Numerical Optimization

Bonus Project

Mohamed Abdelaziz [k12137202]

25 July 2023

Implemented Tasks

1. Forward-Backward Splitting
    - 5 Problems with 5 Starting points with varying sizes as in the project description.
    - Predefined Problem with 5 Starting points as in the project description.

Notes:
* All runs are located in separate file for better readability.

1 Problem 1

- m = 1, n = 2
- M: [[0.976, 4.304]]
- y: [2.055]
- α: 0.001
- τ: 1.0

1. **Starting Point [1.764, 0.4]**
    - Solution: [0.0, 0.424]
    - Iterations: 2807
    - Time Elapsed: 0:00:00.034907
2. **Starting Point [0.979, 2.241]**
    - Solution: [0.0, 0.424]
    - Iterations: 1252
    - Time Elapsed: 0:00:00.014991
3. **Starting Point [1.868, 0.977]**
    - Solution: [0.0, 0.424]
    - Iterations: 2772
    - Time Elapsed: 0:00:00.032909
4. **Starting Point [1.95, 0.151]**
    - Solution: [0.0, 0.424]
    - Iterations: 1828
    - Time Elapsed: 0:00:00.022909
5. **Starting Point [0.103, 0.411]**
    - Solution: [0.0, 0.424]
    - Iterations: 648
    - Time Elapsed: 0:00:00.008009

2 Problem 2

- m = 2, n = 4
- M: [[0.976, 4.304, 2.055, 0.898], [-1.527, 2.918, -1.248, 7.835]]
- y: [9.273, -2.331]
- α: 0.001
- τ: 1.0

1. **Starting Point [1.764, 0.4, 0.979, 2.241]**
   - Solution: [0.0, 2.311, 0.0, -1.137]
   - Iterations: 35264
   - Time Elapsed: 0:00:00.432841
2. **Starting Point [1.868, 0.977, 0.95, 0.151]**
   - Solution: 0.0, 2.311, 0.0, -1.137]
   - Iterations: 27293
   - Time Elapsed: 0:00:00.331114
3. **Starting Point [0.103, 0.411, 0.144, 1.454]**
   - Solution: 0.0, 2.311, 0.0, -1.137]
   - Iterations: 26919
   - Time Elapsed: 0:00:00.328972
4. **Starting Point [0.761, 0.122, 0.444, 0.334]**
   - Solution: 0.0, 2.311, 0.0, -1.137]
   - Iterations: 27612
   - Time Elapsed: 0:00:00.347074
5. **Starting Point [1.494, 0.205, 0.313, 0.854]**
   - Solution: 0.0, 2.311, 0.0, -1.137]
   - Iterations: 27468
   - Time Elapsed: 0:00:00.338060

3 Problem 3

- m = 3, n = 6
- M: [[0.976, 4.304, 2.055, 0.898, -1.527, 2.918], [-1.248, 7.835, 9.273, -2.331, 5.835, 0.578], [1.361, 8.512, -8.579, -8.257, -9.596, 6.652]]
- y: [5.563, 7.4, 9.572]
- α: 0.001
- τ: 1.0

1. **Starting Point [1.764, 0.4, 0.979, 2.241, 1.868, 0.977]**
   - Solution: [0.0, 0.953, 0.0, 0.073, -0.0, 0.318]
   - Iterations: 2912
   - Time Elapsed: 0:00:00.037332
2. **Starting Point [0.95, 0.151, 0.103, 0.411, 0.144, 1.454]**
   - Solution: [0.0, 0.953, 0.0, 0.073, -0.0, 0.318]
   - Iterations: 4777
   - Time Elapsed: 0:00:00.063894
3. **Starting Point [0.761, 0.122, 0.444, 0.334, 1.494, 0.205]**
   - Solution: [0.0, 0.953, 0.0, 0.073, -0.0, 0.318]
   - Iterations: 3011
   - Time Elapsed: 0:00:00.038900
4. **Starting Point [0.313, 0.854, 2.553, 0.654, 0.864, 0.742]**
   - Solution: [0.0, 0.953, 0.0, 0.073, -0.0, 0.318]
   - Iterations: 4537
   - Time Elapsed: 0:00:00.057265
5. **Starting Point [2.27, 1.454, 0.046, 0.187, 1.533, 1.469]**
   - Solution: [0.0, 0.953, 0.0, 0.073, -0.0, 0.318]
   - Iterations: 4338
   - Time Elapsed: 0:00:00.055849

