UNIVERSIDAD AUTÓNOMA GABRIEL RENE MORENO

FACULTAD DE INGENIERÍA EN CIENCIAS DE LA COMPUTACIÓN Y TELECOMUNICACIONES

GRUPO # 10

ECUACIONES DIFERENCIALES DE ORDEN SUPERIOR METODO: COEFICIENTE IDETERMINADO Y EULER

DOCENTE

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* ECUACIONES DIFERENCIALES

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1. $Y'' - 4y' + 3y = e^{5x}$; si $y_{(0)} = 3$; si $y'_{(0)} = 9$

- Llevar el primer término a una EDH

$\mathbf{y'' - 4y' + 3y = 0} \quad \Leftrightarrow \; D^2 - 4D + 3 = 0$

(D-1) (D-3) = 0

D-1=0 D-3=0

D = 1 D=3 $\Leftrightarrow \quad \mathbf{y_h = C_1 e^x + C_2 \boldsymbol{x} e^{3x}}$

- Calcular el segundo término de la ecuación y poder Hallar la solución Particular

$y_p = Axe^{5x} \quad \Leftrightarrow \quad y_p = A\,xe^{5x} \quad \Leftrightarrow \quad \mathbf{y'} = \boldsymbol{A}e^{5x} + \mathbf{5}Axe^{5x}$

$\mathbf{y''} = \mathbf{5}Axe^{5x} + \mathbf{25}A\,e^{5x}$

- Reemplazar valores y encontrar valores:

$\mathbf{5}Axe^{5x} + \mathbf{25}A\,e^{5x} - 4(\boldsymbol{A}e^{5x} + \mathbf{5}Axe^{5x}) + 3(Ax\,e^{5x}) = xe^{5x}$

$(\mathbf{5}\,\boldsymbol{A} + \mathbf{25} - \mathbf{4}\boldsymbol{A} - \mathbf{20}\,\boldsymbol{A} + \mathbf{3}\boldsymbol{A})\,x e^{5x} = xe^{5x} \quad \Leftrightarrow$ 8A=1 $\Leftrightarrow$ A= $\frac{1}{8}$

Y= $\mathbf{C_1 e^x + C_2 \boldsymbol{x} e^{3x} + \frac{1}{8} xe^{5x}}$

- Reemplazando $y_{(0)} = 3$ en Y

3= $\mathbf{C_1 e^0 + C_2 * 0 * e^{3*0}} + \frac{1}{8} * 0 * e^{5*0} \quad \Leftrightarrow$ 3= $\mathbf{C_1} \quad \Leftrightarrow \quad \mathbf{C_1 = 3}$

- Reemplazando $y'_{(0)} = 9$ en Y

$y' = C_1 e^x + C_2 e^{3x} + 3C_2 xe^{3x} + \frac{1}{8} e^{5x} + \frac{5}{8} xe^{5x}$

9 = $C_1 e^0 + C_2 e^{3*0} + 3C_2 * 0 * e^{3*0} + \frac{1}{8} e^{5*0} + \frac{5}{8} * 0 * e^{5*0}$

9 = $C_1 + C_2 + \frac{1}{8} \quad \Leftrightarrow \quad C_2 = 9 - C_1 - \frac{1}{8} \quad \Leftrightarrow \quad C_2 = 9 - 3 - \frac{1}{8}$

$C_2 = 9 - 3 - \frac{1}{8} \quad \Leftrightarrow \quad C_2 = \frac{47}{8} \quad \Leftrightarrow$ **Y=** $\mathbf{3e^x + \frac{47}{8} \boldsymbol{x} e^{3x} + \frac{1}{8} xe^{5x}}$

2. $y'' + 3y' - 10y = xe^{-2x}$

- Llevar el primer término a una EDH

$$\mathbf{y'' + 3y' - 10y = 0}$$

$D^2 + 3D - 10 = 0$

(D+5) (D-2) = 0

D+5 = 0 D-2=0

D=-5 D=2

$$\mathbf{y_h = C_1 e^{-5x} + C_2 e^{2x}}$$

- Calcular el segundo término de la ecuación y poder Hallar la solución Particular

$y_p = (Ax + B)e^{-2x} \iff y_p = (Ax + B)e^{-2x}\, x \iff y_p = (Ax^2 + Bx)e^{-2x}$

- Hallando las derivadas correspondientes, obtenemos:

$$\mathbf{y' = 2Axe^{-2x} + Be^{-2x} - 2Ax^2e^{-2x} - 2Bxe^{-2x}}$$

$$\mathbf{y'' = 2Ae^{-2x} - 8Axe^{-2x} - 4Be^{-2x} + 4Ax^2e^{-2x} + 4Bxe^{-2x}}$$

- Reemplazar valores y encontrar valores:

$2Ae^{-2x} - 8Axe^{-2x} - 4Be^{-2x} + 4Ax^2e^{-2x} + 4Bxe^{-2x}$+3($2Axe^{-2x} + Be^{-2x} - 2Ax^2e^{-2x} - 2Bxe^{-2x}$)- 10 ($(Ax^2 + Bx)e^{-2x}$) $= xe^{-2x}$

$2Ae^{-2x} - 8Axe^{-2x} - 4Be^{-2x} + 4Ax^2e^{-2x} + 4Bxe^{-2x}$+$6Axe^{-2x} + 3Be^{-2x} - 6Ax^2e^{-2x} - 6Bxe^{-2x}$- $(10Ax^2 + 10Bx)e^{-2x} = xe^{-2x}$

$2Ae^{-2x} - 2Axe^{-2x} - Be^{-2x} - 2Ax^2e^{-2x} - 2Bxe^{-2x} - 10Ax^2e^{-2x} - 10Bxe^{-2x} = xe^{-2x}$

$2Ae^{-2x} - 2Axe^{-2x} - Be^{-2x} - 12Ax^2e^{-2x} - 12Bxe^{-2x} = xe^{-2x}$

$(-12Ax^2 - 2Ax - 12Bx + 2A - B)\, e^{-2x} = (x^2 + x + 0)e^{-2x}$

$-12Ax^2 - 2Ax - 12Bx + 2A - B = x^2 + x + 0$

$-12Ax^2 = x^2$ $-2Ax - 12Bx + 2A - B = x + 0$

A = $-\frac{1}{12}$ $-2A - 12B + 2A - B = 1$

$-2(-\frac{1}{12}) - 12B + 2(-\frac{1}{12}) - B = 1$

-13B = 1 $\iff$ B $-\frac{1}{13}$

- Reemplazar los valores de a y b

$y_p = Ax^2e^{-2x} + Bxe^{-2x}$

$y_p = -\frac{1}{12}x^2e^{-2x} - \frac{1}{13}xe^{-2x}$

$$\mathbf{y = C_1e^{-5x} + C_2e^{2x} - \frac{1}{12}x^2e^{-2x} - \frac{1}{13}xe^{-2x}}$$

3. $6y'' - 4y' + 8y = 61e^{2x}senx$; si $y_{(0)} = y'_{(0)} = 4$

$$\mathbf{6y'' - 4y' + 8y = 0}$$

$2(3D^2 - 2D + 4) = 0 \quad \Leftrightarrow \quad 3D^2 - 2D + 4 = 0$ D = $\frac{1}{3} \pm \frac{1}{3}\sqrt{11}x$

$$\mathbf{y_h = e^{\frac{1}{3}x}(C_1\cos\left(\frac{\sqrt{11x}}{3}\right) + C_2\,Sen\left(\frac{\sqrt{11x}}{3}\right))}$$

- Calcular el segundo término de la ecuación y poder Hallar la solución Particular

$y_p = (ASen(x) + B\,Cos(x))e^{2x}$

$$\mathbf{y' = (ASen(x) + B\,Cos(x))2e^{2x} + (A\,Cos(x) - BSen(x))e^{2x}}$$

$$\mathbf{y'' = 3e^{2x}(ASen(x) + B\,Cos(x)) + 4e^{2x}(A\,Cos(x) - BSen(x))}$$

- Reemplazar valores y encontrar valores:

$6(3e^{2x}(ASen(x) + B\,Cos(x)) + 4e^{2x}(A\,Cos(x) - BSen(x))) - 4((ASen(x) + B\,Cos(x))2e^{2x} + (A\,Cos(x) - BSen(x))4e^{2x})+8((ASen(x) + B\,Cos(x))e^{2x})= 61e^{2x}senx$

$18e^{2x}(ASen(x) + B\,Cos(x)) + 24e^{2x}(A\,Cos(x) - BSen(x)) - (ASen(x) + B\,Cos(x))8e^{2x} - (A\,Cos(x) - BSen(x))4e^{2x}+8(ASen(x) + B\,Cos(x))e^{2x}= 61e^{2x}senx$

$18e^{2x}(ASen(x) + B\,Cos(x)) + 20e^{2x}(A\,Cos(x) - BSen(x))= 61e^{2x}senx$

18A+20B=61

18B-20A=0

A $\frac{549}{362}$ B=$\frac{305}{181}$

$$\mathbf{Y = e^{\frac{1}{3}x}\left(C_1\cos\left(\frac{\sqrt{11x}}{3}\right) + C_2Sen\left(\frac{\sqrt{11x}}{3}\right)\right) + \left(\frac{549}{362}Sen(x) + \frac{305}{181}Cos(x)\right)e^{2x}}$$

- Reemplazando $y_{(0)} = 4$ en Y

$$4 = e^{\frac{0}{3}}\left(C_1 \cos\left(\frac{\sqrt{11*0}}{3}\right) + C_2 \,\mathrm{Sen}\left(\frac{\sqrt{11*0}}{3}\right)\right) + \left(\frac{549}{362} Sen(0) + \frac{305}{181} Cos(0)\right) e^{2*0}$$

$$4 = (C_1 + 0) + \left(0 + \frac{305}{181}\right) \quad \Leftrightarrow \quad C_1 = 4 - \frac{305}{181} \quad \Leftrightarrow \quad C_1 = \frac{419}{181}$$

