MGT-541 Temperature Analysis Exercise

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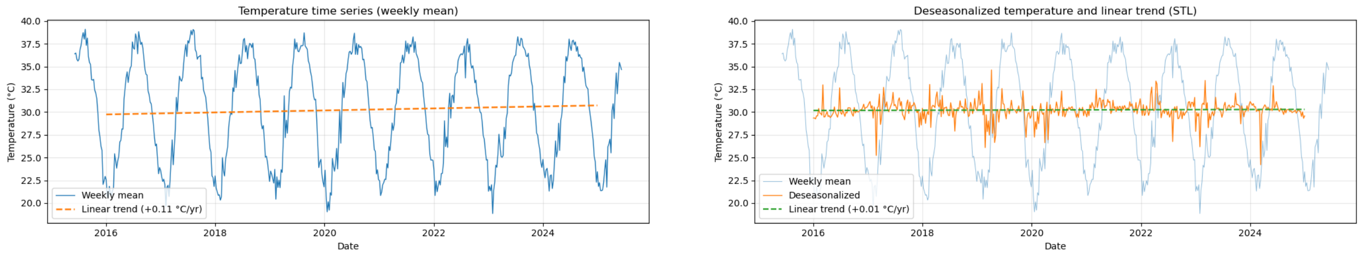
Q1. Should the series be detrended? Methods and rationale

In principle, detrending would be desirable to reduce the risk of triggering payouts due to the long–run warming drift rather than insurable weather contingencies.
Thus, we follow different approaches to understand which one suits better our case, starting from a **simple baseline trend** computed on the whole time series.



(a) Weekly mean temperature (full range). The 2016–2024 window (complete years) is used to estimate the linear trend.

(b) Deseasonalized weekly series (STL) with fitted linear trend (°C/year) over 2016–2024.

Figure 1: Baseline diagnostics: raw weekly series and deseasonalized trend used to inform detrending choices.

We can easily see that fitting a single linear trend on 2016–2024 (period for which we have full data over the years) (Fig. 1a) ignores the strong seasonal structure of the series. Even the trend obtained after removing seasonality via STL (Fig. 1b) is not fully appropriate for our purpose, because the trigger is intrinsically seasonal (daytime heat hours).
We therefore consider **season–specific detrending methods** that explicitly condition on the month of the year.

Notation. Let $T_{y,m,t}$ denote the observed temperature at intramonth index t in year y and calendar month m. Let y_0 be a baseline year (earliest complete year, here $y_0 = 2016$). For any month m, let $S_{y,m}$ be a monthly summary statistic (mean or tail metric) computed from $\{T_{y,m,t}\}_t$.

Method A — Seasonal linear trend per month (OLS). For each month m, we regress a monthly statistic on calendar year and remove the fitted drift:

$$S_{y,m} = \alpha_m + \beta_{1,m}\, y + \varepsilon_{y,m}, \qquad T^{(A)}_{y,m,t} = T_{y,m,t} - \hat{\beta}_{1,m}(y - y_0).$$

Equivalently, one can keep the original temperatures and use a month– and year–specific trigger $\tau_{y,m} = \tau_0 + \hat{\beta}_{1,m}(y - y_0)$ so that payouts are evaluated relative to a drifting threshold instead of a drifting mean against a fixed threshold.
This is simple and season-specific, but with short samples $\hat{\beta}_{1,m}$ can be noisy and outlier-sensitive.

Method B — Tail-focused detrending (heatwave relevance). Method B is structurally identical, but replaces the mean with a tail statistic in order to focus on the **upper part of the distribution** that drives heatwave payouts. This follows the empirical finding that extremes often respond more strongly to climate change than the mean [1].

Concretely, for each (y, m) we define a tail metric, e.g. the empirical 95th percentile

$$S_{y,m} \;:=\; Q_{0.95}\big(T_{y,m,\cdot}\big),$$

or, equivalently, the average of the hottest few percent of intramonth observations.

