Networks NET-21-0101
"Coordinating drones with mothership vehicles: The mothership and multiple drones routing problem with Graphs"

Answer to Reviewers' Comments

February 23, 2022

We wish to thank the Editors and the Reviewers for their valuable comments and advices which allowed us to further improve the quality of our paper.

We revised the manuscript by taking into account all the suggestions of reviewers 1 and 2. We report below our changes inside the colored textboxes.

Editors' Comments The revision has surely improved the paper. However, I still have some major concerns. Therefore, I would like to provide the authors the opportunity to further revise/improve the paper before sending it out to the reviewers. In their revision the authors should specifically focus on:

1. language. Please fix grammatically incorrect sentences. For example page 3: "We also assume that is not needed the mothership to be stopped..."
2. the motivation for studying this problem is still not very convincing. Try to explain early on (introduction) why your problem with graphs is relevant, possibly with practical examples. There are plenty of examples of settings in which drones must visit nodes or edges. Please define and explain the notion of graphs here. Can a graph be modeled as a set of edges that need to be visited (this would simplify notation a lot)?
3. the definition of the different types of overlap is not clear. Is the "overlapping" setting not simply a restricted version of the "partially overlapping"? That is, I assume that non-overlapping is also allowed in the partial overlapping case? This is not clear from the text. It would be hard to justify focusing on strict partial overlaps only. (If my interpretation is correct then the theorem on page 14 is true by definition)
4. simplify and cleanup notation: would it not be possible to drop some of the indexes to improve the readability? The auxiliary variables also do not appear to be adding much value here. Overall, the formulation seems overly complex, and the authors may want to see how to simplify.
5. the authors may want to simplify the presentation of the completely overlapping and partially overlapping problems as they share a lot of common elements which creates a lot of redundancy. I recommend only focusing on the key differences.
6. the authors may want to convert to gray scale for the different figures as to improve the readability and accessibility of the paper. Please incorporate the responses to these additional comments in a new version of the response letter that also includes the responses to the earlier reviews.

Thanks for the comments. We report the answers to the major concerns point by point below:

1. We have revised the whole manuscript carefully and tried to avoid any grammar or syntax error. In addition, we have asked a professional editing service (Roger G. Tilley - World English) to check the English.

2. We have moved the part of the references to practical applications of the problem studied in this paper from Section 2 (Problem Description) to Section 1 (Introduction). Moreover, in the same paragraph, the notion of graph is provided in this context. It is used to represent a part of a network where drones perform inspection activities of different typologies (as shown in the Case Study Section). In addition, in Section 2, we mention one of the most recent applications of drone technology in hybrid systems where drones are supported by boats to rescue immigrants in the sea.
   As regard the possibility of substituting the set of graphs with a set of edges, we point out that it would simplify the notation with the cons of making the model more complex. Indeed, without the graph structures, the selection of the subset of edges to be visited in a drone tour should be found by the model itself. Moreover, the whole structure of the solution would change and the possibility to limit the visit of a drone to a given fraction of the total length of the graph would be lost.

3. We are not completely sure about the concept of non-overlapping mentioned by the associate editor. Conceptually, it is true that the partial overlapping variant is an extension of the complete overlapping one. If a non-overlapping solution represents the situation in which one drone is launched and retrieved before another different drone is launched to visit another target graph, this is a possibility included in the partial overlapping version of the problem. We have added in the revised version of the paper a sentence clarifying this concept (Page 3). However, the two versions of the problem are formulated by using different sets of variables by introducing the concept of stage instead of the concept of operation. For this reason, the relationship between the feasible sets of both problem variants must be formally proved as presented in Theorem 3.1.

4. In the revised version of the manuscript, we have moved the more general case of non-homogeneous fleet of drones, that requires the additional δ index, to the Appendix. We think that it is not possible to further simplify the notation of the formulation for the case of a homogeneous fleet of drones.
   As regards to the auxiliary variables, we are not sure about the specific set of them to which the associate editor refers to. The ones used for linearizing constraints are being introduced because of the request of one of the reviewers to have a self-contained paper.

5. In the revised version of the paper, first we report the common constraints to the formulations of the two problem variants (Section 3). Then, in the two following subsections (3.1 and 3.2), we present the constraints characterizing the two formulations. For each version of the problem, we explain the different assumptions and the concepts of operation and stage behind them. We point out that, even if some subsets of constraints are similar, they are expressed with different set of variables. In order to make the subsections self-content and following the suggestions of the Reviewer 2, we report explicitly them for both problem variants. Indeed, we think that because of the complex notation that the formulations require, this can help to improve the readability of the paper.

6. In the revised version of the manuscript, we have converted to gray scale for the different figures.

Editors' Comments The paper focuses on coordinating and routing a mothership with multiple drones. I have received report from two referees with relevant expertise in terms of the topic and the associated methodology. The feedback of these reviewers is largely consistent. While the reviewers see potential and find the topic interesting (reviewer 1 even uses the word "important"), they both indicate that the paper is difficult to read, and that significant editing/rewriting is required to make the content of the paper more accessible to the reader. Reviewer 1, who comments that the paper can be seen as a "first draft", provides a very extensive list of comments and suggestions to help such a thorough revision. In line with the comments of this reviewer, the authors may want to consider a more informative title for the paper. Moreover, reviewer 2 presents a few fundamental concerns about the relevance of the presented model. The reviewer asks for a more convincing motivation and justification of the underlying assumptions. I agree with these concerns and recommend a major (risky) revision to addresses all comments of the reviewers. The authors may want to use a language editor.

Answer E

Thank you for the feedback. We revised the manuscript following the reviewers' advices. The revision has been very deep. There have been many changes in the content and also in the style to accomplish all the requests from the reviewers. One of the main changes was motivated by the request to include computational results for the asynchronous model (now called partial overlapping model) and to compare them with the synchronous version (now called complete overlapping model). For a clearer comparison, we have included a simplified version of the old synchronous model assuming that all drones are indistinguishable since this was an assumption of the second model in the previous version of the paper. Since, we still think that the old synchronous model is valuable and may be interesting to the readers, we have decided to keep it and its computational results in an Appendix.

