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3.1.) Degrees of Freedom

1.) **2 DOF**

2.) **3 DOF**

3.) **5 DOF**

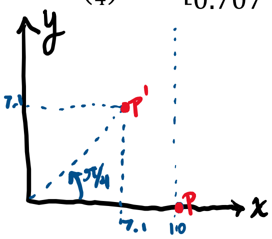
3.2.) Rotation Matrices

$$p = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

1.) $R\left(\frac{\pi}{4}\right) = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) \\ \sin(\pi/4) & \cos(\pi/4) \end{bmatrix} = \begin{bmatrix} \mathbf{0.7071} & \mathbf{-0.7071} \\ \mathbf{0.7071} & \mathbf{0.7071} \end{bmatrix}$

2.) $p' = R\left(\frac{\pi}{4}\right) p = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{7.071} \\ \mathbf{7.071} \end{bmatrix}$

3.)



3.3.) Inverting Homogeneous Transformations

$$let: \quad c = \cos(\theta), s = \sin(\theta)$$

$$H_i^j = \begin{bmatrix} R_i^j & d_i^j \\ \vec{0}^T & 1 \end{bmatrix} = \begin{bmatrix} c & -s & (\Delta x) \\ s & c & (\Delta y) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because \left|H_i^j\right| = c*(c-0) - (-s)*(s-0) + (\Delta x)*(0-0) = c^2 + s^2 = 1 \neq 0, \qquad \exists \left(H_i^j\right)^{-1} \forall\, i,j$$

$$\left(H_i^j\right)^T = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ (\Delta x) & (\Delta y) & 1 \end{bmatrix}$$

$$\left(H_i^j\right)^{-1} = \frac{1}{\left|H_i^j\right|} \begin{bmatrix} \begin{vmatrix} c & 0 \\ (\Delta y) & 1 \end{vmatrix} & -\begin{vmatrix} -s & 0 \\ (\Delta x) & 1 \end{vmatrix} & \begin{vmatrix} -s & c \\ (\Delta x) & (\Delta y) \end{vmatrix} \\ -\begin{vmatrix} s & 0 \\ (\Delta y) & 1 \end{vmatrix} & \begin{vmatrix} c & 0 \\ (\Delta x) & 1 \end{vmatrix} & -\begin{vmatrix} c & s \\ (\Delta x) & (\Delta y) \end{vmatrix} \\ \begin{vmatrix} s & 0 \\ c & 0 \end{vmatrix} & -\begin{vmatrix} c & 0 \\ -s & 0 \end{vmatrix} & \begin{vmatrix} c & s \\ -s & c \end{vmatrix} \end{bmatrix}$$

$$\rightarrow \left(H_i^j\right)^{-1} = \frac{1}{1} \begin{bmatrix} c-0 & -(-s-0) & -s(\Delta y) - c(\Delta x) \\ -(s-0) & c & -\left(c(\Delta y) - s(\Delta x)\right) \\ (0-0) & -(0-0) & c^2 - (-s^2) \end{bmatrix}$$

$$\rightarrow \left(H_i^j\right)^{-1} = \begin{bmatrix} c & s & -c(\Delta x) - s(\Delta y) \\ -s & c & s(\Delta x) - c(\Delta y) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because R_i^j = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}, \left(R_i^j\right)^T = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}, \qquad -\left(R_i^j\right)^T d_i^j = -\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} (\Delta x) \\ (\Delta y) \end{bmatrix} = \begin{bmatrix} -c(\Delta x) - s(\Delta y) \\ s(\Delta x) - c(\Delta y) \end{bmatrix}$$

$$\left(H_i^j\right)^{-1} = \begin{bmatrix} \left(R_i^j\right)^T & -\left(R_i^j\right)^T d_i^j \\ \vec{0}^T & 1 \end{bmatrix}$$

3.4.) Homogeneous Transformations

$$t = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \theta = \frac{\pi}{4}$$

1.) $T_1 = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{5} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