For each month m, we then fit a linear model in the year dimension,

$$S_{y,m} \;=\; \alpha^{(95)}_m \;+\; \beta^{(95)}_m\,(y - y_0) \;+\; \eta_{y,m},$$

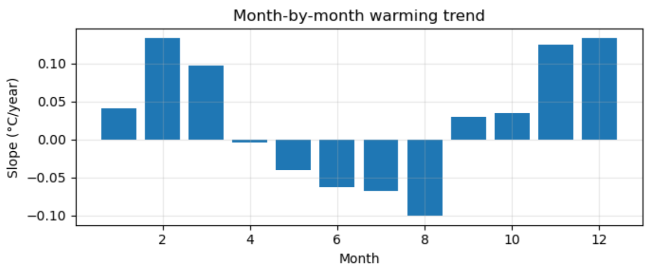
obtaining $\hat{\beta}^{(95)}_m$, the month–specific trend in the upper tail.

We detrend each hourly observation using this tail trend,

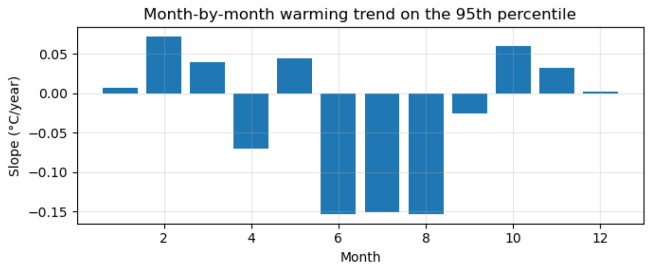
$$T^{(B)}_{y,m,t} \;=\; T_{y,m,t} \;-\; \hat{\beta}^{(95)}_m\,(y - y_0),$$

which is equivalent to using a month– and year–specific threshold $\tau_{y,m} \;=\; \tau_0 \;+\; \hat{\beta}^{(95)}_m\,(y - y_0)$.

Method B therefore keeps the same seasonal structure as Method A, but calibrates the drift on the heat-relevant upper tail rather than on the monthly mean, which is more aligned with the mechanics of a high-temperature trigger.



(a) Method A: month–specific linear slopes $\hat{\beta}_m^{\text{mean}}$ fitted on monthly means.



(b) Method B: month–specific linear slopes $\hat{\beta}_m^{(95)}$ fitted on the 95th percentile.

Figure 2: Season–aware detrending strategies based on monthly means (Method A) and heat–relevant tails (Method B).

Method C — Baseline anchoring by month (no linearity assumption). Here we avoid fitting a time trend and simply re-centre each month to the chosen baseline year $y_0 = 2016$. Let $L_{y,m} = \frac{1}{n_{y,m}} \sum_t T_{y,m,t}$ be the monthly mean in year y, month m, and $L_{y_0,m}$ the corresponding mean in the baseline year. The monthly offset is

$$\Delta_{y,m} = L_{y,m} - L_{y_0,m},$$

and every hourly observation in that month–year is adjusted as

$$T_{y,m,t}^{(C)} = T_{y,m,t} - \Delta_{y,m},$$

which is equivalent to using a month–year specific threshold $\tau_{y,m} = \tau_0 + \Delta_{y,m}$.

This method removes month–year level shifts without assuming linearity in time, is robust on a short sample, but its result depends on the chosen baseline year y_0 (see Fig. 3).