Reviewer 1

In my opinion, the main strengths of the manuscript are right at the beginning and at the end:

1. *It deals with a very interesting problem that is very topical these days: the optimization of route problems with drones. Specifically, this manuscript addresses a problem in which a "mothership" travels along a route and launches multiple drones that perform a task and are later retrieved at another point along the route. This is a very difficult problem.*
2. *It proposes and implements some novel methods that are capable of providing solutions to some instances of such a difficult problem.*

Unfortunately, I find that almost everything between these two points needs a major revision to give the work more coherence, precision and clarity. Although perhaps is unfair, it could be said that the manuscript is a first draft of a hard work that may eventually turn out to be important.

Answer R1.1

Thank you for your comments. We took into account all your suggestions to improve the presentation, coherence and quality of our manuscript. There have been many changes in the content and also in the style to accomplish all the requests from the reviewers. One of the main changes was motivated by the request to include computational results for the asynchronous model (now called partial overlapping model) and to compare them with the synchronous version (now called complete overlapping model). For a clearer comparison, we have included a simplified version of the old synchronous model assuming that all drones are indistinguishable since this was an assumption of the second model in the previous version of the paper. Since, we still think that the old synchronous model is valuable and may be interesting to the readers, we have decided to keep it and its computational results in an Appendix. The rest of the changes are explained one by one in the detailed answers included below.

Title: "multiple drones": however, in the computational results the case of a single drone is also considered.

Answer R1.2

The referee is right but although the title refers to the case of multiple drones, the model can also be applied to the case of a single drone. We think that it is useful to include the case of a single drone to compare the results and complexity of the two cases (single and multiple). Therefore, we have also included the single drone model so as to compare the new results with the one presented in [Amorosi et al., 2021]. We have justified better why this case is also considered in the computational experiments section of the revised version of the manuscript.

Abstract: "weighted distance" The total distance is weighted? I have not seen this on the manuscript.

Answer R1.3

Thanks for the comment. We agree with the reviewer and we have removed the adjective weighted. Indeed, we are not weighting the distance covered by the mothership in the objective function.

Page 2: "synchronous version in which every drone is launched and retrieved in the same stage." ... "the asynchronous situation where one assumes that the mothership can retrieve one drone in a different stage from the one in which it has been launched": I don't know if the term synchronous/asynchronous is appropriate: It can be confused with the idea that the moment the drones arrive at the mothership is synchronized, or not.

Answer R1.4

Although we thought this terminology was clear, we agree with the reviewer that may lead to some confusion. Clearly in both variants of the problem, the mothership and the fleet of drones are synchronized. To clarify this issue we have now defined the concept of **operation***: An operation is the trip followed by the mothership between the launching and the retrieving point of the same set of drones.* The difference between "asynchronous" and "synchronous" cases is the possibility of overlapping two (consecutive) operations or not, respectively. In the "synchronous" ("asynchronous") version, the mothership can not (can) start a new operation before the previous one finished. In order to be more consistent with the actual meaning of the two problem variants, in the revised version of our manuscript we changed the used terminology and we indicated the two versions of the problem distinguishing between *complete overlapping operations* and *partial overlapping operations*. Moreover, in the revised version of the manuscript, we have inserted a new figure that illustrates this difference between the two model variants.

Page 2: "Finally, Section 6 concludes the paper": The "Case Study" section 6 is not mentioned.

Answer R1.5

Thanks for the comment. We have included a sentence that refers to the Case Study section in the revised version of the manuscript and a sentence referring to the Appendix.

Page 2: "base vehicle (mothership) can stop anywhere in a continuous space and has to support the launch/retrieve of a number of drones": The time that is stopped is not taken into account to assess the quality of the solution? In the "Case Study", can the helicopter stop at the intermediate points?

Answer R1.6

Thanks for the suggestion. In our model, we are considering a simplified version that assumes that the time spent by the mothership to retrieve and launch the drones is negligible, like in [Poikonen and Golden, 2020]. We focused on minimizing the time required by the mothership to support drone operations in a coordinated system. We think that this assumption is often considered in the literature, although it could be interesting to study the case that takes into account this stopping time to be minimized in future works.

Page 2: "... operations consisting in visiting given percentages of the length of a set of graphs" and Page 3 "1) traversing a given percentage of the length of each one of its edges or 2) visiting a percentage of the total length of the network.": I have never seen this before: Is it only necessary to travel a percentage of the distance? For example, if only 50% is required, does it not matter which "half" is covered? I think this should be justified in some real situations.

Answer R1.7

Thanks for the question. In our model, when we require to travel a fraction (percentage) of the distance, it does not matter which part of the graph is covered. As we also highlighted in the revised version of the manuscript, many different inspections and monitoring activities can be performed by drones in a systematic and safe way. We can mention, for example, monitoring of the status of bridges and portions of road networks, both for preventive maintenance and for inspection after disasters (e.g. earthquakes). Other kinds of inspection activities, like, for example, video surveillance of urban areas of big cities, can be also modelled by adopting the formulations presented in this paper. In this context, the request of visiting only a given fraction of the target graphs (e.g. borders of a neighborhood) can be due to the necessity of "covering" different areas in a limited amount of time. Another example that we can mention is the traffic flow monitoring. In this case, to verify if the traffic progression is not disrupted, only inspecting a portion of the edge provides valuable information.

Page 2: "The base vehicle can move freely on a continuous space": Is this realistic? please mention some examples (like helicopter case).

Answer R1.8

Thanks for the question and the suggestion. Following this suggestion we have included in the revision the cases of motherships that can move freely in the plane as helicopters or boats and we have highlighted, in the revised version of the manuscript, a reference related to a configuration of the tandem helicopter-drone adopted in real cases. For example, systems consisting in a boat and a fleet of drones are used by coast guards to perform surveillance activities to identify immigrants that need help in the sea (see [AltiGator, 2015]).