2.) $T_2 = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) & 0 \\ \sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0.7071} & \mathbf{-0.7071} & \mathbf{0} \\ \mathbf{0.7071} & \mathbf{0.7071} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

3.) $H_\alpha^0 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0.7071} & \mathbf{-0.7071} & \boldsymbol{\alpha_x} \\ \mathbf{0.7071} & \mathbf{0.7071} & \boldsymbol{\alpha_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

3.1.) $p^\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow p^0 \textit{ should be}: \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix}$

$$\begin{bmatrix} p^0 \\ 1 \end{bmatrix} = H_\alpha^0 \begin{bmatrix} p^\alpha \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ 1 \end{bmatrix} \rightarrow \boldsymbol{p^0} = \begin{bmatrix} \boldsymbol{\alpha_x} \\ \boldsymbol{\alpha_y} \end{bmatrix}$$

3.2.) $q^\alpha = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow q^0 \textit{ should be}: \begin{bmatrix} \alpha_x + \cos(\theta) \\ \alpha_y + \sin(\theta) \end{bmatrix} = \begin{bmatrix} \alpha_x + 0.7071 \\ \alpha_y + 0.7071 \end{bmatrix}$

$$\begin{bmatrix} q^0 \\ 1 \end{bmatrix} = H_\alpha^0 \begin{bmatrix} q^\alpha \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) + \alpha_x \\ \sin(\theta) + \alpha_y \\ 1 \end{bmatrix} \rightarrow \boldsymbol{q^0} = \begin{bmatrix} \boldsymbol{\alpha_x} + \mathbf{0.7071} \\ \boldsymbol{\alpha_y} + \mathbf{0.7071} \end{bmatrix}$$

3.3.) $v^\alpha = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow v^0 \textit{ should be}: \begin{bmatrix} \cos\left(\theta + \frac{\pi}{2}\right) \\ \sin\left(\theta + \frac{\pi}{2}\right) \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} = \begin{bmatrix} -0.7071 \\ 0.7071 \end{bmatrix}$

$$\begin{bmatrix} v_0 \\ 0 \end{bmatrix} = H_\alpha^0 \begin{bmatrix} v^\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{bmatrix} \rightarrow \boldsymbol{v^0} = \begin{bmatrix} \mathbf{-0.7071} \\ \mathbf{0.7071} \end{bmatrix}$$

3.4.) $u^\alpha = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow u^0 \textit{ should be}: \frac{\sqrt{2}}{1} \begin{bmatrix} \cos\left(\theta + \frac{\pi}{4}\right) \\ \sin\left(\theta + \frac{\pi}{4}\right) \end{bmatrix} = \sqrt{2} \begin{bmatrix} \cos(\pi/2) \\ \sin(\pi/2) \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$

$$\begin{bmatrix} u_0 \\ 0 \end{bmatrix} = H_\alpha^0 \begin{bmatrix} u^\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) - \sin(\theta) \\ \sin(\theta) + \cos(\theta) \\ 0 \end{bmatrix} \rightarrow \boldsymbol{u^0} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\sqrt{2}} \end{bmatrix}$$

4.) $H_\beta^0 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \beta_x \\ 0 & 1 & \beta_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \sin(\theta) & \cos(\theta) & \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 0 & 0 & 1 \end{bmatrix}$

$$\rightarrow \boldsymbol{H_\beta^0} = \begin{bmatrix} \mathbf{0.7071} & \mathbf{-0.7071} & \mathbf{0.7071\beta_x - 0.7071\beta_y} \\ \mathbf{0.7071} & \mathbf{0.7071} & \mathbf{0.7071\beta_x + 0.7071\beta_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

4.1.) $p^\beta = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow p^0 \textit{ should be}: \begin{bmatrix} \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \beta_x \sin(\theta) + \beta_y \cos(\theta) \end{bmatrix}$

$$\begin{bmatrix} p^0 \\ 1 \end{bmatrix} = H_\beta^0 \begin{bmatrix} p^\beta \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \sin(\theta) & \cos(\theta) & \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 1 \end{bmatrix}$$

$$\rightarrow \boldsymbol{p^0} = \begin{bmatrix} \mathbf{0.7071\beta_x - 0.7071\beta_y} \\ \mathbf{0.7071\beta_x + 0.7071\beta_y} \end{bmatrix}$$