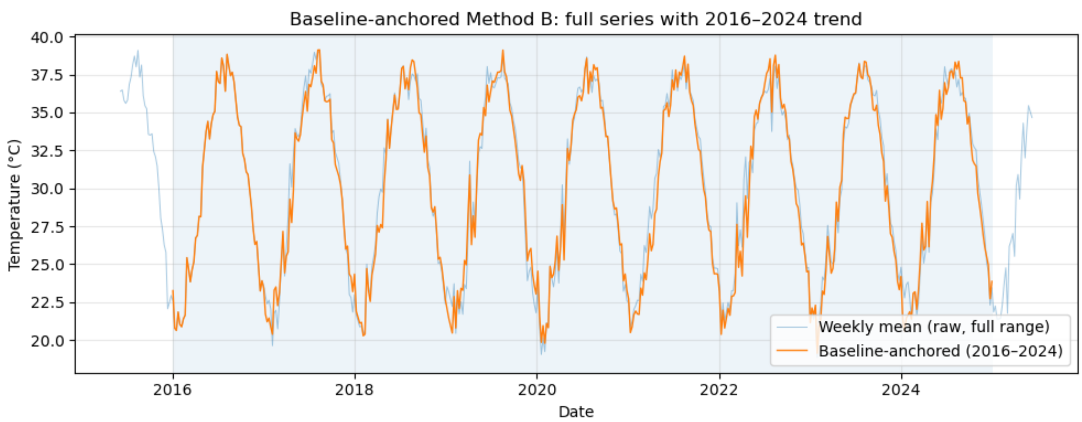


Figure 3: Method C: baseline anchoring by month—subtract month-specific offsets relative to the baseline year y_0.

Discussion. When we apply Methods A and B to this dataset (2015–2025), several summer-month slopes turn out to be slightly negative (Fig. 2). Given the short sample and the strong year–to–year variability, this should not be read as evidence of "no warming", but rather as an indication that *within this window and metric* the estimated drift is statistically non-positive. This is plausible in arid climates, where long-term warming can manifest as a more even increase across the whole year instead of a monotonic rise in peak-summer daytime extremes [2].

Method C, which anchors each month to a 2016 baseline, partially compensates for interannual shifts and in some years delivers lower adjusted payouts than the raw series. However, over the full 2015–2024 horizon it leads to a systematically larger average payout at 35°C and 40°C (see the Adj columns in Table 1), i.e. it effectively makes the contract more generous to the policyholder than the raw data would suggest, at the expense of the insurer.

Baseline decision. In light of (i) the instability and occasional negativity of the estimated monthly trends, and (ii) the fact that any detrending scheme is equivalent to raising the effective trigger over time (or lowering temperatures), thereby mechanically reducing expected payouts, we choose **not** to detrend the series in the baseline analysis. The raw series thus defines the primary pricing and risk view, while Methods A–C are retained as *sensitivity* scenarios to illustrate how alternative detrending choices would reallocate value between insurer and policyholder.

Q2, Q4, Q5. Payout statistics with \$10 M annual cap (2015–2024)

We report annual payouts *after* applying the policy's aggregate annual limit, equal to the Total Sum Insured (TSI = \$10 M). Each year's payout is computed as

$$\text{Payout}_y \;=\; \min\bigl\{\text{Theoretical}_y,\, 10\,\text{M}\bigr\}.$$

Years 2015 and 2025 are excluded because full-year statistics are unavailable (temperature data for these years are only partially observed). Table 1 summarizes the minimum, mean, and maximum annual payouts for each threshold. "Adj" denotes series adjusted for trend (shown for reference only).

Table 1: Payout summary statistics over 2016–2024 **after applying the \$10 M annual cap** (values in millions). Raw = baseline; *Adj using Method C shown for reference only.*

Threshold	Raw Min	Raw Mean	Raw Max	Adj Min	Adj Mean	Adj Max
35°C	10.00 M	10.00 M	10.00 M	10.00 M	10.00 M	10.00 M
40°C	0.50 M	≈ 4.44 M	10.00 M	1 M	≈ 4.83 M	9.50 M
45°C	0.00 M	0.00 M	0.00 M	0.00 M	0.00 M	0.00 M

Note. Table 2 lists *theoretical*, uncapped payouts; the cap is applied here for the contractual figures.

Table 2: Annual payouts by threshold, 2016–2024 (millions). *Adjusted (Adj) values — obtained by detrending the time series using Method C — are shown for reference only and are not considered in the baseline analysis.*