Page 3: "Moreover, at each stage the drones ... determined)": Stage? what are the stages? They have not been defined yet. And are all the drones launched on each stage? No, later in the paper it is seen that it does not, but here the entire sentence is not understood.

Answer R1.9

To clarify this issue we have substituted the concept of "stage" by the one of "operation". This is already explained in Answer R1.4.

Page 3: the sentence "Note also that every drone of the fleet does not have to be launched from the current base vehicle location in all the stages because of the capacity constraint" is not understood.

Answer R1.10

Thanks for the observation. In our model, we assume that the drone endurance does not permit the existence of a single point from where the mothership can launch and retrieve every drone of the fleet so that all operations are done without moving the mothership. On the contrary, the mothership must be close enough to the target graphs to launch and retrieve the drones, and obviously, only some of the available drones can be launched from this point due to their limited endurance. In the revised version, we slightly modified this sentence to the following one "*Note also that every drone of the fleet cannot be launched from the same base vehicle location to make all the tasks because of its limited endurance*".

Page 3: "In this section we present a MINLP..." express MINLP also with words.

Answer R1.11

Done. We have also expressed the meaning of MINLP with words.

Page 3, Table 1: The alpha parameters are presented as percentages although they are between 0 and 1.

Answer R1.12

Done. In the revised version of the manuscript, we have expressed the α parameters as fractions between 0 and 1.

Page 3, equation (alpha-E): Try to explain the equation. /lambda and /rho are not defined yet in the text, only in table 2. They are between 0 and 1, then alpha must be too. It seems that, here, /mu is not necessary because ALL the edges of g are visited.

Answer R1.13

Thanks for the observations. We agree with the reviewer. In the revised version of the manuscript, we have better commented the equation $(\alpha - E)$ and we have explained the meaning of λ and ρ. We have also updated Table 1 by specifying that α is the fraction of the length that must be traversed and it ranges between 0 and 1. Moreover, we have removed the term μ in the constraint $(\alpha - E)$, because all edges must be visited in this case. In order to be consistent, we have also included in the (Time_D^o) constraint the following sentence: *Note that, in the special case where all edges must be visited, the third sum of the left-hand side of the (Time_D^o), reduces to $\sum_{e_g \in E_g} d^{e_g}$ by setting all the μ^{e_g} variables equal to one.*

Page 4, Table 2, second line of "Continuous Decision Variables": Are v_{min}^{eg} and $v_{max}^{e_g}$ variables or parameters, as it says in the text (third line after table 2)? If they are variables, the last inequalities of (alpha-E) are not linear. If they are parameters, what value is given to them?

Answer R1.14

Thanks for the observation. We agree with the reviewer. In the revised version of the manuscript, we have changed in the text the term "parameter values" by "auxiliary variables". In addition, we have explicitly reported the linearization of the last inequality of $(\alpha - E)$ and $(\alpha - G)$.

Page 4, Table 2, lines 3 and 4 of "Continuous Decision Variables": put "the drones", in plural.

Answer R1.15

Done.

Page 4, equations (alpha-E): Try to explain them.

Answer R1.16

Done. In the revised version of the manuscript we have better commented and explained $(\alpha - E)$ and $(\alpha - G)$.

Page 4, line -9: "Let us denote by $\mathcal{T}$ the set of stages/tasks that the mothership and the fleet of drones have to carry out." The stages are not very well defined here. First it is said that they are the tasks to be carried out, but then it is said that $|\mathcal{T}| \leq |\mathcal{G}|$, because in each task at least one drone is launched (which serves a graph). Then, is the number of stages an unknown variable a priori? How do you put in the formulations $t \in \mathcal{T}$? Later (page 7) it is stated that, $|\mathcal{T}| = |\mathcal{G}|+1$, with a fictitious last stage. And in section 3.2 it is explained even better. The authors could try to explain it well from the beginning.

Answer R1.17

Thanks for the observations. We agree with the reviewer. To be consistent with the terminology, in the revised version of the manuscript we have reserved the term operation for the first model and reserved the term stage for the second model where it really makes sense. We assume that in one operation, it is possible to launch from the mothership more that one drone and, thus, to visit more that one graph. For this reason, the number of operations ($|\mathcal{O}|$), that is unknown a priori, is less than or equal to the number of graphs ($|\mathcal{G}|$). Consequently, in the formulations, we indexed the variables and constraints assuming $\mathcal{O} = \mathcal{G}$, although all visits to graphs are forced to be concentrated in the first tasks by means of the valid inequalities explained in Section 4.1.

Page 5, line 1: This path is not well understood. R^{e_g}, L^{e_g} are not defined in the text (they are in Table 2). Why does the path end with an $R^{e_g''}$ and not with the next $L^{e_g''}$? And why is x_L^{t+1} included? (that drone may not come out in stage $t+1$).

Answer R1.18

Thanks for the comment. We agree with the reviewer. We have introduced the continuous variables R^{e_g} and L^{e_g} in the text. We have changed the path followed by the drone so that it exits from the graph g by the point $L^{e_g''}$. In addition, we have inserted a paragraph explaining this path.

Page 5, Fig. 1: Why is it interesting to launch two drones from the same point x_L? Does the launch have a cost of stopping the mothership and then restarting it? Otherwise, it seems, in the figure, that if the first green drone is launched later and the second green drone is picked up earlier, the point $x_L^2 = x_R^1$ could be located lower and the total distance traveled by the mothership would be less. In general, it seems that launching and retrieving the drones separately gives better solutions, unless there is a cost to stop the mothership.

Answer R1.19

Thanks for the comment. The idea of the example is to show the notation of the launching and retrieving points x_L and x_R. The picture printed in the original version of the manuscript showed a feasible solution (and not the optimal solution) of the problem. In the revised version of the manuscript, we have changed the location of the graphs to obtain feasible and optimal solution pictures clarifying the issue.

