24-311
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1.) $\begin{cases} 2x_1 - 6x_2 - 1x_3 = -38, \\ -3x_1 - 1x_2 + 7x_3 = -34, \\ -8x_1 + 1x_2 - 2x_2 = -20 \end{cases}$

a.) $A\vec{x} = \vec{b}$ | $A = \begin{bmatrix} +2 & -6 & -1 \\ -3 & -1 & +7 \\ -8 & +1 & -2 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -38 \\ -34 \\ -20 \end{bmatrix}$, $|A| = 373 \neq 0$ ✓ ← not singular or ill-conditioned

• A is <u>not singular</u> b/c there exists no $\vec{y}$ such that $A\vec{y} = \vec{0}$ and $\vec{y} \neq \vec{0}$.
↑ $A^{-1}A = I_3$ ✓, $|A| \neq 0$

b.) GAUSSIAN ELIMINATION (with partial pivoting)

$\begin{array}{c} R_1: \\ R_2: \\ R_3: \end{array} \left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ -3 & -1 & 7 & -34 \\ -8 & 1 & -2 & -20 \end{array}\right] \xrightarrow[R_3 = R_3 + 4R_1]{} \cancel{\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ -3 & -1 & 7 & -34 \\ 0 & -23 & -6 & -172 \end{array}\right]} \xrightarrow[R_2 = 3R_1 + 2R_2]{} \cancel{\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ 0 & -20 & 11 & -182 \\ 0 & -23 & -6 & -172 \end{array}\right]} \xrightarrow[R_3]{}$

get row with largest $|R_i(1)|$ to top $\xrightarrow[R_1 \text{ swap } R_3]{} \left[\begin{array}{ccc|c} -8 & 1 & -2 & -20 \\ -3 & -1 & 7 & -34 \\ 2 & -6 & -1 & -38 \end{array}\right]$

zero out L_1 of R_3 and R_2 $\xrightarrow[R_2 = 3R_1 + 8R_2, \; R_3 = R_1 + 4R_3]{} \left[\begin{array}{ccc|c} -8 & 1 & -2 & -20 \\ 0 & -11 & 62 & -212 \\ 0 & -23 & -6 & -172 \end{array}\right]$

zero out L_2 of R_3 $\xrightarrow[R_3 = -23R_2 + 11R_3]{} \left[\begin{array}{ccc|c} -8 & 1 & -2 & -20 \\ 0 & -11 & 62 & -212 \\ 0 & 0 & -1492 & 2984 \end{array}\right]$

BACKWARD SUBSTITUTION:

Solve R_3: $-1492x_3 = 2984 \Rightarrow x_3 = \frac{-2984}{1492} = -2$,

solve R_2: $-11x_2 + 62x_3 = -212 \Rightarrow x_2 = \frac{-212 - 62 \cdot -2}{-11} = 8$

solve R_1: $-8x_1 + 1x_2 - 2x_3 = -20 \Rightarrow x_1 = \frac{-20 + 2 \cdot -2 - 8}{-8} = 4$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}}$$

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c.) $A = \begin{bmatrix} 2 & -6 & 1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{bmatrix}$, let $A = LU$

Obtain U:

$$A \xrightarrow[\substack{R_2 = R_2 + f_{21}R_1, \\ R_3 = R_3 + f_{31}R_1, \\ f_{21} = 3/2, \\ f_{31} = 4}]{} \begin{bmatrix} 2 & -6 & 1 \\ 0 & -10 & \frac{11}{2} \\ 0 & -23 & -6 \end{bmatrix} \xrightarrow[\substack{R_3 = R_3 + f_{32}R_2, \\ f_{32} = \frac{-23}{10}}]{} \begin{bmatrix} 2 & -6 & 1 \\ 0 & -10 & 11/2 \\ 0 & 0 & \frac{373}{20} \end{bmatrix} = U$$

Obtain corresponding L:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -f_{21} & 1 & 0 \\ -f_{31} & -f_{32} & 1 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & \frac{23}{10} & 1 \end{bmatrix}$$

$\because \underset{AA^{-1} = I_3}{A = LU}$, $A^{-1} = [x_1 | x_2 | x_3] = X$

let: $\cancel{\begin{cases} L\hat{e}_1 = \vec{d}_1, \\ L\hat{e}_2 = d_2, \\ L\hat{e}_3 = d_3 \end{cases} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & \frac{23}{10} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \vec{d}_1 \Rightarrow}$

$\because LD = I_3$, $D = [d_1 | d_2 | d_3]$ $\begin{cases} L\vec{d}_1 = \hat{e}_1, \\ L\vec{d}_2 = \hat{e}_2, \\ L\vec{d}_3 = \hat{e}_3 \end{cases}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & \frac{23}{10} & 1 \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \vec{d}_1 = \begin{bmatrix} 1 \\ 1.5 \\ 11/20 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & \frac{23}{10} & 1 \end{bmatrix} \begin{bmatrix} d_{21} \\ d_{22} \\ d_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \vec{d}_2 = \begin{bmatrix} 0 \\ 1 \\ -23/10 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & \frac{23}{10} & 1 \end{bmatrix} \begin{bmatrix} d_{31} \\ d_{32} \\ d_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \vec{d}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\because UX = D$, $X = A^{-1}$

$$\begin{cases} U\vec{x}_1 = d_1, \\ U\vec{x}_2 = d_2, \\ U\vec{x}_3 = d_3 \end{cases} \Rightarrow \begin{bmatrix} 2 & -6 & 1 \\ 0 & -10 & 11/2 \\ 0 & 0 & 373/20 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 11/20 \end{bmatrix} \Rightarrow x_1 = \begin{bmatrix} -0.0134 \\ -0.1662 \\ -0.0295 \end{bmatrix},$$

$$U \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -23/10 \end{bmatrix} \Rightarrow x_2 = \begin{bmatrix} -0.0349 \\ -0.0322 \\ 0.1233 \end{bmatrix},$$

$$U \begin{bmatrix} x_{31} \\ x_{32} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow x_3 = \begin{bmatrix} -0.1153 \\ -0.0295 \\ -0.0536 \end{bmatrix},$$

$$\therefore \boxed{A^{-1} = \begin{bmatrix} -0.0134 & -0.0349 & -0.1153 \\ -0.1662 & -0.0322 & -0.0295 \\ -0.0295 & 0.1233 & -0.0536 \end{bmatrix}}$$

check: $A \cdot A^{-1} = \begin{bmatrix} 0.9999 & 0.0001 & 0 \\ -0.0001 & 1 & 0.0002 \\ 0 & 0.0004 & 1.0001 \end{bmatrix} \cong I_3$ ✓

just ~~rounding~~ truncation errors