3. Written Questions

3.1 Piazza

Code: `RoboKin2018Piazza`

3.2 REL

Code: `RoboKin2018REL`

3.3 Matrices

3.3.1.) 2, 3, 6

3.3.2.)

1.)

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 3a + 1b + 7c \\ 3d + 1e + 7f \end{bmatrix} = \begin{bmatrix} \boldsymbol{3a + b + 7c} \\ \boldsymbol{3d + e + 7f} \end{bmatrix}$$

2.)

$$\begin{bmatrix} 9 & 3 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 9*2+3*4 & 9*1+3*3 \\ 0*2+1*4 & 0*1+1*3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{30} & \boldsymbol{18} \\ \boldsymbol{4} & \boldsymbol{3} \end{bmatrix}$$

3.)

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} * \begin{bmatrix} 9 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2*9+1*0 & 2*3+1*1 \\ 4*9+3*0 & 4*3+3*1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{18} & \boldsymbol{7} \\ \boldsymbol{36} & \boldsymbol{15} \end{bmatrix}$$

4.)

$$R = \begin{bmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{bmatrix}$$

a.)

$$\boldsymbol{R^{-1}} = \begin{bmatrix} \boldsymbol{\cos(x)} & \boldsymbol{\sin(x)} \\ \boldsymbol{-\sin(x)} & \boldsymbol{\cos(x)} \end{bmatrix}$$

b.)

$$\boldsymbol{R^{T}} = \begin{bmatrix} \boldsymbol{\cos(x)} & \boldsymbol{\sin(x)} \\ \boldsymbol{-\sin(x)} & \boldsymbol{\cos(x)} \end{bmatrix}$$

c.) Because R is an orthogonal matrix, $R^{-1} = R^{T}$.

5.)

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 15 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

a.)

$$\det(A) = \begin{vmatrix} 1 & 3 \\ 4 & 15 \end{vmatrix} = 1*15 - 4*3 = \boldsymbol{3}$$

b.)

$$rank(A) = \mathbf{2} \quad is \; \boldsymbol{full\; rank}.$$

c.)

$$\det(B) = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 1 * 8 - 4 * 2 = \mathbf{0}$$

d.)

$$\text{rank(B)} = \mathbf{1} \quad is \; \boldsymbol{not\; full\; rank}$$

e.) These matrices exemplify the fact that 2x2 matrices which are not full rank have determinants equal to zero because the second row is a scalar multiple of the first (same principle extends to higher dimensions when any rows are not linearly independent of the others).

6.)

$$\text{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \text{y} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

a.)

$$\text{x} \cdot \text{y} = 1 * 4 + 2 * 5 + 3 * 6 = \mathbf{32}$$

b.)

$$\text{x} \times \text{y} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \hat{k} = \begin{bmatrix} \mathbf{-3} \\ \mathbf{6} \\ \mathbf{-3} \end{bmatrix}$$

c.)

$$\|\text{x}\| = \sqrt{1^2 + 2^2 + 3^2} = \mathbf{3.742}$$

$$\|\text{y}\| = \sqrt{4^2 + 5^2 + 6^2} = \mathbf{8.775}$$

7.)

$$\begin{bmatrix} 1 & 1 \\ 2/3 & 1 \end{bmatrix} = \frac{1}{1 * 1 - 2/3 * 1} \begin{bmatrix} 1 & -1 \\ -2/3 & 1 \end{bmatrix} = 3 * \begin{bmatrix} 1 & -1 \\ -2/3 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{-3} \\ \mathbf{-2} & \mathbf{3} \end{bmatrix}$$

3.4 Calculus

3.4.1.)

$$\frac{d}{dx}[x * \cos(x)] = \boldsymbol{\cos(x) - x * \sin(x)}$$

3.4.2.)

$$\frac{\partial}{\partial x}[x \sin(y) + y^2 \cos(x)] = \boldsymbol{\sin(y) - y^2 \sin(x)}$$

$$\frac{\partial}{\partial y}[x\sin(y) + y^2\cos(x)] = \boldsymbol{x\cos(y) + 2y\cos(x)}$$

4. Feedback

Completed.

5. Code Questions

Completed.