1 Dynamic Response (Chapter 3)

1.1 Problem 26

Mason's rule $\rightarrow$ system has gain: $G(s) = \frac{\frac{K}{s(s+2)}}{1+\frac{K}{s(s+2)}} = \frac{K}{s^2+2s+K}$

$$\rightarrow \omega_n = \sqrt{K}, \quad 2\zeta\omega_n = 2 \rightarrow \zeta = \sqrt{1/K}$$

Overshoot of no more than 10% in response to a unit step: $0.1 > M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$
Lookup table $\rightarrow \zeta = \sqrt{1/K} \gtrsim 0.595$

$$\boldsymbol{\rightarrow K = 2.82}$$

1.2 Problem 27

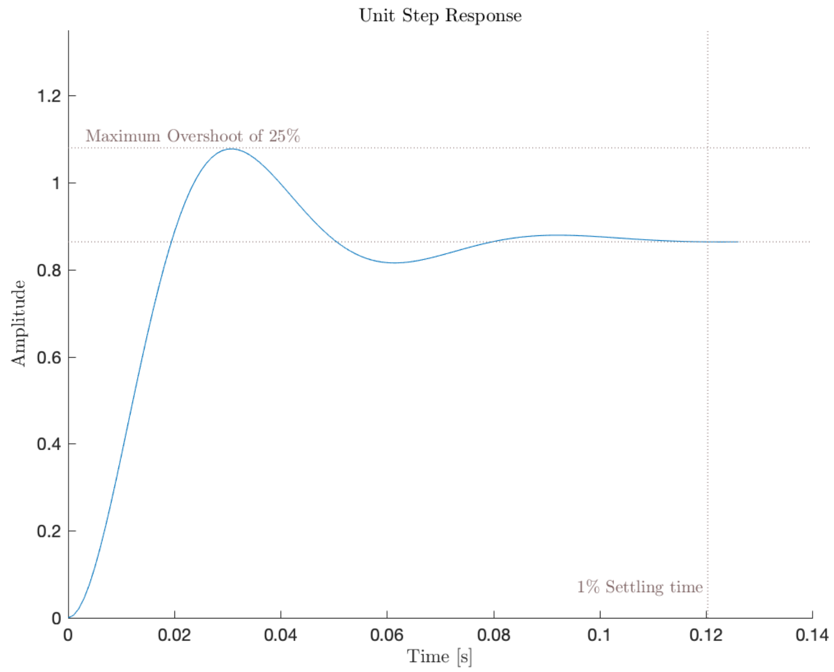
Find: $G_{compensator} = G_c = \frac{K}{s+a}$ so that $M_p < 0.25$ and $t_{s,1\%} < 0.1s$
Mason's rule $\rightarrow$ system has gain: $G(s) = \frac{\frac{100K}{(s+a)(s+25)}}{1+\frac{100K}{(s+a)(s+25)}} = \frac{100K}{(s+a)(s+25)+100K} = \frac{100K}{s^2+(25+a)s+25a+100K}$

$$\rightarrow \omega_n = \sqrt{25a+100K}, \quad 2\zeta\omega_n = (25+a) \rightarrow \zeta = \frac{a+25}{10\sqrt{4K+a}}$$

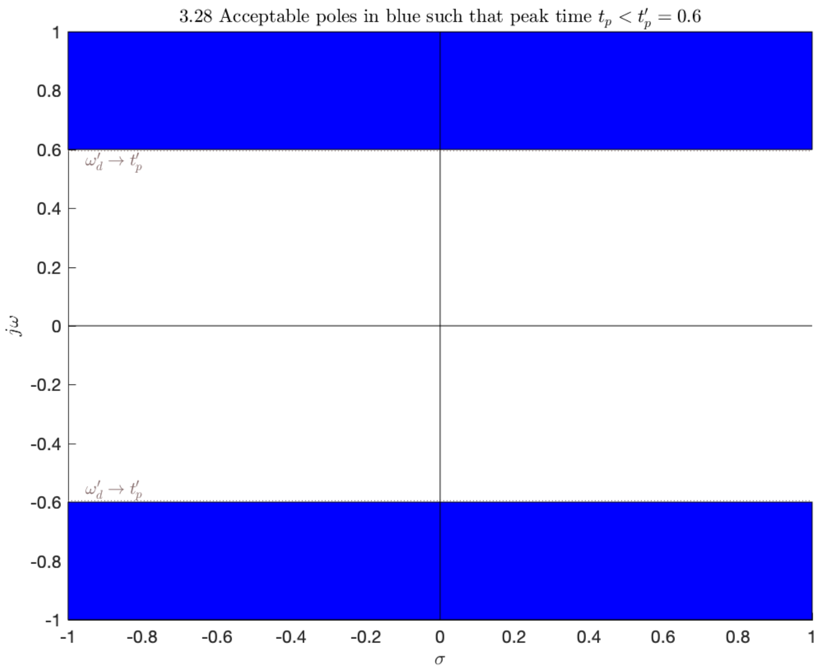
$$\because 0.1 < t_{s,1\%} = \frac{4.6}{\zeta\omega_n} \rightarrow 46 > \frac{a+25}{2} \rightarrow a = 67$$

$$\because 0.25 \geq M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Lookup table $\rightarrow \zeta = \frac{a+25}{10\sqrt{4K+a}} \gtrsim 0.41 \rightarrow K = 109$
$\boldsymbol{\therefore G_c = \frac{109}{s+67}}$ **with one pole at** $\boldsymbol{\sigma = -67}$
Matlab Verification:

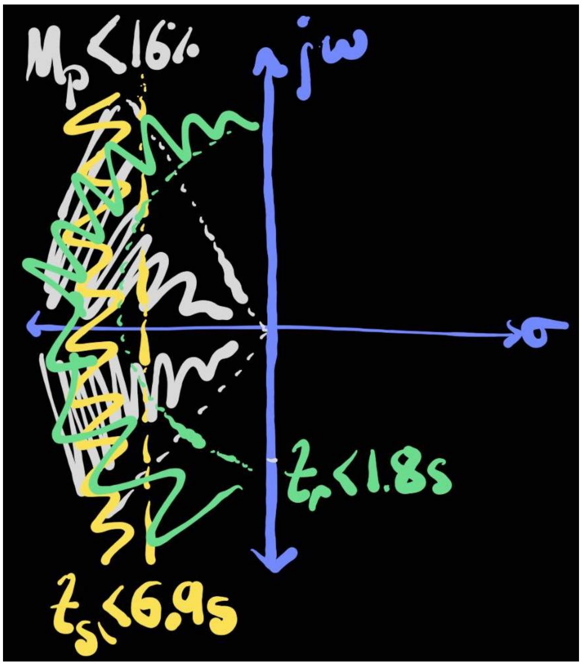


1.3 Problem 28



1.4 Problem 30

a. Sketch of pole locations:



b.

$$t_r \approx \frac{1.8}{\omega_n} = 1.8s \rightarrow \omega_n = 1\,\frac{rad}{s}$$

$$t_s = \frac{4.6}{\zeta\omega_n} = 6.9s \rightarrow \zeta = 0.667$$

$$\because M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\boldsymbol{M_p = 0.0601 = 6\%}$$

1.5 Problem 31

a. "Simple estimate" from figure:
ζ is angle from vertical axis to any point in the angular window:

$$\theta_1 < \sin^{-1}(\zeta) < \theta_2$$

$$\rightarrow\ 30^\circ < \sin^{-1}(\zeta) <\ 60^\circ$$

$$\rightarrow\ 0.5 < \zeta <\ 0.866$$

ω_n is length of line from origin to any point in the region. Therefore:

$$\sqrt{2^2 + 1.5^2} < \omega_n <\ \sqrt{4^2 + 2.75^2}$$

$$2.5 < \omega_n <\ 4.85$$

b. Find K, K_I where $K_\alpha = \alpha = 2$:

Mason's rule $\rightarrow$ system has gain: $G(s) = \frac{\left(1+\frac{K_I}{s}\right)K\left(\frac{K_\alpha}{s+\alpha}\right)}{1+\left(1+\frac{K_I}{s}\right)K\left(\frac{K_\alpha}{s+\alpha}\right)} = \frac{2Ks+2KK_I}{s^2+(2+2K)s+2KK_I}$

$$\rightarrow \omega_n = \sqrt{2KK_I}, \quad 2\zeta\omega_n = (2+2K) \rightarrow \zeta = \frac{(2K+2)}{\sqrt{8KK_I}}$$

Choose mid-range values: let $\omega_n = 3.68,\ \zeta = 0.68$

Simultaneous solve $\rightarrow$ $\boldsymbol{K = 1.50}$, $\boldsymbol{K_I = 4.51}$

1.6 Problem 33

Second order system has a homogeneous EOM:

$$\ddot{x} + \tfrac{b}{m}\dot{x} + \tfrac{k}{m}x = 0 \rightarrow \omega_n = \sqrt{\tfrac{k}{m}},\ 2\zeta\omega_n = \tfrac{b}{m} \rightarrow \zeta = \frac{b}{2\sqrt{km}}$$

From observing the system's step response to $u(t) = 2$:

$$M_p \approx \frac{1}{2}\frac{0.1125-0.1}{0.1} = 0.0625 \xrightarrow{table} \zeta \approx 0.66, \quad t_p \approx 2s, \quad t_r = t_{r10\%,90\%} \approx (1.25-0.25) = 1s$$

Solve simultaneous set of equations:

$$\begin{cases} \zeta = 0.66 = \frac{b}{2\sqrt{km}} \\ t_p = 2s = \frac{pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}} \\ t_r = 1s \approx \frac{1.8}{\omega_n} = \sqrt{\frac{81m}{25k}} \end{cases}$$

$\rightarrow$ No solution...