4.2.) $q^\beta = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow q^0 \textit{ should be}: \begin{bmatrix} \beta_x \cos(\theta) - \beta_y \sin(\theta) + \cos(\theta) \\ \beta_x \sin(\theta) + \beta_y \cos(\theta) + \sin(\theta) \end{bmatrix}$

$$\begin{bmatrix} q^0 \\ 1 \end{bmatrix} = H_\beta^0 \begin{bmatrix} q^\beta \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \sin(\theta) & \cos(\theta) & \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) + \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \sin(\theta) + \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 1 \end{bmatrix}$$

$$\rightarrow \boldsymbol{q^0} = \begin{bmatrix} \mathbf{0.7071 + 0.7071\beta_x - 0.7071\beta_y} \\ \mathbf{0.7071 + 0.7071\beta_x + 0.7071\beta_y} \end{bmatrix}$$

4.3.) $v^\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow v^0 \textit{ should be}: \begin{bmatrix} \cos\left(\theta + \frac{\pi}{2}\right) \\ \sin\left(\theta + \frac{\pi}{2}\right) \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} = \begin{bmatrix} -0.7071 \\ 0.7071 \end{bmatrix}$

$$\begin{bmatrix} v^0 \\ 0 \end{bmatrix} = H_\beta^0 \begin{bmatrix} v^\beta \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \sin(\theta) & \cos(\theta) & \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{bmatrix} \rightarrow \boldsymbol{v^0} = \begin{bmatrix} \mathbf{-0.7071} \\ \mathbf{0.7071} \end{bmatrix}$$

4.4.) $u^\beta = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow u^0 \textit{ should be}: \frac{\sqrt{2}}{1} \begin{bmatrix} \cos\left(\theta + \frac{\pi}{4}\right) \\ \sin\left(\theta + \frac{\pi}{4}\right) \end{bmatrix} = \sqrt{2} \begin{bmatrix} \cos(\pi/2) \\ \sin(\pi/2) \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$

$$\begin{bmatrix} u^0 \\ 0 \end{bmatrix} = H_\beta^0 \begin{bmatrix} u^\beta \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \beta_x \cos(\theta) - \beta_y \sin(\theta) \\ \sin(\theta) & \cos(\theta) & \beta_x \sin(\theta) + \beta_y \cos(\theta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) - \sin(\theta) \\ \sin(\theta) + \cos(\theta) \\ 0 \end{bmatrix}$$

$$\rightarrow \boldsymbol{u^0} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\sqrt{2}} \end{bmatrix}$$

5.) $H_\beta^\alpha = H_0^\alpha H_\beta^0 = (H_\alpha^0)^{-1} H_\beta^0$

$$\textit{let}: c = \cos(\theta), s = \sin(\theta)$$

$$\rightarrow H_\beta^\alpha = \begin{bmatrix} c & -s & \alpha_x \\ s & c & \alpha_y \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} c & -s & \beta_x c - \beta_y s \\ s & c & \beta_x s + \beta_y c \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because (H_\alpha^0)^{-1} = \begin{bmatrix} [R_\alpha^0]^T & -[R_\alpha^0]^T d_\alpha^0 \\ \vec{0}^T & 1 \end{bmatrix}$$

