    Associate Editor  
    Comments to the Author:  
    The paper focusses on coordinating and routing a mothership with multiple drones. I have received report from two referees with relevant expertise in terms of the topic and the associated methodology. The feedback of these reviewers is largely consistent. While the reviewers see potential and find the topic interesting (reviewer 1 even uses the word “important”), they both indicate that the paper is difficult to read, and that significant editing/rewriting is required to make the content of the paper more accessible to the reader. Reviewer 1, who comments that the paper can be seen as a “first draft”, provides a very extensive list of comments and suggestions to help such a thorough revision. In line with the comments of this reviewer, the authors may want to consider a more informative title for the paper. Moreover, reviewer 2 presents a few fundamental concerns about the relevance of the presented model. The reviewer asks for a more convincing motivation and justification of the underlying assumptions. I agree with these concerns and recommend a major (risky) revision to addresses all comments of the reviewers. The authors may want to use a language editor.  
  
  
    Reviewer(s)' Comments to Author:  
  
    Reviewer: 1  
  
    Comments to the Author  
    Review on the manuscript ID-21-0101, "Coordinating drones with mothership vehicles: The mothership and multiple drones routing problem with Graphs".  
  
    In my opinion, the main strengths of the manuscript are right at the beginning and at the end:  
    (1) It deals with a very interesting problem that is very topical these days: the optimization of route problems with drones. Specifically, this manuscript addresses a problem in which a "mothership" travels along a route and launches multiple drones that perform a task and are later retrieved at another point along the route. This is a very difficult problem.  
    (2) It proposes and implements some novel methods that are capable of providing solutions to some instances of such a difficult problem.  
  
    Unfortunately, I find that almost everything between these two points needs a major revision to give the work more coherence, precision and clarity. Although perhaps is unfair, it could be said that the manuscript is a first draft of a hard work that may eventually turn out to be important.  
  
    I have spent a lot of time reading this manuscript (which is too often difficult to read) and I hope that the following comments will help the authors to improve the work and obtain a first-rate article:  
  
  
    Title: "multiple drones": however, in the computational results the case of a single drone is also considered.  
  
    Abstract: "weighted distance" The total distance is weighted? I have not seen this on the manuscript.  
  
    Page 2: "synchronous version in which every drone is launched and retrieved in the same stage." ... "the asynchronous situation where one assumes that the mothership can retrieve one drone in a different stage from the one in which it has been launched": I don't know if the term synchronous/asynchronous is appropriate: It can be confused with the idea that the moment the drones arrive at the mothership is synchronized, or not.  
  
    Page 2: "Finally, Section 6 concludes the paper":  The "Case Study" section 6 is not mentioned.  
  
    Page 2: "base vehicle (mothership) can stop anywhere in a continuous space and has to support the launch/retrieve of a number of drones": The time that is stopped is not taken into account to assess the quality of the solution? In the "Case Study", can the helicopter stop at the intermediate points?  
  
    Page 2: "... operations consisting in visiting given percentages of the length of a set of graphs" and Page 3 "1) traversing a given percentage of the length of each one of its edges or 2) visiting a percentage of the total length of the network.": I have never seen this before: Is it only necessary to travel a percentage of the distance? For example, if only 50% is required, does it not matter which "half" is covered? I think this should be justified in some real situations.  
  
    Page 2: "The base vehicle can move freely on a continuous space": Is this realistic? please mention some examples (like helicopter case).  
  
    Page 3: "Moreover, at each stage the drones ... determined)": Stage? what are the stages? They have not been defined yet. And are all the drones launched on each stage? No, later in the paper it is seen that it does not, but here the entire sentence is not understood.  
  
    Page 3: the sentence "Note also that every drone of the fleet does not have to be launched from the current base vehicle location in all the stages because of the capacity constraint" is not understood.  
  
    Page 3: "In this section we present a MINLP..." express MINLP also with words.  
  
    Page 3, Table 1: The alpha parameters are presented as percentages although they are between 0 and 1.  
  
