,i,k,u} - \sum_{j=1}^{NBT} \sum_{u=1}^{t} fSB_{i,j,k,u} \quad \forall i, k, t \tag{8}$$

$$vB_{j,k,t} = vB_{j,k}^0 + \sum_{i=1}^{NST} \sum_{u=1}^{t} fSB_{i,j,k,u} - \sum_{l=1}^{NCDU} \sum_{u=1}^{t} fBD_{j,l,k,u} \quad \forall j, k, t \tag{9}$$

$$\sum_{i=1}^{NST} \sum_{t=1}^{SCH} FVS_{v,i,t} = VV_v^0 \quad \forall v \tag{10}$$

$$\sum_{l=1}^{NCDU} \sum_{t=1}^{SCH} FBD_{j,l,t} = DM_j \quad \forall j \tag{11}$$

The constraints (5)–(7) define, respectively, the material balance for the vessels, storage tanks and charging tanks at each time t. (8) and (9) determine the material balance for each component k in the storage tanks and charging tanks. Recall that the vessels regard a fixed percentage $yV_{v,k}$ for each component k contained in the vessel v. On the other hand, composition of a flow sent from a storage tank i to a charging tank j must follow the same percentages as oil blend that remains stored in the storage tank i. This behaviour would be ensured by the following non-linear constraints.

$$vS_{i,k,t} FSB_{i,j,t} = VS_{i,t} fSB_{i,j,k,t} \quad \forall i, j, k, t$$

These constraints (and similar ones for the flows sent from the charging tanks) have been relaxed by Yüzgeç et al. (2010) causing the false split issue mentioned in the introduction, which we address in this paper.

At last, (10) and (11) defines, respectively, the total volume of crude oil pushed by each vessel to the system, and the total volume demanded by the CDUs that should be supplied by each charging tank.

Flow control:

$$FVS_{vi}^{\min} XW_{v,t} \leq FVS_{v,i,t} \leq FVS_{vi}^{\max} XW_{v,t} \quad \forall v, i, t \tag{12}$$

$$FVS_{v,i,t} \leq FVS_{v,i}^{\max}(1 - DSB_{i,j,t}) \quad \forall v, i, j, t \tag{13}$$

$$FSB_{i,j,t} \leq FSB_{i,j}^{\max}\left(1 - \sum_{l=1}^{NCDU} DBD_{j,l,t}\right) \quad \forall i, j, t \tag{14}$$

$$FSB_{i,j}^{\min} DSB_{i,j,t} \leq FSB_{i,j,t} \leq FSB_{i,j}^{\max} DSB_{i,j,t} \quad \forall i, j, t \tag{15}$$

$$FBD_{j,l}^{\min} DBD_{j,l,t} \leq FBD_{j,l,t} \leq FBD_{j,l}^{\max} DBD_{j,l,t} \quad \forall j, l, t \tag{16}$$

$$VS_i^{\min} \leq VS_{i,t} \leq VS_i^{\max} \quad \forall i, t \tag{17}$$

$$VB_j^{\min} \leq VB_{j,t} \leq VB_j^{\max} \quad \forall j, t \tag{18}$$

The constraints (12)–(16) define the lower and upper bounds for the flows between the stages and ensure that each storage or charging tank cannot send and receive lots simultaneously. Finally, (17) and (18) establishes the lower and upper bounds for the total inventory level contained in storage and charging tanks. This control is not applied to the vessels because they perform only unloading operations.

$$fVS_{v,i,k,t} = FVS_{v,i,t} yV_{v,k} \quad \forall v, i, k, t \tag{19}$$

$$FSB_{i,j,t} yS_{i,k}^{\min} \leq fSB_{i,j,k,t} \leq FSB_{i,j,t} yS_{i,k}^{\max} \quad \forall i, j, k, t \tag{20}$$

$$FBD_{j,l,t} yB_{j,k}^{\min} \leq fBD_{j,l,k,t} \leq FBD_{j,l,t} yB_{j,k}^{\max} \quad \forall j, l, k, t \tag{21}$$

$$VS_{i,t} yS_{i,k}^{\min} \leq vS_{i,k,t} \leq VS_{i,t} yS_{i,k}^{\max} \quad \forall i, k, t \tag{22}$$

$$VB_{j,t} yB_{j,k}^{\min} \leq vB_{j,k,t} \leq VB_{j,t} yB_{j,k}^{\max} \quad \forall j, k, t \tag{23}$$

Analogously to the previous constraints, the groups (19)–(21) and (22) and (23) define for each component k, respectively, the lower and upper bounds for the flows between the stages and for the inventory level contained in the storage and charging tanks.

Charging operations:

$$\sum_{l=1}^{NCDU} DBD_{j,l,t} \leq 1 \quad \forall j, t \tag{24}$$

$$\sum_{j=1}^{NBT} DBD_{j,l,t} = 1 \quad \forall l, t \tag{25}$$

$$Z_{j,j',l,t} \geq DBD_{j',l,t} + DBD_{j,l,t-1} - 1 \quad \forall j, j' \| j \neq j', l \quad t = 2, \ldots, SCH \tag{26}$$

The constraints (24) state that the charging tanks cannot feed more than one CDU at each time t, and (25) establishes that necessarily each CDU must be charged by one charging tank at any time t along the horizon. At last, (26) defines the changeover occurrence from a charging tank j to a charging tank j' for the CDUs along the time horizon.

2.1.3. Objective function

The optimization objective (27) is to find the minimum cost schedule regarding, respectively, unloading waiting times for berthing, unloading, tank inventories (at the terminals and refineries), and charging tank changeovers.

$$\sum_{v=1}^{NV} CUnl_v(TLv - TFv + 1) + \sum_{v=1}^{NV} CWait_v(TFv - TArrv)$$
$$+ \sum_{i=1}^{NST} \sum_{t=1}^{SCH} CInvST_i[(VS_{i,t} + VS_{i,t-1})/2]$$
$$+ \sum_{j=1}^{NBT} \sum_{t=1}^{SCH} CInvBT_j[(VB_{j,t} + VB_{j,t-1})/2]$$
$$+ \sum_{t=2}^{SCH} \sum_{j=1}^{NBT} \sum_{j'=1}^{NBT} \sum_{l=1}^{NCDU} CSetup_{j,j',l} Z_{j,j',l,t} \tag{27}$$

3. Issues of the literature models

Quesada and Grossmann (1995), Lee et al. (1996) and Yüzgeç et al. (2010) affirmed that the nonlinear mixing equations can be reformulated into linear ones since the scheduling system involves only mixing operations without splitting operations. However, that statement leads to an imprecision in the mathematical model that will be named in this paper a false split and will be detailed as follows.

The existence of this issue was verified in two dimensions: stages and times, since there is no constraint in the original formulation prohibiting their occurrences in both cases. Later in this section, we propose a correction to handle with this issue through a new mathematical model which avoids the non-linearity using an aggregated inventory capacity for the storage tanks combined to a disaggregation algorithm for the flows between stages.

