

Ratios, Proportions & Percentages

Why This Matters

Ratios, proportions, and percentages are how we **compare** and **scale** quantities. They answer questions like:

- "How does this compare to that?"
- "If I double this, what happens to that?"
- "What's 15% of 200?"
- "Is this a good deal?"

As a developer, you encounter these constantly:

- API rate limits (requests per second)
- Image aspect ratios (16:9)
- CPU usage (75%)
- Progress bars (60% complete)
- Scaling layouts (responsive design)

Let's build intuition from scratch.

The Big Picture: Comparison and Scaling

Ratios: Compare two quantities

Coffee to water: 1:16
(1 part coffee, 16 parts water)

Proportions: Maintain ratios when scaling

If 1:16 works for 1 cup, then 2:32 works for 2 cups

Percentages: Special ratio comparing to 100

$75\% = 75 \text{ out of } 100 = 3 \text{ out of } 4$

All three concepts are deeply related.

1. Ratios: Comparing Quantities

What They Are

A **ratio** compares two quantities by division.

Notation: $a : b$ or a/b or "a to b"

Examples:

Pizza slices: 3 pepperoni : 5 cheese
Speed: 60 miles : 1 hour (60 mph)
Recipe: 2 eggs : 1 cup flour

Mental Model: Parts of a Whole

If you have a ratio of 3:5 (red:blue):

```
Red: ■■■  
Blue: ■■■■■  
Total: 8 parts (3 red + 5 blue)
```

Interpreting Ratios

3:5 means:

- For every 3 reds, there are 5 blues
- Red is 3/8 of the total
- Blue is 5/8 of the total
- The relationship is 3/5 or 0.6 (reds per blue)

Programming Analogy: Aspect Ratios

```
const aspectRatio = {  
  width: 16,  
  height: 9  
};  
  
// Maintain ratio when scaling  
function scale(aspectRatio, newWidth) {  
  const ratio = aspectRatio.width / aspectRatio.height;  
  return {  
    width: newWidth,  
    height: newWidth / ratio  
  };  
}  
  
scale(aspectRatio, 1920); // { width: 1920, height: 1080 }
```

Simplifying Ratios

Just like fractions, ratios can be simplified:

```
6:8 → 3:4 (divide both by 2)  
10:15 → 2:3 (divide both by 5)
```

Method: Find the GCD (greatest common divisor) and divide both sides.

Ratios with More Than Two Parts

```
Red:Green:Blue = 2:3:5
```

```
Total parts: 2 + 3 + 5 = 10
```

```
If you have 100 items:  
Red: 20 (2/10 of 100)
```

Green: 30 (3/10 of 100)

Blue: 50 (5/10 of 100)

Unit Ratios

A **unit ratio** has one side equal to 1:

$$3:5 \rightarrow 1:1.67 \text{ (divide by 3)} \\ \text{or } 0.6:1 \text{ (divide by 5)}$$

This is useful for "per unit" comparisons:

- Miles per gallon (mpg)
- Requests per second
- Price per item

2. Proportions: Keeping Ratios Constant

What They Are

A **proportion** is an equation stating that two ratios are equal.

$$\frac{a}{b} = \frac{c}{d}$$

Read as: "a is to b as c is to d"

Example: Recipe Scaling

Original recipe (serves 4):

$$2 \text{ eggs : 4 servings}$$

How many eggs for 10 servings?

$$\frac{2 \text{ eggs}}{4 \text{ servings}} = \frac{x \text{ eggs}}{10 \text{ servings}}$$

Cross-multiply to solve:

$$2 \times 10 = 4 \times x \\ 20 = 4x \\ x = 5 \text{ eggs}$$

Cross-Multiplication (Why It Works)

$$\frac{a}{b} = \frac{c}{d}$$

Multiply both sides by bd:

$$a \times d = b \times c$$

This gives you a simple equation to solve.