$$\rightarrow H_\beta^\alpha = \begin{bmatrix} c & \mathrm{s} & -c\alpha_x - s\alpha_y \\ -s & c & s\alpha_x - c\alpha_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c & -\mathrm{s} & \beta_x \mathrm{c} - \beta_y s \\ s & c & \beta_x s + \beta_y c \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow H_\beta^\alpha = \begin{bmatrix} c^2+s^2 & 0 & -c\alpha_x - s\alpha_y + s^2\beta_x + cs\beta_y + c^2\beta_x - cs\beta_y \\ 0 & c^2+s^2 & s\alpha_x - c\alpha_y + cs\beta_x + c^2\beta_y - cs\beta_x + s^2\beta_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -c\alpha_x - s\alpha_y + \beta_x \\ 0 & 1 & s\alpha_x - c\alpha_y + \beta_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \boldsymbol{H_\beta^\alpha} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & -\boldsymbol{\cos(\theta)\,\alpha_x - \sin(\theta)\,\alpha_y + \beta_x} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{\sin(\theta)\,\alpha_x - \cos(\theta)\,\alpha_y + \beta_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \boldsymbol{-0.7071\alpha_x - 0.7071\alpha_y + \beta_x} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{0.7071\alpha_x - 0.7071\alpha_y + \beta_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

6.)

$$H_\alpha^0 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \boldsymbol{H_\alpha^0} = \begin{bmatrix} \mathbf{0.7071} & \mathbf{-0.7071} & \boldsymbol{\alpha_x} \\ \mathbf{0.7071} & \mathbf{0.7071} & \boldsymbol{\alpha_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

$$\because (H_\alpha^0)^{-1} = \begin{bmatrix} [R_\alpha^0]^T & -[R_\alpha^0]^T d_\alpha^0 \\ \vec{0}^T & 1 \end{bmatrix}$$

$$(H_\alpha^0)^{-1} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \alpha_x \\ \sin(\theta) & \cos(\theta) & \alpha_y \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & -\cos(\theta)\,\alpha_x - \sin(\theta)\,\alpha_y \\ -\sin(\theta) & \cos(\theta) & \sin(\theta)\,\alpha_x - \cos(\theta)\,\alpha_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \left(\boldsymbol{H_\alpha^0}\right)^{-\mathbf{1}} = \begin{bmatrix} \mathbf{0.7071} & \mathbf{0.7071} & \boldsymbol{-0.7071\alpha_x - 0.7071\alpha_y} \\ \mathbf{-0.7071} & \mathbf{0.7071} & \boldsymbol{0.7071\alpha_x - 0.7071\alpha_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

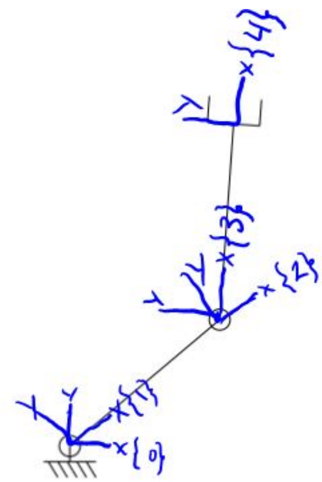
$$let: c = \cos(\theta)\,, s = \sin(\theta)$$

$$\therefore ((H_\alpha^0)^{-1} H_\alpha^0) = \begin{bmatrix} c & \mathrm{s} & -c\alpha_x - s\alpha_y \\ -s & c & s\alpha_x - c\alpha_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c & -\mathrm{s} & \alpha_x \\ s & c & \alpha_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c^2+s^2 & -\mathrm{cs}+\mathrm{cs} & c\alpha_x + s\alpha_y - c\alpha_x - s\alpha_y \\ -cs+cs & c^2+s^2 & -s\alpha_x + c\alpha_y + s\alpha_x - c\alpha_y \\ 0 & 0 & 1 \end{bmatrix}$$

