Демидович

7.1. $2x^7 = \int 2x^7 dx = 2\int x^7 dx \; 2\cdot\frac{x^{7+1}}{8} + C = \frac{1}{4}x^8 + C$

7.2. $4\sqrt[3]{x} = 4\int x^{\frac{1}{3}} dx = 4\cdot\frac{x^{\frac{1}{3}+1}}{\frac{4}{3}} + C = 4\cdot\frac{3}{4}x^{\frac{4}{3}} + C = 3\sqrt[3]{x^4} + C$

7.3 $\frac{3}{x} + \frac{5}{x^2} = \int(3x^{-1} + 5x^{-2})dx = 3\int x^{-1}dx + 5\int x^{-2}dx = 3\ln|x| + 5\cdot\frac{1}{x} + C$

7.4. $\frac{x^3+5x^2-1}{x} = \frac{x^3}{x} + \frac{5x^2}{x} - \frac{1}{x} = x^2 + 5x - x^{-1} = \int(x^2+5x-x^{-1})dx$

$= \int x^2 dx + \int 5x dx - \int x^{-1} dx = \frac{x^3}{3} + \frac{5}{2}x^2 - \ln|x| + C$

7.5. $\frac{(\sqrt{x}+1)^3}{x\sqrt{x}} = \frac{(\sqrt{x})^3 + 3(\sqrt{x})^2\cdot 1 + 3\sqrt{x}\cdot 1^2 + 1^3}{\sqrt{x\cdot x^2} = \sqrt{x^3}} =$

$= \cancel{\frac{(x^{\frac{1}{2}})^3}{x^{\frac{3}{2}}}} + \frac{3(x^{\frac{1}{2}})^2}{x^{\frac{3}{2}}} + \frac{3x^{\frac{1}{2}}}{x^{\frac{3}{2}}} + \frac{1}{x^{\frac{3}{2}}} = \int(3x^{-\frac{1}{2}} + 3x^{-1} + x^{-\frac{3}{2}})dx =$

$= 3\int x^{-\frac{1}{2}}dx + 3\int x dx + \int x^{-\frac{3}{2}}dx = 3\cdot\frac{x^{-\frac{1}{2}+1}}{\frac{1}{2}} + 3\cdot\ln|x| + \frac{x^{-\frac{3}{2}+1}}{-\frac{1}{2}} + C =$

$= 6\sqrt{x} + 3\ln|x| + \frac{2}{\sqrt{x^3}} + C$

7.6 $1 - 2\sin^2\frac{x}{2} = \Big|\ \sin 2\alpha =$

7.7. $\int \frac{1}{\sqrt{a+bx}} dx = \int (a+bx)^{-\frac{1}{2}} dx =$

$= \frac{1}{b} \int (a+bx)^{-\frac{1}{2}} d(a+bx) = \frac{2}{b} (a+bx)^{\frac{1}{2}} + C$

$\int x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} + C$

$d(a+bx) = (a+bx)' dx$

$d(a+bx) = b dx$

$dx = \frac{d(a+bx)}{b}$

7.8. $\int e^{2-3x} dx =$

$= -\frac{1}{3} \int e^{2-3x} d(2-3x) = -\frac{1}{3} e^{2-3x} + C$

$\int e^x dx = e^x + C$

$d(2-3x) = \underbrace{(2-3x)'}_{-3} dx$

$dx = -\frac{1}{3} d(2-3x)$

Прием с экспонентой

$\int \frac{e^x - 1}{e^x + 1} dx =$

$= \frac{1}{t} \int \frac{t-1}{t+1} dt = \int \frac{t-1}{t(t+1)} dt = I$

$I = \int \left(-\frac{1}{t} + \frac{2}{t+1}\right) dt = -\int \frac{1}{t} dt + 2 \cdot \int \frac{1}{t+1} \underbrace{dt}_{d(t+1)}$