The first dimension, the false split occurred in time, can be defined as the non-homogeneous division of the components contained in a tank between a sending lot and the volume that remains in the tank. Fig. 2 shows a false splitting case that occur in a solution for the example 3 used in Lee et al. (1996) and Yüzgeç et al. (2010). In the left side of this figure, it is represented that the content of the storage tank $i = 1$ at time $t = 2$ is composed of 1.32% of component $k = 1$. However, the lot sent from this tank at time $t = 3$ contains 3% of this component, and the remaining percentage of this component is reduced to 1.31% to satisfy the mass conservation constraint. In a true split operation, the percentage of component 1 present in the lot sent at time $t = 3$ should be exactly 1.32%.

As it can be seen, the ranges between the lower and upper bounds for the inventory levels and lot sizes are respected, but they do not imply in a prohibition of false splitting operations. Thus the nonlinear mixing equations cannot be reformulated into linear ones without allowing the false splitting occurrence.

The second dimension, the false split occurred between stages, can be defined as the non-homogeneous division of the components contained in a source tank between multiple sending lots to different destination tanks in the next stage. As can be seen in Fig. 3, the same flexibility is given by the model since there is no constraint which ensures a homogeneous division of the sending lots of components. The source storage tank presents at time $t = 0$ a crude oil composition formed by 46% of component $k = 1$, 32% of $k = 2$ and 22% of $k = 3$. Despite of the source composition, the two different lots $FSB_{1,2,1}$ and $FSB_{1,6,1}$ sent at time $t = 1$ to the charging tanks $j = 1$ and $j = 6$ present both compositions different from the source, and also from each other, respectively being for $k = 1, 2, 3$: 40%–30%–30% and 50%–20%–30%. So there is a clearly non-homogenous division between the source stage (storage tanks of the waterway terminal) and the destiny stage (charging tanks of the refinery).

Thus, we propose a more consistent way to handle with these flow splitting issues without using non-linear inequalities. It considers an aggregated inventory capacity for the storage tanks combined to a disaggregation algorithm for the flows among stages.

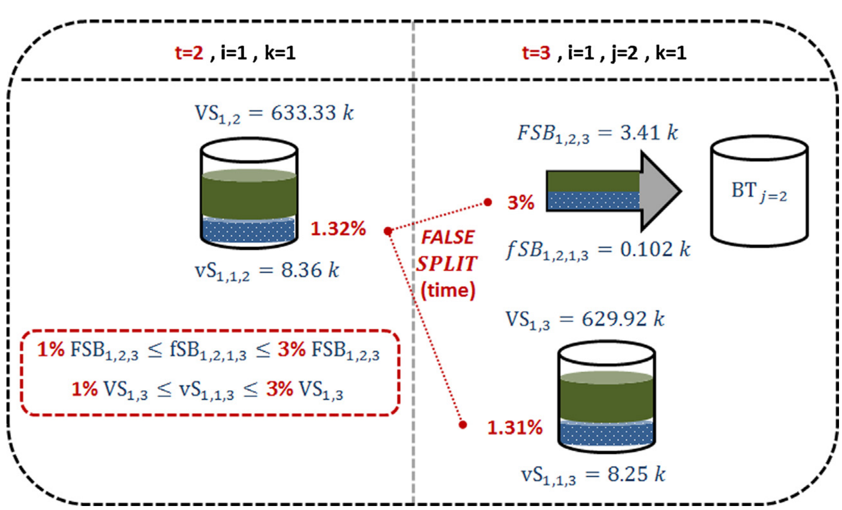

Fig. 2. Split in time, Example 3 in Lee et al. (1996) and Yüzgeç et al. (2010).

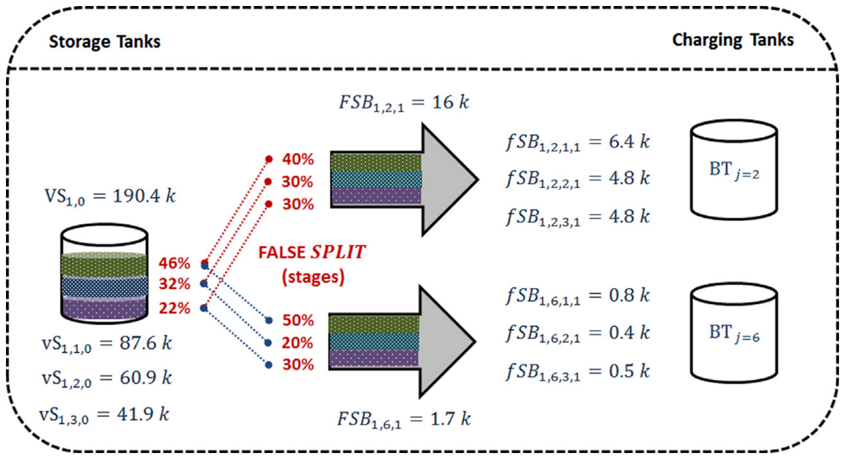

Fig. 3. Split between stages, Created instance # 112.

The capacities and initial inventory levels are summed for all the storage tanks in order to compose the input data for the aggregated storage tank, becoming the unique tank in the terminal stage. After solving the new MILP model, the aggregated flows of components $fVS_{v,i,k,t}$ and $fSB_{i,j,k,t}$ are defined by the solution and an algorithm is executed in order to disaggregate these flows into $fVS_{v,di,k,t}$ and $fSB_{di,j,k,t}$ where $di \in \{1, \ldots, DNST\}$ is the index of each disaggregated storage tank in the terminal. As consequence, the inventory levels of the components $vS_{di,k,t}$ are also defined.

The proposed disaggregation handles the split issue by classifying the disaggregated storage tanks into three different types: mixing, dedicated and pure. The basic idea is to align the sending and receiving flows with the available capacity of the disaggregated storage tanks ensuring that there are no false split occurrences.

We assume there is one tank dedicated to each component k, any member of identical mixing tanks are allowed to keep any composition needed to minimize the loss of storage capacity implied by the use of dedicated tanks, and one pure tank is used as a dedicated being able to change its component k when its inventory level is empty. If it is possible to receive the whole volume by the dedicated tanks, it is possible to compose any sending lot composition. However, it may cause quite great loss of capacity. So, besides the dedicated tanks, we also create the mixing tanks, all of them keeping compositions with the same percentages and aligned to the composition demanded by the charging tanks, focusing on mitigate the loss of capacity issue. A second refinement strategy is the usage of the pure tank, which is a dedicated tank that can change its component k when it becomes empty, increasing the range of the compositions that can be assembled by the dedicated and mixing tanks. It helps when a component has a stored volume much grater than the others.