Page 5, line -9: "... the drones return to the point x_R^1 from where they are launched again to ...". If the drones land at one point and then leave the same point, there is no time to recharge the batteries. Unless the mothership remains stationary at that point until the drones reload.

Answer R1.20

Thanks for the observation. As mentioned in the Answer R1.6, in this model we are making the simplification to assume that recharging the batteries of the drones is done in no time and the aim is to minimize the total time required by the mothership. Your suggestion could be very interesting to be considered as an extension of this model, by including the recharging batteries time (as a release time) in the objective function. In the revised version of the manuscript, we have included this idea as further research in the Concluding Remarks section.

Page 5, line -1: "The optimal order to visit the edges of each graph in its corresponding stage". It is not necessary to optimize the order of visiting the edges: it is enough that the total distance traveled is less than the autonomy of the drone.

Answer R1.21

The reviewer is right. Since we are only taking into account in the objective function the time of travelling by the mothership (and not the one travelled by the drones), it is not needed to find the best route to visit the target graphs: it is only required to prevent the existence of subtours in the paths followed by the drones. In the new version of the manuscript, we have changed this sentence to be consistent.

Page 6, lines 1, 2 –inequalities (1) and (2)– and 8: "Inequalities (1) and (2) state that for each stage at most one drone can be launched and retrieved for performing an operation.": "at most"? Can not several drones be launched on a stage? On page 4 you talk about "at least one drone".

Answer R1.22

Thanks for the observation. We agree with the reviewer. In the Appendix of the revised version of the manuscript, we have rewritten inequalities (1) and (2) by changing:

$$\sum_{g\in\mathcal{G}}\sum_{e_g\in E_g}\sum_{d\in\mathcal{D}} u^{e_g t\delta} \leq 1, \forall t\in\mathcal{T} \Longrightarrow \sum_{g\in\mathcal{G}}\sum_{e_g\in E_g} u^{e_g o\delta} \leq 1, \forall o\in\mathcal{O}, \forall\delta\in\mathcal{D}.$$

$$\sum_{g\in\mathcal{G}}\sum_{e_g\in E_g}\sum_{\delta\in\mathcal{D}} v^{e_g t\delta} \leq 1, \forall t\in\mathcal{T} \Longrightarrow \sum_{g\in\mathcal{G}}\sum_{e_g\in E_g} v^{e_g o\delta} \leq 1, \forall o\in\mathcal{O}, \forall\delta\in\mathcal{D}.$$

These changes describe the idea that, in each operation, each drone can visit at most a target graph. We further clarify the meaning of these constraints in the revised version of the manuscript. However, we point out that the corrected constraints were already implemented in the Python code related to the model and the results presented in the manuscript.

Page 6, lines 19 and 20: remove the sentence "since they would allow free jumps of the drone between different routes at no time." It is not understood and is unnecessary.

Answer R1.23

Done. In the revised version of the manuscript, we have rephrased the sentence to clarify the necessity of the subtour elimination constraints. The corrected sentence states *Please, note that the subtour elimination constraints are needed to avoid the presence of disconnected paths on the edges of the graph.*

Page 7, line 7: The sentence "The coordination between the drones and the mothership must ensure that the time spent by the drone d to visit the graph g at the stage t is less than or equal to the time that the mothership needs to move from the launching point to the retrieving point during the stage t" is not congruent with the one in page 3 "However, this does not mean that the mothership and all drones must arrive at a rendezvous location at the same time: the fastest arriving vehicle may wait for the others at the rendezvous location." The flight time of each drone has to be less than its endurance.

Answer R1.24

Thanks for carefully reading the manuscript. We agree with the reviewer. In the revised version of the manuscript, we have rewritten the sentence in page 3 with the one referred in page 7. The idea of the coordination model is that the time spent by the drones to perform their tasks must be lower than or equal to the time spent by the mothership to go from the launching point to the retrieving point as stated in equations (DCW-CO) and (DCW-PO).

Page 7, equation (DCW): It deserves an explanation.

Answer R1.25

Done. In the revised version of the manuscript, we have inserted a paragraph that explains the meaning of this constraint that plays an important role in modelling the coordination system.

Page 7 line -19: "constraint will become an equality and we can model...": An equality? the LHS depends on g and the drone and the RHS does not!

Answer R1.26

Thanks for the comment. We point out that the RHS also depends on g and δ by means of variables $u^{e_g t\delta}$. In the revised version of the manuscript, we have changed the constraint in the following way:

$$time^o_\delta \geq \frac{1}{v_\delta}\left(\sum_{e_g\in E_g} u^{e_g o\delta} d_L^{e_g o\delta} + \sum_{e_g, e'_g\in E_g} z^{e_g e'_g} d^{e_g e'_g} + \sum_{e_g\in E_g} \mu^{e_g} d^{e_g} + \sum_{e_g\in E_g} v^{e_g o\delta} d_R^{e_g o\delta}\right) - \\ - N_\delta\Big(1 - \sum_{e_g\in E_g} u^{e_g o\delta}\Big),$$

$$time^o_D \geq time^o_\delta, \;\; \forall \delta\in\mathcal{D},$$

$$time^o_D \leq time^o_M.$$

These constraints define the time spent by the drone δ to perform operation o, the maximum time spent by a drone to visit a graph during operation o and the coordination in terms of time between the mothership and drones, respectively. You can find more details in the new Appendix section of the manuscript.

Page 8, line -3: explain better the sentence "... reducing the available capacity so that is possible to traverse the required percentage of these graphs."

Answer R1.27

Thanks for carefully reading the paper. In the revised version of the manuscript, we have rewritten the sentence to justify the assumption of Theorem 3.2 by which it is possible to reduce the visit of graphs to the visit of points by reducing the endurance of the drone.

Page 9, Theorem 2.1: Please explain the meaning and the implications of this theorem.