- Reemplazando $y'_{(0)} = 4$ en Y

$$y' = \frac{1}{3} e^{\frac{1}{3}x}\left(C_1 \cos\left(\frac{\sqrt{11x}}{3}\right) + C_2 \,\mathrm{Sen}\left(\frac{\sqrt{11x}}{3}\right)\right) + e^{\frac{1}{3}x}\left(-C_1 \,\mathrm{Sen}\left(\frac{\sqrt{11x}}{3}\right)\left(\frac{\sqrt{11}}{3}\right) + \mathrm{Cos}\left(\frac{\sqrt{11x}}{3}\right)\left(\frac{\sqrt{11}}{3}\right)\right) + \left(\frac{549}{362} Sen(x) + \frac{305}{181} Cos(x)\right) 2e^{2x} + \left(\frac{549}{362} Cos(x) - \frac{305}{181} Sen(x)\right) e^{2x}$$

$$4 = \frac{1}{3} e^{\frac{1}{3}*0}\left(C_1 \cos\left(\frac{\sqrt{11*0}}{3}\right) + C_2 \,\mathrm{Sen}\left(\frac{\sqrt{11*0}}{3}\right)\right) + e^{\frac{1}{3}*0}\left(-C_1 \,\mathrm{Sen}\left(\frac{\sqrt{11*0}}{3}\right)\left(\frac{\sqrt{11}}{3}\right) + C_2 \,\mathrm{Cos}\left(\frac{\sqrt{11*0}}{3}\right)\left(\frac{\sqrt{11}}{3}\right)\right) + \left(\frac{549}{362} Sen(0) + \frac{305}{181} Cos(0)\right) 2e^{2*0} + \left(\frac{549}{362} Cos(0) - \frac{305}{181} Sen(0)\right) e^{2*0}$$

$$4 = \frac{1}{3}(C_1 + 0) + \left(-0 + C_2 \frac{\sqrt{11}}{3}\right) + \left(0 + \frac{305}{181}\right) 2 + \left(\frac{549}{362} - 0\right)$$

$$4 = \frac{1}{3} C_1 + C_2 \frac{\sqrt{11}}{3} + \frac{610}{181} + \frac{549}{362} \quad \Leftrightarrow \quad 4 - \frac{1769}{362} - \frac{1}{3} C_1 = C_2 \frac{\sqrt{11}}{3} \Leftrightarrow \quad -\frac{321}{362} - \frac{1}{3}\left(\frac{419}{181}\right) = C_2 \frac{\sqrt{11}}{3}$$

$$-\frac{321}{362} - \frac{1}{3}\left(\frac{419}{181}\right) = C_2 \frac{\sqrt{11}}{3} \quad \Leftrightarrow \quad -\frac{1801}{1086} = C_2 \frac{\sqrt{11}}{3} \quad \Leftrightarrow \quad -\frac{1801*3}{1086*\sqrt{11}} = C_2$$

$$-\frac{5403}{3601} = C_2 \qquad C_2 = -\frac{5403}{3601}$$

$$\mathbf{Y = e^{\frac{1}{3}x}\left(\frac{419}{181} \cos\left(\frac{\sqrt{11x}}{3}\right) - \frac{5403}{3601} Sen\left(\frac{\sqrt{11x}}{3}\right)\right) + \left(\frac{549}{362} Sen(x) + \frac{305}{181} Cos(x)\right) e^{2x}}$$

4. $y'' + 4y' + 4y = 2x + 6$

$D^2 + 4D + 4 = 0$

$(D + 2)(D + 2) = 0$

$r = -2 \qquad r = -2$

$y_h = C_1 e^{-2x} + C_2 x e^{-2x}$

Solución particular Q(x)= $2x + 6$

$y \rightarrow y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

Reemplazamos en la ecuación principal

$4A + 4(Ax + B) = 2x + 6$

$4Ax + 4A + 4B = 2x + 6$

$4A = 2 \qquad 4B + 4A = 6$

$A = \frac{1}{2} \qquad B = 1$

Reemplazamos en y_p

$$y_p = \frac{x}{2} + 1$$

$y_g = y_h + y_p$

$y \;\; = C\ e^{-2x} + C\ xe^{-2x} \quad x$

$-$

1

2 $+ \, _2 + 1$

5. $y''' + y'' = 8x^2$

$D^3 + D^2 = 0$

$D^2(D + 1) = 0$

$r = \pm 0 \qquad r = -1$

$y_h = C_1x + C_2 + C_3e^{-x}$

6. $y'' - 2y' + y = x^3 + 4x$
$D^2 - 2D + 1 = 0$ $(D - 1)(D - 1) = 0$
$r_1 = 1 \qquad r_2 = 1$
$y_h = C_1e^x + C_2xe^x$
Solución particular $Q(x)= 3x^3 + 4x$
$y \rightarrow y_p = Ax^3 + 2Bx^2 + Cx + E$
$y'_p = 3Ax^2 + 2Bx + C$
$y''_p = 6Ax + 2B$