We also assume that initial inventory of the storage tanks are compatible with this policy. This can occur if, for example, the same policy was adopted in the planning operations before the beginning of the time horizon. Finally, we assume that the minimum inventory levels for all disaggregated tanks are zero.

4. New modeling and formulation proposals

Here, we describe some modifications that have been implemented in both the model and the formulation proposed by Yüzgeç et al. (2010). These modifications are:

1 *Reformulation of the constraints that model the operation times of the vessels*: this modification is an improvement with respect to the previous formulation that does not change the model assumptions.
2 *Controlled components considered as crude oil types instead of individual substances*: this modification only changes the way the instances are generated except that new constraints must be included to keep the sum of percentages of the controlled components equal to 100%. In the reported experiments such constraints are added to both the previous and the new formulation.
3 *Constraints on the lots received by the charging tanks*: this modification is a new assumption that avoids false split on the flows from the charging tanks to the CDUs. In order to meet this assumption, new constraints are included to restrict the composition of the lots received by the charging tanks. Such constraints are also added to both the previous and the new formulation.
4 *Symmetry breaking for identical CDUs*: this modification is an improvement with respect to the previous formulation that does not change the model assumptions. It is only applied when the CDUs are identical.
5 *Reformulation for the changeover variables*: this modification is an improvement with respect to the previous formulation that does not change the model assumptions. It is based on the work of Pochet and Wolsey (2006).
6 *Disaggregation algorithm*: this is a post-processing step that can be applied to the solution obtained by either formulations.

Note that only the modifications 2 and 6 represent changes in the model assumptions. However, these changes can be incorporated in the model of Yüzgeç et al. (2010) by adapting the input instances: considering crude oil types for the modification 2 and aggregating the storage tanks for the modification 6. Moreover, the original instances that consider controlled components as individual substances can be solved by the modified formulation. This allows a direct comparison of the formulation performances.

4.1. Operation times of the vessels

The use of the integer variables TF_v and TL_v in (3) can be avoided by using the following constraints (28)–(30).

$$XF_{v,t} = 0 \qquad \forall v,\, t = 1, \ldots, SCH \| t < TArr_v \tag{28}$$

$$\sum_{u=t+1}^{SCH} XF_{v+1,u} \geq \sum_{u=t}^{SCH} XL_{v,u} \qquad \forall t,\, v = 1, \ldots, NV - 1 \tag{29}$$

$$\sum_{u=t+1}^{SCH} XL_{v,u} \geq \sum_{u=t}^{SCH} XF_{v,u} \qquad \forall v, t \tag{30}$$

The constraints expressed over the binary variables strengthen the linear relaxation of the proposed formulation, facilitating the branch-and-bound solving process.

4.2. Controlled components as crude oil types

The instances in Lee et al. (1996) and Yüzgeç et al. (2010) use the concentration ranges of key components as input data, highlighting the sulfur as an important component to be controlled to assure the crude oil quality. The examples used in these articles regard two controlled components at most, each one with less than 10 percent of the total volume of the tanks or lots.

However, the operational level decisions on the lot scheduling for the crude oil supply process usually focus on ensuring the total composition in the charging tanks as an adequate blend of the different crude oil types. The sulfur concentration, for example, is an inherent feature of the crude oil type k that composes the blend in the tank, and the performed mixture is going to settle the yields of the derivatives from the distillation process. This operational policy isolates the exogenous uncertainty on the component concentrations since the received volumes of crude oil types can be planned with more accuracy. Final adjustments of the oil blend that may be necessary due to specific combinations of component concentrations are performed at the charging tanks level using refinery internal procedures that are out of the scope of this model.

The original model represents as a controlled component k the concentration of some substance which is an inherent feature of a crude oil molecule from a particular crude oil stream (sulfur, for example). In the proposed model, the controlled component is a crude oil type (light or heavy, for example), which aggregates different crude oil streams, each one with its particular inherent components. In this case, the chemical issue is treated previously, out of the model, when the assignment of each crude oil stream to the correspondent crude oil type (an aggregation of streams) is performed. The proposed model focuses only on the Logistics problem.

Thus the new model regards the controlled components as the crude oil types that compose the demanded blend by the distillation process, and the composition of the total volume in the storage tanks is assured by (31).

$$\sum_{k=1}^{NCE} vS_{i,k,t} = VS_{i,t} \quad \forall i, t \tag{31}$$

4.3. Constraints on the lots received by the charging tanks

In order to avoid false split on the flows from the charging tanks to the CDUs, we consider additional constraints on the lots received by these tanks as new model assumptions. As both the lots sent to the CDUs and the remaining inventory must meet the same range for each component, we assume that these ranges must also be applied to the sum of all volumes received from the storage tanks at a given time period. Although some feasible solutions may be lost due to this assumption, we believe that it is not very restrictive since the composition ranges at the charging tanks are usually tight. An important advantage of this assumption is the suppression of the charging tank composition control. This suppression is allowed since we ensure that the initial inventory level and, as described in (32), the sum of the receiving lots are within the output percentage range for each crude oil type k. This assumption implies that the lots sent to the crude distillation units follow the component percentage ranges for the charging tanks. As a result, the variables $vB_{j,k,t}$ and $fBD_{j,l,k,t}$ and the constraints (9), (21) and (23) are suppressed in the new formulation since it is possible to calculate the charging tank compositions in a post processing step.

$$\sum_{i=1}^{NST} FSB_{i,j,t}\, yB_{j,k}^{\min} \leq \sum_{i=1}^{NST} fSB_{i,j,k,t} \leq \sum_{i=1}^{NST} FSB_{i,j,t}\, yB_{j,k}^{\max} \quad \forall j, k, t \tag{32}$$

4.4. Symmetry breaking for identical crude distillation units

Since the crude distillation units are identical in all instances proposed by Lee et al. (1996) and Yüzgeç et al. (2010) and also in the new instances described in Section 5, we propose that the index l corresponding to the CDUs is suppressed by replacing (7), (11), (14), (16), (24), (25) and (27) by the constraints (33)–(38) as follows:

$$VB_{j,t} = VB_j^0 + \sum_{i=1}^{NST}\sum_{u=1}^{t} FSB_{i,j,u} - \sum_{u=1}^{t} FBD_{j,u} \quad \forall j, t \tag{33}$$

$$\sum_{t=1}^{SCH} FBD_{j,t} = DM_j \quad \forall j \tag{34}$$

$$FSB_{i,j,t} \leq FSB_{i,j}^{\max}(1 - DBD_{j,t}) \quad \forall i, j, t \tag{35}$$

$$FBD_j^{\min} DBD_{j,t} \leq FBD_{j,t} \leq FBD_j^{\max} DBD_{j,t} \quad \forall j, t \tag{36}$$

$$\sum_{j=1}^{NBT} DBD_{j,t} = NCDU \quad \forall t \tag{37}$$

$$\begin{aligned} &\sum_{v=1}^{NV} CUnl_v(TLv - TFv + 1) + \sum_{v=1}^{NV} CWait_v(TFv - TArrv) \\ &\quad + \sum_{i=1}^{NST}\sum_{t=1}^{SCH} CInvST_i[(VS_{i,t} + VS_{i,t-1})/2] \\ &\quad + \sum_{j=1}^{NBT}\sum_{t=1}^{SCH} CInvBT_j[(VB_{j,t} + VB_{j,t-1})/2] + \sum_{t=2}^{SCH}\sum_{j=1}^{NBT}\sum_{j'=1}^{NBT} CSetup_{j,j'} Z_{j,j',t} \end{aligned} \tag{38}$$

Note that the constraints (26) are not easily adapted to the suppression of the index l. This adaptation is addressed later in the reformulation of the changeover variables described in the next subsection.