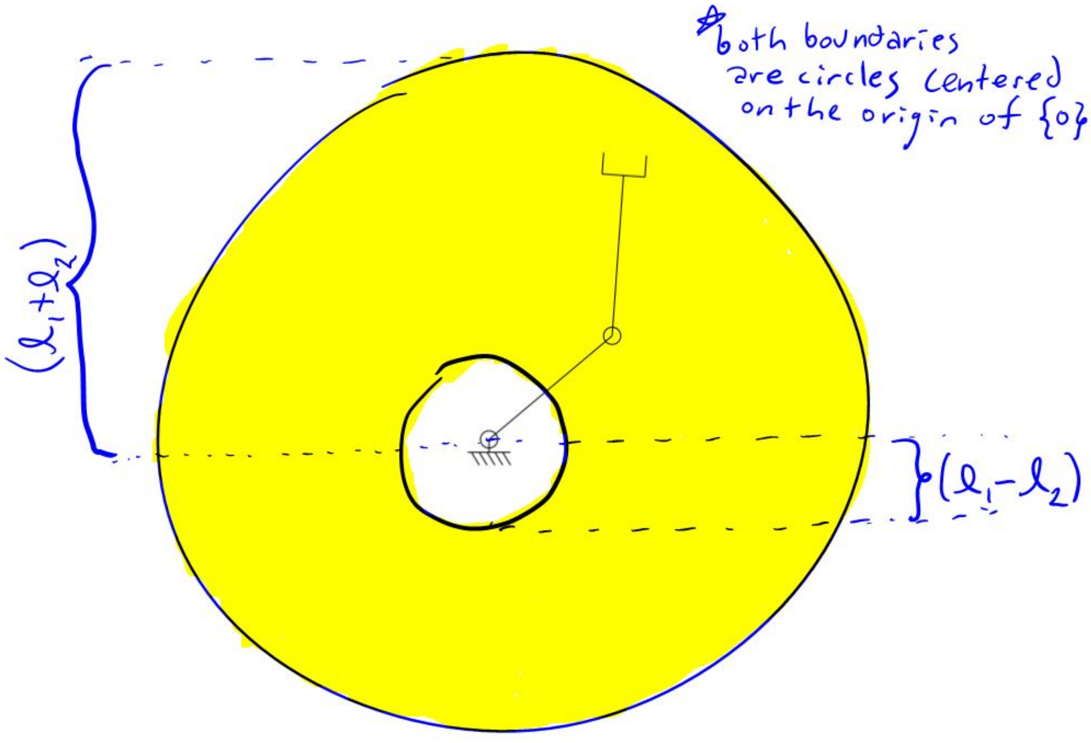
$$\rightarrow \left(\left(\boldsymbol{H_\alpha^0}\right)^{-\mathbf{1}} \boldsymbol{H_\alpha^0}\right) = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

3.5.) Workspace and Frames

1-5.)



6.)



3.6.) Forward Kinematics of an RR Arm

1.) $\boldsymbol{H_1^0} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.) $\boldsymbol{H_2^1} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \boldsymbol{\ell_1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

3.) $\boldsymbol{H_3^2} = \begin{bmatrix} \mathbf{cos}(\boldsymbol{\theta_2}) & -\mathbf{sin}(\boldsymbol{\theta_2}) & \mathbf{0} \\ \mathbf{sin}(\boldsymbol{\theta_2}) & \mathbf{cos}(\boldsymbol{\theta_2}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

4.) $\boldsymbol{H_4^3} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \boldsymbol{\ell_2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

5.) $\boldsymbol{H_4^0} = \boldsymbol{H_1^0 H_2^1 H_3^2 H_4^3}$

6.)

6.1.) $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\ell_1} + \boldsymbol{\ell_2} \\ \mathbf{0} \end{bmatrix}$

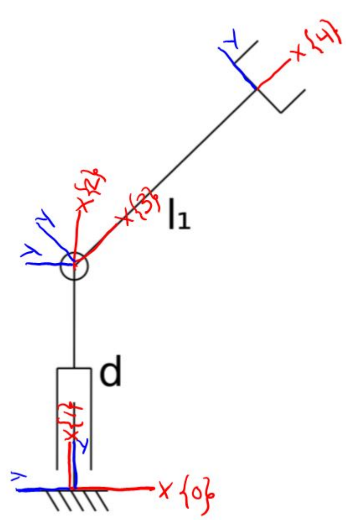
6.2.) $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\ell_1} \\ \boldsymbol{\ell_2} \end{bmatrix}$