Reemplazamos en la ecuación principal

$6Ax + 2B - 2(3Ax^2 + 2Bx + C) + (Ax^3 + 2Bx^2$
$+ Cx + E) = x^3 + 0x^2 + 4x + 0$
$6Ax + 2B - 6Ax^2 - 4Bx - 2C + Ax^3 + 2Bx^2 +$
$Cx + E = x^3 + 0x^2 + 4x + 0$
$Ax^3 + x^2(-6A + B) + x(6A - 4B + C) + (2B -$
$2C + E) = x^3 + 0x^2 + 4x + 0$
$A = 1 -6A + B = 0 \qquad 6A - 4B + C = 4$ $2B -$
$2C + E = 0$

$B = 6C = 22 \quad E = 32$

Reemplazamos en y_p

$y_p = x^3 + 6x^2 + 22x + 32$
$y_g = y_h + y_p$

$y_g = C_1e^x + C_2xe^x + x^3 + 6x^2 + 22x + 32$

7. $y'' - 2y' - 3y = 4e^{x} - 9$

$(D2 - 2D - 3)\ y = 0;$

$(D - 3)(D + 1) = 0\ ;\ D = 3;\ D1 = -1$

$Yh = C1e^{2x} + C2e^{-x}$

$Yp = (Ay + B)\ 4ex$

$X^{2} = (Ay + B)\ 4ex;\ yp\ (Ay^{3} + Bx^{2})4ex$

$Y' = 4x\ (Ay^{2} + (B + BA)\ x + 2b)\ ex$

$Y'' = 4\ (Ax3\ (B + 6B)\ x^{2} + 4B + 4)\ x + 120$

$(4Ax^{3} + 4Bx^{2} + 24x^{2} + 16Bx + 24\ Ax + 8\ B)\ e^{3} - 2[4Ax + 4Bx^{2} + 4Ay2 + 80x]e^{x}$

$-12(Ax^{3} + By^{2})\ ex = 4ey^{-9}$

SOLUCION

$(-16Ay^{3} + 16Ax + 8B)\ ex = 4ex^{-9}$

8. $y'' + 5y' = x - 2$

$(D2 - 5\ D) = 0\ \ ;\ D2 - 5D = 0$

$D\ 8\ D + 5) = 0;\ D = 5;\ D2 = 0$

$Yh = (C1\ e^{5x} + C2\ e\ ;\ y = C_1\ e^{xy} + C_2$

$Yp = Ax + B$

$Y' = A;\ Y'' = 0$

$D - 5A = x - 2;\ -5A = 2;\ A = 0/5$

$Yp = -e\ /\ 5\ x;\ y = yh + yp$

$Y = C1\ E^{5x} + C2\ -2\ /5\ x$

Para $y\ (0) = 0$

$0 = C1\ e\ 0 + C2 - 2/5;\ C1 + C2 = 0$

Para $y'\ (0) = 2$

$2 = 5C1e - 2/5;\ 3A = 12/5$

SOLUCION

$Y = 12\ e^{5x} - 12 - 2/5x$

$Y' = 60e^{5X} - 2/5$

9. $Y'' + 5Y' - 6Y = 10e2X$

$(D2 + 5D - 6)\ Y = 0;\ D2 + 5D - 6 = 0$

$(D+6)(D-1) = 0;\ D1 = 12e5x;\ D2 = 1$

Yh = C1 e-6X + C2 ex

$$Y_P = \frac{10}{(D+6)(B-1)} e^{2x_2} \ ; \ Y_P = \frac{10}{(2+6)(2+1)} e^{2x_2}$$

$$Y_P = \frac{10e2x}{8}$$

y = yh + yp ; y = C1 + e -1x + C2 ex= $\frac{10e2x}{8}$

para y = 1

$$1 = c1^{e^0} + C_2 + \frac{10e^{2x}}{8} \ ; 1 + C_2 = -\frac{2}{4} C_1 = \frac{5}{8}$$