It is important to highlight that the need for suppression of the index l is only necessary when the instances use identical CDUs because it creates a symmetry that complicates the MILP model solving process. When considering distinct CDUs, it is expected that the formulation with the index l presents a similar performance to the formulation without the index l for identical CDUs due to the elimination of this symmetry. In hybrid cases, where we have some groups of identical CDUs that are distinct between each others, we can use a unique index l for each group.

Concerning the storage and charging tanks, the initial storage level eliminates the symmetry on the supplying process. So, only the crude distillation units present actual symmetry.

4.5. Reformulation for the changeover variables

The proposed reformulation focus on the segment of the problem between the charging tanks and the crude distillation units by the following three reasons: the changeover cost is significantly greater than the other costs; the changeover occurrence is a consequence of the relation between the continuous supply to the CDUs and the available inventory level in the charging tanks; the blend demands to be supplied along the time horizon are assigned to the charging tanks.

Pochet and Wolsey (2006) presents a reformulation for the pigment sequencing problem, which is a multi-item lot sizing problem with single machine, Wagner–Whitin and sequence-dependent changeover costs, without initial inventory level, and at most one item produced per period. The multi-item approach can be decomposed in formulations with single-item to the variant DLS-CC-SC (discrete lot sizing with constant capacity and start-up costs) of the problem.

The reformulation proposed in this paper is an adaptation of the one proposed by Pochet and Wolsey (2006). It is based on replacing the changeover dynamics presented in Eq. (26) and represented by the changeover variables $Z_{j,j',l,t}$ by a new start-up and switch-off dynamics represented by the new variables $SU_{j,t}$ and $SO_{j,t}$ described as follows:

- $SU_{j,t}$: Binary variables which denote the start-up occurrence on charging a crude distillation unit by a tank j at time t. A start-up occurs at time t when $DBD_{j,t-1}=0$ and $DBD_{j,t}=1$.
- $SO_{j,t}$: Binary variables which denote the switch-off occurrence on charging a crude distillation unit by a tank j at time t. A switch-off occurs at time t when $DBD_{j,t}=1$ and $DBD_{j,t+1}=0$.

Eq. (26) is replaced by (39)–(43).

$$\sum_{j=1}^{NBT} Z_{j,j',t} \geq DBD_{j',t} \quad \forall j', t \tag{39}$$

$$\sum_{j'=1}^{NBT} Z_{j,j',t} \geq DBD_{j,t-1} \quad \forall j, t \tag{40}$$

$$\sum_{j=1}^{NBT} DBD_{j,0} = NCDU \tag{41}$$

$$DBD_{jprime,t} - SU_{jprime,t} = Z_{jprime,jprime,t} \quad \forall jprime, t \tag{42}$$

$$DBD_{jprime,t-1} - SO_{jprime,t-1} = Z_{jprime,jprime,t} \quad \forall j', t \tag{43}$$

The constraints (39) and (40) ensure that a non-zero value assigned to a $DBD_{j,t}$ variables implies to non-zero values assigned to the corresponding changeover variables. The constraints (41) guarantee that all identical CDUs are being served in the beginning of the horizon. Finally, (42) and (43) keep the consistency among the $DBD_{j,t}$ variables, the changeover variables and the start-up and switch-off variables.

4.6. Disaggregation algorithm

Let VS^0_{di} and $VS^{\max}_{di}$ be the initial and maximum inventory level of each storage tank di, as well as $yS^0_{di,k}$ the initial percentage of each crude oil type k.

The main framework of the disaggregation algorithm is detailed in Algorithm 1, and the calculation for the sending and receiving disaggregated lots are detailed, respectively, in Algorithms 2 and 3.

The parameter $descAgVS$ is a discounted percentage on the waterway terminal aggregated capacity $VS^{\max}_i$ applied as a security factor to avoid the inventory infeasibility on performing the disaggregation (inventory overflow or fault). This means that the value of $VS^{\max}_i$ used when optimizing the MILP model is $(100-descAgVS)\%$ of the original one.

Algorithm 1: Disaggregation Algorithm - Main

Data: $fVS_{v,i,k,t}$, $fSB_{i,j,k,t}$, $FVS_{v,i,t}$, $FSB_{i,j,t}$, VS^0_{di} , VS^{max}_{di} , $yS^0_{di,k}$, $yS^{min}_{di,k}$, $yS^{max}_{di,k}$, $yB^{max}_{j,k}$

Result: $fVS_{v,di,k,t}$, $fSB_{di,j,k,t}$, $vS_{di,k,t}$

Calculate $vS_{di,k,0}$ for each disaggregated tank di based on VS^0_{di} and $yS^0_{di,k}$. ;

for $k \leftarrow 1$ **to** NCE **do**
  $yAvB^{max}_k \leftarrow$ the average of $yB^{max}_{j,k}$ for all j ;

$i \leftarrow$ index of the unique aggregated storage ;

$mixPercST_{i,k,t} \leftarrow 0$, $kP_t \leftarrow 0$;

for $t \leftarrow 1$ **to** SCH **do**
  for $di \leftarrow 1$ **to** $DNST$ **do**
    for $k \leftarrow 1$ **to** NCE **do**
      $vS_{di,k,t} \leftarrow vS_{di,k,t-1}$;
  Calculate the percentage composition of the mixing tanks $mixPercST_{i,k,t-1}$ for each k ;
  // Calculate the disaggregated sending lots fSB$_{di,j,k,t}$
  Execute **Algorithm 2** (t , i , $mixPercST_{i,k,t}$, $fSB_{i,j,k,t}$, $FSB_{i,j,t}$, $fSB_{di,j,k,t}$, $vS_{di,k,t}$) ;
  // Caculate the disaggregated receiving lots fVS$_{v,di,k,t}$
  Execute **Algorithm 3** (t , i , kP_t, $yAvB^{max}_k$, $fVS_{v,i,k,t}$, $FVS_{v,i,t}$, VS^{max}_{di} , $fVS_{v,di,k,t}$, $vS_{di,k,t}$) ;

We highlight that kP_t is the variable that controls the component k in the pure tank at each time t. This control ensures that the component in the tank can only be changed by another one after the tank becomes completely empty (see the "if-else" command in Algorithm 3). Furthermore, the variable $mixPercST_{i,k,t}$ is used to store the percentage composition of the mixing tanks at each time t, since this composition will lead the dynamics of the sending and receiving lots.

