Survival Analysis on Li-ion Batteries

Using R version 4.5.0

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Introduction

For this project, I wanted to find a data set that applies survival analysis on renewable energies. I've found this paper where they use survival analysis to predict the EOL of Toyota battery cells and its data set is openly available.

The aim of this document is to reproduce some of the results in [1] and generate several models to predict the lifetime of a cell after collecting its voltage curves. Battery degradation is directly related to the internal resistance, the voltage and capacity of the batteries. The method proposed in [1] uses the voltage charge and discharge curves on the first 50 cycles as covariates. The voltage curves will change its form as the batteries degrade. Moreover, using the voltage as feature eases implementation, as it is monitored by the BMS and it could be easily retrieved, thus showing the practical approach of the method.

We have focused this document on studying the models using the voltage charge curves' variables. The proposed models in this report are :

- 1.Cox Proportional Hazard
- 2. AFT Weibull
- 3. Survival Random Forest

The results with this models are going to be compared to the one used in the paper, which is an **XGBoost** implementation of the CoxPH model.

ETL

To load the data, we have gone to the data source and use the treatment scripts in python and translate them into R and also to have the same data sets as in [1].

The raw data set has a size of 2.6 GB and contains the internal resistance, Voltage, capacity (Qr and Qd) and temperatures measured during the deep cycling of 8 battery batches with 46-48 cells (373 cells in total). From this initial 373 cells, 362 are left after the data treatment.

During the data treatment we had to collect the main features and the time-to-event (T) and the indicator (δ) as well as the voltage charge and discharge features used in [1] for training the models. The internal resistance, temperature and capacity data where omitted. The raw data is processed and two data sets are converted from python to R and made available:

- 1) **toyota_structured_data.rds** which contains the voltage values on the charging/discharging cycles for each cell. Available here.
- 2) **data_treated.rds** which contains the voltage features (signature paths) on charge and discharge with labels $ch_{feature_number}$ and $dch_{feature_number}$ along with the EOL time (T) and the censor indicator (δ) for each cell. Available here

The raw data set contains the internal resistance, voltage, capacity (Qr and Qd) and temperatures measured during the deep cycling of 8 battery batches with 46-48 cells (373 cells in total). From this initial 373 cells, 362 are left after the data treatment.

We have modified the scripts to import as well the features used in the paper. These features are the path signatures of the voltage curves. By using path signatures on the charge and discharge voltage curves, we can compress the information of these curves into a few features. Particularly, when using a signature of the form $Sig_i(t) = (t, V(t))$ with a signature depth k = 2, we obtain **6 features for each cycle**. Nonetheless, the author of the paper wants to express it in a way where we compress the voltage of the first 50 cycles for each cell. Thus, when recalculating the paths of the form $P_i = (cycle_i, Sig_i(t))$ for the first 50 cycles and with signature depth of k = 2, we obtain 36 features (6²) for each cell.

Therefore, we observe 36 features for each cell on charging and 36 features for discharging.

```
suppressPackageStartupMessages({
    library(tidyverse)
library(survival)
library(survivalROC)
library(gtsummary)
library(broom)
library(ggsurvfit)
library(ggrepel)
})

df = readRDS("toyota_structured_data.rds")
data = as.data.frame(readRDS(file="data_treated.rds"))
```

EDA

The cell's fully charge voltage is around 3.5 V and with a capacity of 1.1 Ah. The EOL (End Of Life) of the cell is estimated when the capacity has fallen down below 80% of the batteries capacity, i.e. 0.88 Ah. The EDA plots could be found in the Annex.

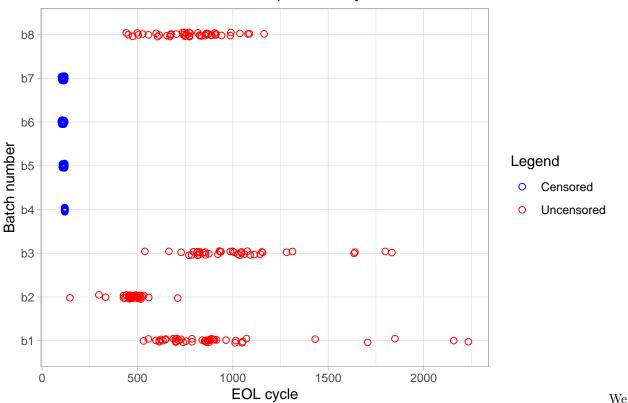
The experiments were done with different charging and discharging policies, so we can assume a random behavior when studying the survival risk.