$= -\ln|t| + 2 \cdot \ln|t+1| + C =$

$= -\ln|\underbrace{e^x}_{>0}| + 2\ln|\underbrace{e^x+1}_{>0}| + C =$

$= -\ln e^x + 2\ln e^x + 1 + C =$

$= -x + 2\ln|e^x + 1| + C$

1. Замена

$e^x = t, \quad x = \ln t$

$dx = \frac{1}{t} dt$

$\frac{t-1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$

$t - 1 = A(t+1) + Bt$

метод частных значений

пусть $t = -1$

$t = -1: \; -2 = -B, \; B = 2$

$t = 0: \; -1 = A, \; A = -1$

$\frac{t-1}{t(t+1)} = \frac{-1}{t} + \frac{2}{t+1}$

$\int \frac{dt}{t} = \ln|t| + C$

Сканировано с CamScanner

Пример 2

$$\int \sqrt{1-e^{x}} \cdot \underbrace{e^{x}dx}_{-d(1-e^{x})} =$$

$d(1-e^{x}) = (1-e^{x})' dx$

$d(1-e^{x}) = (\underbrace{1'}_{0} - e^{x'}) dx$

$d(1-e^{x}) = -e^{x}dx$

$e^{x}dx = -d(1-e^{x})$

$$= -\int (1-e^{x})^{\frac{1}{2}} \cdot d(1-e^{x}) =$$

$$= -\frac{(1-e^{x})^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{2}{3}(1-e^{x})^{\frac{3}{2}} + C$$

Пример 3.

$$\int \sqrt{e^{x}-1}\, dx =$$

Замена

$\sqrt{e^{x}-1} = t$, $\quad dx = d(\ln(t^2+1))$

$e^{x}-1 = t^2$ $\quad dx = \frac{1}{t^2+1} \cdot 2t\, dt$

$e^{x} = t^2+1$

$x = \ln(t^2+1)$ $\quad dx = \frac{2t}{t^2+1}dt$

$\underbrace{\ln e^{x}}_{x} = \ln(t+1)$

$x = \ln(t+1)$

$$= \int t \cdot \frac{2t}{t^2+1} dt = 2\int \frac{t^2}{t^2+1} dt =$$

$$= 2\int \frac{t^2+1-1}{t^2+1} dt =$$

$$= 2\int \frac{t^2+1}{t^2+1} dt - 2\int \frac{1}{t^2+1} dt =$$

$$= 2\int 1 dt - 2\int \frac{1}{t^2+1} dt = 2t - 2\,\mathrm{arctg}\, t + C =$$

$$= 2\sqrt{e^{x}-1} - 2\,\mathrm{arctg}\sqrt{e^{x}-1} + C$$

Пример 4

$$\int e^{2x} dx = \frac{1}{2}\int e^{2x} \cdot d(2x) = \frac{1}{2}e^{2x} + C$$

$d(2x) = (2x)' dx$

$dx = \frac{1}{2} d(2x)$

Пример 5

$$\int e^{-x} dx = -\int e^{-x} d(-x) = -e^{-x} + C$$

$d(-x) = (-x)' dx$

$dx = \frac{d(-x)}{-1} = -d(-x)$

$a = e$
$\ln(e) = 1$ $\Big| \Rightarrow \int e^{x} dx = e^{x} + C$

Список интегралов с экспонентой.

1. $\int e^{cx} = \frac{1}{c} e^{cx}$

2. $\int a^{cx} dx = \frac{1}{c \ln(a)} a^{cx}, \; a>0, a \neq 1$

3. $\int x e^{cx} dx = \frac{e^{cx}}{c^2} (cx-1)$

4. $\int x^2 e^{cx} dx = e^{cx} \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3} \right)$

5. $\int x^n e^{cx} dx = \frac{1}{c} x^n e^{cx} - \frac{n}{c} \int x^{n-1} e^{cx} dx$

6. $\int \frac{e^{cx} dx}{x} = \ln|x| + \sum_{i=1}^{\infty} \frac{(cx)^i}{i \cdot i!}$

7. $\int e^{ax+b} \cancel{+} C = \frac{1}{a} e^{ax+b} + C$

$n^{m/n} = \sqrt[n]{k^m}$

7.9. $\int \frac{1}{\sqrt[3]{5^x}} dx = \int 5^{-\frac{x}{3}} dx = (*)$ $(-)$

Пусть $u = -\frac{x}{3}$, тогда $du = (-\frac{x}{3})' dx$

$du = -\frac{1}{3} \cdot 1\, dx \Rightarrow \cancel{-\frac{1}{3}dx}\ dx = du : (-\frac{1}{3}) = -3du$

$= (*) \int 5^u \cdot (-\frac{1}{3}) dx = -\frac{1}{3} \int 5^u du = \left| \int a^u du = \frac{a^u}{\ln(a)} \right|_u =$

$= -\frac{1}{3} \cdot \frac{5^u}{\ln(5)} + C = -\frac{1}{3} \frac{5^{-\frac{x}{3}}}{\ln(5)} + C = -\frac{3}{\ln(5) \cdot 5^{\frac{x}{3}}} + C$

7.6.

