1 Part D Background and Theory

a. Symbol Directory:

Experimental Quantities:		
y	Position Perpendicular to Plate	mm
x	Position Along Plate from Leading Edge	mm
x_c	Centerstream X-Position	$8.0mm$
x_d	Downstream X-Position	$12.6mm$
δx	Uncertainty in X-Position Measurements	$0.05mm$
h_s	Static Pressure Head	m
h_p	Pitot Tube Pressure-Head	m
T_{amb}	Ambient Temperature	$22.8°C$
ρ_a	Density of Ambient Air	${}^{kg}/_{m^3}$
U_∞	Measured Free-Stream Velocity	${}^{m}/_{s}$

Computed Quantities:		
$\delta\mathbf{X}$	Notation for Uncertainty in Some Measurement "**X**"	
Δh	Difference in Static and Pitot Pressure Heads	m
P_{dyn}	Dynamic Pressure	Pa
V_c	Computed Flow Velocity at Pitot Tube Opening	${}^{m}/_{s}$
Re	Reynolds Number	
δ	Boundary Layer Thickness	mm
a, b, c	Velocity Curve Regression Parameters	
SE_V	Standard Error of Velocity Regression	${}^{m}/_{s}$

Constants and Thermophysical Properties:		
g	Acceleration due to Gravity	$9.807\frac{m}{s^2}$
ρ_f	Pitot Tube Fluid (water) Density	$^*997\frac{kg}{m^3}$
ν_a	Kinematic Viscosity of Air	$^*15.36 \times 10^{-6}\frac{m^2}{s}$

**Value taken from* Principles of Heat and Mass Transfer by Incropera, et al. Textbook at Ambient Temperature

b. Basic Calculations:

Flow Velocity From the assignment

$$V_{av} = \frac{Q}{A_c} = \frac{4Q}{\pi d^2} \tag{1}$$

Reynolds Number As given in equation 8 of the assignment, the Reynolds Number can be determined by:

$$[Re] = \frac{\rho V_{av} d}{\mu} = \frac{V_{av} d}{\nu} \tag{2}$$

Head Loss By definition, head loss can be computed as:

$$\Delta h = h_1 - h_2 \tag{3}$$

c. System Calculations:

Head Loss and Flow Rate Measurement: If the head loss over a certain very short region which experiences a change in cross sectional area is known, the Bernoulli and Venturi principles can be applied to measure the volumetric flow rate through that area as outline in equation 11 of the assignment:

$$Q = C_d A_0 \sqrt{\frac{2g(\Delta h)}{1-(\frac{A_0}{A_1})^2}} = \frac{C_d \pi d_0^2}{4}\sqrt{\frac{2g(\Delta h)}{1-(\frac{d_0}{d_1})^4}} \tag{4}$$

where C_d is the discharge coefficient, Δh is the head loss, and A_0 and A_1 are the cross sectional areas of the throat of the Venturi tube or Orifice and upstream pipe respectively.

Uncertainties: Per the specific request of prompt D.2, the uncertainty in calculated flow rates, Q, due to uncertainties in the measurements of d_0 and d_1, can be determined by combining partial uncertainties of the equation for Q given in equation 11 of the assignment as follows:

$$\delta Q^2 = \left[\frac{\partial Q}{\partial d_0}\right]^2 \delta d_0^2 + \left[\frac{\partial Q}{\partial d_1}\right]^2 \delta d_1^2$$

$$\delta Q = \sqrt{\left(\frac{\pi\, C_d\, d_0 \sqrt{\frac{2(\Delta h)\, g}{1-\frac{{d_0}^4}{{d_1}^4}}}}{2} + \frac{\pi\sqrt{2}\, C_d\, (\Delta h)\, {d_0}^5\, g}{2\, {d_1}^4 \left(\frac{{d_0}^4}{{d_1}^4}-1\right)^2 \sqrt{\frac{(\Delta h)\, g}{1-\frac{{d_0}^4}{{d_1}^4}}}}\right)^2 {\delta d_0}^2 - \left(\frac{{C_d}^2\, (\Delta h)\, {d_0}^{12}\, g\, \pi^2}{2\, {d_1}^{10}\left(\frac{{d_0}^4}{{d_1}^4}-1\right)^3}\right) {\delta d_1}^2}$$

$$\rightarrow \delta Q = \frac{\pi\sqrt{2}}{2}\sqrt{-\frac{{C_d}^2\, (\Delta h)\, {d_0}^2\, {d_1}^2\, g\, \left({d_0}^{10}\, {\delta d_1}^2 + {d_1}^{10}\, {\delta d_0}^2\right)}{\left({d_0}^4 - {d_1}^4\right)^3}} \tag{5}$$

where it is given that $\delta d_0 = \delta d_1 = 0.2mm = 2\times 10^{-4}m$ for both the Orifice and Venturi tube diameter measurements.

Relative Difference: To evaluate the calculated volumetric flow rates, a relative difference was computed for each. For some arbitrary volumetric flow rate computed under a scenario X, the difference relative to the measured flow rate, Q_m, was calculated as:

$$Q_{mX} = \frac{Q_m - Q_X}{Q_X} \times 100\% \tag{6}$$