4 Problem 4

- m = 4, n = 8
- M: [[0.976, 4.304, 2.055, 0.898, -1.527, 2.918, -1.248, 7.835], [9.273, -2.331, 5.835, 0.578, 1.361, 8.512, -8.579, -8.257], [-9.596, 6.652, 5.563, 7.4, 9.572, 5.983, -0.77, 5.611], [-7.635, 2.798, -7.133, 8.893, 0.437, -1.707, -4.709, 5.485]]
- y: [-0.877, 1.369, -9.624, 2.353]
- α: 0.001
- τ: 1.0

1. **Starting Point [1.764, 0.4, 0.979, 2.241, 1.868, 0.977, 0.95, 0.151]**
   - Solution: [0.512, -0.0, -0.778, 0.0, -0.038, -0.0, -0.133, -0.0]
   - Iterations: 4334
   - Time Elapsed: 0:00:00.056611
2. **Starting Point [0.103, 0.411, 0.144, 1.454, 0.761, 0.122, 0.444, 0.334]**
   - Solution: [0.512, -0.0, -0.778, 0.0, -0.038, -0.0, -0.133, -0.0]
   - Iterations: 3019
   - Time Elapsed: 0:00:00.039896
3. **Starting Point [1.494, 0.205, 0.313, 0.854, 2.553, 0.654, 0.864, 0.742]**
   - Solution: [0.512, -0.0, -0.778, 0.0, -0.038, -0.0, -0.133, -0.0]
   - Iterations: 3708
   - Time Elapsed: 0:00:00.049868
4. **Starting Point [2.27, 1.454, 0.046, 0.187, 1.533, 1.469, 0.155, 0.378]**
   - Solution: [0.512, -0.0, -0.778, 0.0, -0.038, -0.0, -0.133, -0.0]
   - Iterations: 3451
   - Time Elapsed: 0:00:00.044063
5. **Starting Point [0.888, 1.981, 0.348, 0.156, 1.23, 1.202, 0.387, 0.302]**
   - Solution: [0.512, -0.0, -0.778, 0.0, -0.038, -0.0, -0.133, -0.0]
   - Iterations: 3320
   - Time Elapsed: 0:00:00.069609

5 Problem 5

- m = 5, n = 10
- M: [[0.976, 4.304, 2.055, 0.898, -1.527, 2.918, -1.248, 7.835, 9.273, -2.331], [5.835, 0.578, 1.361, 8.512, -8.579, -8.257, -9.596, 6.652, 5.563, 7.4], [9.572, 5.983, -0.77, 5.611, -7.635, 2.798, -7.133, 8.893, 0.437, -1.707], [-4.709, 5.485, -0.877, 1.369, -9.624, 2.353, 2.242, 2.339, 8.875, 3.636], [-2.81, -1.259, 3.953, -8.795, 3.335, 3.413, -5.792, -7.421, -3.691, -2.726]]
- y: [1.404, -1.228, 9.767, -7.959, -5.822]
- α: 0.001
- τ: 1.0

1. **Starting Point [1.764, 0.4, 0.979, 2.241, 1.868, 0.977, 0.95, 0.151, 0.103, 0.411]**
    - Solution: [1.409, -0.0, -0.0, 0.0, 0.071, 0.0, 0.56, 0.0, -0.021, -0.439]
    - Iterations: 4876
    - Time Elapsed: 0:00:00.072365
2. **Starting Point [0.144, 1.454, 0.761, 0.122, 0.444, 0.334, 1.494, 0.205, 0.313, 0.854]**
    - Solution: [1.409, -0.0, -0.0, 0.0, 0.071, 0.0, 0.56, 0.0, -0.021, -0.439]
    - Iterations: 4406
    - Time Elapsed: 0:00:00.062095
3. **Starting Point [2.553, 0.654, 0.864, 0.742, 2.27, 1.454, 0.046, 0.187, 1.533, 1.469]**
    - Solution: [1.409, -0.0, -0.0, 0.0, 0.071, 0.0, 0.56, 0.0, -0.021, -0.439]
    - Iterations: 3831
    - Time Elapsed: 0:00:00.053385
4. **Starting Point [0.155, 0.378, 0.888, 1.981, 0.348, 0.156, 1.23, 1.202, 0.387, 0.302]**
    - Solution: [1.409, -0.0, -0.0, 0.0, 0.071, 0.0, 0.56, 0.0, -0.021, -0.439]
    - Iterations: 8370
    - Time Elapsed: 0:00:00.109859
5. **Starting Point [1.049, 1.42, 1.706, 1.951, 0.51, 0.438, 1.253, 0.777, 1.614, 0.213]**
    - Solution: [1.409, -0.0, -0.0, 0.0, 0.071, 0.0, 0.56, 0.0, -0.021, -0.439]
    - Iterations: 3940
    - Time Elapsed: 0:00:00.053343