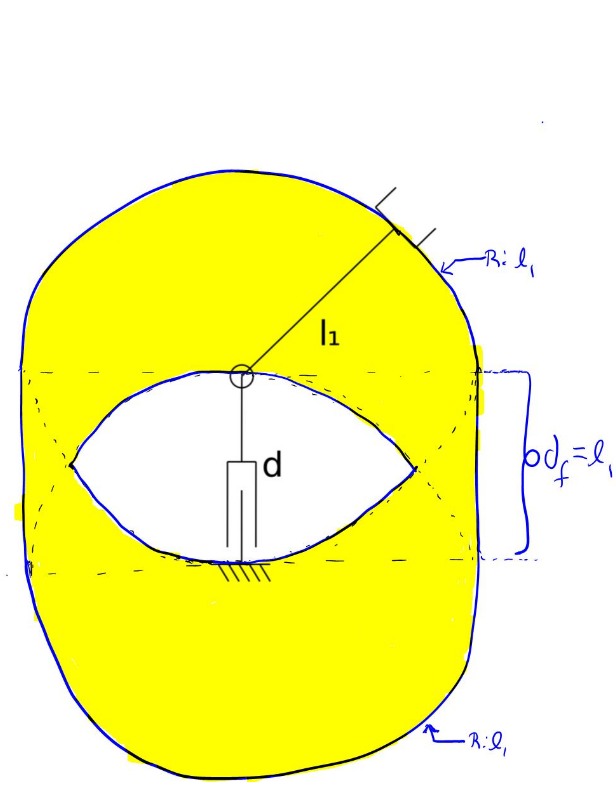
6.3.) $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\ell_2} \\ \boldsymbol{\ell_1} \end{bmatrix}$

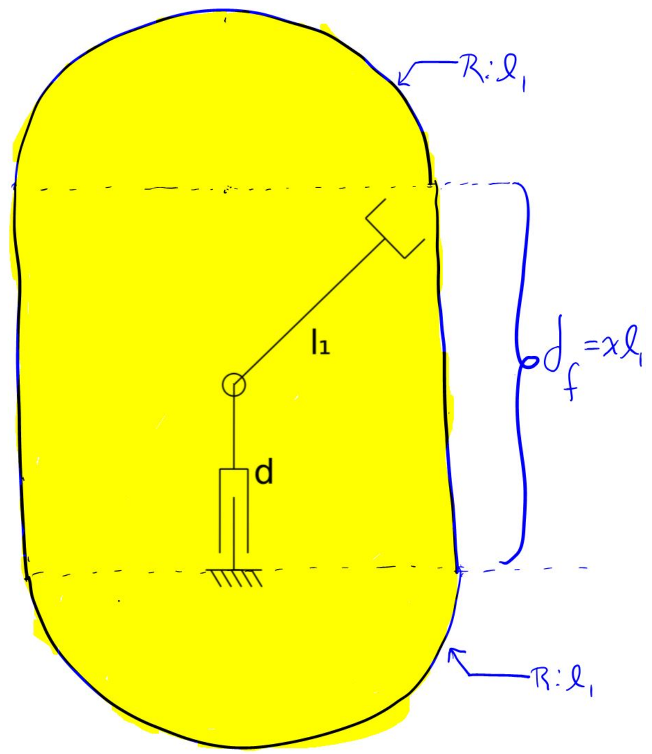
6.4.) $\because \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ell_1 \cos(\theta_1) + \ell_2 \cos(\theta_1 + \theta_2) \\ \ell_1 \sin(\theta_1) + \ell_2 \sin(\theta_1 + \theta_2) \end{bmatrix}, \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ell_1 \cos\left(\frac{\pi}{3}\right) + \ell_2 \cos\left(\frac{5\pi}{6}\right) \\ \ell_1 \sin\left(\frac{\pi}{3}\right) + \ell_2 \sin\left(\frac{5\pi}{6}\right) \end{bmatrix}$

$$\rightarrow \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \frac{\boldsymbol{\ell_1}}{\mathbf{2}} - \frac{\sqrt{\mathbf{3}}\boldsymbol{\ell_2}}{\mathbf{2}} \\ \frac{\sqrt{\mathbf{3}}\boldsymbol{\ell_1}}{\mathbf{2}} + \frac{\boldsymbol{\ell_2}}{\mathbf{2}} \end{bmatrix}$$

3.7.) Workspace and Frames of a PR Arm







$x > 2$

(here, x is depicted infinitesimally larger than 2).