Visual: Proportion as Scaling

Original:

■■ (eggs)

■■■■ (servings)

Scaled:

■■■■■ (eggs)

■■■■■■■■ (servings)

The ratio stays the same: 1 egg per 2 servings

Example: Map Scale

Map scale: 1 inch = 50 miles

If two cities are 3.5 inches apart on the map, what's the real distance?

$$\frac{1 \text{ inch}}{50 \text{ miles}} = \frac{3.5 \text{ inches}}{x \text{ miles}}$$

$$1 \times x = 50 \times 3.5$$

$$x = 175 \text{ miles}$$

Programming Analogy: Responsive Scaling

```
// Original dimensions
const original = { width: 800, height: 600 };

// Scale to new width, maintain proportion
function scaleProportionally(original, newWidth) {
  const ratio = original.height / original.width;
  return {
    width: newWidth,
    height: newWidth * ratio
  };
}

scaleProportionally(original, 1200); // { width: 1200, height: 900 }
```

3. Percentages: Ratios Out of 100

What They Are

Percent means "per hundred" (Latin: *per centum*).

$$75\% = 75/100 = 0.75$$

Percentages are just a convenient way to express fractions and ratios.

Mental Model: Parts per 100

Think of percentages as **standardized ratios** where the whole is always 100:

75% means:
■■■■■■■■■■ (10 rows)
■■■■■■■■■■
■■■■■■■■■■
■■■■■■■■■■
■■■■■■■■■■
■■■■■■■■■■
■■■■■■■■■■
■■■■■■○○ ← 75 filled out of 100
■■■■■○○○○○
○○○○○○○○○○

Converting Between Forms

Percent	Decimal	Fraction
50%	0.50	1/2
25%	0.25	1/4
75%	0.75	3/4
10%	0.10	1/10
100%	1.00	1/1
150%	1.50	3/2

Percent to Decimal: Divide by 100

$$75\% = 75/100 = 0.75$$

Decimal to Percent: Multiply by 100

$$0.75 = 0.75 \times 100\% = 75\%$$

Percent to Fraction: Put over 100 and simplify

$$75\% = 75/100 = 3/4$$

Three Types of Percentage Problems

Type 1: Find the Percentage of a Number

"What is 20% of 150?"

$$20\% \times 150 = 0.20 \times 150 = 30$$

Method:

1. Convert percent to decimal
2. Multiply

Programming:

```
const percent = 20;
const number = 150;
const result = (percent / 100) * number; // 30
```

Type 2: Find What Percent One Number Is of Another

"30 is what percent of 150?"

$$30/150 = 0.2 = 20\%$$

Method:

1. Divide the part by the whole
2. Convert to percent (multiply by 100)

Programming:

```
const part = 30;
const whole = 150;
const percent = (part / whole) * 100; // 20
```

Type 3: Find the Whole When Given a Percentage

"30 is 20% of what number?"

$$\begin{aligned} 30 &= 0.20 \times x \\ x &= 30 / 0.20 = 150 \end{aligned}$$

Method:

1. Set up equation: part = percent × whole
2. Solve for whole: whole = part / percent

Programming:

```
const part = 30;
const percent = 20;
const whole = part / (percent / 100); // 150
```

4. Percentage Increase and Decrease

Percentage Increase

Formula:

$$\text{Percentage Increase} = (\text{New} - \text{Old}) / \text{Old} \times 100\%$$

Example: Price went from \$50 to \$65

$$\text{Increase} = 65 - 50 = 15$$

$$\text{Percentage} = 15/50 \times 100\% = 30\%$$

The price increased by 30%.

Percentage Decrease

Formula:

$$\text{Percentage Decrease} = (\text{Old} - \text{New}) / \text{Old} \times 100\%$$

Example: Price dropped from \$80 to \$60

$$\text{Decrease} = 80 - 60 = 20$$

$$\text{Percentage} = 20/80 \times 100\% = 25\%$$

The price decreased by 25%.

Adding/Subtracting Percentages

Increase by 20%:

$$\text{New Value} = \text{Old Value} \times 1.20$$

If old = 100:

$$\text{New} = 100 \times 1.20 = 120$$

Decrease by 20%:

$$\text{New Value} = \text{Old Value} \times 0.80$$

If old = 100:

$$\text{New} = 100 \times 0.80 = 80$$

General Formula:

Increase by x%: multiply by $(1 + x/100)$

Decrease by x%: multiply by $(1 - x/100)$

Common Mistake: Percentages Don't Add Symmetrically

If you increase by 50% then decrease by 50%, you don't get back to the original:

```
Start: 100
Increase by 50%:  $100 \times 1.5 = 150$ 
Decrease by 50%:  $150 \times 0.5 = 75$  (NOT 100!)
```

Why? The 50% decrease is calculated from the new base (150), not the original (100).