Para y'=1

Y' = -6xe0 + 6ex + 5e^5 /2

SOLUCION

Y = $\frac{5}{8} e - 6x + \frac{2}{7} \ ex + \frac{10ex}{8}$

Y '=$\frac{5}{2} e2x - \frac{3x}{7} - \frac{pi2x}{14}$

5. $y''' + y'' = 8x^2$

$D^3 + D^2 = 0$

$D^2(D+1) = 0$

$r = \pm 0 \qquad r = -1$

$y_h = C_1x + C_2 + C_3e^{-x}$

Solución particular Q(x)= $8x^2$

$y \to y_p = x^2(Ax^2 + Bx + c)$

$y''_p = 12Ax^2 + 6Bx + 2C$

$y'''_p = 24Ax + 6B$

Reemplazamos en la ecuación principal

$24Ax + 6B + 12Ax^2 + 6Bx + 2C = 8x^2$

$x^2 12A + x(24A + 6B) + (6B + 2C) = 8x^2 + 0x + 0$

$12A = 8 \qquad 24A + 6B = 0 \qquad 6B + 2C = 0$

$A = \frac{2}{3} \qquad B = -\frac{8}{3} \qquad C = 8$

Reemplazamos en y_p

$y_p = x^2(\frac{2}{3}x^2 + 8)$

$y_g = y_h + y_p$

$y_g = C_1x + C_2 + C_3e^{-x} + x^2(\frac{2}{3}x^2 - \frac{8}{3}x + 8)$

4. $y'' + 4y' + 4y = 2x + 6$

$D^2 + 4D + 4 = 0$

$(D+2)(D+2) = 0$

$r = -2 \qquad r = -2$

$y_h = C_1 e^{-2x} + C_2 x e^{-2x}$

Solución particular Q(x)= $2x + 6$

$y \to y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

Reemplazamos en la ecuación principal

$4A + 4(Ax + B) = 2x + 6$

$4Ax + 4A + 4B = 2x + 6$

$4A = 2 \qquad 4B + 4A = 6$

$A = \frac{1}{2} \qquad B = 1$

Reemplazamos en y_p

$$y_p = \frac{x}{2} + 1$$

$y_g = y_h + y_p$

$$y_g = C_1 e^{-2x} + C_2 x e^{-2x} + \frac{x}{2} + 1$$

6. $y'' - 2y' + y = x^3 + 4x$

$D^2 - 2D + 1 = 0$

$(D-1)(D-1) = 0$

$r_1 = 1 \qquad r_2 = 1$

$y_h = C_1 e^x + C_2 x e^x$

Solución particular Q(x)= $3x^3 + 4x$

$y \rightarrow y_p = Ax^3 + 2Bx^2 + Cx + E$

$y'_p = 3Ax^2 + 2Bx + C$

$y''_p = 6Ax + 2B$

Reemplazamos en la ecuación principal

$6Ax + 2B - 2(3Ax^2 + 2Bx + C) + (Ax^3 + 2Bx^2 + Cx + E) = x^3 + 0x^2 + 4x + 0$

$6Ax + 2B - 6Ax^2 - 4Bx - 2C + Ax^3 + 2Bx^2 + Cx + E = x^3 + 0x^2 + 4x + 0$

$Ax^3 + x^2(-6A + B) + x(6A - 4B + C) + (2B - 2C + E) = x^3 + 0x^2 + 4x + 0$

$A = 1 \qquad -6A + B = 0 \qquad 6A - 4B + C = 4 \qquad 2B - 2C + E = 0$

$B = 6 \qquad C = 22 \qquad E = 32$

Reemplazamos en y_p

$y_p = x^3 + 6x^2 + 22x + 32$

$y_g = y_h + y_p$

$y_g = C_1 e^x + C_2 x e^x + x^3 + 6x^2 + 22x + 32$

13. $(D^2 + 5)Y = \cos(\sqrt{15}\,x)$

$D2 + 5 = 0$

$D = \sqrt{5} \Rightarrow D = +5$

$Yh = c1 \cos\sqrt{5}\,x + \text{sen}\sqrt{5}$

$YP = A\cos\sqrt{15} + B\cos\sqrt{15}$

$YP = \frac{x\,sen\sqrt{5}}{2\sqrt{5}}$

$Y = yh + yp$

$Y = Yh = c1\cos\sqrt{5}\,x + \text{sen}\sqrt{5} + \frac{x\,sen\sqrt{5}}{2\sqrt{5}}$

14 . $(D^3 + D^2 + D + 1) = e^{X} + e^{-X} + \text{sen } X$

$D^3 + D^2 + D + 1 = 0$

$Yh = C1 + C2\,X + CeX$

$YP = AeX + e^{-X} + \text{senx} + Be^{X} + e^{-x}\cos X$

$Yp = \frac{1}{3}ex + e - x - \cos x$

$Y = Yh + YP$

$Y = \frac{1}{3}ex + e - x - \cos x + AeX + e^{-X} + \text{senx} + Be^{X} + e^{-x}\cos X$

