Numerical Optimization - Bonus Project (individual)

Deadlines: Sunday, July 30, 23:59.
General description:
You can either choose to program the SQP method or the forward-backward (F-B) splitting method for nonsmooth problems. The former is more difficult, but for the letter one you need to do some additional studying, since we covered this topic only briefly (see the exercises classes on June 21 and 28).

Ragarding the SQP, you can get additional points for implementing FULL QUASI-NEWTON APPROXIMATIONS from Section 18.3 and/or the WARM START strategies from Section 18.4 (for example, you can initialize the working set for each QP subproblem to be the final active set from the previous SQP iteration).
Problems to solve:

- SQP (20 problems/runs): 4 problems with 2-5 variables, for each problem choose 5 starting points:

  1. problem from Exercise 18.3 from the Nocedal-Wright book with additional starting points $x_0 = (0,0,0,0,0)$, $x_0 = (1,0,3,0,0)$ and 2 more starting points of your choice (you can start near the solution if you are experiencing troubles with remote starting points).
  2. $$\text{(Rosenbrock function)} \quad f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2, \tag{1}$$
     with constraints $x_1^2 + x_2^2 \leq 1$ and $x_2 \geq 0$ and the following starting points x_0
     $$(0.8, 0.6), \quad (1,0), \quad (1,1), \quad (0,0), \quad (-1,1).$$
  3. $$f(x) = 150(x_1 x_2)^2 + (0.5x_1 + 2x_2 - 2)^2, \tag{2}$$
     with constraints
     $$\begin{aligned} 0 &\leq c_1(x) = (x_1 - 1/2)^2 + (x_2 - 1)^2 - 5/16 \\ 0 &\leq c_2(x) = (x_1 + 1)^2 + (x_2 - 3/8)^2 - 73/64 \\ 0 &\leq c_3(x) = -(x1 + 1)^2 - (x2 - 1)^2 + \sqrt{2} \end{aligned}$$
     and the following starting points x_0
     $$(0.1, 0.74), \quad (0,0), \quad (0.5, 0.5), \quad (0, 0.76), \quad (-0.2, 0.8).$$
  4. the same problem as you had to solve with the active set method for Project 2, but with the feasible set given by the standard norm: $\|x\|_2^2 := x_1^2 + \ldots + x_n^2 \leq 1$. Consider the case $m = 5$ with 2 starting points and the special case (where $m = 10$) given via $\tilde{M}$ with 3 starting points (the same as you used from Project 2). For comparison, write also the corresponding solutions you got for $\|x\|_1 \leq 1$ using the active set method.

- F-B (20 problems/runs):
  choose the same objective function as you had to solve with the active set method for Project 2, but instead of the constraint $\|x\|_1 \leq 1$, add the term $\tau\|x\|_1$ to the objective, where $\tau > 0$ is a number you can choose. This makes the problem unconstrained, but nonsmooth.

  To solve this problem, implement the forward-backward splitting method (see the exercises classes on June 21 and 28 as well as the article uploaded to Moodle). Perform the same 20 runs as you did for the active set method (the same data and starting points).

  You can get additional points if you will try to tune the parameter τ in such a way that the corresponding solutions are "similar" to the solutions of the active set method (I also don't know how to choose τ to achieve this - perhaps you can try $\|\lambda\|_\infty := \max_{i=1,\ldots,m} |\lambda_i|$, that is the size of the largest multiplier you got from the active set method).