Programming: Applying Percentage Changes

```
function applyPercentChange(value, percentChange) {
    return value * (1 + percentChange / 100);
}

applyPercentChange(100, 20); // 120 (20% increase)
applyPercentChange(100, -20); // 80 (20% decrease)
```

5. Real-World Applications

Sales and Discounts

Original price: \$120, 25% off

```
Discount =  $120 \times 0.25 = \$30$ 
Sale price =  $120 - 30 = \$90$ 
```

Or directly:

```
Sale price =  $120 \times 0.75 = \$90$ 
```

Sales Tax

Item costs \$50, 8% tax

```
Tax =  $50 \times 0.08 = \$4$ 
Total =  $50 + 4 = \$54$ 
```

Or directly:

```
Total =  $50 \times 1.08 = \$54$ 
```

Tip Calculation

Bill is \$80, 18% tip

```
Tip =  $80 \times 0.18 = \$14.40$ 
Total =  $80 + 14.40 = \$94.40$ 
```

Interest (Simple)

\$1000 invested at 5% annual interest for 3 years

```
Interest per year =  $1000 \times 0.05 = \$50$ 
Total interest =  $50 \times 3 = \$150$ 
```

```
Final amount = 1000 + 150 = $1150
```

CPU/Memory Usage

```
const totalMemory = 16000; // MB
const usedMemory = 12000; // MB
const percentUsed = (usedMemory / totalMemory) * 100; // 75%
```

Progress Bars

```
function updateProgress(completed, total) {
  const percent = (completed / total) * 100;
  progressBar.style.width = `${percent}%`;
  label.textContent = `${Math.round(percent)}%`;
}
```

Image Scaling (Aspect Ratio)

```
// Maintain 16:9 aspect ratio
function calculateHeight(width) {
  return width * (9 / 16);
}

calculateHeight(1920); // 1080
```

API Rate Limits

```
Rate limit: 100 requests per minute
Used: 73 requests
Percentage used: 73/100 = 73%
Remaining: 27%
```

6. Proportional Reasoning in Programming

Scaling Coordinates

```
// Scale canvas from 800x600 to 1600x1200 (2x)
function scalePoint(point, scaleFactor) {
  return {
    x: point.x * scaleFactor,
    y: point.y * scaleFactor
  };
}

scalePoint({ x: 100, y: 150 }, 2); // { x: 200, y: 300 }
```

Normalized Values (0 to 1)

```
// Normalize value to 0-1 range
function normalize(value, min, max) {
  return (value - min) / (max - min);
}

normalize(50, 0, 100); // 0.5 (50%)
normalize(75, 0, 100); // 0.75 (75%)

// Convert back
function denormalize(normalized, min, max) {
  return normalized * (max - min) + min;
}

denormalize(0.5, 0, 100); // 50
```

Animation Easing (Proportional Speed)

```
// Linear interpolation (lerp)
function lerp(start, end, t) {
  // t is a percentage (0 to 1)
  return start + (end - start) * t;
}

lerp(0, 100, 0.5); // 50 (halfway)
lerp(0, 100, 0.25); // 25 (quarter way)
```

Resource Allocation

```
// Distribute resources proportionally
function allocate(resources, ratios) {
  const total = ratios.reduce((sum, r) => sum + r, 0);
  return ratios.map(r => (r / total) * resources);
}

allocate(100, [1, 2, 3]); // [16.67, 33.33, 50]
```

Common Mistakes & Misconceptions

✗ "50% of X plus 50% more is 100% of X"

No! $50\% + 50\% \text{ of } 50\% = 75\%$

```
Start: 100
Add 50%:  $100 \times 1.5 = 150$ 
This is 50% more, not 100% more
```

"Percentages can't be over 100%"

They absolutely can. 200% means twice as much.