Answer R1.28

Done. We have inserted a sentence in the revised version of the manuscript that explains the meaning and implications of Theorem 3.2 on the relationship between the two models presented in the manuscript. Moreover, a new theorem stating the relationships between models is included.

Page 9, set of inequalities in Theorem 2.1: Before, v_D and v_M were the speeds of the drone and mothership. Now who is v_C? Should it be v_M?

Answer R1.29

Done. We have corrected these typos in the new version of the manuscript.

Page 9, line -8: "In this problem, we assume that the fleet has more than one drone since otherwise ...": However, the computational results manage the case with one drone.

Answer R1.30

As mentioned in the Answer R1.2, the idea of including the case of a single drone is to compare the complexity of the multiple drones model with the one presented in [Amorosi et al., 2021] with only one drone. In the computational experiments section of the revised version of the manuscript, we have included an explanation that motivates this fact. It is also meant to show that using more drones reduce the objective function value.

Page 10, line -11: "To linearize the first term of the objective function in AMMDRPG, ...": Is the objective function in AMMDRPG (page 7) not linear?

Answer R1.31

Thanks for the observation. We agree with the reviewer. We have corrected this sentence, because this term does not appear in the objective function. In this model, since we are only minimizing the time required by the mothership, the objective function is linear.

Page 10, lines -10 to -8: there are undefined variables.

Answer R1.32

Thanks for the observation. In the Appendix of the revised version of the manuscript, we have inserted in Table 8 the auxiliary variables $p_L^{e_g o\delta}$, $p^{e_g e'_g}$ and $p_R^{e_g o\delta}$, that model the product of the continuous and binary variables:

- $p_L^{e_g o\delta} = d_L^{e_g o\delta} u^{e_g o\delta}$.
- $p_R^{e_g o\delta} = d_R^{e_g o\delta} v^{e_g o\delta}$.
- $p^{e_g e'_g} = d^{e_g e'_g} z^{e_g e'_g}$.

These variables appear in the constraint (DCW-CO) and their definitions are also included in the explanation of this constraint.

Page 10, line -6: please explain better the sentence "every launching or rendezvous point is inside the circle whose diametrically opposite points are described below."

Answer R1.33

In the revised version of the manuscript, we have rewritten this sentence to clarify the estimation of this bound. The new sentence is: *Note that, among all graph nodes and the origin and the destination points, it is possible to identify the pair of points at maximum distance. From this pair of points, we can build a circle whose diameter is the segment joining them. Hence, because we are minimizing the distance travelled by the mothership, every launching or rendezvous point is inside this circle.*

Page 11, line -3: How is $\mathcal{L}(e_g, e'_g)$ computed? Because it could be a CPP (easy) but it could also be a CPP at 50% on each edge or at 50% in total length. It would be interesting to explain how it is computed.

Answer R1.34

Depending on the variant of the problem that we are considering, the computation of $\mathcal{L}(e_g, e'_g)$ is performed (i) by solving a CPP at α^{e_g} fraction of the length of each edge e_g or (ii) by solving a CPP at α^{e_g} fraction of the total length of the graph g. In both cases, we impose that the path starts from e_g and ends at e'_g. In order to better clarify this issue, in the revised version of the manuscript, we have adopted the same example of Figure 2 (where half of each edge is required) to execute the matheuristic and we have commented the results explaining how the partial solutions are obtained in each step of the procedure.

Page 12, STEP 2: Is it true that grouping graphs to be visited in the same stage produces better results than considering one graph per phase?

Answer R1.35

Thanks for this interesting question. The aim of the model is to minimize the time spent by the mothership. Hence, grouping the graphs to be visited in the same operation reduces this time, because more target graphs are visited in parallel. Moreover, the second computational experiment shows that the use of more than one drone produces an improvement in terms of the objective value of the problem.

Page 12, line 5: n_D is undefined.

Answer R1.36

Thanks for carefully reading the paper. In the revised version of the manuscript, we have modified the notation by substituting n_D and $\#drones$ with $|\mathcal{D}|$, where $\mathcal{D}$ is the set of drones.

Page 12, line 7: "search for point P": How is that search done?

Answer R1.37

In the revised version of the manuscript, we have explained that P is obtained by solving a feasibility location problem (satisfying the endurance constraint) to quickly obtain a solution.

Page 12, line 9: "If such a point exists": Is that point unique? What if it is not unique? If the point does not exist, what is done by the algorithm?

Answer R1.38

Thanks for the question. As mentioned in the Answer R1.37, the idea is to solve a feasibility problem. Hence, if such a point exists, it could not be unique. If the point does not exist, then the clusters can not be grouped in terms of the endurance constraint. In the revised version of the manuscript, we have included the case of no existence of this point P, although it was already included in the matheuristic code written in Python.

Page 12, line 9: The parameter "maxit" is not mentioned anywhere else in the paper.

Answer R1.39

The parameter *maxit* is an input of the matheuristic that sets the maximum number of iterations for STEP 2. However, in order to better explain the matheuristic input and output and its main steps, in the revised version of the paper, we adopted the LaTeX *algorithm* environment to describe it.

Page 12, STEP 3: "Compute a reference point", "seeks for the minimization": How is it done? Also, does the origin point intervene but not the point of destination?

Answer R1.40

Thanks for the question. In the revised version of the manuscript, we have specified which problem we are solving in STEP 3. As the goal consists in minimizing the total time traveled by the mothership, the idea is to determine reference points, that are associated with each cluster, satisfying the endurance constraint, by minimizing the distance between each pair of them and the distances between them and *orig* and *dest*. More precisely, we solve the problem of minimizing the sum of the distances between each pair of reference points (including *orig* and *dest*) subject to the endurance constraint (for each cluster). We point out that the *dest* point was already included in the Python implementation of this step of the matheuristic.

Page 12, STEP 4: "Compute the TSP ..." It is supposed to be a tiny TSP. And are origin and destination not considered? Wouldn't it be better to calculate an open TSP from origin to destination visiting the reference points?