15. x2 y''- 3xy' + 4y = x + x^2 + x^2 ln x

A^2 D^2 - AD + 4

D^2 –D + 4 – e^x + e ^{2X} + t

(D - 4) (D – 1) = 0

Yh= C1 e4x +C2 e-x

YP= $\frac{et_{+e^{2t}+t}}{D^2-3D+4}$; A =3

Yh = $\frac{e^{3t}+t}{3^2-9+4}$ => $\frac{e^{3t}+t}{4}$

Y = yh + yp

Y = C1 e^{4x} +C2 e^{-x} + $\frac{e^{3t}+t}{4}$

$16. x^3y''' + xy' = 3x^4$

$x^3y''' + xy' = 3x^4$ /x

$x^2y''' + y' = 3x^3$ $\qquad cv:$

$u = y'$

$u''x^2 + u = 3x^3$

$por\ ec. de\ euler: \quad x = e^v$

$de\ la\ ec. caracteristica: \lambda a_0(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1) \dots = 0$

$calculamos: \ (\lambda - 1)\lambda + 1 = 0$

$u'' - u' + u = 3e^{3v}$

$Aplicando\ el\ metodo\ de\ coeficientes\ constantes$
$Solucion\ general:$

$$\bar{u} = C_1\ e^{\frac{v}{2}}\ sen\left(\frac{\sqrt{3}v}{2}\right) + C_2 e^{\frac{v}{2}} \cos\left(\frac{\sqrt{3}v}{2}\right)$$

$Para\ la\ solucion\ particular\ usamos\ el\ metodo\ de\ coeficientes\ indeterminados$
$Solucion\ particular:$
$3e^{3v}$

$u_0 = Ae^v$

$u'_0 = 3Ae^{3v}$
$u''_0 = 9Ae^{3v}$

Sustituimos: $\quad 7A\cancel{e^{3v}} = 3\cancel{e^{3v}}$

Coeficientes: $\quad 7A = 3 \qquad A = \frac{3}{7}$

$$u_0 = \frac{3e^{3v}}{7}$$

Resolvemos sustituyendo: $\quad u = sol. general + sol. particular$

Reemplazando:

$$u = C_1 e^{\frac{v}{2}} sen\left(\frac{\sqrt{3}v}{2}\right) + C\ e^{\frac{v}{2}} \cos\left(\frac{\sqrt{3}v}{2}\right) + \frac{3e^{3v}}{7}$$

Deshacemos $v = \ln x$

$$u = C_1\sqrt{x}\ sen\left(\frac{\sqrt{3}\ln x}{2}\right) + C\sqrt{x}\ cos\left(\frac{\sqrt{3}\ln x}{2}\right) + \frac{3x^3}{7}$$

$Deshacemos\colon\ u = y' \quad y\ multiplicamos\ x7$

$$7y' = 7C_1\sqrt{x}\ sen\left(\frac{\sqrt{3}\ln x}{2}\right) + 7C\sqrt{x}\ cos\left(\frac{\sqrt{3}\ln x}{2}\right) + 3x^3$$

Agrupamos el diferencial:

$$7dy = \left(7C_1\sqrt{x}\ sen\left(\frac{\sqrt{3}\ln x}{2}\right) + 7C\sqrt{x}\ cos\left(\frac{\sqrt{3}\ln x}{2}\right) + 3x^3\right)$$

$$\int dy = \int \sqrt{x}\left(C_1\sqrt{x}\ sen\left(\frac{\sqrt{3}\ln x}{2}\right) + C\sqrt{x}\ cos\left(\frac{\sqrt{3}\ln x}{2}\right)\right) + \frac{3x^3}{7}dx$$

Resolviendo:

$$y = \frac{C_1x^{\frac{3}{2}}sen\left(\frac{\sqrt{3}\ln x}{2}\right)}{2} + \frac{C_1x^{\frac{3}{2}}sen\left(\frac{\sqrt{3}\ln x}{2}\right)}{2\sqrt{3}} - \frac{C_1x^{\frac{3}{2}}cos\left(\frac{\sqrt{3}\ln x}{2}\right)}{2\sqrt{3}} + \frac{C_1x^{\frac{3}{2}}cos\left(\frac{\sqrt{3}\ln x}{2}\right)}{2} + \frac{3x^4}{28} + C_2$$

$$y = C\left[\frac{x^{\frac{3}{2}}sen\left(\frac{\sqrt{3}\ln x}{2}\right)}{2\sqrt{3}} + \frac{x^{\frac{3}{2}}cos\left(\frac{\sqrt{3}\ln x}{2}\right)}{2}\right] + C_1x^{\frac{3}{2}}\left[\frac{x^{\frac{3}{2}}sen\left(\frac{\sqrt{3}\ln x}{2}\right)}{2} - \frac{x^{\frac{3}{2}}cos\left(\frac{\sqrt{3}\ln x}{2}\right)}{2\sqrt{3}}\right] + \frac{3x^4}{28} + C_2\ ;\ x = 0$$

$17. x^2y'' - 2xy' + 2y = ln^2x + \ln x^2$

Reducción a Homogénea (Ecuación de Cauchy Euler)

