cwcolomb

1.) For $\{(x_i, y_i)\}$,

LEAST SQUARES COST FUNCTION FOR QUADRATIC REGRESSION: (curve being fit)

$$J(a_0, a_1, a_2) = \sum_{i=1}^{n} [y_i - a_0 - a_1 x_i - a_2 x_i^2]^2 \quad \text{for } f(x) = a_2 x^2 + a_1 x + a_0$$

MINIMIZING COST FUNCTION:

for data:

i	x_i	y_i
1	0	0
2	1	2
3	2	3
4	3	1.5
5	4	0.5

SET UP SYSTEM OF EQUATIONS TO FIND MIN OF J:

let:

$$\begin{cases} \left[\frac{\partial J}{\partial a_0}\right] = -2\sum_{i=1}^{n} [y_i - a_0 - a_1 x_i - a_2 x_i^2] = 0 \\ \left[\frac{\partial J}{\partial a_1}\right] = -2\sum_{i=1}^{n} x_i [y_i - a_0 - a_1 x_i - a_2 x_i^2] = 0 \\ \left[\frac{\partial J}{\partial a_2}\right] = -2\sum_{i=1}^{n} x_i^2 [y_i - a_0 - a_1 x_i - a_2 x_i^2] = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \sum_{i=1}^{n} y_i = a_0 \sum_{i=1}^{n} 1 + a_1 \sum_{i=1}^{n} x_i + a_2 \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} y_i x_i = a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 + a_2 \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} y_i x_i^2 = a_0 \sum_{i=1}^{n} x_i^2 + a_1 \sum_{i=1}^{n} x_i^3 + a_2 \sum_{i=1}^{n} x_i^4 \end{cases}$$

$$\Rightarrow \begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \\ \sum y_i x_i^2 \end{bmatrix}$$

$\therefore$ Given data:

i	x_i	y_i
1	0	0
2	1	2
3	2	3
4	3	1.5
5	4	0.5

$$\rightarrow \begin{cases} n = 5, \\ \sum_{i=1}^{5} x_i = 10, \\ \sum_{i=1}^{5} x_i^2 = 30, \\ \sum_{i=1}^{5} x_i^3 = 100, \\ \sum_{i=1}^{5} x_i^4 = 354, \\ \sum_{i=1}^{5} y_i = 7, \\ \sum_{i=1}^{5} y_i x_i = 14.5 \\ \sum_{i=1}^{5} y_i x_i^2 = 35.5, \end{cases} \Rightarrow \begin{bmatrix} 5 & 10 & 30 \\ 10 & 30 & 100 \\ 30 & 100 & 354 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 14.5 \\ 35.5 \end{bmatrix}$$

Solve system of equations:

let $G = \left[\begin{array}{ccc|c} 5 & 10 & 30 & 7 \\ 10 & 30 & 100 & 14.5 \\ 30 & 100 & 354 & 35.5 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 5 & 10 & 30 & 7 \\ 10 & 30 & 100 & 14.5 \\ 30 & 100 & 354 & 35.5 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 5 & 10 & 30 & 7 \\ 0 & 10 & 40 & 0.5 \\ 30 & 100 & 354 & 35.5 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 6R_1} \left[\begin{array}{ccc|c} 5 & 10 & 30 & 7 \\ 0 & 10 & 40 & 0.5 \\ 0 & 40 & 174 & -6.5 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left[\begin{array}{ccc|c} 5 & 10 & 30 & 7 \\ 0 & 10 & 40 & 0.5 \\ 0 & 0 & 14 & -8.5 \end{array}\right]$$

$$\therefore \begin{cases} a_2 = \frac{-8.5}{14} = \frac{-17}{28}, \\ a_1 = \frac{1}{10}\left(0.5 - 40 \cdot \frac{-17}{28}\right) = \frac{347}{140}, \\ a_0 = \frac{1}{5}\left(7 - 10 \cdot \frac{347}{140} - 30\,\frac{-17}{28}\right) = \frac{3}{35} \end{cases} \Rightarrow \boxed{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -17/28 \\ 347/140 \\ 3/35 \end{bmatrix}}$$

Regressed Quadratic Curve $\therefore \boxed{f(x) = \frac{3}{35}x^2 + \frac{347}{140}x - \frac{17}{28}}$