6 Pre-defined Problem

Note: in the runs file for the pre-defined problem, I only exported the first 5000 iterations to be able to upload the file in moodle ¿ 20 MB.

- m = 5, n = 10
- M: [[1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1]]
- y: [1, -2, 3, -4, 5, -5, 4, -3, 2, -1]
- α: 0.001
- τ: 1.0

1. **Starting Point [1.764, 0.4, 0.979, 2.241, 1.868, 0.977, 0.95, 0.151, 0.103, 0.411, 0.144, 1.454, 0.761, 0.122, 0.444, 0.334, 1.494, 0.205, 0.313, 0.854]**
   - Solution: [0.0, 0.0, -0.746, -0.254, 1.445, 0.555, -1.257, -1.743, 1.846, 2.154, -2.442, -1.558, 1.82, 1.18, -0.972, -1.028, 1.0, 0.0, -0.0, -0.0]
   - Iterations: 12083
   - Time Elapsed: 0:00:00.183522
2. **Starting Point [2.553, 0.654, 0.864, 0.742, 2.27, 1.454, 0.046, 0.187, 1.533, 1.469, 0.155, 0.378, 0.888, 1.981, 0.348, 0.156, 1.23, 1.202, 0.387, 0.302]**
   - Solution: [0.0, 0.0, -0.746, -0.254, 1.445, 0.555, -1.257, -1.743, 1.846, 2.154, -2.442, -1.558, 1.82, 1.18, -0.972, -1.028, 1.0, 0.0, -0.0, -0.0]
   - Iterations: 12411
   - Time Elapsed: 0:00:00.188481
3. **Starting Point [1.049, 1.42, 1.706, 1.951, 0.51, 0.438, 1.253, 0.777, 1.614, 0.213, 0.895, 0.387, 0.511, 1.181, 0.028, 0.428, 0.067, 0.302, 0.634, 0.363]**
   - Solution: [0.0, 0.0, -0.746, -0.254, 1.445, 0.555, -1.257, -1.743, 1.846, 2.154, -2.442, -1.558, 1.82, 1.18, -0.972, -1.028, 1.0, 0.0, -0.0, -0.0]
   - Iterations: 11465

- Time Elapsed: 0:00:00.177527

4. **Starting Point [0.672, 0.36, 0.813, 1.726, 0.177, 0.402, 1.63, 0.463, 0.907, 0.052, 0.729, 0.129, 1.139, 1.235, 0.402, 0.685, 0.871, 0.579, 0.312, 0.056]**
   - Solution: [0.0, 0.0, -0.746, -0.254, 1.445, 0.555, -1.257, -1.743, 1.846, 2.154, -2.442, -1.558, 1.82, 1.18, -0.972, -1.028, 1.0, 0.0, -0.0, -0.0]
   - Iterations: 10944
   - Time Elapsed: 0:00:00.165656
5. **Starting Point [1.165, 0.901, 0.466, 1.536, 1.488, 1.896, 1.179, 0.18, 1.071, 1.054, 0.403, 1.222, 0.208, 0.977, 0.356, 0.707, 0.011, 1.786, 0.127, 0.402]**
   - Solution: [0.0, 0.0, -0.746, -0.254, 1.445, 0.555, -1.257, -1.743, 1.846, 2.154, -2.442, -1.558, 1.82, 1.18, -0.972, -1.028, 1.0, 0.0, -0.0, -0.0]
   - Iterations: 11584
   - Time Elapsed: 0:00:00.175971