$\int 1 - \sin^2(\frac{x}{2}) dx = \int \cos^2(\frac{x}{2}) \frac{dx}{2} = \left| u = \frac{x}{2};\ du = (\frac{x}{2})' dx = \cancel{\frac{1}{2}dx} \right| =$

$\Rightarrow dx = du : \frac{1}{2} = 2du$

$= 2 \int \cos^2 u\, du = \left| \cancel{\int \cos^2(u) du} \right| =$ понизим степень $\cos^2 u = \frac{1}{2} \cdot (1 + \cos 2u)$ $=$

$= 2 \cdot \frac{1}{2} \int (1 + \cos 2u) du = \int 1 du + \int \cos 2u\, du =$ $\int 1 du = u + C$ $\int \cos 2u\, du$. $(*)$

Пусть $2u = t$, тогда $dt = (2u)' du \Rightarrow du = \frac{dt}{2}$ $\Big| \Rightarrow \int \cos 2u\, du =$ $= \frac{1}{2}\int \cos t\, dt = \frac{1}{2}\sin t + C$

$\int \cos(t) dt = \frac{\sin(t)}{2} = \left| t = 2u \right| = \frac{\sin 2u}{2}$

$(*) = u + \frac{1}{2}\sin t + C = \frac{x}{2} + \frac{1}{2}\sin 2u + C = \frac{x}{2} + \frac{1}{2}\sin\frac{x \cdot 2}{2} =$

$\cancel{\int \cos^2(u) du = \frac{\sin 2u}{2} + u}$

$\frac{1}{2}\int \cos(2u)du + \frac{1}{2}\int 1\, du = \frac{1}{2} \cdot \frac{\sin 2u}{2} + \frac{u}{2} + C$ $\left| \begin{array}{l} \text{обр. замена} \\ u = \frac{x}{2} \end{array} \right| \Rightarrow 2\int \cos^2 u\, du = \frac{\sin(2u)}{2} + u =$

$= \frac{\sin(x)}{2} + \frac{x}{2} + C = \frac{\sin(x) + x}{2} + C$

$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow 1 - \sin^2\alpha = \cos^2\alpha$ и $1 - \cos^2\alpha = \sin^2\alpha$

$\sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$; $\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$

7.10

$$\int \frac{1}{\cos^2 4x} dx = \left| \int \frac{1}{\cos^2 x} dx = tg\, x + C \right| = (*)$$

Пусть $4x = u$, тогда $du = y' dx = (4x)' dx = 4dx$

$dx = \frac{du}{4}$ и $du = 4dx$

$$(*) = \frac{1}{4} \int \frac{1}{\cos^2 u} \cdot du = \frac{1}{4} \cdot tg\, u + C = \frac{1}{4} tg\, 4x + C$$

7.11

$$\int \frac{x^3+1}{x-1} = (*)$$

Пусть $x-1 = u$, тогда $du = (x-1)' dx = 1 \cdot dx = dx$

$x = u+1$ $\quad dx = du$

$x^3 = (u+1)^3$

$$(*) = \int \frac{(u+1)^3+1}{u} du = \left| \begin{matrix}(a+b)^3 = \\ = a^3+3a^2b+3ab^2+b^3\end{matrix} \right| = \int \left(\frac{u^3}{u} + \frac{3u^2}{u} + \frac{3u}{u} + \frac{1+1}{u} \right) du =$$

$$= \int u^2 du + 3 \int u\, du + 3 \int du + 2 \int \frac{1}{u} du =$$

$$= \frac{u^{2+1}}{3} + 3 \cdot \frac{u^{1+1}}{2} + 3u + 2 \cdot \ln(u) + C =$$

$$= \frac{u^3}{3} + \frac{3}{2} u^2 + 3u + 2\ln(u) + C = \left| u = (x-1) \right| =$$

$$= \frac{1}{3}(x-1)^3 + \frac{3}{2}(x-1)^2 + 3(x-1) + 2\ln(x-1) + C =$$

$$= \frac{1}{3} \cdot (x^3 - 3x^2 + 3x - 1) + \frac{3}{2}(x^2 - 2x + 1) + 3(x-3 + \ln(x-1) + C =$$

$$= \frac{x^3}{3} - x^2 + x - \frac{1}{3} + \frac{3x^2}{2} - 3x + \frac{3}{2} + 3x - 3 + \ln(x-1) + C = \frac{x^3}{3} + \frac{x^2}{2} + x - \frac{11}{6} + \ln(x-1) + C$$

