

# Algebra Foundations

## Why This Matters

Algebra is the language of *generalization*. Instead of solving "3 + 5 = ?" one at a time, algebra lets you solve "x + 5 = 8, what is x?" for *any* similar problem.

If you've ever written a function with parameters, you've done algebra:

```
function calculate(x, y) {  
  return x + y; // x and y are variables (algebra!)  
}
```

Algebra is about:

- Using **variables** as placeholders
- Writing **equations** that describe relationships
- **Solving** for unknowns systematically

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## The Big Picture: What Problem Does Algebra Solve?

**Arithmetic:** Specific calculations

```
5 + 3 = 8  
10 × 2 = 20
```

**Algebra:** General patterns and unknowns

```
x + 3 = 8 (find x)  
2y = 20 (find y)  
a + b = c (relationship between a, b, c)
```

Algebra lets you:

- Describe patterns
- Work backwards from results to causes
- Solve for unknowns
- Express general rules

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## 1. Variables: Placeholders for Values

### What They Are

A **variable** is a symbol (usually a letter) that stands for a number you don't know yet, or a number that can change.

Common variable names: x, y, z, a, b, c, n, t

**Programming Analogy:**

```
let x = 5;           // x is a variable
let y = x + 3;       // y depends on x
```

In math, variables work the same way—they're containers for values.

## Variables as Function Parameters

When you write:

```
function double(n) {
  return n * 2;
}
```

`n` is a variable. It could be 5, 10, or 100. Same in algebra:

$$f(n) = 2n$$

This means "whatever `n` is, double it."

## Constants vs Variables

- **Variable:** Can change (`x`, `y`, `n`)
- **Constant:** Fixed value (5,  $\pi$ , -2)

Example:

$$y = 2x + 3$$

- `x` and `y` are variables
- `2` and `3` are constants

## Multiple Variables

You can have more than one:

```
area = length × width
A = l × w
```

Both `l` and `w` are variables. `A` (area) depends on them.

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## 2. Expressions vs Equations

### Expression

A combination of numbers, variables, and operations **without an equals sign**.

Examples:

```
3x + 5
2a - 7
x2 + 2x + 1
```

Think of expressions as **incomplete statements** or **formulas**. They evaluate to a value but don't claim anything.

#### Programming Analogy:

```
const expression = x * 2 + 5; // just a value, not a comparison
```

### Equation

Two expressions set equal to each other **with an equals sign**.

Examples:

```
3x + 5 = 14
2a - 7 = 3
y = x2 + 2x + 1
```

Equations make a **claim**: "These two things are equal."

#### Programming Analogy:

```
const equation = (3*x + 5 === 14); // true or false
```

### Key Difference

- **Expression**: "What is  $3x + 5$  when  $x = 2$ ?" → Evaluate it
- **Equation**: "When does  $3x + 5 = 14$ ?" → Solve it

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## 3. Evaluating Expressions

### What It Means

**Substitute** a value for the variable and **compute** the result.

Example:

```
Expression: 3x + 5
If x = 2, then:
3(2) + 5 = 6 + 5 = 11
```

### Step by Step

```
Evaluate: 2x2 - 3x + 1 when x = 4
```

```
Step 1: Substitute x = 4
2(4)2 - 3(4) + 1
```

```
Step 2: Exponents first
2(16) - 3(4) + 1
```

```
Step 3: Multiply
```

$32 - 12 + 1$

Step 4: Add/subtract left to right

$20 + 1 = 21$

Answer: 21

#### Programming Analogy:

```
function evaluate(x) {  
  return 2*x**2 - 3*x + 1;  
}  
  
console.log(evaluate(4)); // 21
```

## 4. Order of Operations (Why It Exists)

### The Problem

What does  $2 + 3 \times 4$  equal?