Algorithm 2: Disaggregation Algorithm – Sending Lots

Data: t , i , $mixPercST_{i,k,t}$, $fSB_{i,j,k,t}$, $FSB_{i,j,t}$, $fSB_{di,j,k,t}$, $vS_{di,k,t}$
Result: $fSB_{di,j,k,t}$, $vS_{di,k,t}$

for $j \leftarrow 1$ **to** NBT **do**
  if $FSB_{i,j,t} > 0$ **then**
    $mixVS_{i,t} \leftarrow \sum_{k=1}^{NCE} \sum_{\substack{di \mid di \\ is\ mixing \\ tank}} vS_{di,k,t}$;
    $MinkLot \leftarrow \min\left\{\min\left\{\frac{fSB_{i,j,k,t}}{mixPercST_{i,k,t-1}} \mid k \in \{1,\ldots,NCE\}\right\} , mixVS_{i,t}\right\}$;
    for $k \leftarrow 1$ **to** NCE **do**
      $dedSendDist_k \leftarrow fSB_{i,j,k,t}$;
      foreach $di \parallel di$ *is a mixing tank* **do**
        $fSB_{di,j,k,t} \leftarrow \frac{MinkLot \times vS_{di,k,t}}{mixVS_{i,t}}$;
        $vS_{di,k,t} \leftarrow vS_{di,k,t} - fSB_{di,j,k,t}$;
        $dedSendDist_k \leftarrow dedSendDist_k - fSB_{di,j,k,t}$;
      foreach $di \parallel di$ *is a pure tank* **do**
        $fSB_{di,j,k,t} \leftarrow \min\left\{dedSendDist_k , vS_{di,k,t}\right\}$;
        $vS_{di,k,t} \leftarrow vS_{di,k,t} - fSB_{di,j,k,t}$;
        $dedSendDist_k \leftarrow dedSendDist_k - fSB_{di,j,k,t}$;
      foreach $di \parallel di$ *is a dedicated tank for the crude oil* k **do**
        $fSB_{di,j,k,t} \leftarrow dedSendDist_k$;
        $vS_{di,k,t} \leftarrow vS_{di,k,t} - fSB_{di,j,k,t}$;
        `// Possible infeasibility based on the inventory fault`

Algorithm 3: Disaggregation Algorithm – Receiving Lots

Data: t , i , kP_t, $yAvB_k^{max}$, $fVS_{v,i,k,t}$, $FVS_{v,i,t}$, VS_{di}^{max} , $fVS_{v,di,k,t}$, $vS_{di,k,t}$
Result: $fVS_{v,di,k,t}$, $vS_{di,k,t}$

for $di \leftarrow 1$ **to** $DNST$ **do**
- $VS_{di,t} \leftarrow \sum_{k=1}^{NCE} vS_{di,k,t}$;

for $v \leftarrow 1$ **to** NV **do**
- **if** $FVS_{v,i,t} > 0$ **then**
  - **for** $k \leftarrow 1$ **to** NCE **do**
    - $volRecDist_k \leftarrow fVS_{v,i,k,t}$;
  - **foreach** $di \parallel$ *di is a dedicated tank* **do**
    - $k \leftarrow$ *index of the crude oil type stored in the tank di* ;
    - $fVS_{v,di,k,t} \leftarrow \min\left\{fVS_{v,i,k,t}\ ,\ VS_{di}^{max} - VS_{di,t}\right\}$;
    - $volRecDist_k \leftarrow volRecDist_k - fVS_{v,di,k,t}$;
    - $vS_{di,k,t} \leftarrow vS_{di,k,t} + fVS_{v,di,k,t}$;
  - $kV \leftarrow \arg\max_k \left\{ fVS_{v,i,k,t} + \sum_{di \mid di \text{ is mixing tank}} vS_{di,k,t} \right\}$;
  - $mixLotMaxPct \leftarrow$ *minimum volume received in a mixing tank to increase the pecentage of the mixed crude oil* kV *to at least* $yAvB_{kV}^{max}$;
  - $mixLotMaxPct \leftarrow \min\left\{ mixLotMaxPct\ ,\ volRecDist_{kV}\ ,\ \sum_{di \mid di \text{ is mixing tank}} (VS_{di}^{max} - VS_{di,t}) \right\}$;
  - **foreach** $di \parallel$ *di is a mixing tank* **do**
    - $fVS_{v,di,kV,t} \leftarrow mixLotMaxPct \times \frac{VS_{di}^{max} - VS_{di,t}}{\sum_{di \mid di \text{ is mixing tank}} (VS_{di}^{max} - VS_{di,t})}$;
    - $vS_{di,k,t} \leftarrow vS_{di,k,t} + fVS_{v,di,kV,t}$;
    - $volRecDist_{kV} \leftarrow volRecDist_{kV} - fVS_{v,di,kV,t}$;
  - **foreach** $di \parallel$ *di is the pure tank* **do**
    - **if** $\sum_{k=1}^{NCE} vS_{di,k,t} = 0$ **then**
      - $kP_t \leftarrow \arg\max_k \left\{fVS_{v,i,k,t}\right\}$;
    - **else**
      - $kP_t \leftarrow kP_{t-1}$;
    - $fVS_{v,di,kP_t,t} \leftarrow \min\left\{volRecDist_{kP_t}\ ,\ VS_{di}^{max} - VS_{di,t}\right\}$;
    - $vS_{di,kP_t,t} \leftarrow vS_{di,kP_t,t} + fVS_{v,di,kP_t,t}$;
    - $volRecDist_{kP_t} \leftarrow volRecDist_{kP_t} - fVS_{v,di,kP_t,t}$;
  - **foreach** $di \parallel$ *di is a mixing tank* **do**
    - **for** $k \leftarrow 1$ **to** NCE **do**
      - $fVS_{v,di,k,t} \leftarrow fVS_{v,di,k,t} + \left(volRecDist_{kP_t} \times \frac{VS_{di}^{max} - VS_{di,t}}{\sum_{di \mid di \text{ is mixing tank}} (VS_{di}^{max} - VS_{di,t})} \right)$;
      - $vS_{di,k,t} \leftarrow vS_{di,k,t} + fVS_{v,di,k,t}$;
      - `// Possible infeasibility based on the inventory overflow`

Concerning the aggregated sending lots, the disaggregation algorithm starts by calculating the maximum volume that can be sent from the mixing tanks such that the percentages of the controlled components can be fixed later by sending additional volumes. This value is denoted by *MinkLot*, which is limited by the sum of all volumes stored in the mixing tanks (denoted by $mixVS_{i,t}$). The variable $dedSendDist_k$ stores, for each component k, the aggregated volumes sent from the aggregated storage tank i to be distributed among the disaggregated tanks di following the priorities set by the Algorithm 2. It can be noticed that the sending lots are always proportional to the stored volumes of each component in each mixing tank. This ensures that the percentages of components sent from each mixing tank follow its current composition, avoiding false splits operations. Then, for each controlled component, such percentages are completed by sending additional volumes from the pure tank, and then, from the dedicated tanks. The comment in the last line of Algorithm 2 indicates that infeasibilities may be generated at this point due to the lack of volume in the dedicated tanks.