	35°C		40°C		45°C	
Year	**Raw**	**Adj**	**Raw**	**Adj**	**Raw**	**Adj**
2016	229.50 M	229.50 M	9.50 M	9.50 M	0.00 M	0.00 M
2017	276.50 M	219.00 M	12.50 M	9.00 M	0.00 M	0.00 M
2018	227.00 M	242.50 M	5.00 M	6.50 M	0.00 M	0.00 M
2019	249.00 M	236.00 M	7.00 M	4.50 M	0.00 M	0.00 M
2020	233.00 M	260.50 M	1.00 M	1.00 M	0.00 M	0.00 M
2021	232.50 M	230.00 M	2.50 M	3.50 M	0.00 M	0.00 M
2022	180.50 M	294.50 M	1.00 M	4.00 M	0.00 M	0.00 M
2023	246.50 M	244.00 M	3.50 M	4.00 M	0.00 M	0.00 M
2024	231.50 M	262.50 M	0.50 M	1.50 M	0.00 M	0.00 M

Q3. Limited data and extreme events: extending the series and estimating the 1-in-200 payout

Set-up. Under the 40°C trigger the historical series already hits the contractual cap of 10M per year. To analyse a more conservative design and to quantify a rare extreme event, we therefore focus on a **higher trigger** at 45°C and ask what the *1-in-200* annual payout would be in a warmer future climate.

Because the available record is short and contains no exceedances of the 45°C trigger, we first **extend the temperature series** synthetically to represent two possible contract horizons (20 and 100 years). On each extended scenario we then run a **Monte Carlo simulation** of annual payouts and define the 1-in-200 event as the 99.5th percentile of the simulated annual payout distribution (i.e. the payout level that is exceeded with probability 0.5% in any given year).

Methodology (historical calibration → stochastic extension). We generate synthetic hourly years that are consistent with the observed seasonality, variability, and persistence, while superimposing a *scenario* warming drift derived from UAE climate projections. In particular, we use a linear warming rate of

$$\beta_{\text{UAE}} \approx 0.05\ ^{\circ}\text{C/year},$$

chosen as a simple approximation to the upper range of projected temperature increase in the UAE under high-emission scenarios, rather than a pure extrapolation of the short historical sample [3]. The calibration is performed on full years 2016–2024, daytime hours only, to match the source data.

1. **Seasonal mean by month–hour.** We first remove the imposed linear warming trend back to the base year $y_0 = 2016$ using β_{UAE}, and estimate a month–hour climatology

$$\mu_{m,h} \;=\; \mathbb{E}[T \mid \text{month} = m,\ \text{hour} = h]$$

on the detrended series. This captures the typical diurnal and seasonal cycle for Dubai.

2. **Scenario warming trend.** For any target year y, the conditional mean temperature at hour t is then

$$\mu_t \;=\; \mu_{m(t),h(t)} \;+\; \beta_{\mathrm{UAE}}\,\big(y - y_0\big),$$

i.e., the calibrated seasonal cycle shifted upward according to the externally imposed UAE warming scenario.

3. **Heteroskedastic volatility by month–hour.** Around this mean we compute residuals

$$R_t \;=\; T_t - \mu_{m(t),h(t)} - \beta_{\mathrm{UAE}}\big(y(t) - y_0\big),$$

and estimate a month–hour specific standard deviation

$$\sigma_{m,h} \;=\; \mathrm{sd}\big(R_t \mid \text{month} = m,\ \text{hour} = h\big),$$

so that hotter months and hours are allowed to be more volatile than cooler ones.

4. **Persistence and heavy tails.** We model standardized residuals with a *gap-aware* AR(1)—i.e., the AR step is raised to the number of hours between consecutive kept timestamps—to capture hour-to-hour thermal persistence on a daytime-only grid. To avoid understating extremes, the innovations ε_t are modeled as Student-t_ν with unit variance (rather than Gaussian) to allow for **heavier tails** and occasional **large deviations**.
Heavy-tail models are indeed widely recommended when analyzing climate extremes to keep tail risk from being biased low [4, 5].

$$u_t \;=\; \phi^{k_t} u_{t-1} \;+\; \sqrt{1-\phi^{2k_t}}\;\varepsilon_t,$$

where k_t is the time gap in hours between two consecutive (daytime) observations, and ϕ is the hour-to-hour persistence estimated from hot months.

5. **Hourly generator for a target climate year.** For a given target year y we build a daytime-only hourly grid and, for each hour t,

$$T_t \;=\; \mu_t \;+\; \sigma_{m(t),h(t)}\, u_t.$$

This produces synthetic hourly temperatures that replicate the observed seasonality, variability, and persistence, but shifted upward according to the UAE warming scenario β_{UAE}.