- If you go left to right:  $(2 + 3) \times 4 = 20$  ❌
- If you do multiplication first:  $2 + (3 \times 4) = 14$  ✓

**We need rules so everyone gets the same answer.**

### PEMDAS (or BODMAS)

The order you must follow:

1. **P**arentheses (or **B**rackets)
2. **E**xponents (or **O**rders)
3. **M**ultiplication and **D**ivision (left to right)
4. **A**ddition and **S**ubtraction (left to right)

### Why This Order?

It's a convention, but it makes sense:

- **Parentheses:** Explicit grouping (you decide)
- **Exponents:** Repeated multiplication (compact)
- **Multiplication/Division:** More "binding" than addition
- **Addition/Subtraction:** Least binding

### Examples

#### Example 1

$2 + 3 \times 4$

Step 1: Multiply first

$2 + 12$

Step 2: Add  
14

### Example 2

$(2 + 3) \times 4$

Step 1: Parentheses first  
 $5 \times 4$

Step 2: Multiply  
20

### Example 3

$10 - 2 \times 3 + 4$

Step 1: Multiply  
 $10 - 6 + 4$

Step 2: Left to right  
 $4 + 4 = 8$

### Example 4

$2 + 3^2 \times (4 - 1)$

Step 1: Parentheses  
 $2 + 3^2 \times 3$

Step 2: Exponent  
 $2 + 9 \times 3$

Step 3: Multiply  
 $2 + 27$

Step 4: Add  
29

### Programming Note

```
console.log(2 + 3 * 4);           // 14 (same rules!)
console.log((2 + 3) * 4);         // 20 (parentheses change it)
```

Programming languages follow the same order of operations.

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## 5. Solving Equations: Undoing Operations

### What Does "Solve" Mean?

To solve an equation means to find the value(s) of the variable that make the equation true.

Example:

$$x + 5 = 12$$

What value of  $x$  makes this true?

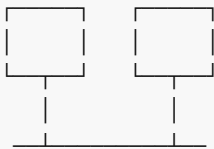
$$x = 7 \quad (\text{because } 7 + 5 = 12)$$

## The Golden Rule: Do the Same Thing to Both Sides

Whatever you do to one side, do to the other.

Equations are like balanced scales:

$$x + 5 = 12$$



If you add/subtract/multiply/divide one side, do it to the other to keep the balance.

## Strategy: Undo Operations in Reverse Order

Think of solving equations as **reversing a series of operations**.

If the equation does:  $x \rightarrow \text{add } 5 \rightarrow \text{multiply by } 2$  Then to solve:  $\text{divide by } 2 \leftarrow \text{subtract } 5 \leftarrow x$

**Programming Analogy:**

```
// Building the equation
let result = (x + 5) * 2;

// Solving (undo in reverse)
// result / 2 = x + 5
// (result / 2) - 5 = x
let x = (result / 2) - 5;
```

## 6. Solving Linear Equations

### Type 1: Addition/Subtraction

$$x + 5 = 12$$

Goal: Isolate  $x$  (get  $x$  alone)

Step 1: Subtract 5 from both sides

$$x + 5 - 5 = 12 - 5$$

$$x = 7$$

Check:  $7 + 5 = 12$  ✓

## Type 2: Multiplication/Division

$$3x = 15$$

Goal: Isolate  $x$

Step 1: Divide both sides by 3

$$3x / 3 = 15 / 3$$

$$x = 5$$

Check:  $3(5) = 15$  ✓

## Type 3: Multiple Steps

$$2x + 7 = 15$$

Step 1: Subtract 7 from both sides

$$2x = 8$$

Step 2: Divide both sides by 2

$$x = 4$$

Check:  $2(4) + 7 = 8 + 7 = 15$  ✓

## Type 4: Variables on Both Sides

$$5x + 3 = 2x + 12$$

Step 1: Subtract  $2x$  from both sides

$$3x + 3 = 12$$

Step 2: Subtract 3 from both sides

$$3x = 9$$

Step 3: Divide by 3

$$x = 3$$

Check:  $5(3) + 3 = 15 + 3 = 18$

$$2(3) + 12 = 6 + 12 = 18$$
 ✓

## Type 5: Fractions

$$x/4 = 3$$

Step 1: Multiply both sides by 4

$$x = 12$$

Check:  $12/4 = 3$  ✓

More complex:

$$(x + 2)/3 = 5$$

Step 1: Multiply both sides by 3

$$x + 2 = 15$$

Step 2: Subtract 2

$$x = 13$$

Check:  $(13 + 2)/3 = 15/3 = 5$  ✓

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## 7. Working with Negative Coefficients

### Example 1

$$-x = 5$$

Step 1: Multiply both sides by -1

$$x = -5$$

Check:  $-(-5) = 5$  ✓

### Example 2

$$-3x + 4 = 10$$

Step 1: Subtract 4

$$-3x = 6$$

Step 2: Divide by -3

$$x = -2$$

Check:  $-3(-2) + 4 = 6 + 4 = 10$  ✓

**Remember:** Dividing or multiplying by a negative flips the sign.

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## 8. Distributing (The Distributive Property)

### What It Means

$$a(b + c) = ab + ac$$

You **distribute** the multiplication over the addition.

