Facility location concerns with the optimal placement of one or several new facilities/plants to satisfy the customers’ demands. In discrete facility location problems, the positions of both the customers and the potential new facilities are part of the input as well as the travel costs between them. On the other hand, in continuous facility location problems, although the (geographical) coordinates (in a given *d*-dimensional space) of the customers are provided, the information about the potential location of the facilities is unknown, in the sense that the facilities can be located at any place of the given space. Both the discrete and the continuous versions of these facility location problems have been widely studied in the literature (see the monographs [[15](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR15), [33](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR33)] and the references therein). Several versions of these facility location problems have been analyzed, by considering different objective functions [[45](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR45)], by fixing either the number of facilities to be located (as in the *p*-median or *p*-center problems) [[20](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR20)] or the maximum capacities for the facilities (capacitated facility location) [[25](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR25), [38](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR38)], or assuming uncertainty in the demands of the customers (see [[2](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR2), [9](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR9), [10](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR10)] for a recent review), amongst many others.

In this paper, we propose a unified framework for facility location problems in which the underlying problem is a discrete facility location problem. However, because of locational imprecision or inaccuracy, the new facilities are allowed to be located not only in the exact location of the potential facilities, but in certain regions around each of them, the *neighborhoods*. In case the initial placements of the potential facilities are exact enough, that is, their neighborhoods are singletons (with a single element which coincides with the initial placement of the potential facilities), the problem becomes the discrete location version of the problem. On the other hand, if the neighborhoods are large enough, the problem turns into the continuous location version of the problem, allowing the facilities to be located in the entire space. Otherwise, different shapes and sizes for the neighborhoods allow one to model how imprecise the provided locational information is. The goal is, apart from the discrete location decision of the problem (placement of facilities among the given set and allocations customers-plants), to find the optimal location of the open facilities in the neighborhoods. The main difference between this problem and its underlying discrete facility location problem, is that in the former, the travel distances between facilities and customers are assumed to be known, while in the neighborhood version of the problem, as in the continuous case, those distances depend on the place where the facility is located in the neighborhood. Hence, in this problem, the matrix of travel costs is not provided, but a distance measure to compute the travel costs between customers and facilities is given. This problem, as far as we know, has not been fully investigated in Location Analysis, although some attempts have been presented in [[8](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR8)] and [[21](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR21)] where sensitivity analyses were performed by allowing the customers to *move* around disk-shaped neighborhoods on the plane. Also, this problem can be seen as a constrained version of the classical multifacility location problem, which have been only partially studied in the literature (see [[6](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR6)]). This framework will be called *Facility Location with Neighborhoods*, a terminology borrowed from the neighborhood versions of the Minimum Spanning Tree problem [[4](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR4), [14](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR14)] and the Traveling Salesman problem [[11](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR11), [16](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR16)].

The importance of analyzing this family of problems comes from its wide range of applications. It is well known that discrete facility location problems are useful in many real-world applications (see [[27](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR27), [32](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR32)], amongst many others). However, in many situations, as for instance in the design of telecommunication networks, where a set of servers must be located to supply connection to a set of customers, the exact location of a server may not be exactly provided. In contrast, a region where the decision maker wishes to locate each of the facilities (a corridor, a room, or any other bounded shape) can be *easily* given. In such a case, a robust worst-case decision would not reflect reality, since the decision maker does not explicitly provide the location of the facility because a lack of certainty but because it allows locational flexibility to the decision. An optimal design may be obtained if the new facilities are allowed to be located in adequately chosen neighborhoods.

In this paper, we provide suitable mathematical programming formulations for the neighborhood versions of a widely studied family of objective functions in facility location problems: ordered median (OM) functions. In these problems, *p* facilities are to be located by minimizing a flexible objective function that allows one to model different classical location problems. For instance, OM problems allow modeling location problems in which the customers support the median (*p*-median) or the maximum (*p*-center) travel costs, among many other robust alternatives. OM problems were introduced in Location Analysis by Puerto and Fernández [[39](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR39)] and several papers have analyzed this family of objective functions in facility location: discrete problems [[23](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR23), [26](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR26), [37](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR37)], continuous problems [[3](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR3), [5](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR5)], network/tree location problems [[22](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR22), [44](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR44), [47](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR47)], hub location problems [[41](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR41)], stochastic facility location problems [[49](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR49)], multiobjecive location [[19](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR19)], etc (see [[43](https://link.springer.com/article/10.1007/s10589-019-00077-x#ref-CR43)] for a recent overview on the recent developments on ordered median location problems). In particular, we analyze the neighborhood version of OM location problems for the so-called monotone case. We study the still general case in which the neighborhoods are second-order cone representable regions. These sets allow one to model as particular cases polyhedral neighborhoods or ℓτℓτ-norm balls. The distance measure to represent travel costs between customers and facilities are assumed to be ℓνℓν-norm based distances. Within this framework we present four different mixed integer second order cone optimization (MISOCO) models.