№7.15

Пример

$$\int \frac{dx}{x^2-x^4} = \int \frac{dx}{x^2(1^2-x^2)} = \int \frac{1-x^2+x^2}{x^2(1-x^2)}dx =$$

$$= \int \frac{1}{x^2}dx + \int \frac{1}{1-x^2}dx = \int x^{-2}dx + \int \frac{dx}{1-x^2} = -\frac{1}{x} + \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C$$

$$* \int \frac{1}{x^2}dx = \int \frac{dx}{x^2} = \left| \int x^n dx = \frac{x^{n+1}}{n+1} \right| = \int x^{-2}dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

№7.15

$$\int\left(3x^2+2x+\frac{1}{x}\right)dx = 3\int x^2dx + 2\int x\,dx + \int x^{-1}dx =$$

$$= \frac{2x^2}{2} + \frac{3x^3}{3} + \ln|x| + C$$

№7.16

$$\int \frac{2x+3}{x^4}dx = \int \frac{2x}{x^4}dx + \int \frac{3}{x^4}dx = 2\int x^{-3}dx + 3\int x^{-4}dx = \frac{2\cdot x^{-2}}{(-2)} + \frac{3\cdot x^{-3}}{(-3)} + C =$$

$$= -\frac{1}{x^2} - \frac{1}{x^3} + C$$

№7.17

$$\int \sqrt{mx}\,dx = \int (mx)^{\frac{1}{2}}dx = m^{\frac{1}{2}}\int x^{\frac{1}{2}}dx = m^{\frac{1}{2}} \cdot \frac{x^{\frac{1}{2}+1}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{m\,x^3} + C$$

№ 7.18

$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}} dx = \frac{x^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} + C =$

$= \frac{x^{1-\frac{1}{n}}}{1-\frac{1}{n}} + C = \frac{n \cdot x^{\frac{n-1}{n}}}{n-1} + C$

№ 7.19

$\int \left(\frac{1}{\sqrt[3]{x^2}} - \frac{x+1}{\sqrt[4]{x^3}} \right) dx = \int x^{-\frac{2}{3}} dx - \int (x+1) \cdot x^{-\frac{3}{4}} dx =$

$= \int x^{-\frac{2}{3}} dx - \int x^{\underbrace{1+(-\frac{3}{4})}_{\frac{1}{4}}} dx - \int x^{-\frac{3}{4}} dx = \cancel{\frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{\frac{1}{4}}}{\frac{1}{4}} - x}$

$= \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{\frac{5}{4}}}{\frac{5}{4}} - \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + C = \cancel{3\sqrt[3]{x} - \frac{4}{5\sqrt[4]{x^5}} - \frac{4}{x^4} + C}$

$= 3 \cdot \sqrt[3]{x} - \frac{4\sqrt[4]{x^5}}{5} - 4\sqrt[4]{x} + C$

$(x+1) \cdot x^{-\frac{3}{4}} = x^{1+(-\frac{3}{4})} + x^{-\frac{3}{4}} = x^{\frac{1}{4}} + x^{-\frac{3}{4}}$

$\int x^{\frac{1}{4}} dx = \frac{x^{\frac{1}{4}+1}}{\frac{5}{4}} = \frac{4}{5} \cdot x^{\frac{5}{4}} = \frac{4\sqrt[4]{x^5}}{5}$

$\int x^{-\frac{3}{4}} dx = \frac{x^{-\frac{3}{4}+1}}{\frac{1}{4}} = 4 \cdot x^{\frac{1}{4}} = 4\sqrt[4]{x}$