On the other hand, the aggregated receiving lots prioritize the dedicated tanks since they are responsible for fixing the percentages of the controlled components in the sending operations, as it can be observed in the 4th "for" loop of Algorithm 3. The variable $volRecDist_k$ stores, for each component k, the aggregated volumes received by the aggregated storage tank i to be distributed among the disaggregated tanks di following the priorities set by this algorithm. In order to avoid excessive unbalances in the mixing tank compositions, it calculates the most representative component kV as the one with the maximum available volume. Then, the received volume is allocated at first to the mixing tanks until the component kV reaches the input threshold $yAvB_{kV}^{max}$ calculated previously. The value of this volume is denoted by *MixLotMaxPct*. This mechanism intend to avoid extreme compositions in the mixing tanks caused by the reduction of the percentage of the crude oil kV due to the use of the pure tank. Finally the pure tank is chosen and, if still necessary, the mixing tanks are used again until their capacity limits. The comment in the last line of Algorithm 3 indicates that infeasibilities may be generated at this point due to the surplus of volume in the mixing tanks.

Both lack and surplus violations lead to an impossibility in generating a feasible solution using the proposed procedure. The presented method is a result of successive improvements introduced until all the 48 tested instances could be solved without any of these infeasibilities. If such happen for other instances, new improvements concerning the instances or the algorithm should be necessary.

As illustration example for the execution of the disaggregation algorithm, the instance # 115 is depicted in Fig. 4.

At time $t = 10$, it can be noticed that the sending lots from the mixing tanks were complemented, respectively, by the crude oil type $k = 2$ from the pure tank (which becomes empty) and by the components $k = 1$ and $k = 2$ from the dedicated tanks. Thus, it can be concluded that the lot size from the mixing tanks has been defined by the component $k = 3$.

Then, at time $t = 13$, there was an aggregated receiving lot to the storage tanks concerning the crude oil type $k = 3$. The disaggregation algorithm priority established that the dedicated tank $di = 3$ should receive this flow, but it was full at this time. So the mixing tanks received the flow until it reached 30% of the total volume (upper bound $yAvB_k^{max}$), and finally the flow remainder was unloaded to the pure tank. It is important to highlight that the pure tank was filled at time $t = 9$ by the crude oil type $k = 2$ and, after it became empty, it could be filled by $k = 3$ at time $t = 13$, what presented the flexibility given by the pure tank concerning the terminal inventory capacity.

5. Experimental results

In this section, we present the experiments performed to show the effectiveness of the proposed algorithms and reformulations, and compare its performance to the original formulation of Yüzgeç et al. (2010). All the tests were run on a PC with a Intel Core i5 2.50 gigahertz CPU, 6 gigabytes RAM under Windows 7 OS using ILOG CPLEX 12.5. We used as modeling tool the UFFLP MIP Library integrated to Visual Basic.NET development platform.

The tested instances were created based on the order of magnitude of some values described in the Petrobras case study contained in Leiras (2010) and some input data contained in Yüzgeç et al. (2010). The created instances were divided in two complexity criteria: the number of charging tanks and CDUs, and the number of time periods. Two groups were created for the first criteria (6 tanks + 1 CDU, and 8 tanks + 2 CDUs) and 4 groups for the second one (15, 20, 25 and 30 time periods), generating 8 groups each one composed by 6 instances, giving a total of 48 instances.

All the input data related to the costs are equal to the ones contained in Yüzgeç et al. (2010). Moreover, the instances identified as infeasible by the MILP model (with the aggregated storage tank) were discarded. The remaining rules and informations of the creation process for the tested instances are detailed in Appendix A.

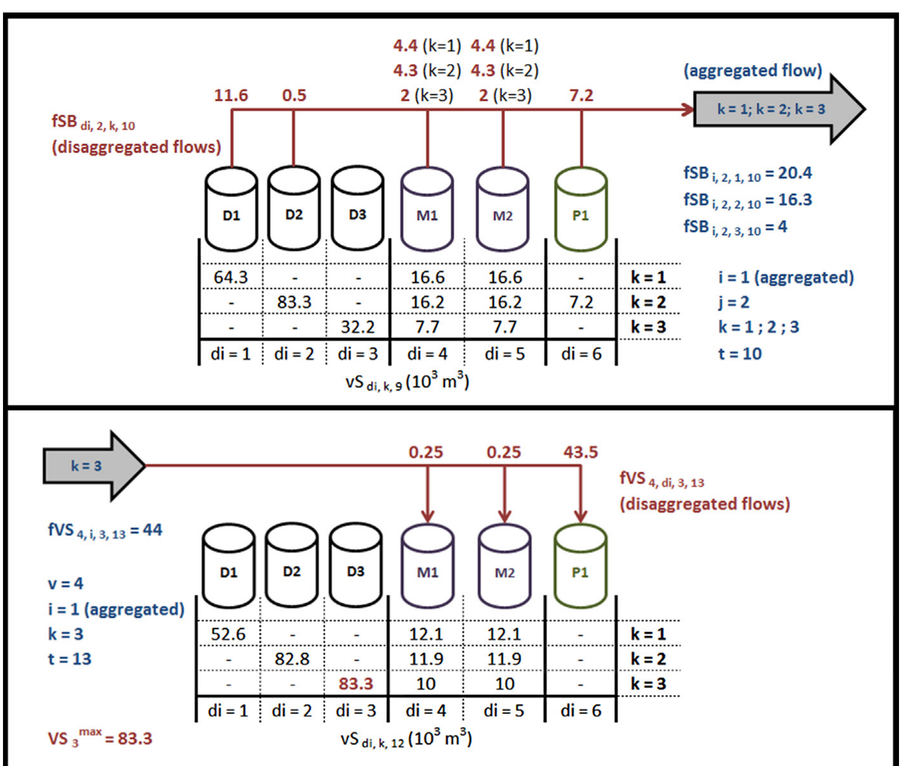

Fig. 4. Results – Disaggregation algorithm, Created instance # 115.

Table 1
Results – proposed reformulation × original formulation – all instances.