    Page 3, equation (alpha-E): Try to explain the equation. /lambda and /rho are not defined yet in the text, only in table 2. They are between 0 and 1, then alpha must be too. Ii seems that, here, /mu is not necessary because ALL the edges of g are visited.  
  
    Page 4, Table 2, second line of "Continuous Decision Variables": Are v^eg\_min and v^eg\_max variables or parameters, as it says in the text (third line after table 2)? If they are variables, the last inequalities of (alpha-E) are not linear. If they are parameters, what value is given to them?  
  
    Page 4, Table 2, lines 3 and 4 of "Continuous Decision Variables": put "the drones", in plural.  
  
    Page 4, equations (alpha-E): Try to explain them.  
  
    Page 4, line -9: "Let us denote by T the set of stages/tasks that the mothership and the fleet of drones have to carry out." The stages are not very well defined here. First it is said that they are the tasks to be carried out, but then it is said that | T | <= | G |, because in each task at least one drone is launched (which serves a graph). Then, is the number of stages an unknown variable a priori? How do you put in the formulations t \in T?  
    Later (page 7) it is stated that, | T | = | G | +1, with a fictitious last stage. And in section 3.2 it is explained even better. The authors could try to explain it well from the beginning.  
  
    Page 5, line 1: This path is not well understood. R^eg, L^eg are not defined in the text (they are in Table 2). Why does the path end with an R'' and not with the next L''? And why is x\_L^{t+1} included? (that drone may not come out in stage t+1).  
  
    Page 5, Fig. 1: Why is it interesting to launch two drones from the same point L? Does the launch have a cost of stopping the mothership and then restarting it? Otherwise it seems, in the figure, that if the first green drone is launched later and the second green drone is picked up earlier, the point x2L = x1R could be located lower and the total distance traveled by the mothership would be less. In general, it seems that launching and retrieving the drones separately gives better solutions, unless there is a cost to stop the mothership.   
  
    Page 5, line -9: "... the drones return to the point x1R from where they are launched again to ...". If the drones land at one point and then leave the same point, there is no time to recharge the batteries. Unless the mothership remains stationary at that point until the drones reload.  
  
    Page 5, line -1: "The optimal order to visit the edges of each graph in its corresponding stage". It is not necessary to optimize the order of visiting the edges: it is enough that the total distance traveled is less than the autonomy of the drone.  
  
    Page 6, lines 1, 2 --inequalities (1) and (2)-- and 8: "Inequalities (1) and (2) state that for each stage at most one drone can be launched and retrieved for performing an operation.": "at most"? Can not several drones be launched on a stage? On page 4 you talk about "at least one drone".  
  
    Page 6, lines 19 and 20: remove the sentence "since they would allow free jumps of the drone between different routes at no time." It is not understood and is unnecessary.   
  
    Page 7, line 7: The sentece "The coordination between the drones and the mothership must ensure that the time spent by the drone d to visit the graph g at the stage t is less than or equal to the time that the mothership needs to move from the launching point to the retrieving point during the stage t" is not congruent with the one in page 3 "However, this does not mean that the mothership and all drones must arrive at a rendezvous location at the same time: the fastest arriving vehicle may wait for the others at the rendezvous location." The flight time of each drone has to be less than its endurance.   
  
    Page 7, equation (DCW): It deserves an explanation.  
  
    Page 7 line -19: "constraint will become an equality and we can model...": An equality? the LHS depends on g and the drone and the RHS does not!  
  
    Page 8, line -3: explain better the sentence "... reducing the available capacity so that is possible to traverse the required percentage of these graphs."  
  
    Page 9, Theorem 2.1: Please explain the meaning and the implications of this theorem.  
  
    Page 9, set of inequalities in Theorem 2.1: Before, v\_D and v\_M were the speeds of the drone and mothership. Now who is v\_C? Should it be v\_M?  
  
    Page 9, line -8: "In this problem, we assume that the fleet has more than one drone since otherwise ...": However, the computational results manage the case with one drone.  
  
    Page 10, line -11: "To linearize the first term of the objective function in AMMDRPG, ...": Is the objective function in AMMDRPG (page 7) not linear?   
  
    Page 10, lines -10 to -8: there are undefined variables.  
  