### Visual



$3(x + 2)$  means "3 groups of  $(x + 2)$ "

Group 1:  $x + 2$

Group 2:  $x + 2$

Group 3:  $x + 2$

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Total:  $3x + 6$

## Examples

$$5(x + 3) = 5x + 15$$

$$2(3x - 4) = 6x - 8$$

$$-3(2x + 1) = -6x - 3$$

## Why It's Useful

It lets you simplify expressions and solve equations:

$$3(x + 2) = 21$$

Step 1: Distribute

$$3x + 6 = 21$$

Step 2: Subtract 6

$$3x = 15$$

Step 3: Divide by 3

$$x = 5$$

$$\text{Check: } 3(5 + 2) = 3(7) = 21 \checkmark$$

## Programming Analogy

```
// Without distributing
const result = 3 * (x + 2);

// Distributed (equivalent)
const result = 3*x + 6;
```

# 9. Combining Like Terms

## What Are Like Terms?

Terms with the **same variable and exponent**.

**Like terms:**

$3x$  and  $5x$  (both have  $x$ )

$2y^2$  and  $-7y^2$  (both have  $y^2$ )

**NOT like terms:**

3x and 5y      (different variables)

2x and 2x<sup>2</sup>    (different exponents)

**How to Combine**

Add or subtract the **coefficients** (numbers in front):

3x + 5x = 8x

7y - 2y = 5y

4x<sup>2</sup> + 3x<sup>2</sup> = 7x<sup>2</sup>

**Example**

Simplify: 3x + 5 + 2x - 3

Step 1: Group like terms  
(3x + 2x) + (5 - 3)

Step 2: Combine  
5x + 2

**Why It Matters**

Combining like terms simplifies equations:

5x + 3x - 2 = 14

Step 1: Combine like terms  
8x - 2 = 14

Step 2: Add 2  
8x = 16

Step 3: Divide by 8  
x = 2

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**10. Inequalities: Ranges, Not Points**

**What They Are**

Inequalities use symbols like < , > , ≤ , ≥ instead of = .

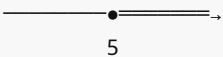
Symbol	Meaning
<	Less than
>	Greater than
≤	Less than or equal to

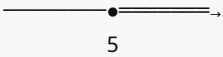
$\geq$	Greater than or equal to
--------	--------------------------

## Examples

$x > 5$       ( $x$  is greater than 5)  
 $y \leq 10$      ( $y$  is less than or equal to 10)  
 $-3 < z$       ( $z$  is greater than -3)

## Visual: Number Line

$x > 5$ :  
  
 (5 not included, everything to the right)

$x \geq 5$ :  
  
 (5 IS included, everything to the right)

- **Open circle (o):** Not included ( $<$ ,  $>$ )
- **Filled circle (●):** Included ( $\leq$ ,  $\geq$ )

## Solving Inequalities

Works just like equations... with ONE exception.

### Normal Operations

$$x + 3 > 7$$

Subtract 3:  
 $x > 4$

### THE EXCEPTION: Multiplying/Dividing by Negatives

When you multiply or divide by a negative number, **FLIP** the inequality sign.

$$-2x > 6$$

Divide by -2 (and flip the sign):  
 $x < -3$

**Why?** Because multiplying by a negative reverses order:

$5 > 3$  (true)  
 Multiply both by -1:  
 $-5 < -3$  (still true, but flipped)

## Example with Multiple Steps

$$-3x + 4 \leq 10$$

Step 1: Subtract 4

$$-3x \leq 6$$

Step 2: Divide by -3 (flip sign!)

$$x \geq -2$$

## Compound Inequalities

You can have two inequalities at once:

$$1 < x < 5$$

This means:  $x$  is between 1 and 5

$$(x > 1 \text{ AND } x < 5)$$

Visual:



## Programming Analogy

```
if (x > 5) {  
  console.log("x is greater than 5");  
}  
  
if (x >= 5 && x <= 10) {  
  console.log("x is between 5 and 10 (inclusive)");  
}
```

## Common Mistakes & Misconceptions

### ✗ "Variables are always $x$ "

Variables can be any letter:  $y$ ,  $z$ ,  $a$ ,  $n$ ,  $t$ ,  $\theta$ . Choose meaningful names like `time`, `distance`.