From the 8 batches, 4 were driven to the EOL (b1, b2, b3 and b8) and 4 were censored i.e. they left the experiment without arriving to EOL (b4, b5, b6 and b7). See the following figure (see Figure 3 in [1]).

```
BATCHES = c("b1", "b2", "b3", "b4", "b5", "b6", "b7", "b8")
x \text{ batches} = c()
y_eol = c()
censorship = c()
tmp batch = c()
for (label in BATCHES) {
  for (i in 1:length(df)) {
    if (df[[i]][["summary_data"]][["batch_name"]] == label)
      x_batches = rbind(x_batches, label)
      y_eol = rbind(y_eol,df[[i]][["summary_data"]][["end_of_life"]])
        if (label %in% c("b4","b5","b6","b7"))
          {censorship = rbind(censorship, "Censored") }
        else {censorship = rbind(censorship, "Uncensored")
                                                                   }
      }
  }
}
df_eol = data.frame(batches=x_batches, eol=y_eol, censorship=censorship)
ggplot(data = df_eol, aes(x=eol, y=batches)) +
  geom_jitter(height=0.05, width = 0, size = 2, shape = 1, aes(colour = censorship)) +
  labs(x = "EOL cycle",
```

```
y = "Batch number",
color = "Legend",
title = "EOL distribution and censorship summary of cells in each batch") +
scale_color_manual(values = c("blue", "red")) +
theme_light()
```

EOL distribution and censorship summary of cells in each batch



observe that the censored batches never pass the 200 cycles and that most of the EOL happens between 500 and 1000 cycles. The mean is 440 cycles and the median is 119 cycles.

```
summary(data$time_to_failure)
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 101.0 113.0 119.0 439.9 754.5 2235.0
```

EDA: survival analysis

We have a total of 174 batteries arriving to EOL, with a median time-of-failure of 771 cycles.

```
fit.KM <- survfit(Surv(time_to_failure, failure) ~ 1, data = data)
print(fit.KM)

## Call: survfit(formula = Surv(time_to_failure, failure) ~ 1, data = data)
##

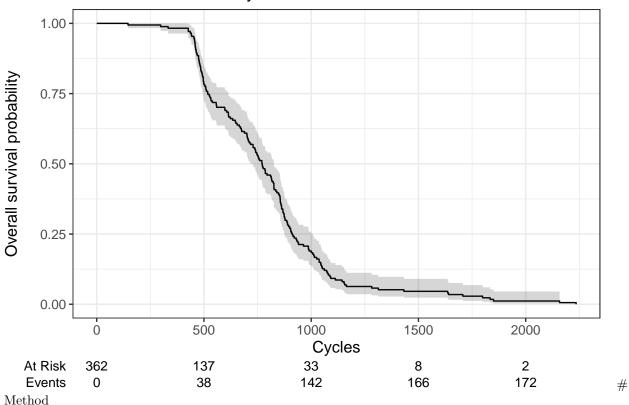
## n events median 0.95LCL 0.95UCL
## [1,] 362 174 771 717 826

fit.ggKM <- survfit2(Surv(time_to_failure, failure) ~ 1, data = data)</pre>
```

The survival curve of the Toyota batteries. We could also plot the CFD function, but to reduce space we are going to omit it. The CFD could be plotted by giving the argument type = "risk".

```
ggsurvfit(fit.ggKM, type = "survival") +
labs(
    x = "Cycles",
    y = "Overall survival probability",
    title = "Survival function on Toyota Batteries"
    ) +
add_confidence_interval() +
add_risktable()
```

Survival function on Toyota Batteries



As we are going to compare the results w.r.t. the paper, we need to follow the same methods.

Regarding the data treatment, we are going to split the data and also apply a scale on the data based on the Inter Quantile Range (Robust Scaler in ScikitLearn).

Regarding the results comparison, we are going to compare each model by the **C-index** (concordance index) and the integrated* **Brier Score** as well as the **Cumulative Dynamic AUC** for $t \in [500, 1000]$ cycles.