$x^{\frac{1}{2}} = \sqrt{x}$; $x^{\frac{1}{3}} = \sqrt[3]{x}$

№7.20

$$\int\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{ax}}\right)^2 dx=\int\frac{(\sqrt{a})^2+2\sqrt{ax}+(\sqrt{x})^2}{\sqrt{ax}}dx=$$

$$=\int\frac{a}{\sqrt{ax}}dx+\int\frac{2\sqrt{ax}}{\sqrt{ax}}dx+\int\frac{x}{\sqrt{ax}}dx=\int\sqrt{a}\,x^{-\frac{1}{2}}dx+2\int dx+\frac{1}{\sqrt{a}}\int x^{\frac{1}{2}}dx=$$

$$=\sqrt{a}\cdot\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+2x+\frac{1}{\sqrt{a}}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=2\sqrt{a}\cdot x^{\frac{1}{2}}+2x+\frac{2\sqrt{x^3}}{3\sqrt{a}}+C$$

№7.21

$$\int\frac{x^3+2}{x}dx=\int\frac{x^3}{x}dx+\int\frac{2}{x}dx=\int x^2dx+2\int x^{-1}dx=$$

$$=\frac{x^3}{3}+2\ln|x|+C$$

№7.22

$$\int 2^x e^x dx=2^x e^x-\int e^x 2^x\ln 2\,dx$$

$$\int u\,dv=uv-\int v\,du$$

$$u=2^x\Rightarrow du=(2^x)'dx=2^x\ln 2\,dx$$

$$dv=e^x dx\Rightarrow v=\int e^x dx=e^x+C$$

Сканировано с CamScanner

N
7.23

$\int 2^x(1+3x^2\cdot 2^{-x})dx = \int 2^x dx + \int 2^x\cdot 3x^2\cdot 2^{-x}dx =$

$= \int 2^x dx + 3\int x^2 dx = \left|\int a^x dx = \frac{a^x}{\ln a} + C \ \left(\begin{matrix}a>0\\ a\neq 1\end{matrix}\right)\right| =$

$= \frac{2^x}{\ln 2} + \frac{3x^3}{3} + C = \frac{2^x}{\ln 2} + x^3 + C$

N 7.24.

$\int \cancel{2\cos}(2x+3\cos x)dx = \int 2x dx + \int 3\cos x dx = 2\int x dx + 3\int \cos x dx = \frac{2x^2}{2} + 3\sin x + C$

$= x^2 + 3\sin x + C$

N 7.25

$\int \frac{2-\sin x}{\sin^2 x}dx = \int \frac{2}{\sin^2 x}dx - \int \frac{\sin x}{\sin^2 x}dx = 2\int \frac{1}{\sin^2 x}dx - \int \frac{1}{\sin x}dx =$

$= -2\mathrm{ctg}\, x - \ln|\mathrm{tg}\, x/2| + C$

$\int \frac{dx}{\sin x} = \left| t = \mathrm{tg}\frac{x}{2} \Rightarrow \frac{x}{2}\ \mathrm{arctg}\, t \Rightarrow x = 2\,\mathrm{arctg}\, t \Rightarrow dx = \frac{2dt}{1+t^2}\right.$

$t = \mathrm{tg}\frac{x}{2} = \frac{\sin x/2}{\cos x/2} \Rightarrow t^2 = \frac{\sin^2 x/2}{\cos^2 x/2} = \frac{1-\cos^2 x/2}{\cos^2 x/2} \Rightarrow t^2\cos^2 x/2 = 1-\cos^2 x/2$

~~$\int \sin^\alpha x \cos^\beta x\, dx$ α и β - нечет. и >0; α и β - четн. и >0~~

$(t^2+1)\cos^2 x/2 = 1$

$\cos^2 x/2 = \frac{1}{t^2+1}$

$\sin x = 2\sin x/2 \cos x/2 = \cos^2 x/2 \cdot 2 \cdot \frac{\sin x/2}{\cos x/2} =$

$= \underbrace{2\cos^2 x/2}_{t^2+1} \cdot \underbrace{\mathrm{tg}\, x/2}_{t} = 2\cdot\frac{t}{t^2+1}$

$\sin x = \frac{2t}{1+t^2}$; $dx = \frac{2dt}{1+t^2}$ $\Big|\Rightarrow \int \frac{dx}{\sin x} = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}} = \int \frac{dt}{t} = \ln|t| + C =$

$= \ln|\mathrm{tg}\, x/2| + C$

$\mathrm{tg}\, x/2 = \frac{\sin x/2}{\cos x/2} = \frac{2\sin x/2 \cdot \cos x/2}{\underbrace{2\cos^2 x/2}_{1+\cos x}} = \frac{\sin x}{1+\cos x}$

