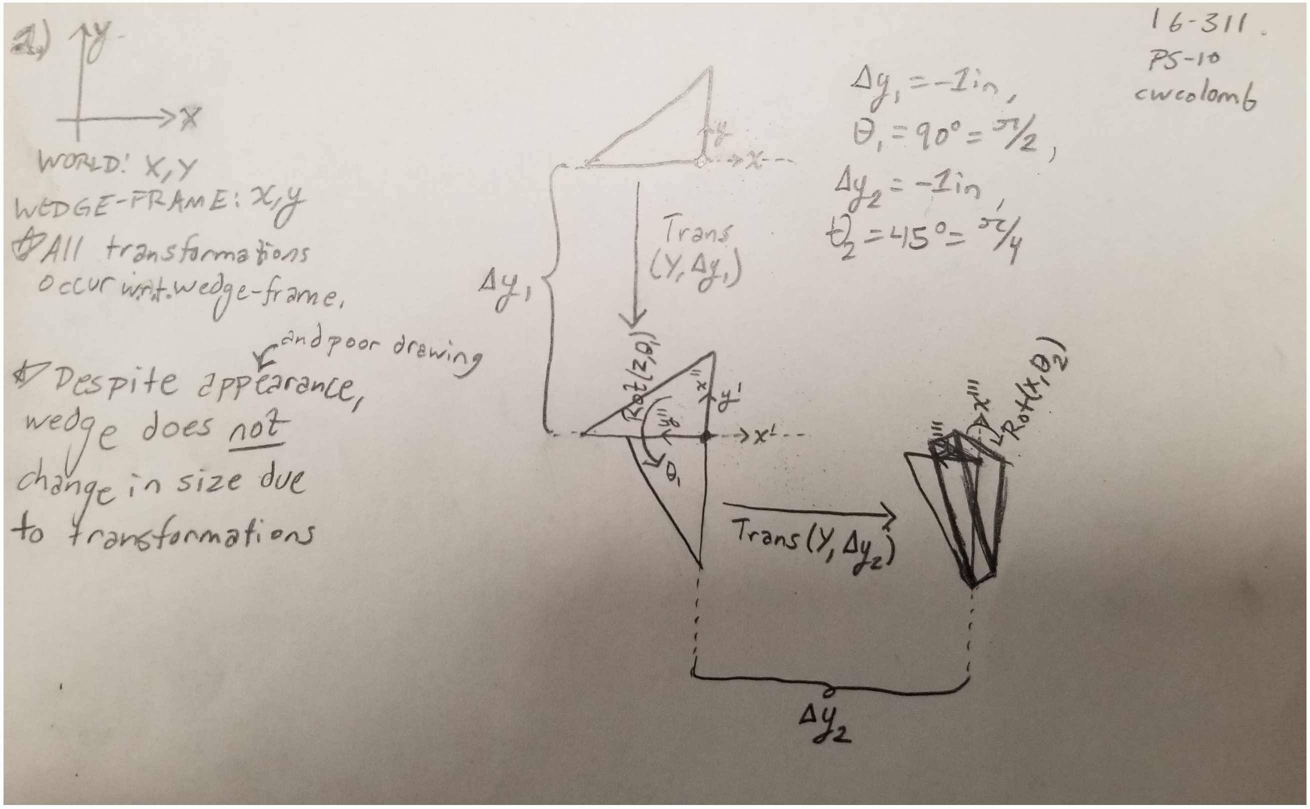
2.)



3.)

a.)

$$H = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 5 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b.)

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of -2 units in the x direction, 5 units in the y, and 9 in the z.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about X-axis of $\boldsymbol{\theta} = -\frac{\boldsymbol{\pi}}{\mathbf{4}} = \mathbf{-45}°$, because $\cos(\theta) = \frac{\sqrt{2}}{2}, \sin(\theta) = \frac{-\sqrt{2}}{2}$

4.)

$$H = R_z(\theta_1) * T_{x1}(l_1) * R_z(\theta_2) * T_{x2}(s)$$

$$\rightarrow H = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & l_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & s \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow H = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & l_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & s * \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & s * \sin(\theta_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow H = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & s * \cos(\theta_2) + l_1 \\ \sin(\theta_2) & \cos(\theta_2) & s * \sin(\theta_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow H = \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1)\sin(\theta_2) - \sin(\theta_1)\cos(\theta_2) & \cos(\theta_1)(s\cos(\theta_2) + l_1) - s\sin(\theta_1)\sin(\theta_2) \\ \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2) & \sin(\theta_1)(s\cos(\theta_2) + l_1) + s\cos(\theta_1)\sin(\theta_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos(\theta_1)\cos(\theta_2) + l_1\cos(\theta_1) - s\sin(\theta_1)\sin(\theta_2) \\ s\sin(\theta_1)\cos(\theta_2) + l_1\sin(\theta_1) + s\cos(\theta_1)\sin(\theta_2) \\ 1 \end{bmatrix}$$

$$\because \theta = \theta_1 + \theta_2$$

$$\boldsymbol{\rho} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}\cos(\boldsymbol{\theta_1})\cos(\boldsymbol{\theta_2}) + \boldsymbol{l_1}\cos(\boldsymbol{\theta_1}) - \boldsymbol{s}\sin(\boldsymbol{\theta_1})\sin(\boldsymbol{\theta_2}) \\ \boldsymbol{s}\sin(\boldsymbol{\theta_1})\cos(\boldsymbol{\theta_2}) + \boldsymbol{l_1}\sin(\boldsymbol{\theta_1}) + \boldsymbol{s}\cos(\boldsymbol{\theta_1})\sin(\boldsymbol{\theta_2}) \\ \boldsymbol{\theta_1} + \boldsymbol{\theta_2} \end{bmatrix}$$