3.8.) Forward Kinematics of a PR Arm

1.) $H_1^0 = \begin{bmatrix} cos\left(\frac{\pi}{2}\right) & \text{-}\sin\left(\frac{\pi}{2}\right) & 0 \\ \sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \boldsymbol{H_1^0} = \begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

2.) $\boldsymbol{H_2^1} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \boldsymbol{d} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

3.) $\boldsymbol{H_3^2} = \begin{bmatrix} \mathbf{cos}(\boldsymbol{\theta}) & -\mathbf{sin}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{sin}(\boldsymbol{\theta}) & \mathbf{cos}(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

4.) $\boldsymbol{H_4^3} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \boldsymbol{\ell_1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

5.) $\boldsymbol{H_4^0} = \boldsymbol{H_1^0 H_2^1 H_3^2 H_4^3}$

6.)

6.1.) $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\ell_1} \end{bmatrix}$

6.2.) $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\ell_1} \\ \mathbf{3} \end{bmatrix}$

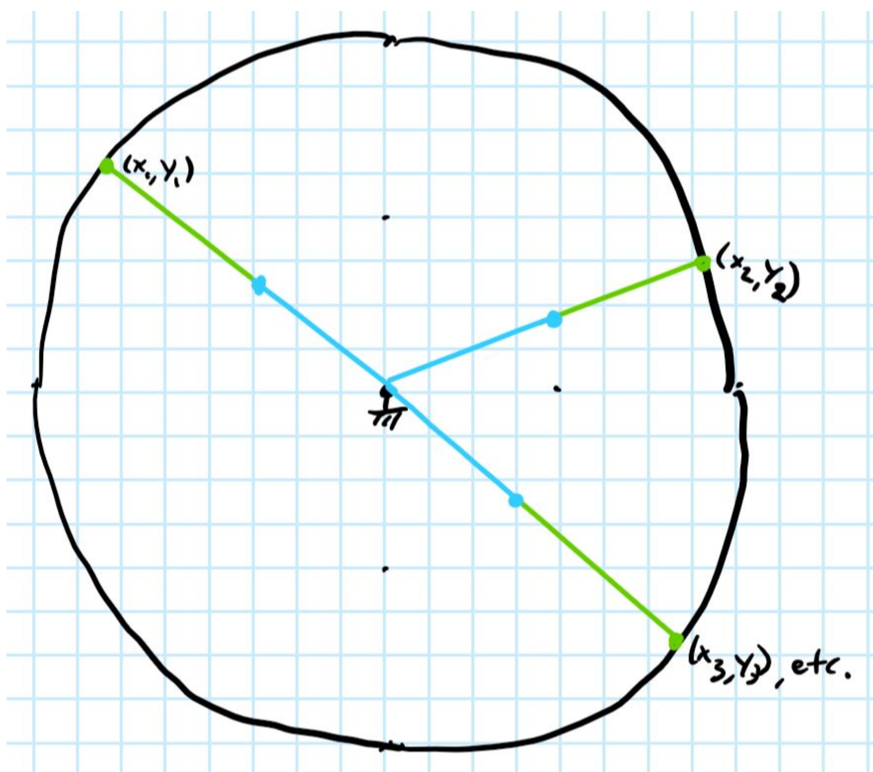
6.3.) $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\ell_1} \\ \mathbf{1} \end{bmatrix}$

6.4.) $\because \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ell_1 \cos(\theta) \\ d + \ell_1 \sin(\theta) \end{bmatrix}, \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ell_1 \cos\left(\frac{\pi}{4}\right) \\ 3 + \ell_1 \sin\left(\frac{\pi}{4}\right) \end{bmatrix}$