# Tanks and CDUs	# Periods	# Solved: ORIG	# Solved: REF	Avg final gap: ORIG (%)	Avg final gap: REF (%)	Avg increase: linear relaxation (%)	Avg reduction: final gap (%)	Avg reduction: best integer solution (%)
	15	0	3	50.8	12.5	50.8	79.2	1.5
6 tanks	20	0	1	59.0	18.6	65.4	70.7	1.2
1 CDU	25	0	0	65.7	27.6	32.3	57.8	4.4
	30	0	0	63.2	23.1	46.0	63.4	6.2
Subtotal		0	4	59.7	20.4	48.6	67.8	3.3
	15	0	0	77.5	16.1	100.1	79.6	21.6
8 tanks	20	0	0	79.3	16.1	109.0	80.2	27.1
2 CDUs	25	0	0	79.9	35.3	57.5	55.9	34.1
	30	0	0	79.7	36.0	38.7	55.0	40.2
Subtotal		0	0	79.1	25.9	76.3	67.7	30.8
TOTAL		0	4	69.4	23.2	62.5	67.7	17.1

The complexity of the instances was gradually increased to evaluate the impact in the computational efficiency of solving the problem. A time limit of 1800 s was set for the solving process. This time limit was chosen because, observing the evolution of the gaps during the resolution of several instances, we conclude that most of the instances that were not solved would probably require much more time to reach the optimality. So, for a scheduling perspective, a solution time higher than 1800 s would be unreal as the demands for new scenarios can appear within hours and additional time should be given for data updating before the search for the new solution.

Then, after the MILP solution, the disaggregation algorithm has been executed for the feasible instances. Using *descAgVS* = 20%, all the 48 tested instances were feasible also concerning the disaggregation algorithm. This value was set experimentally. Larger values turn some MIP models into infeasible, and smaller values cause surplus infeasibilities in the disaggregation algorithm.

The results of the comparison between the original formulation (ORIG) and the developed reformulation (REF) can be seen in Table 1. In this table, the presented gaps are the final gaps between the best lower and upper bounds reported by the MILP solver. The final gap is a measure of how far the solver is from proving the optimality of the current solution. The solved instances present final gap zero and solutions with optimal cost, but the unsolved instances present solutions which can be far from the optimal cost by at most the final gap value (the maximum cost difference). So, the average reduction in the final gap is the percentage reduction concerning the formulations ORIG and REF. The average increase in the linear relaxation represent the percentage increase between these formulations concerning the objective function value with integrality constraints relaxed. Finally, the average reduction in the best integer solution is the percentage reduction between the results of these formulations concerning the best integer solution obtained at the final gap.

Out of the 48 tested instances, no one could be solved to optimality by the original formulation, against 4 by the proposed reformulation. Furthermore, the reformulation allowed, in comparison with the original formulation, a 67.7% average reduction on the final gap, a 62.5% average increase on the linear relaxation lower bound, and a 17.1% average reduction on the best integer solution.

It can be highlighted that the improvement on the linear relaxation and the best integer solutions were significantly greater for the instances with 8 tanks – 2 CDUs than for the instances with 6 tanks – 1 CDU. These better results can be explained by the reformulation emphasis on the problem segment between the charging tanks and the crude distillation units, which benefits the solving process for more complex instances concerning the number of charging tanks and CDUs. It can also be noticed an average reduction of the best integer solution of 30.8% in the 8 tanks – 2 CDUs group against a reduction of only 3.3% in the 6 tanks – 1 CDU group.

Table 2 reports the root node relaxation gaps for the four instances where the optimal solutions are known. These gaps are calculated as the difference between the root lower bound and the optimal value, divided by the latter. By this table, it can be noticed a reduction of 24.2% on the average gap of the root node by the reformulation. Furthermore, the average solving times are 26.1% lower than the time limit of 1800 s.

Concerning the disaggregation algorithm to handle with the false splitting issue, all the 48 tested instances were feasible on performing this algorithm. This could be achieved only by setting the parameter *descAgVS* = 20%, which has been experimentally established.

6. Conclusions

This work presented a modeling study based on a reformulation for the crude distillation unit charging problem with oil blends and sequence-dependent changeover costs. It also presented a disaggregation algorithm which allowed for dealing with the flow splitting issue in a more consistent way than the previous references.

An amount of 48 instances were created with realistic sizes. No one could be solved by the original formulation, but 4 instances could be solved by the proposed reformulation. The most complex instances presented larger improvements since the reformulation emphasis is on the problem segment between the charging tanks

Table 2
Results – proposed reformulation × original formulation – solved instances.

ID	Avg gap root node: ORIG (%)	Avg gap root node: REF (%)	Avg solving time: REF (s)
103	68.7	52.5	1623.7
104	69.2	56.0	995.3
105	71.7	55.6	1669.0
107	67.6	46.1	1031.3
Total	69.3	52.5	1329.8

and the crude distillation units, which generates better linear relaxations and benefits the solving process for them.

Furthermore, regarding the security parameter experimentally established, the developed disaggregation algorithm generated feasible solutions for all the 48 tested instances without any false split occurrence.

However, for another group of instances, other value may be demanded for this parameter. So, as further research, the drawback of the infeasibility caused by the lack or surplus of crude oil in the disaggregation algorithm may be handled by the usage of the infeasible results in the post-processing routine as a starting point for a new iteration in the Logistic Model, in order to accelerate the achievement of a feasible solution or find out a more adjusted parameter value for the tested instances.

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Appendix A. Detailed description of instance generation

Here, we give a detailed description on the generation of the new instances used in the experiments reported in Section 5. For that, we use the following functions: $U(X, Y)$ to denote a random number generated from a uniform distribution between X and Y; $D(X, Y, p)$ to denote a discrete distribution where the parameter X has the probability p and Y has the probability $1 - p$; and $N(X, Y)$ to denote a random number generated by a normal distribution with mean X and standard-deviation Y.

The instance generation rules follow below for the two groups of instances: 6 tanks + 1 CDU, and 8 tanks + 2 CDUs.