Split data into a training and a testing set

```
set.seed(42)
test_split = 0.2
n_obs = length(data$failure)
i.training <- sample.int(n_obs, size = ceiling(n_obs*(1-test_split)), replace = FALSE)
i.testing <- setdiff(seq_len(n_obs), i.training)
d_train <- data[i.training,]
d_test <- data[i.testing,]</pre>
```

Cox Proportional model

We are going to start with the full model to make a variable selection with two methods, one with the step function (AIC criterion) and another with the penalized method (elasticNet method).

```
CPH_ch_full <- coxph(Surv(time_to_failure, failure) ~ ch_1+ch_2+ch_3+ch_4+ch_5+ch_6+ch_7+ch_8+ch_9+ch_1
```

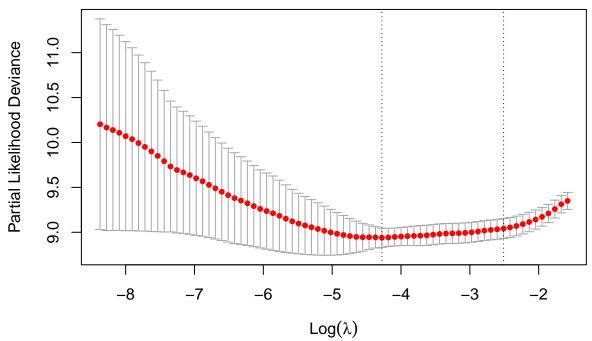
The variables selected by the step function are: V2, V6, V10, V12, V14, V18, V22, V24, V26, V28, V30, V32 and V34.

```
CPH_ch_step <- step(CPH_ch_full, trace = F)</pre>
```

We have balanced the penalized regression with $\alpha=0.9$ to introduce a little Ridge term on the ElasticNet cost function.

```
suppressPackageStartupMessages(library(glmnet))
y_train = Surv(d_train$time_to_failure, d_train$failure)
X_train_ch = as.matrix(d_train[,-c(1,2,3,40:76)])
X_train_dch = as.matrix(d_train[,-c(1,2,3,4:39)])
CPH_ch_pen <- glmnet(X_train_ch, y_train, family = "cox", alpha = 0.9)
CPH_ch_pen.cv10 <- cv.glmnet(X_train_ch, y_train, family = "cox")
plot(CPH_ch_pen.cv10)</pre>
```

17 16 17 18 16 15 12 12 12 10 9 8 6 4 4 3



going to select the higher variables selected, which is for the $\lambda_m in$. We can observe that the variables selected by the penalized method are different from the step AIC method. The selected variables with $\lambda_{min} = 0.01387614$ are V4,V5,V6,V10,V18,V22,V23,V24,V30,V32 and V35.

Without using the testing data set yet, the penalized model has a higher concordance index. Logically, the step model has higher AIC score.

```
cph_ch.mod <- CPH_ch_pen_lamdbda_min

y_test = Surv(d_test$time_to_failure, d_test$failure)

X_test_ch = d_test[,-c(1,2,3,40:76)]

X_test_dch = d_test[,-c(1,2,3,4:39)]

# Select the variables from the penalized model

X_test_ch_sel = X_test_ch[,c(4,5,6,10,18,22,23,24,30,32,35)]</pre>
```

We have used the *survex* library (R version 4.5.0) to calculate the Integrated Brier Score and The Integrated Cumulative Dynamic AUC between 500 and 1000 cycles.

To save space, we have compared the step and the penalized models, and we have obtained better scores (higher C-index and AUC and lower IBS) for the penalized selected CPH model.

```
cph_ch.mod$sch_residual <- cox.zph(cph_ch.mod)
cph_ch.mod$sch_residual</pre>
```

```
##
           chisq df
## ch_4
          0.3351 1 0.563
## ch_5
          5.8575 1 0.016
## ch_6
          2.2880 1 0.130
## ch_10
          2.7902 1 0.095
## ch_18
          3.2567 1 0.071
## ch_22
          1.7803 1 0.182
          5.7751 1 0.016
## ch_23
```

```
## ch_24 0.0154 1 0.901

## ch_30 4.6380 1 0.031

## ch_32 0.9548 1 0.329

## ch_35 2.7863 1 0.095

## GLOBAL 22.2864 11 0.022
```