Monte Carlo for the $1:200$ payout. Using the AR(1)+Student-t generator described above, we simulate $N =$ 50,000 independent synthetic years under the 45°C trigger. For each synthetic year we compute the annual payout using the same window rule as in Q2. Annual payouts are capped at 10M.

The 1:200 payout is defined as the 99.5$^{\text{th}}$ percentile of the simulated annual payout distribution:

$$\text{Payout}_{1200} \;=\; \text{Quantile}_{0.995}\big(\,\text{Payout}^{(1)},\ldots,\text{Payout}^{(N)}\,\big).$$

Table 3 reports the results for a mid-century climate (2045) and for a longer-run scenario (2125), both under the imposed UAE warming drift β_{UAE} [3].

Table 3: Monte Carlo results for the 45°C trigger (values in $, capped at 10,000,000 per year).

Target year	***N***	**Mean**	**Median**	**p99**	**p99.5** (1:200)
2045	50,000	5,660	0	0	500,000
2125	50,000	6,836,840	6,500,000	10,000,000	10,000,000

Discussion. In the mid-century climate (2045), the mean annual payout under the 45°C trigger remains modest (of the order of a few thousand euros), with a 1:200 **payout of about $500,000**, well below the contractual cap. Even under the imposed UAE warming scenario, seven consecutive daytime hours above 45°C remain a relatively rare event over the next ∼20 years.

In the long-run scenario (2125), instead, the distribution shifts markedly: the mean approaches 6.8M and both the 99th and 99.5th percentiles **hit the cap of $10M**. In this regime, full payouts become frequent in the tail, suggesting that a 45°C trigger may become too "cheap" for the insurer if a warming path consistent with β_{UAE} is realised, and that a future contract revision (higher threshold or different payoff structure) would likely be required.

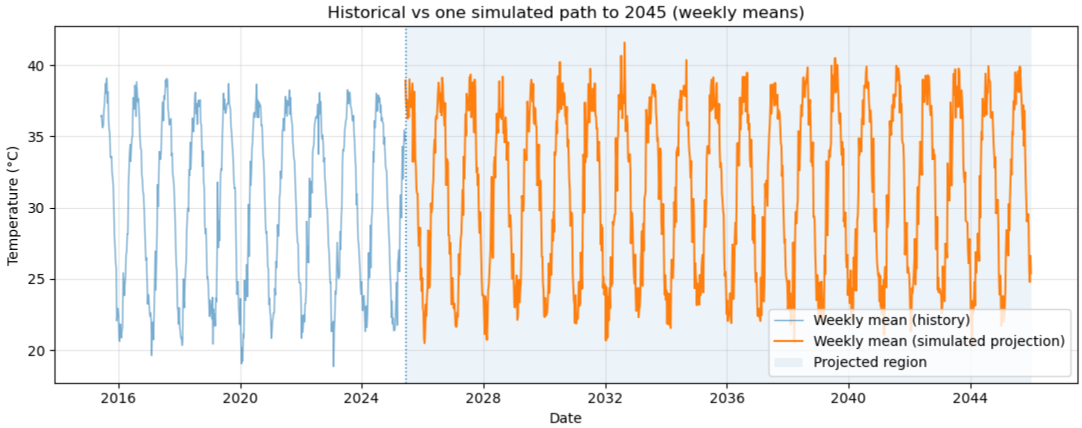


Figure 4: Historical weekly mean temperatures (2015–2024) and one simulated weekly-mean projection up to 2045, generated with the calibrated AR(1)+Student-t model and the UAE warming scenario $\beta_{\mathrm{UAE}} \approx 0.05$ °C/year.

References

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[4] Daniel S. Wilks. *Statistical Methods in the Atmospheric Sciences*. 3rd. Oxford: Academic Press, 2011. URL: https://www.academia.edu/6594847/Statistical_Methods_in_the_Atmospheric_Sciences (visited on 11/14/2025).

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