Answer R1.41

Thanks for the suggestion. In the revised version of the manuscript, we have better explained that we are solving an open TSP over the reference points computed in STEP 3 and *orig* and *dest* points. We remark that the corrected computation of the TSP was already implemented in the Python code related to the matheuristic and the results presented in the manuscript.

Page 12, STEP 4: The parameter "maxseed" is not mentioned anywhere else in the paper.

Answer R1.42

The parameter *maxseed* is an input of the matheuristic that sets the maximum number of iterations to repeat the clustering procedure (from STEP 2 to STEP 4). However, in order to better explain the matheuristic input and output and its main steps, in the revised version of the paper, we adopted the LaTeX *algorithm* environment to describe it.

Page 12, STEP 4: "go to STEP 2" -¿ save the reference points, if they are better, and go to STEP 2.

Answer R1.43

Thanks for the observation. We agree with the reviewer. We have better explained the flow of the algorithm in the revised version of the manuscript.

Page 12, STEP 5: "Set the values of the binary variables u and v ...": How?

Answer R1.44

Thanks for the question. In the revised version of the manuscript, we have explained how these binary variables are set. In STEP 5, we take the best solution obtained by the clustering procedure from STEP 2 to STEP 4 over the *maxseed* iterations. Based on this solution, we associate each reference point, generated in STEP 3, with one operation o following the order of visits of the reference points provided by STEP 4. Successively, for each cluster, among the graphs belonging to it, the point R^{e_g} on edge e_g, generated in STEP 1, is used to set the corresponding variable $u^{e_g o}$ equals to one. Similarly, by using the point $L^{e'_g}$, generated in STEP 1, the corresponding variable $v^{e'_g o}$ is set equal to one. Note that the same matheuristic can be adopted to initialize the partial overlapping model thanks to the results in Theorem 3.1.

Page 12, line -1: "One cluster contains graphs g_1 and g_3 (in red), while graphs g_2 and g_4 represent distinct clusters.": It seems strange that the two graphs furthest from the origin form a cluster and the two closest to the origin are different clusters.

Answer R1.45

To have a more understandable example, in the revised version of the manuscript, we have used another example reported earlier in the paper (Figure 2) also for illustrating the matheuristic.

Page 13, line 1: "..according with STEP 3, produces the points ..." Nowhere is it explained how they are computed.

Answer R1.46

Thanks for the observation. In the revised version of the manuscript, we have described how the reference points are computed in STEP 3. See also Answer R1.40.

Page 13, line 5: "the tour of the mothership along the origin point, P1, P2 and the destination point, ...": but not a TSP between the reference points, as the algorithm text says.

Answer R1.47

In the revised version of the manuscript, we have clarified that, in STEP 4, an open TSP over the reference points, *orig* and *dest* points is computed.

Page 13, line 9: "the values of the variables $u^{e_g td}$ and $v^{e_g td}$": the values of these variables before step 5 and after step 5 are not the same for all graphs in the figures.

Answer R1.48

Thanks for the observation. We point out that, in STEP 5, the variables $u^{e_g o}$ and $v^{e_g o}$ are provided to Gurobi only as initial partial solution of the model. Thus, during the resolution procedure, the solver can find a better solution with different values of these variables. As mentioned also in Answer R1.45, in the revised version of the manuscript, we have changed the illustrative example.

Page 14, line 4: "Note that in this example the drones do not visit the full 100% of each graph, but only a pre-specified percentage of each one of them": However, in STEP 1 it seems that 100% is considered.

Answer R1.49

Thanks for the observation. We agree with the reviewer. As mentioned in Answer R1.45, in the revised version of the manuscript, we have substituted the illustrative example and we have solved the CPP at half of the length of each edge.

Page 15, line 5: "This set consists of 5 instances of respectively 5 and 10 target graphs, ...": This phrase is not understood. It seems to mean that each row in Table 4 corresponds to 5 instances.

Answer R1.50

In the revised version of the manuscript, we have rewritten this sentence. We have generated one set of 5 instances with 5 graphs and one set of 5 instances with 10 graphs.

Page 15, line 7: "... of 10 nodes.": in the table they say up to 12. And really for it to add up to 100% it should be up to 12.

Answer R1.51

We had forgotten to mention the remaining 20% of graphs with 12 nodes. In the revised version of the manuscript, we have described also the missing 20% of graphs with 12 nodes.

Page 15, line 9: "We consider in our experiments that the number of drones varies between 1 and 3": The title of the paper specifies multiple drones.

Answer R1.52

We refer the reviewer to Answer R1.30.

Page 15, Table 3: "random variable": How is it selected?

Answer R1.53

In the revised version of the manuscript, we have specified that the α parameters are uniform random sampled in the interval $(0, 1)$.

Page 15, line 15: "... an initial solution computed by the matheuristic ...": Are these solutions computed by the matheuristic reported somewhere in the paper?

Answer R1.54

Thanks for the question. The solutions computed by the matheuristic were not reported in the paper. In the revised version of the manuscript, we have inserted a boxplot figure that reports the relative gap of the feasible solution obtained with the matheuristic with respect to the best feasible solution obtained with the exact method by initializing it with the matheuristic solution.

Page 15, line 19: "... a percentage of each edge (e) and the percentage ...": Why are the values of those percentages not put in Table 4?

Answer R1.55

We think that providing the random value of the α parameter for each one of the edges of each graph of the 20 instances does not contribute to give more information about the results obtained in the experiments, since we are reporting the average results for each configuration applied to each instance.

Page 15, line 20: "The fourth column ...": There is no fourth column in the table: for each number of drones there are 3 columns.

Answer R1.56

We point out that Table 4, now Table 9 in the Appendix, contains 4 columns that reports, respectively, the number of graphs, the endurance of the drones, the model version (a given fraction of each graph edge (e) or of each graph must be visited (g)) and the cardinality of the set of drones. This last column contains three subcolumns reporting, for each cardinality of set $\mathcal{D}$, respectively, the average percentage gap without initialization (wi), the average percentage gap with initialization (i) and the running time of the matheuristic (TimeH).

