24-311 PS-10 cucolomb

2.) Let: $I = \int_0^3 x^2 e^x \, dx$

a.) ANALYTICAL: let: $u = x^2$, $dv = e^x dx$ where $\int_0^3 u\,dv = uv\Big|_0^3 - \int_0^3 v\,du$

$\Rightarrow du = 2x\,dx$, $v = e^x$

$$\therefore \int_0^3 x^2 e^x dx = x^2 e^x \Big|_0^3 - \int_0^3 2x e^x dx = \left[x^2 e^x\right]_0^3 - \left[2xe^x\right]_0^3 + \left[2e^x\right]_0^3$$

$$= 9e^3 - 6e^3 + 2(e^3 - 1)$$

$$\Rightarrow I = \boxed{5e^3 - 2} = 98.428$$

b.) TRAPEZOID RULE:

$$\because I = \int_0^3 x^2 e^x dx, \quad I \approx (3-0) \cdot \frac{[x^2e^x]_3 + [x^2e^x]_0}{2}$$

$$\approx \frac{3}{2}(9e^3 + 0)$$

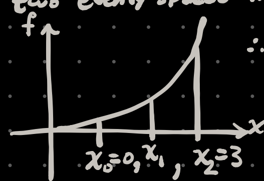
$$\Rightarrow \boxed{I \approx \frac{27}{2} e^3} = 271.15, \quad \varepsilon = \left(\frac{98.428 - 271.15}{98.428}\right) \cdot 100\%$$

$$\Rightarrow \boxed{\varepsilon = -175.5\%}$$

c.) SIMPSON'S 1/3 RULE:

$$\because I = \int_0^3 x^2 e^x dx = \int_0^3 f(x)\,dx \;\Big|\; f(x) = x^2 e^x,$$

two evenly spaced intervals



$\therefore x_1 = \frac{3+0}{2} = \frac{3}{2}$

$$I \approx \frac{(3-0)}{2} \cdot \frac{1}{3} \cdot \left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$\approx \frac{1}{2}\left(0 + 4 \cdot \frac{9}{4} e^{3/2} + 9e^3\right)$$

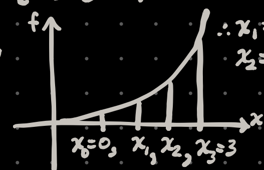
$$\Rightarrow I \approx \boxed{9e^{3/2} + 9e^3} = 221.11, \quad \varepsilon = \left(\frac{98.428 - 211.11}{98.428}\right) \cdot 100\%$$

$$\Rightarrow \boxed{\varepsilon = -124.6\%}$$

d.) SIMPSON'S 3/8 RULE:

$$\because I = \int_0^3 x^2 e^x dx = \int_0^3 f(x)\,dx \;\Big|\; f(x) = x^2 e^x,$$

two evenly spaced intervals



$\therefore x_1 = 0 + \frac{1}{3}(3-0) = 1$, $x_2 = 0 + \frac{2}{3}(3-0) = 2$

$$I \approx \frac{(3-0)}{8} \cdot \left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

$$\approx \frac{3}{8}\left(0 + 3 \cdot 1 \cdot e^1 + 3 \cdot 4 \cdot e^2 + 9e^3\right)$$

$$\Rightarrow I \approx \boxed{\frac{3}{8}\left(9e^3 + 12e^2 + 3e\right)} = 104.10, \quad \varepsilon = \left(\frac{98.428 - 104.10}{98.428}\right) \cdot 100\%$$

$$\Rightarrow \boxed{\varepsilon = -5.762\%}$$