- **6 tanks + 1 CDU**:
  - $SCH \in \{15, 20, 25, 30\}$; $NV = 6$; $NST = 1$ (aggregated); $DNST = NBT = 6$; $NCE = 3$; $NCDU = 1$;
  - 2 vessels for each component k ($yV_{v,k} = 1$) with random order of arrivals;
  - $TArr_v = TArr_{v-1} + \alpha$, where $\alpha \leftarrow U\left(1, \left\lfloor \frac{SCH}{NV} \right\rfloor\right)$;
  - $VV_v^0 \leftarrow D(60k, 100k, 0.5)$;
  - $DM_j \leftarrow U\left(0.9 \times \frac{\sum_{v=1}^{NV} VV_v^0}{NBT}\ 1.1 \times \frac{\sum_{v=1}^{NV} VV_v^0}{NBT}\right)$;
  - $VS_i^{\min} = VB_j^{\min} = 0$;
  - $VB_j^{\max} \leftarrow D(60k, 80k, 0.5)$; $VS_{di}^{\max} \leftarrow D(60k, 80k, 0.5)$;
  - $VB_j^0 \leftarrow U(0.2 \times DM_j\ 0.5 \times DM_j)$;
  - $VS_{di}^0 \leftarrow U(0.2 \times DM_j\ 0.5 \times DM_j)$, if di is not a pure tank, and 0 otherwise;
  - $yS_{di,k}^0 = 1$, if di is dedicated to component k, and 0 if is dedicated to other components;
  - For the mixing tanks, $yS_{di,k}^0$ are set so as to meet the values of $yS_{i,k}^0$ given by Table A.4;
  - $FVS_{v,i}^{\min} = FSB_{i,j}^{\min} = 0$;
  - $FVS_{v,i}^{\max} \leftarrow N(\bar{FVS}^{\max}, 0.1 \times \bar{FVS}^{\max})$, where $\bar{FVS}^{\max}$ follows Table A.3;
  - $FSB_{i,j}^{\max} \leftarrow N(\bar{FSB}^{\max}, 0.1 \times \bar{FSB}^{\max})$, where $\bar{FSB}^{\max}$ follows Table A.3;
  - $FBD_j^{\min} \leftarrow N(\bar{FBD}^{\min}, 0.1 \times \bar{FBD}^{\min})$, where $\bar{FBD}^{\min}$ follows Table A.3;
  - $FBD_j^{\max} \leftarrow N(\bar{FBD}^{\max}, 0.1 \times \bar{FBD}^{\max})$, where $\bar{FBD}^{\max}$ follows Table A.3;
  - The composition parameters for the controlled components follow Table A.4.
- **8 tanks + 2 CDUs**:
  - $SCH \in \{15, 20, 25, 30\}$; $NV = 6$; $NST = 1$ (aggregated); $DNST = NBT = 8$; $NCE = 4$; $NCDU = 2$;
  - 1 vessel for each component k ($yV_{v,k} = 1$), and 2 vessels containing 2 different types of crude oil k. The order of arrivals is random. These two components k' and k' are chosen randomly, and their percentages are respectively given by $yV_{v,k'} \leftarrow U(0.2, 0.6)$ and $yV_{v,k''} = 1 - yV_{v,k'}$.
  - $TArr_v = TArr_{v-1} + \alpha$, where $\alpha \leftarrow U\left(1, \left\lfloor \frac{SCH}{NV} \right\rfloor\right)$;
  - $VV_v^0 \leftarrow D(50k, 80k, 0.5)$;
  - $DM_j \leftarrow U\left(0.9 \times \frac{\sum_{v=1}^{NV} VV_v^0}{NBT}\ 1.1 \times \frac{\sum_{v=1}^{NV} VV_v^0}{NBT}\right)$;
  - $VS_i^{\min} = VB_j^{\min} = 0$;
  - $VB_j^{\max} \leftarrow D(50k, 80k, 0.5)$; $VS_{di}^{\max} \leftarrow D(50k, 80k, 0.5)$;
  - $VB_j^0 \leftarrow U(0.4 \times DM_j\ 0.6 \times DM_j)$;
  - $VS_{di}^0 \leftarrow U(0.4 \times DM_j\ 0.6 \times DM_j)$, if di is not a pure tank, and 0 otherwise;
  - $yS_{di,k}^0 = 1$, if di is dedicated to component k, and 0 if is dedicated to other components;
  - For the mixing tanks, $yS_{di,k}^0$ are set so as to meet the values of $yS_{i,k}^0$ given by Table A.6;
  - $FVS_{v,i}^{\min} = FSB_{i,j}^{\min} = 0$;
  - $FVS_{v,i}^{\max} \leftarrow N(\bar{FVS}^{\max}, 0.1 \times \bar{FVS}^{\max})$, where $\bar{FVS}^{\max}$ follows Table A.5;
  - $FSB_{i,j}^{\max} \leftarrow N(\bar{FSB}^{\max}, 0.1 \times \bar{FSB}^{\max})$, where $\bar{FSB}^{\max}$ follows Table A.5;
  - $FBD_j^{\min} \leftarrow N(\bar{FBD}^{\min}, 0.1 \times \bar{FBD}^{\min})$, where $\bar{FBD}^{\min}$ follows Table A.5;
  - $FBD_j^{\max} \leftarrow N(\bar{FBD}^{\max}, 0.1 \times \bar{FBD}^{\max})$, where $\bar{FBD}^{\max}$ follows Table A.5;
  - The composition parameters for the controlled components follow Table A.6.

Table A.4
Composition parameters for the controlled components in the instance creation process (6 tanks + 1 CDU).

k	1	2	3
$yS_{i,k}^0$	$U(0.4, 0.6)$	$U(0.2, 0.4)$	$1 - yS_{i,1}^0 - yS_{i,2}^0$
$yS_{i,k}^{\min}$	0	0	0
$yS_{i,k}^{\max}$	1	1	1
$yB_{j,k}^{\min}$	0.4	0.2	0.1
$yB_{j,k}^{\max}$	0.6	0.4	0.3
$yB_{j,k}^0$	$U(0.45, 0.55)$	$U(0.25, 0.35)$	$1 - yB_{j,1}^0 - yB_{j,2}^0$

Table A.3
Mean values for the flow parameters in the instance creation process (6 tanks + 1 CDU).

SCH	15	20	25	30
$\bar{FVS}^{\max}$	96k	72k	58k	48k
$\bar{FSB}^{\max}$	64k	48k	38k	32k
$\bar{FBD}^{\min}$	16k	12k	10k	8k
$\bar{FBD}^{\max}$	40k	30k	24k	20k

Table A.5
Mean values for the flow parameters in the instance creation process (8 tanks + 2 CDUs).

SCH	15	20	25	30
$\bar{FVS}^{\max}$	64k	48k	38k	32k
$\bar{FSB}^{\max}$	32k	24k	19k	16k
$\bar{FBD}^{\min}$	16k	12k	10k	8k
$\bar{FBD}^{\max}$	24k	18k	15k	12k

Table A.6
Composition parameters for the controlled components in the instance creation process (8 tanks + 2 CDUs).

k	1	2	3	4
$yS^0_{i,k}$	$U(0.1, 0.3)$	$U(0.1, 0.3)$	$U(0.1, 0.3)$	$1 - yS^0_{i,1} - yS^0_{i,2} - yS^0_{i,3}$
$yS^{\min}_{i,k}$	0	0	0	0
$yS^{\max}_{i,k}$	1	1	1	1
$yB^{\min}_{j,k}$	0.2	0.2	0.1	0.1
$yB^{\max}_{j,k}$	0.4	0.4	0.3	0.3
$yB^0_{j,k}$	$U(0.1, 0.2)$	$U(0.1, 0.2)$	$U(0.1, 0.2)$	$1 - yB^0_{j,1} - yB^0_{j,2} - yB^0_{j,3}$

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