$x^2y'' - 2xy' + 2y = 0$

$y = x^r$

$x > 0$

$y' = rx^{r-1}$

$y'' = r(r-1)x^{r-2}$

Reemplazando

$x^2r(r-1)\,x^{r-2} - 2rx^{r-1} + 2x^r = 0$

$(r-2)(r-1) = 0$

$r_1 = 2$
$r_2 = 1$

Remplazando en $y = x^r$

$y_1 = x^2$
$y_2 = x$

$y_h = C_1x^2 + C_2x$

Solución particular

$y_{p=}u_1x^2 + u_2x$

$y1 = x^2$
$y2 = x$

$$f(x) = \frac{sol. no\ homogenea}{coeficiente\ de\ la\ variable\ de\ segundo\ orden}$$

$$f(x) = \frac{ln^2x + lnx^2}{x^2}$$

$$w = \begin{vmatrix} y1 & y2 \\ y1' & y2' \end{vmatrix} = \begin{vmatrix} x^2 & x \\ 2x & 1 \end{vmatrix} = (x^2 - 2x^2)$$

$w = -x^2$

Luego integrar con las formulas

$$u_1 = -\int \frac{y2f(x)}{w}dx$$

$$u_2 = \int \frac{y1f(x)}{w}dx$$

$$u_1 = -\int \frac{x\left(\frac{ln^2x + lnx^2}{x^2}\right)}{-x^2}dx$$

$$u_1 = -\frac{1}{2}x^{-2}ln^2x - \frac{3}{4}x^{-2}lnx - \frac{3}{4}x^{-2}$$

$$u_2 = \int \frac{x^2\left(\frac{ln^2x + lnx^2}{x^2}\right)}{-x^2}dx$$

$$u_2 = -x^{-1}\,(lnx)^2$$

Reemplazando

$$y_{p=}u_1x^2 + u_2x$$

$$y_p = x^2\left(-\frac{1}{2}x^{-2}ln^2x - \frac{3}{4}x^{-2}lnx - \frac{3}{4}x^{-2}\right) + x(-x^{-1}\,(lnx)^2)$$

$$y_{p=} -\frac{3}{2}ln^2x - \frac{3}{4}lnx - \frac{3}{4}$$

Solución general

$$y_g = C_1x^2 + C_2x - \frac{3}{2}ln^2x - \frac{3}{4}lnx - \frac{3}{4}$$

19. $\boldsymbol{(x+1)^2y'' + (x+1)y' - y = ln(x+1) + x - 1}$

$\mathrm{u} = \mathrm{x} + 1 \;\; \rightarrow \;\; \mathrm{x} = \mathrm{u} - 1$ (1) aplicamos ...

$u^2y'' + uy' - y = \ln(u) + u - 2$

Por la Ec. de Euler

Aplicamos la sustitucion de : $\mathrm{u} = e^v$ (2)

$Calculamos\ (\lambda - 1)\lambda + \lambda - 1 = 0$

$Elaborando: \lambda^2 - 1 = 0 \;\; \rightarrow \;\; y'' - y = e^v + v - 2$

Aplicando Ec. Lineal con coef. constantes
encontramos las raices donde:
$k \rightarrow$ *multiplicidad de la raiz*
$r \rightarrow$ *suma de la raiz*

$\mathrm{k} = 1 \quad \mathrm{r} : \; \mathrm{c} * \mathrm{e}^{\mathrm{v}}$

$\mathrm{k} = 1 \quad \mathrm{r} : \; \dfrac{c_1}{e^v}$

$Sol.General \;\; \rightarrow \;\; \bar{\mathrm{y}} = \mathrm{c} * \mathrm{e}^{\mathrm{v}} + \dfrac{\mathrm{c}_1}{\mathrm{e}^{\mathrm{v}}}$

Aplicando Coeficientes Indefinidos

$$\left.\begin{array}{l} \text{Sol. Privada en "v-2"} \quad : \mathrm{Y}_{\mathrm{o}} = 2 - \mathrm{v} \\ \text{Sol. Privada en " } \mathrm{e}^{\mathrm{v}} \text{ "} \quad : \dfrac{\mathrm{v} * \mathrm{e}^{\mathrm{v}}}{2} \end{array}\right\} \qquad \mathrm{Y} = \bar{\mathrm{Y}} + \mathrm{Y}_{\mathrm{o}} + \mathrm{Y}_1$$

$$\rightarrow \mathrm{Y} = \frac{v * e^{v}}{2} + c * e^{v} + \frac{c_1}{e^{v}} - v + 2$$

$$deshacemos : v = \ln(u) \ \rightarrow y = \frac{u\ln(u)}{2} - \ln(u) + c * u + \frac{c_1}{u} + 2$$

$$Desfactorizamos\ y\ factorizamos :$$

$$y = \frac{xln(x+1)}{2} - \frac{\ln(x+1)}{2} + \frac{c_1}{x+1} + cx + c + 2$$

$$\boxed{\therefore \boldsymbol{y = \frac{ln(x+1)(x-1)}{2} + \frac{c_1}{x+1} + c(x+1) + 2}}$$