    Page 10, line -6: please explain better the sentence "every launching or rendezvous point is inside the circle whose diametrically opposite points are described below."  
  
    Page 11, line -3: How is "L(eg,e'g)" computed? Because it could be a CPP (easy) but it could also be a CPP at 50% on each edge or at 50% in total length. It would be interesting to explain how it is computed.  
  
    Page 12, STEP 2: Is it true that grouping graphs to be visited in the same stage produces better results than considering one graph per phase?  
  
    Page 12, line 5: "n\_D" is undefined.  
  
    Page 12, line 7: "search for point P": How is that search done?  
  
    Page 12, line 9: "If such a point exists": Is that point unique? What if it is not unique? If the point does not exist, what is done by the algorithm?  
  
    Page 12, line 9: The parameter "maxit" is not mentioned anywhere else in the paper.  
  
    Page 12, STEP 3: "Compute a reference point", "seeks for the minimization": How is it done? Also, does the origin point intervene but not the point of destination?  
  
    Page 12, STEP 4: "Compute the TSP ..." It is supposed to be a tiny TSP. And are origin and destination not considered? Wouldn't it be better to calculate an open TSP from origin to destination visiting the reference points?  
  
    Page 12, STEP 4: The parameter "maxseed" is not mentioned anywhere else in the paper.  
  
    Page 12, STEP 4: "go to STEP 2" -> save the reference points, if they are better, and go to STEP 2.  
  
    Page 12, STEP 5: "Set the values of the binary variables u and v ...": How?  
  
    Page 12, line -1: "One cluster contains graphs g1 and g3 (in red), while graphs g2 and g4 represent distinct clusters.": It seems strange that the two graphs furthest from the origin form a cluster and the two closest to the origin are different clusters.  
  
    Page 13, line 1: "..according with STEP 3, produces the points ..." Nowhere is it explained how they are computed.  
  
    Page 13, line 5: "the tour of the mothership  along the origin point, P1, P2 and the destination point, ...": but not a TSP between the reference points, as the algorithm text says.   
  
    Page 13, line 9: "the values of the variables u^egtd and v^egtd": the values of these variables before step 5 and after step 5 are not the same for all graphs in the figures.  
  
    Page 14, line 4: "Note that in this example the drones do not visit the full 100% of each graph, but only a pre-specified percentage of each one of them": However, in STEP 1 it seems that 100% is considered.  
  
    Page 15, line 5: "This set consists of 5 instances of respectively 5 and 10 target graphs, ...": This phrase is not understood. It seems to mean that each row in Table 4 corresponds to 5 instances.   
  
    Page 15, line 7: "... of 10 nodes.": in the table they say up to 12. And really for it to add up to 100% it should be up to 12.  
  
    Page 15, line 9: "We consider in our experiments that the number of drones varies between 1 and 3": The title of the paper specifies multiple drones.  
  
    Page 15, Table 3: "random variable": How is it selected?  
  
    Page 15, line 15: "... an initial solution computed by the matheuristic ...": Are these solutions computed by the matheuristic reported somewhere in the paper?  
  
    Page 15, line 19: "... a percentage of each edge (e) and the percentage ...": Why are the values of those percentages not put in Table 4?  
  
    Page 15, line 20: "The fourth column ...": There is no fourth column in the table: for each number of drones there are 3 columns.  
  
    Page 15, line 22: "We report respectively average percentage gap ...": Gap on what? Is it the gap between the feasible solution of the exact method and the lower bound of the model, or is the gap between the best feasible solution obtained with the exact method and the (feasible) solution from the matheuristic?  
    In the first case, why is the feasible solution of the exact method not compared with those of the matheuristic?  
  
    Page 15, line -2: ".. the objective function values of the problem ...": Obtained with the matheuristic or with the exact algorithm?  
  
    Page 16, line 1: "... instances with three target graphs ...": aren't they 5 or 10?  
  
    Page 16, line 2: "(capacity) in {10,20,30,40,50,60}": the value 10 is not in the table.   
  
    Page 17, line 6: the speed "of the helicopter is 50 km/h": A low speed for a helicopter.  
  