Пример

$$\int x(x-3)^{14}dx = \left| \begin{array}{l} x-3=t \\ x=t+3 \\ \cancel{d(x-3)=(x-3)'dx} \\ dx=d(t+3) \\ dx=dt \end{array} \right| = \int (t+3)t^{14}dt = \int t^{15}dt + 3\int t^{14}dt =$$

$$= \frac{t^{16}}{16} + \frac{3t^{15}}{15} + C = \frac{t^{16}}{16} + \frac{t^{15}}{5} + C$$

Пример

$$\int \frac{dx}{e^x+1} = \left| \begin{array}{l} e^x=t \\ x=\ln t \\ dx=d(\ln t) \\ dx=(\ln t)'dt=\frac{1}{t}dt \end{array} \right| = \int \frac{\frac{1}{t}dt}{t+1} = \int \frac{dt}{t(t+1)} = \int \frac{1}{t^2+t}dt =$$

$$= \int \frac{1}{\left(t+\frac{1}{2}\right)^2 - \frac{1}{2^2}}dt = (*)$$

Соберем полн. квадрат

$$a^2+2ab+b^2=(a+b)^2$$

$$t^2+t = \left(t^2 + \underbrace{2t}_{2a}\cdot\underbrace{\frac{1}{2}}_{b} + \frac{1}{4}\right) - \frac{1}{4} = \left(t+\frac{1}{2}\right)^2 - \frac{1}{2^2} \qquad \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$= (*) = -\int \frac{1}{\frac{1}{2^2} - \left(t+\frac{1}{2}\right)^2}dx = -\frac{1}{2\cdot\frac{1}{2}}\ln\left|\frac{\frac{1}{2}+t+\frac{1}{2}}{\frac{1}{2}-t-\frac{1}{2}}\right| + C = -\ln\left|\frac{t+1}{-t}\right| + C =$$

$$= -\ln\left|\frac{e^x+1}{e^x}\right| + C = \left| P\cdot\ln A = \ln A^P \right| = \ln\left|\frac{e^{-x}}{e^x+1}\right| + C = \left| \begin{array}{l} e^x>0 \\ e^x+1>0 \\ \frac{-e^x}{e^x+1}<0 \\ \left|\frac{-e^x}{e^x+1}\right| = \frac{e^x}{e^x+1} \end{array} \right.$$

$$= \ln\left(\frac{e^x}{e^x+1}\right) + C$$

Пример

$$\int \frac{x\,dx}{\sqrt{1+x}} = \left| \begin{array}{l} \sqrt{1+x} = t \\ 1+x = t^2 \\ x = t^2-1 \end{array} \right.$$

$dx = d(t^2-1)$

$dx = (t^2-1)'dt$

$dx = 2t\cdot dt$

~~$d(t^2-1) = (t^2-1)dx$~~

~~$d(t^2-1) = 2t\,dx$~~

~~$dx = \frac{d(t^2-1)}{2t}$~~

$$= \int \frac{(t^2-1)\,2t\,dt}{t} = 2\int\left(\frac{t^3}{t} - \frac{t}{t}\right)dt = 2\int\frac{t^3}{t}dt - 2\int\frac{t}{t}dt = 2\int t^2dt - 2\int dt = \frac{2t^3}{3} - 2t + C =$$

$$= \frac{2(\sqrt{1+x})^3}{3} - 2\sqrt{1+x} + C$$

Метод замены переменной

Пр.

$dy = y'dx$

$$\int \cos^3 x \sin x\,dx = \int \underbrace{(\cos x)}_{t}{}^3 \sin x\,dx = \left| \begin{array}{l} \cos x = t \\ d(\cos x) = dt \\ (\cos x)'\cdot dx = dt \\ -\sin x\,dx = dt \quad |\cdot(-1) \\ \sin x\,dx = -dt \end{array} \right| =$$

$$= \int t^3(-dt) = -\int t^3 dt =$$

$$= -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

Пр.