### ✗ " $3x$ means $3 + x$ "

**No.**  $3x$  means  $3 \times x$ . If you see a number next to a variable, it's multiplication.

### ✗ "You can't have negative solutions"

Negative solutions are totally valid:  $x = -5$  is a perfectly good answer.

### ✗ "Both sides of an equation must look the same"

No. They must *equal* the same value, but can look different:

$$2x + 3 = 11 \quad (\text{left and right look different but equal 11 when } x=4)$$

## ✗ "Dividing both sides by the variable"

Be careful:

```
2x = 3x → Don't divide by x!
```

Instead, subtract 3x:

```
-x = 0
```

```
x = 0
```

Dividing by x assumes  $x \neq 0$ , which might not be true.

## ✗ "Forgetting to flip inequality when multiplying/dividing by negative"

```
-2x > 6 → x < -3 (not x > -3)
```

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## Real-World Examples

### Shopping (Linear Equations)

You have \$20. Apples cost \$2 each. How many can you buy?

```
2x = 20
```

```
x = 10 apples
```

### Speed and Distance

```
Distance = Speed × Time
```

```
d = st
```

If you travel at 60 mph for 2.5 hours:

```
d = 60 × 2.5 = 150 miles
```

### Temperature Conversion

Fahrenheit to Celsius:

```
C = (F - 32) × 5/9
```

If F = 68:

```
C = (68 - 32) × 5/9 = 36 × 5/9 = 20°C
```

### Programming: Loop Conditions

```
for (let i = 0; i < 10; i++) { // i < 10 is an inequality
  console.log(i);
}
```

Budget Constraints

You want to spend no more than \$100:  
 $\text{cost} \leq 100$

Tiny Practice

Solve these equations:

- 1.  $x + 7 = 15$
- 2.  $3x = 27$
- 3.  $2x - 5 = 13$
- 4.  $5x + 3 = 2x + 12$
- 5.  $-4x = 20$
- 6.  $3(x + 2) = 18$

Solve these inequalities:

- 7.  $x + 5 > 12$
- 8.  $-2x \leq 10$
- 9.  $3x - 1 < 8$

Simplify:

- 10.  $5x + 3x - 2$
- 11.  $2(3x + 4) - 5$

► Answers

Summary Cheat Sheet

Key Concepts

Concept	Definition	Example
Variable	Placeholder for a value	$x, y, z$
Expression	Numbers, variables, operations (no $=$ )	$3x + 5$
Equation	Two expressions set equal	$3x + 5 = 14$
Solving	Finding values that make equation true	$x = 3$

Order of Operations: PEMDAS

- 1. **P**arentheses
- 2. **E**xponents
- 3. **M**ultiply/**D**ivide (left to right)
- 4. **A**dd/**S**ubtract (left to right)

Solving Equations

- 1. **Simplify** both sides (distribute, combine like terms)
- 2. **Isolate** the variable (undo operations in reverse)

3. **Do the same** to both sides
4. **Check** your answer

### Special Rules

- **Distributive Property:**  $a(b + c) = ab + ac$
- **Combining Like Terms:**  $3x + 5x = 8x$
- **Inequality Flip:** When multiplying/dividing by negative, flip the sign

### Programming Connections

```
// Variables
let x = 5;

// Expressions
let result = 2*x + 3;

// Equations (checking)
if (2*x + 3 === 13) { /* true when x=5 */ }

// Inequalities
if (x > 3) { /* condition */ }
```

---

### Next Steps

You now understand how to work with variables, expressions, and equations. You can solve for unknowns and express general relationships.

Next, we'll explore **Ratios, Proportions, and Percentages**—how to compare quantities and scale values.

**Continue to:** [02-ratios-proportions-percentages.md](#)