    Page 17, line 7: "an endurance equal to 2 hours,": For every trip? For each drone (which makes 2 trips)? For the total set of the 6 trips?  
  
    Page 17,line 9: "... located in an area of the city where it is possible to assume the take-off and landing of an helicopter.": Therefore, it appears that the helicopter does not land to launch or pick up the drones. Then, it stays still flying at the points where it retrieves a drone waiting for it (or them) to arrive. But this waiting time does not appear in the objective function, which only considers the distance. I think that the definition of the problem should be more precise, especially in what happens at the launching and retrieving points, and its implications in the objective function.  
  
    Page 17, line 12: the solution ... "... with a percentage gap equal to 83%": Is it a feasible solution obtained by Gurobi that is 83% away from the lower bound? Is this solution better than the one provided by the matheuristic?  
  
    Page 17, line 20: "... at point x1R, where they are retrieved by the helicopter.": Does the helicopter stop to pick up the two drones?  
  
    Page 17, line 27: "11.27 km.": Looking at figure 11, if the helicopter does 11.3 km, the drones do, at most, about 15 km in each flight, which at a speed of 100 km/h is far from the 2 hours of endurance. At a speed of 100 km/h, it seems that a single drone could do all the flights leaving from and arriving to the same starting point, and the helicopter (mothership) would travel 0 km, which is the optimal solution.  
  
    Page 17, line -7: "This papers": This paper.  
  
    Page 18, line -12: "Cordoba, by illustrating the solution obtained by adopting the problem formulation, in its synchronized version, and its solution by means of the initialization provided by the proposed matheuristic.": rewrite this sentence.  
  
  
  
  
    Reviewer: 2  
  
    Comments to the Author  
    This paper considers a vehicle routing problem in which one large vehicle (a "mothership") deploys and retrieves several smaller vehicles ("drones").  The drones are tasked to visit remote locations, modeled as graphs.  Two versions of this problem are presented; one in which all drones are launched and retrieved in the same "stage", and a relaxed version in which the drones may be launched and retrieved in different stages.  A heuristic is proposed to solve the first version of the problem.    
  
  
    Unfortunately, I had a difficult time reading this paper, due to numerous grammatical errors as well as general composition issues.  A significant editing effort is required to enable the reader to really make sense of this research.  
  
    Beyond the general writing, I have a few major concerns:  
  
    1.  Regarding the research topic itself, the notion of mothership/drone problems is interesting and could \*\*potentially\*\* have novel real-world applications.  However, as currently written, the research motivation seems to be more of an academic exercise, rather than an actual problem or real relevance.  Some other thoughts on this point:  
  
        - If the aim is to conduct surveillance of remote areas, allowing/requiring only a percentage of the graphs to be covered seems strange; there's no motivation to conduct any surveillance beyond the minimum.  Instead, it seems that an objective of maximizing coverage would be more appropriate.    
  
        - By contrast, if complete coverage of each graph is desired (as appears to be true in the case study), then it seems that the individual graph tours can be pre-computed.  Then, the problem becomes to simply route the mothership to be sufficiently close to starting/ending points of these graphs.  Furthermore, this might allow the consideration of the mothership's speed as a decision variable.  
  
        - Forcing the mothership to move at a constant speed seems limiting (and also prohibits the opportunity for the mothership to simply stay in one place).  
  
  
    2.  There is a synchronization model presented in Section 2.2, and a relaxed version in Section 2.3.  I would argue that the version in Section 2.3 is more realistic, as it doesn't require the drones to be launched and recovered in the same "stage" (as an aside, the notion of a "stage" needs to be more clearly explained).  As a result, I don't think there's any reason to include the overly-restrictive version from Section 2.2.  Unfortunately, the heuristic, numerical study, and case study all use the model from Section 2.2.  
  
  
    3.  In the experimental results of Section 5, it is not clear how the performance of the heuristic was assessed.  Are the gaps reported in Table 4 measures of Gurobi's self-reported optimality gaps (given the 2-hour runtime limit), or are these gaps between the heuristic and Gurobi's best solutions?