By checking the Schoenfeld global residuals we CAN NOT assume that the hazard rates are proportional for a confidence level of 0.05

AFT parametric model

We are going to fit an AFT model, and calculate the scores. The object $aft_ch.pred$ has the predictions on the test set and has a dimension of dim = (72,51). The score times has 51 time points (from 500 to 1000) and we have 72 voltage charge curve observations in the test set. We find the best score with parameters dist="weibull" and an Accelerated Failure Time parameter param="lifeAcc". All the possible parameters in the ldatools package where tested. The fitting of the shape converges, and thus we can use this model if needed.

```
suppressPackageStartupMessages({
  library(eha) # for using the "aftreg" function
  library(ldatools)
 library(SurvMetrics)})
aft_ch.mod <- aftreg(formula = Surv(time_to_failure, failure) ~ ch_4 + ch_5 + ch_6 + ch_10 + ch_18 + ch
                            data = d_train,
                            dist = "weibull",
                            param = "lifeAcc",
                            model = T, x = T, y=T)
cat("AFT model fitting optimization converged:", aft_ch.mod$convergence)
## AFT model fitting optimization converged: TRUE
new data = data.frame(X test ch sel,time to failure=d test$time to failure,failure=d test$failure)
# from ldatools library, we calculate the survival curves
aft_ch.pred <- predictSurvProb(aft_ch.mod, newdata = new_data, times = score_times)
# from SurvMetrics we use the Cindex function on t c [500, 1000]
c_indexes <- c()</pre>
for (t_idx in 1:dim(aft_ch.pred)[2]) {
 C_idx <- Cindex(object = y_test, predicted = aft_ch.pred[,t_idx], t_star = score_times[t_idx])</pre>
  c_indexes <- append(c_indexes, C_idx)</pre>
}
aft_ch.mod$C_idx <- mean(c_indexes)</pre>
aft_ch.mod$IBS <- integrated_brier_score(y_test, surv = aft_ch.pred, times = score_times)
aft_ch.mod$ICD_AUC <- integrated_cd_auc(y_test, surv = aft_ch.pred, times = score_times)
aft_ch.mod$AIC <- AIC(aft_ch.mod)
```

Survival Random Forest Model

First, we've strated training a Survival Random Forest model with all the charge variables, giving us the following table:

```
srf_full_ch <- c(C_idx=0.8255741, IBS=0.08753846, IDC_AUC=0.7698247)
srf_full_ch

## C_idx IBS IDC_AUC
## 0.82557410 0.08753846 0.76982470</pre>
```

However, as we want to compare the previous models, we are going to select the same variables as the penalized Cox PH model, so we can compare the models on the same variables selected.

The combination of parameters splitrule="logrank" and ntree=500 gave the best scores. On the other hand, we have used the variable importance of the Random Forest (see anex in github code) to make a variable selection, but the model is not better than the proposed here.

Results and discussion

```
score_table <- c(cph_ch.mod$C_idx, cph_ch.mod$IBS, cph_ch.mod$ICD_AUC, cph_ch.mod$AIC)
score_table <- rbind(score_table, c(aft_ch.mod$C_idx, aft_ch.mod$IBS, aft_ch.mod$ICD_AUC, aft_ch.mod$AI
score_table <- rbind(score_table, c(srf_ch.mod$C_idx, srf_ch.mod$IBS, srf_ch.mod$ICD_AUC, srf_ch.mod$AI
score_table <- rbind(score_table, c(0.794, 0.112, 0.9015, NA))
row.names(score_table) <- c("CPH", "AFT weibull", "Survival Random Forest", "XGBoost (Paper mean values
colnames(score_table) <- c("C-idx", "IBS", "ICD_AUC", "AIC")
score_table</pre>
##

C-idx IBS ICD_AUC AIC
```

```
## C-idx IBS ICD_AUC AIC
## CPH 0.7553236 0.13803045 0.4630719 1012.299
## AFT weibull 0.7529410 0.14532658 0.4659791 1876.491
## Survival Random Forest 0.8249478 0.08738262 0.7755177 NA
## XGBoost (Paper mean values) 0.7940000 0.11200000 0.9015000 NA
```

As we can observe in the table, the models CPH (non-parametric) and AFT weibull (parametric) perform similarly. However, the AIC on the CPH model is much lower than on the AFT model. We have proposed an AFT model because the proportional hazard test was not passed on CPH, thus, if we had between the two, we would choose the AFT model over the CPH.