$$\int x\underbrace{\sqrt{x+5}}_{t}\,dx = \left| \begin{array}{l} \sqrt{x+5} = t \\ x+5 = t^2 \\ x = t^2-5 \\ dy = y'dx \\ \cancel{d(t^2-5) = (t^2-5)} \\ d(t^2-5) = dx \\ (t^2-5)'\cdot dt = dx \\ 2t\,dt = dx \end{array} \right|$$

$$= \int (t^2-5)\,t\cdot 2t\,dt$$

$$= 2\int(t^4-5t^2)dt = 2\int t^4dt - 2\int 5t^2dt =$$

$$= \frac{2t^5}{5} - \frac{10t^3}{3} + C = \frac{2(\sqrt{x+5})^5}{5} - \frac{10(\sqrt{x+5})^3}{3} + C$$

Пр.

$$\int \frac{e^{2x}dx}{4+e^{4x}} = \int \frac{e^{2x}dx}{2^2+\underbrace{(e^{2x})^2}_{t}} = \int \frac{\frac{1}{2}dt}{2^2+t^2} = \frac{1}{2}\int \frac{dt}{2^2+t^2} =$$

$$= \frac{1}{2}\cdot\frac{1}{2}\, arctg\, \frac{t}{2} + C =$$

$$= \frac{1}{4}\, arctg \frac{e^{2x}}{2} + C$$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\, arctg\, \frac{x}{a}$ $\qquad dy = y' \cdot dx$

$e^{2x} = t$

$d(e^{2x}) = dt$

$(e^{2x})' \cdot dx = dt$

$e^{2x}(2x)'dx = dt$

$e^{2x}\cdot 2dx = dt$

$e^{2x}dx = \frac{dt \cdot 1}{2}$

Пр.

$$\int \frac{dx}{x(\underbrace{\ln x + 1}_{t})} = $$

$(\ln x)' = \frac{1}{x}$

$\ln(x+1) = t$

$d(\ln(x+1)) = dt$

$(\ln(x+1))' \cdot dx = dt$

$(\ln x)' + (1)' dx = dt$

$\frac{1}{x} dx = dt$

$$\cancel{= \int t\,dt = \frac{t^2}{2} + C} = \frac{1}{2} \ln x+1 + C$$

$$= \int \frac{dt}{t} = \int t^{-1}dt = \ln|t| + C =$$

$$= \ln|\ln x + 1| + C$$

Пр.

$$\int \frac{dx}{\sqrt{1+e^x}} =$$

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C$

$\sqrt{1+e^x} = t$

$1+e^x = t^2$

$e^x = t^2 - 1$

$d(e^x) = d(t^2-1)$

$(e^x)'dx = (t^2-1)'dt$

$e^x \cdot dx = 2t \cdot dt$

$dx = \frac{2t\,dt}{e^x} = \frac{2t\,dt}{t^2-1}$

$$= \int \frac{\frac{2t}{t^2-1}dt}{t} = \int \frac{2t}{t(t^2-1)}dt =$$

$$= 2\int \frac{1}{t^2-1}dt = -2\cdot\int \frac{1}{1-t^2}dt =$$

$$= -2\,\frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C = -\ln\left|\frac{\sqrt{1+e^x}+1}{1-\sqrt{1+e^x}}\right|$$

$$\int_0^1 e^{2x}dx = \frac{1}{2}\int_0^1 e^{2x}\underbrace{d(2x)}_{\substack{(2x)'dx \\ 2dx}} = \frac{1}{2}e^{2x}\Big|_0^1 = \qquad \Big| \int e^x dx = e^x + C$$

$$= \frac{1}{2}(e^2 - e^0) = \frac{1}{2}(e^2 - 1)$$

$$\int_{\frac{\sqrt{3}}{4}}^{\frac{3}{4}} \frac{dx}{9+16x^2} = \cancel{\int_{\frac{\sqrt{3}}{4}}^{\frac{3}{4}} \frac{dx}{3^2+(4x)^2} = \frac{1}{3}} \qquad \Big| \int \frac{dx}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C$$

$$= \int_{\frac{\sqrt{3}}{4}}^{\frac{3}{4}} \frac{dx}{16(\frac{9}{16}+x^2)} = \frac{1}{16}\int_{\frac{\sqrt{3}}{4}}^{\frac{3}{4}} \frac{dx}{(\frac{3}{4})^2+x^2} = \frac{1}{16}\cdot\frac{1}{\frac{3}{4}}\operatorname{arctg}\frac{x\,4}{3}\Big|_{\frac{\sqrt{3}}{4}}^{\frac{3}{4}} =$$