Page 15, line 22: "We report respectively average percentage gap ...": Gap on what? Is it the gap between the feasible solution of the exact method and the lower bound of the model, or is the gap between the best feasible solution obtained with the exact method and the (feasible) solution from the matheuristic? In the first case, why is the feasible solution of the exact method not compared with those of the matheuristic?

Answer R1.57

Thanks for the question and the suggestion. In the revised version of the manuscript, we have specified that we are reporting the gap between the feasible solution of the exact method and the lower bound of the model provided by Gurobi. Moreover, we have inserted a boxplot figure that reports the relative gap of the feasible solution obtained with the matheuristic with respect to the best feasible solution obtained with the exact method by initializing it with the matheuristic solution.

Page 15, line -2: ".. the objective function values of the problem ...": Obtained with the matheuristic or with the exact algorithm?

Answer R1.58

Clarified. In the revised version of the manuscript, we have specified that these values are the ones obtained with the exact algorithm.

Page 16, line 1: "... instances with three target graphs ...": aren't they 5 or 10?

Answer R1.59

Thanks for the question. To be clearer, in the revised version of the manuscript, we have included computational experiments comparing the complete and partial overlapping versions of the problem. These results refer to the exact resolution of the models by Gurobi with and without initialization by the matheuristic solution. They have been performed on instances whose characteristics are reported in Table 4. A second set of tests comparing the objective value for different configurations of drone endurance and cardinality of set $\mathcal{D}$ is considered. They have been performed on instances whose characteristics are summarized in the new Table 6 we have inserted in the revised version of the manuscript.

Page 16, line 2: "(capacity) in 10,20,30,40,50,60": the value 10 is not in the table.

Answer R1.60

Thanks for the observation. In the corrected version of the manuscript, we have modified the structure of the computational experiments section to separate the two different experiments that we are considering in the paper.

Page 17, line 6: the speed "of the helicopter is 50 km/h": A low speed for a helicopter.

Answer R1.61

The case study was an illustrative example of application in a realistic framework. For this application we choose a generic speed for the helicopter but the same application can be solved with any speed for the helicopter and drones. In the new version we choose 30 km/h because we model surveillance operations over a city. We wanted to emphasize that the operations are done at slow motion to reduce the noise and cost and to improve reliability. These are realistic values as shown in [Administration, 2019].

Page 17, line 7: "an endurance equal to 2 hours,": For every trip? For each drone (which makes 2 trips)? For the total set of the 6 trips?

Answer R1.62

Thanks for the question. In the revised version of the manuscript, we have clarified the meaning of drone endurance. This is the drone autonomy in terms of time to travel from a launching point, visiting a graph and coming back to a retrieving point.

Page 17,line 9: "... located in an area of the city where it is possible to assume the take-off and landing of an helicopter.": Therefore, it appears that the helicopter does not land to launch or pick up the drones. Then, it stays still flying at the points where it retrieves a drone waiting for it (or them) to arrive. But this waiting time does not appear in the objective function, which only considers the distance. I think that the definition of the problem should be more precise, especially in what happens at the launching and retrieving points, and its implications in the objective function.

Answer R1.63

Thanks for the comment. In the revised version of the manuscript, we have better explained how the system under consideration works. Referring to the case study, the helicopter takes off from the origin, carrying the fleet of drones. Then, the helicopter flies to the first launching point from where two drones are launched to visit two of the target graphs. In the meanwhile, the helicopter flies to the retrieving point where it picks up both drones, but without stopping or landing. Then, the helicopter flies to the second launching point from where only one drone is launched to visit one of the remaining target graphs. While the drone performs its visit, the helicopter flies to the second retrieving point and so on. When the entire set of target graphs has been visited and all drones have been picked up by the helicopter, this latter flies to the destination where, finally, it lands.

Page 17, line 12: the solution ... "... with a percentage gap equal to 83%": Is it a feasible solution obtained by Gurobi that is 83% away from the lower bound? Is this solution better than the one provided by the matheuristic?

Answer R1.64

Thanks for the question. In the revised version of the manuscript, we reported the solutions found within four hours by solving the two versions of the model with the initialization provided by the matheuristic. We point out that it is possible to find a feasible solution only when we initialize the exact resolution through the matheuristic solution, without any improvement over the running time.

Page 17, line 20: "... at point x_R^1, where they are retrieved by the helicopter.": Does the helicopter stop to pick up the two drones?

Answer R1.65

We refer the reviewer to answers R1.6 and R1.20.

Page 17, line 27: "11.27 km.": Looking at figure 11, if the helicopter does 11.3 km, the drones do, at most, about 15 km in each flight, which at a speed of 100 km/h is far from the 2 hours of endurance. At a speed of 100 km/h, it seems that a single drone could do all the flights leaving from and arriving to the same starting point, and the helicopter (mothership) would travel 0 km, which is the optimal solution.

Answer R1.66

In the revised version of the manuscript, we changed the parameters related with the drone endurance, the number of drones and the drone and mothership speeds. These parameters ensure that it is not possible to visit all the target graphs in a single trip without the mothership support.

Page 17, line -7: "This papers": This paper.

Answer R1.67

Fixed.

Page 18, line -12: "Cordoba, by illustrating the solution obtained by adopting the problem formulation, in its synchronized version, and its solution by means of the initialization provided by the proposed matheuristic.": rewrite this sentence.

Answer R1.68

We have rewritten this sentence to be more precise in the revised version of the manuscript.

Reviewer 2

This paper considers a vehicle routing problem in which one large vehicle (a "mothership") deploys and retrieves several smaller vehicles ("drones"). The drones are tasked to visit remote locations, modeled as graphs. Two versions of this problem are presented; one in which all drones are launched and retrieved in the same "stage", and a relaxed version in which the drones may be launched and retrieved in different stages. A heuristic is proposed to solve the first version of the problem.