20.- $\boldsymbol{x^2y'' + xy' - y = \frac{1}{x+1}}$

$y = x^m$

Derivamos

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

Reemplazamos en ecuación principal

$x^2m(m-1)x^{m-2} + xmx^{m-1} - x^m = 0$

$m(m-1)x^m + mx^m - x^m = 0$

$x^m[m(m-1) + m - 1] = 0$

$x^m = 0$; No se cumple

m(m-1)+m-1=0

$m^2 - m + m - 1 = 0$

$m^2 = 1$

$$\left.\begin{array}{l} r_1 = 1 \\ r_1 = 1 \end{array}\right\} \neq$$

$$\boxed{Y_{H=}C_1x + C_2x^{-1}}$$

Variacion de parámetro para hallar "Yp"

$Y'' + P(x)y' + Q(x)y = F(x)$

$$x^2y'' + xy' - y = \frac{1}{x+1}$$

$$\frac{x^2}{x^2}y'' + \frac{x}{x^2}y' - \frac{y}{x^2} = \frac{1}{x^2(x+1)}$$

$$\boxed{Y_P = N_1Y_1 + N_2Y_2}$$

$$N'_1 = \frac{W_1}{W} = -\frac{Y_2 F(x)}{W}$$

$$N'_2 = \frac{W_2}{W} = \frac{Y_1 F(x)}{W}$$

$$Y_1 = x \quad ; \quad Y_2 = x^{-1} \quad ; F(x) = \frac{1}{x^2(x+1)}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y'_3 & y'_2 \end{vmatrix} \; ; \; W_1 = \begin{vmatrix} 0 & y_2 \\ F(x) & y'_2 \end{vmatrix} \quad ; W_2 = \begin{vmatrix} y_1 & 0 \\ y'_1 & F(x+1) \end{vmatrix}$$

$$W = \begin{vmatrix} x & x^{-1} \\ 1 & -x^{-2} \end{vmatrix} = -x^{-1} - x^{-1} = -2x^{-1}$$

$$W_1 = \begin{vmatrix} 0 & x^{-1} \\ \frac{1}{x^2(x+1)} & -x^{-2} \end{vmatrix} = 0 - \frac{x^{-1}}{x^2(x+1)} = \frac{1}{x^3(x+1)}$$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & \frac{1}{x^2(x+1)} \end{vmatrix} = \frac{x}{x^2(x+1)} - 0 = \frac{1}{x(x+1)}$$

$$N_1 = \frac{1}{2} \int \frac{dx}{x^3 + x^2}$$

$$\frac{1}{x^3 + x^2} \quad ; funciones\ parciales$$

$$\frac{1}{x^3 + x^2} = \frac{A}{x} + \frac{B}{x} + \frac{C}{x+1}$$

$$1 = x(x+1)A(x+1)B + x^2 C$$

$Para \quad x = 1$

$1 = (-1)\cancel{(-1+1)}A + \cancel{(-1+1)}B + (-1^2)C$

$\boxed{1 = C}$

$Para \quad x = 0$

$1 = 0\cancel{(0+1)}A + (0+1)B + \cancel{0^2C}$

$\boxed{1 = B}$

$Para \quad x = 2$

$1 = 2(3)A + 3B + 4C$

$1 = 6A + 3 + 4$

$-6 = 6A$

$\boxed{A = -1}$

$$N_1 = \frac{1}{2}\int \frac{dx}{x^3 + x^2} = \frac{1}{2}\left[\int \left[-\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\right] dx\right]$$

$$N_1 = \frac{1}{2}\left[-\int \frac{dx}{x} + \int x^{-2}dx + \int \frac{dx}{x+1}\right]$$

$$N_1 = \frac{1}{2}[-ln - x^{-1} + \ln(x+1)]$$

$$N_2 = \frac{1}{2}\int \frac{dx}{x+1}$$

$N = X + 1$

$d_n = d_x$

$$N_2 = -\frac{1}{2}\int \frac{dv}{v}$$

$$N_2 = -\frac{1}{2}\ln\ v$$

$$N_2 = -\frac{1}{2}\ln(x+1)$$

$\boxed{Y_P = N_1Y_1 + N_2Y_2}$

$$Y_p = \left[-\frac{lnx}{2} - \frac{x^{-1}}{2} + \frac{\ln\,(x+1)}{2}\right]x + \left(-\frac{1}{2}\right)\ln(x+1)\frac{1}{x}$$

$$\boxed{Y_P = \frac{-xlnx}{2} - \frac{1}{2} + \frac{xln(x+1)}{2} - \frac{\ln\,(x+1)}{2x}}$$

$$Y_G = Y_H + Y_P$$

$$\boxed{Y_G = c_1x + c_2x^{-1} - \frac{1}{2}\left[xln(x) + 1 - xln(x+1) + \frac{\ln\,(x+1)}{x}\right]}$$