On the other hand, looking at the SRF (Survival Random Forest) model, we see that outperform the rest of the models and even the results published in the paper for the C-index and the IBS scores.

We observe a big difference in the AUC score, we observe the lowest for the CPH and AFT and that the SRF gets closer to the XGBoost. We can conclude that the model in the paper is more robust than those proposed in this document.

Conclusion

For simplification and to be able to compare models, we have respected the variable selection done at the beginning of the **Method's chapter**. However, in the original paper [1], all the voltage charge variables are

used for training an XGBoost model. A model using all the variables and Random Survival Forest was also trained and we have seen that it has better scores for the C-index and the IBS (on one test set) that the scores on shown in the paper (mean value on 100 test sets).

We have seen that a parametric model as AFT, has its limitations as it depends on the distribution.

On the other hand, if we use a Cox PH and we **do not have proportional hazards** (our case), we could truncate or stratify the data. Regarding stratification, it was a big limitation because we can not easily classify the path signatures of the voltage curves (*strat*). Regarding truncation we could think about truncating the data between 300 and 1200 cycles, where most of the event occur, and see if we pass a proportional hazard test.

In conclusion, we have reproduce the methodology in [1] and obtained very good results with our Survival Random Forest model for the charging voltage curve model. Other models were proposed, but since Survival Random Forest outperforms the rest and its more flexible in terms of dealing with non-linearities, we could propose it for solving this problematic.

NB: The same methodology can be followed to produce models based on the discharge voltage curves.

Bibliography

- [1] Rasheed Ibraheem et al., Robust survival model for the prediction of Li-ion battery lifetime reliability and risk functions
- [2] David Kuhajda, Using Survival Analysis to Evaluate Medical Equipment Battery Life
- [3] Rasheed Ibraheem et al., Early prediction of Lithium-ion cell degradation trajectories using signatures of voltage curves up to 4-minute sub-sampling rates
- [4] Attia Peter M. et al., Statistical Learning for Accurate and Interpretable Battery Lifetime Prediction
- [5] Attia Peter M. et al., Data-driven prediction of battery cycle life before capacity degradation

Code availability

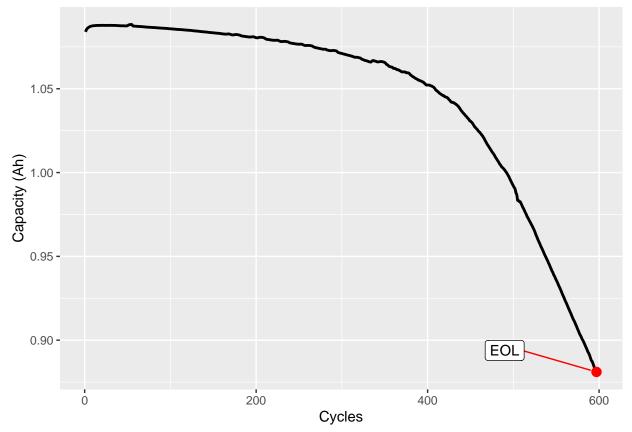
The experiments carried out on this R markdown document can be found in https://github.com/zBotta/surv An-batteries. The objective of this code was purely academic in order to address a course project on Survival Analysis.

Annex

EDA: Extra plots

In the figure below, we observe the evolution of the capacity of cell 45 form the batch 1 (b1c45). The capacity decreases as it has been cycled, until arriving to its EOL at 597 cycles.