Unfortunately, I had a difficult time reading this paper, due to numerous grammatical errors as well as general composition issues. A significant editing effort is required to enable the reader to really make sense of this research.

Answer R2.1

Thanks for your suggestions. In the revised version of the manuscript, we did our best to improve the English and the grammar and the whole presentation of our work. There have been many changes in the content and also in the style to accomplish all the requests from the reviewers. One of the main changes was motivated by your request to include computational results for the asynchronous model (now called partial overlapping operations model) and to compare them with the synchronous version (now called complete overlapping model). For a clearer comparison, we have included a simplified version of the old synchronous model assuming that all drones are indistinguishable since this was an assumption of the second model in the previous version of the paper. Since, we still think that the old synchronous model is valuable and may be interesting to the readers, we have decided to keep it and its computational results in an Appendix. The rest of the changes are explained one by one in the detailed answers included below.

*Regarding the research topic itself, the notion of mothership/drone problems is interesting and could **potentially** have novel real-world applications. However, as currently written, the research motivation seems to be more of an academic exercise, rather than an actual problem or real relevance.*

Answer R2.2

We have tried to improve the motivation of the paper. In the revised version of the manuscript, we further highlighted the different real-world applications that can be modelled by adopting the formulations presented in our work. Many different inspections and monitoring activities can be performed by drones in a more systematic and safer way than traditional systems. We can mention, for example, monitoring of the status of bridges and portions of road networks, both for preventive maintenance and for inspection after disasters (e.g. earthquakes). Other kinds of inspection activities, like, for example, video surveillance of urban areas of big cities, can be also modelled by adopting the formulations presented in this paper. Moreover, in the Case Study section of our manuscript, we considered a system consisting of one helicopter supporting a fleet of drones to perform inspection activities in an urban context. The same typology of system (helicopter-drones) has been already successively implemented in real-world. In the military field, this system has been used by the US Army that left the helicopter at the edge of dangerous airspace and released drones, which penetrated the enemy territory and sent back intelligence, surveillance and reconnaissance information (see reference [FG,]).

If the aim is to conduct surveillance of remote areas, allowing/requiring only a percentage of the graphs to be covered seems strange; there's no motivation to conduct any surveillance beyond the minimum. Instead, it seems that an objective of maximizing coverage would be more appropriate.

Answer R2.3

Thanks for the observation. The request of visiting only a given fraction of the target graphs (e.g. borders of a neighborhood) can be found, for example, in the case of traffic flow monitoring. To verify if the traffic progression is not disrupted, only inspecting a portion of the edge provides valuable information. In general, this model variant can be justified by the necessity of "covering" different areas in a limited amount of time. In such a case, the request of visiting only a given fraction of each target graph implies the possibility to cover a more extended area by monitoring/surveillance activities.

By contrast, if complete coverage of each graph is desired (as appears to be true in the case study), then it seems that the individual graph tours can be pre-computed. Then, the problem becomes to simply route the mothership to be sufficiently close to starting/ending points of these graphs. Furthermore, this might allow the consideration of the mothership's speed as a decision variable.

Answer R2.4

Thanks for the observation. Depending on the considered applications, as also mentioned in Answer R2.3, we consider both the possibility of visiting the whole graph or visiting only a fraction of it. Indeed, the STEP 1 of the matheuristic seeks for a feasible route for each individual graph where a fraction of the length of each edge or each graph is visited. The possibility of considering the mothership's speed as a decision variable is very interesting. However, we point out that the current version of the model is already very complex and, indeed, it is not possible to solve it to optimality in a reasonable time. We will keep this idea for future further research on this topic. In the revised version of the manuscript, we added it in the Concluding Remarks section.

Forcing the mothership to move at a constant speed seems limiting (and also prohibits the opportunity for the mothership to simply stay in one place).

Answer R2.5

Thanks for the comment. Since the possibility to consider the mothership's speed as a decision variable would make the formulation more complex, we assume the mothership speed to be constant. See also Answer R2.4.

There is a synchronization model presented in Section 2.2, and a relaxed version in Section 2.3. I would argue that the version in Section 2.3 is more realistic, as it doesn't require the drones to be launched and recovered in the same "stage" (as an aside, the notion of a "stage" needs to be more clearly explained). As a result, I don't think there's any reason to include the overly-restrictive version from Section 2.2. Unfortunately, the heuristic, numerical study, and case study all use the model from Section 2.2.

Answer R2.6

We agree with the reviewer that the version presented in Section 2.3 (now, Section 3.2) is more realistic than the model presented in Section 2.2 (now, Section 3.1). However, we approached the mathematical modelling of this synchronized system under analysis first formulating the version presented in Section 2.2 (now, Section 3.1). This was a fundamental building block to derive the formulation presented in Section 2.3 (now, Section 3.2). Moreover, Theorem 3.2 provides sufficient conditions to obtain the same solution for both problem variants. Following your request to include computational results for the asynchronous model (now called partial overlapping model) and to compare them with the synchronous version (now called complete overlapping model), we have included a simplified version of the old synchronous model assuming that all drones are indistinguishable since this was an assumption of the second model in the previous version of the paper. Since, we still think that the old synchronous model is valuable and may be interesting for the readers, we have decided to keep it and its computational results in an Appendix. Moreover, in the revised version of the manuscript we have also included experimental results on the second model to be compared in performance, gap and solution time with the first one.

In the experimental results of Section 5, it is not clear how the performance of the heuristic was assessed. Are the gaps reported in Table 4 measures of Gurobi's self-reported optimality gaps (given the 2-hour runtime limit), or are these gaps between the heuristic and Gurobi's best solutions?

Answer R2.7

Thanks for the question and the suggestion. In the revised version of the manuscript, we have specified that we are reporting the gap between the feasible solution of the exact method and the lower bound of the model provided by Gurobi. Moreover, we have inserted a boxplot figure that reports the relative gap of the feasible solution obtained with the matheuristic with respect to the best feasible solution obtained with the exact method by initializing it with the matheuristic solution.

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