The current limitations of the on-the-shelf solvers to solve mixed integer nonlinear problems, and the difficulty of solving even the underlying problem (the classical *p*-median problem is NP-hard), makes the resolution of the problem under study a hard challenge. For that reason, we also develop two math-heuristic algorithms based on different location-allocation schemes, which are able to solve larger problems.

Our paper is organized in five sections. In Sect. [2](https://link.springer.com/article/10.1007/s10589-019-00077-x#Sec2) we introduce the problem and some general properties are stated. Sect. [3](https://link.springer.com/article/10.1007/s10589-019-00077-x#Sec3) is devoted to provide four different mixed integer non linear programming formulations of the problem. At the end of the section, we run some computational experiments in order to compare the four formulations. In Sect. [4](https://link.springer.com/article/10.1007/s10589-019-00077-x#Sec10) the two math-heuristic approaches are described, and the results of some computational experiments are reported. Finally, some conclusions are presented in Sect. [6](https://link.springer.com/article/10.1007/s10589-019-00077-x#Sec14).

Location analysis is a classical branch of operations research that studies the best way to place some facilities to meet demand of customers. In location analysis, problems can be classified in discrete or continuous facility location problems. The first is considered when there is a finite number of candidates to allocate facilities. Continuous facility location problems arise if facilities can be placed anywhere in some continuous regions. Both versions are widely investigated in the literature (see ) due to have wide-spread applications in transportation, logistics or telecommunication. For these problems, lot of variants have been studied in terms of the objective functions to optimize, the number of facilities that must be allocated or the maximum capacity that facilities have, in order to supply the demand and so forth.

In (paper victor), the ordered k-median problem with neighbourhoods is presented as a single source uncapacitated facility continuous location problem that extends its respective underlying discrete location problem. In this problem, facilities are allowed to be allocated in certain regions called *neighbourhoods*. If neighbourhoods are points, the problem reduces to the single source uncapacited facility discrete location problem, that have been already studied in the literature. Otherwise, the continuous version is considered. In this continuous version, different shapes and sizes for the neighbourhoods allow one to model how imprecise the provided locational information is. This problem also has interest on drone delivery and inspection problems. Neighbourhoods can represent regions that the drone must reach and where the customers are willing to pick up the orders (they can be seen as uniform probability densities) in the delivery industry. Moreover, they can be also used for modelling some areas that must be inspected by the drone (whenever visiting a point of these areas is enough to consider them as inspected). This framework will be called Facility Location with Neighbourhoods, a terminology borrowed from the neighbourhood versions of the Minimum Spanning Tree problem [4, 14] and the Traveling Salesman problem [11, 16].

This paper extends the k-median problem with neighbourhoods by including a set of barriers, represented by line segments, that the delivery cannot cross in the continuous space. These barriers simulate buildings in urban areas that drones cannot cross. The resulting problem keeps geometric components from the p-median problem with neighbourhoods that must be exploited to partially overcome the difficulties of the solution approaches and algorithms in the network design among neighbourhoods with barriers. The use of barriers in location problems has been studied (see Klamroth (2002)). However, the combination of both elements has attracted less attention in Location Analysis.

Our goal in this paper is to deal with the k-median problem with Neighbourhoods and barriers that is called the Hampered k-Median with Neighbourhoods (\KMPN). We present exact mathematical programming formulations assuming linear barriers and second-order cone (SOC) representable neighbourhoods. These formulations are modelled by using a geodesic shortest-path representation. These assumptions lead to quadratically-constrained mixed-integer formulations. Solving this family of formulations in addition to the classic k-median, that is NP-hard, makes the resolution of the problem under study a hard challenge. On-the-shelf solvers can deal only with small-size instances. This fact motivates the design of a matheuristic that provides good quality solutions for medium-size instances.

The paper is organized in five sections. In Sect. 2 we introduce the problem and some general properties are stated. Sect. 3 is devoted to provide four different mixed integer non linear programming formulations of the problem. At the end of the section, we run some computational experiments in order to compare the four formulations. In Sect. 4 the two math-heuristic approaches are described, and the results of some computational experiments are reported. Finally, some conclusions are presented in Sect. 6.