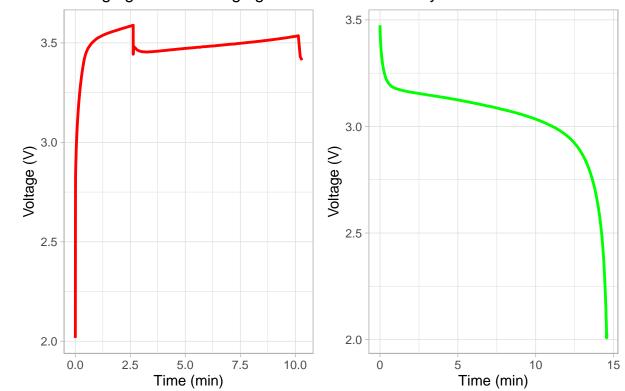
```
aes(eol_cycle, eol_capacity),
label="EOL",
size = 4,
box.padding = 1.5,
point.padding = 0.5,
force = 100,
segment.color = "red",
direction = "x")
```



Below the charge and discharge voltage curves of cell 45 form the batch 1 (b1c45) at cycle 50. Plotting all curves becomes difficult as the time stamp is different for each charge-discharge experiment. During the data treatment, data has been interpolated to respect a constant time stamp.

Characteristic	70% Percentile
Overall	878 (856, 933)

Charging and Discharging curves for b1c45 at cycle 50



Use cases of battery survival curve

In a real world scenario applied on batteries, the survival curves could be used for:

1) Estimating the amount of batteries that a battery producer should have in stock to replace their installed battery packs. For instance, if we have a contract with our clients stating that we had to change the batteries when at least 70% of the batch cells arrive to its EOL, we could estimate after how many cycles we would need to start having a new battery on stock.

The batteries experiencing at least 878 cycles with a 95%CI, would need to be replaced.

```
library(gtsummary )
tbl_survfit(fit.KM , probs = c(0.7))
```

2) As seen in paper [2], we could estimate the maintenance shifts of a given battery fleet to ensure a certain capacity (survival rate).

Variable selection with SRF

```
# RF variable selection
srf_ch.mod <- holdout.vimp(Surv(time_to_failure, failure) ~ ch_1+ch_2+ch_3+ch_4+ch_5+ch_6+ch_7+ch_8+ch_
# taking 1 sd
one_sd_vimp = sd(na.omit(srf_ch.mod$importance))
abs(na.omit(srf_ch.mod$importance)) > one_sd_vimp
## ch_1 ch_2 ch_3 ch_5 ch_6 ch_7 ch_8 ch_9 ch_10 ch_11 ch_12 ch_13 ch_14
## TRUE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE TRUE FALSE
## ch_16 ch_17 ch_18 ch_19 ch_20 ch_21 ch_23 ch_24 ch_25 ch_26 ch_27 ch_28 ch_30
## FALSE FALSE FALSE FALSE TRUE FALSE FALS
## ch_31 ch_33 ch_34 ch_35
## TRUE FALSE FALSE FALSE
The selected variables are V2, V3, V4, V6, V7, V8, V11, V15, V21, V23, V25, V35, V36.
srf ch vimp.mod <- rfsrc(Surv(time to failure, failure) ~ ch 2 + ch 3 + ch 4 + ch 6 + ch 7 + ch 8 + ch
X_{\text{test\_ch\_vimp}} = X_{\text{test\_ch}}[,c(2,3,4,6,7,8,11,15,21,23,25,35,36)]
srf_vimp.explainer <- survex::explain(srf_ch_vimp.mod, verbose = F)</pre>
surv_pred <- srf_vimp.explainer$predict_survival_function(model = srf_vimp.explainer$model,</pre>
                                                                                                                        newdata = X_test_ch_vimp,
                                                                                                                        times = score_times)
risk <- srf_vimp.explainer$predict_function(model = srf_vimp.explainer$model,
                                                                                        newdata = srf_vimp.explainer$data)
srf_ch_vimp.mod$C_idx <- c_index(y_true = srf_vimp.explainer$y, risk = risk)</pre>
srf_ch_vimp.mod$IBS = integrated_brier_score(y_test, surv = surv_pred, times = score_times)
srf_ch_vimp.mod$ICD_AUC = integrated_cd_auc(y_test, surv = surv_pred, times = score_times)
srf_ch_vimp.mod$AIC <- NA</pre>
srf_ch_vimp.mod$C_idx
## [1] 0.8055324
srf_ch_vimp.mod$IBS
## [1] 0.0974398
srf_ch_vimp.mod$ICD_AUC
## [1] 0.7122457
srf_ch_vimp.mod$AIC
```

[1] NA