$$= \frac{1}{12}\operatorname{arctg}\frac{4x}{3}\Big|_{\frac{\sqrt{3}}{4}}^{\frac{3}{4}} = \frac{1}{12}\cdot\left(\operatorname{arctg}\frac{4}{3}\cdot\frac{3}{4} - \operatorname{arctg}\frac{4}{3}\cdot\frac{\sqrt{3}}{4}\right) =$$

$$= \frac{1}{12}\Big(\underbrace{\operatorname{arctg}1}_{\frac{\pi}{4}} - \underbrace{\operatorname{arctg}\frac{\sqrt{3}}{3}}_{\frac{\pi}{6}}\Big) = \frac{1}{12}\left(\frac{\pi}{4}^{3} - \frac{\pi}{6}^{2}\right) = \frac{1}{12}\left(\frac{3\pi - 2\pi}{12}\right) = \frac{1}{12}\cdot\frac{\pi}{12} = \frac{\pi}{144}$$

$$\int_0^{-3} \frac{dx}{\sqrt{25+3x}} = -\int_{-3}^{0} (25+3x)^{-\frac{1}{2}}dx = \left| \begin{array}{l} d(25+3x) = (25+3x)'dx = 3dx \\ \cancel{dx} = \frac{1}{3}d(25+3x) = dx \end{array} \right| =$$

$$= -\frac{1}{3}\int_{-3}^{0} (25+3x)^{-\frac{1}{2}} d(25+3x) = -\frac{1}{3}\cdot\frac{(25+3x)^{-\frac{1}{2}+1}}{\frac{1}{2}}\Big|_{-3}^{0} = -\frac{2}{3}\left(25+3x\right)^{\frac{1}{2}}\Big|_{0}^{-3} =$$

$$= -\frac{2}{3}\Big(\sqrt{25+3\cdot 0} - \sqrt{\underbrace{25+3(-3)}_{-9}}\Big) = -\frac{2}{3}(5-4) = -\frac{2}{3}$$

7.26

$$\int \frac{3-2ctg^2x}{\cos^2x}dx = \int \frac{3}{\cos^2x}dx - \int \frac{2ctg^2x}{\cos^2x}dx = 3\int \frac{1}{\cos^2x}dx - 2\int \frac{\cos^2x}{\sin^2x\cos^2x}dx =$$

$$= 3\int \frac{1}{\cos^2x}dx - 2\int \frac{1}{\sin^2x}dx = 3tgx + 2ctgx + C$$

№ 7.27

$$\int \frac{\cos 2x}{\cos^2x\sin^2x}dx = \int \frac{\cos^2x-\sin^2x}{\cos^2x\sin^2x}dx = \int \frac{1}{\frac{\cos^2x}{\sin^2x}}dx - \frac{1}{\frac{\sin^2x}{\cos^2x}}dx =$$

$$= -ctgx - tgx + C$$

№ 7.28

$$\int \sin^2\frac{x}{2}dx = \int \frac{1-\cos 2\frac{x}{2}}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos x\,dx = \frac{x}{2} - \frac{1}{2}\sin x + C$$

№ 7.29

$$\int tg^2x\,dx = \int \frac{\sin^2x}{\cos^2x}dx = \int \frac{\frac{1-\cos^2x}{2}}{\cos^2x}dx = \int \frac{1}{2}(1-\cos^2x)\cdot\frac{1}{\cos^2x}dx =$$

$$= \frac{1}{2}\int \frac{1}{\cos^2x}dx - \frac{1}{2}\int \frac{\cos^2x}{\cos^2x}dx = \frac{tgx}{2} - \frac{x}{2} + C$$

Подведение под знак диф-ла $\quad \frac{1}{3}\cdot\frac{2}{3}$

№ 7.44.

$$\int \sqrt{3+x}\,dx = \left| \begin{array}{l} \int x^n dx = \frac{x^{n+1}}{n+1} \\ d(3x+1) = (3x+1)'dx \\ d(3x+1) = 3dx \\ \frac{1}{3}d(3x+1) = dx \end{array} \right| = \frac{1}{3}\int (3+x)^{\frac{1}{2}}d(3x+1) = \frac{1}{3}\frac{(3x+1)^{\frac{1}{2}+1}}{\frac{3}{2}} + C = \frac{2}{9}\sqrt{(3x+1)^3} + C$$