100% = the whole thing
200% = twice the whole thing
50% = half the whole thing

"Ratios and fractions are different"

They're the same thing:

Ratio 3:4 = Fraction $\frac{3}{4}$ = Decimal 0.75 = Percent 75%

"You can compare percentages without knowing the base"

"20% increase" means nothing without knowing what it's 20% of:

20% of 100 = 20
20% of 1000 = 200

"0.5 is 5%"

No! 0.5 is 50%. To convert decimal to percent, multiply by 100.

Mental Math Tricks

Common Percentages

10%: Divide by 10 (move decimal left)

10% of 340 = 34

5%: Half of 10%

5% of 340 = 17

20%: Double 10%

20% of 340 = 68

25%: Divide by 4

25% of 340 = 85

50%: Divide by 2

50% of 340 = 170

1%: Divide by 100

1% of 340 = 3.4

Tip Calculation Shortcut

15% tip:

10% = move decimal left one place

5% = half of 10%

15% = 10% + 5%

Bill: \$80

10% = \$8

5% = \$4

15% = \$12

Visual Examples

Ratio Visualization

Ratio 2:3 (apples to oranges)

Apples: 

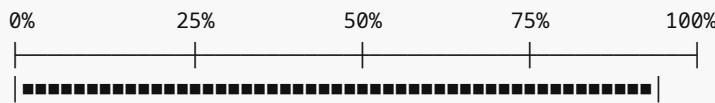
Oranges: 

Total: 5 fruits

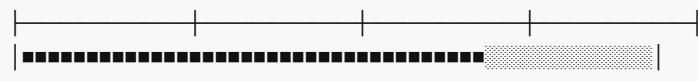
Apples: $2/5 = 40\%$

Oranges: $3/5 = 60\%$

Percentage Bar



75% filled:



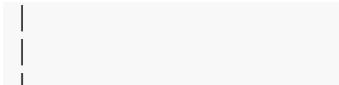
Proportion Scaling

Original (4:3 aspect ratio):



Scaled 2x (maintains ratio):





Tiny Practice

1. Simplify the ratio: 12:18
2. Solve the proportion: $3/5 = x/20$
3. Convert to percent: 0.85
4. Convert to decimal: 42%
5. What is 30% of 250?
6. 15 is what percent of 60?
7. 45 is 15% of what number?
8. Increase 80 by 25%
9. Decrease 120 by 40%
10. If a recipe calls for 3 cups flour for 12 cookies, how many cups for 20 cookies?

► Answers

Summary Cheat Sheet

Ratios

$$a:b = a/b = \text{"a to b"}$$

Simplify: divide both by GCD

Compare: convert to decimals or unit ratios

Proportions

$$\frac{a}{b} = \frac{c}{d} \implies a \times d = b \times c \quad (\text{cross-multiply})$$

Use for: scaling, unit conversion, recipe adjustment

Percentages

$$\text{Percent} = (\text{Part} / \text{Whole}) \times 100$$

To decimal: divide by 100

To fraction: put over 100, simplify

Percentage Operations

Operation	Formula	Example
X% of Y	$(X/100) \times Y$	20% of 150 = 30
X is what % of Y	$(X/Y) \times 100$	30 is 20% of 150
X is Y% of what	$X / (Y/100)$	30 is 20% of 150

Increase by X%	$\text{Value} \times (1 + X/100)$	100 increased by 20% = 120
Decrease by X%	$\text{Value} \times (1 - X/100)$	100 decreased by 20% = 80

Programming Patterns

```
// Ratio as object
const ratio = { a: 16, b: 9 };

// Proportion solving
const x = (b * c) / a;

// Percentage calculation
const percent = (part / whole) * 100;

// Apply percentage change
const newValue = value * (1 + percent / 100);

// Normalize to 0-1
const normalized = (value - min) / (max - min);
```

Next Steps

You now understand how to compare, scale, and work with ratios and percentages. These concepts are fundamental for understanding growth, change, and relationships between quantities.

Next, we'll explore **Powers, Roots, and Exponents**—how repeated multiplication works and why it's useful.

Continue to: [03-powers-roots-exponents.md](#)