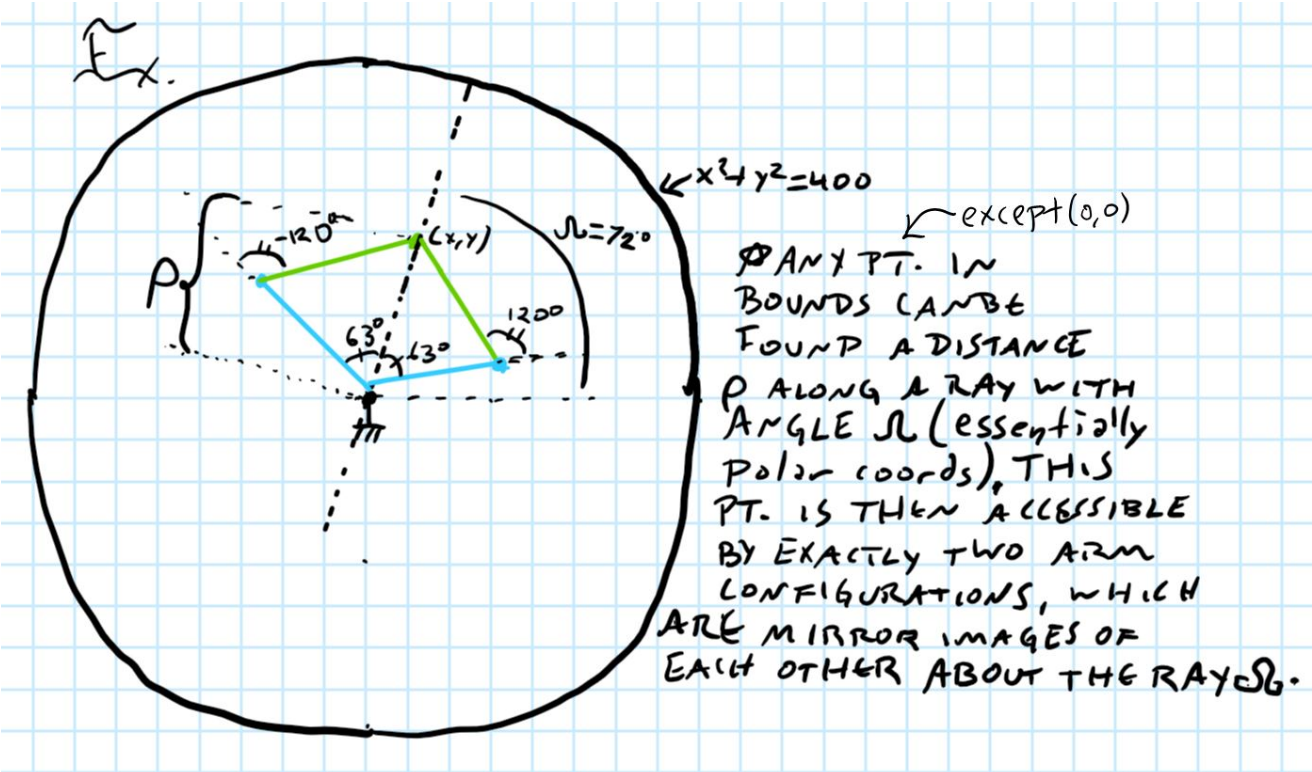
$$\rightarrow \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \dfrac{\sqrt{\mathbf{2}}\ell_1}{\mathbf{2}} \\ \mathbf{3} + \dfrac{\sqrt{\mathbf{2}}\boldsymbol{\ell_1}}{\mathbf{2}} \end{bmatrix}$$

3.9.) Singularities

1.) Any point on the circle $x^2 + y^2 = (10 + 10)^2$ (the boundary where $\theta_2 = 0$).



2.) Any point in the region $0 < x^2 + y^2 < (10+10)^2$ (any points besides (0,0) within, but not on, the outer the boundary).



3.) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, where $\theta_2 = \pm\pi$ and θ_1 can occupy any value.