1.

a) A area total de um grafico é 1

$$\int_0^1 \frac{1}{3}\,dx + \int_1^2 k\,dx = 1$$

Base: $1-0=1$
Altura $= 1/3$
Area $= 1 \times \frac{1}{3}$

Base : $2-1=1$
Atura $= k$
Area $= k$

ou $[kx]_1^2 = 2k - 1k = k$

$$\frac{1}{3} + k = 1 \Leftrightarrow k = \frac{2}{3}$$

b)

$$\begin{cases} 0 & x<0 \\ \frac{x}{3} & 0 \leq x < 1 \\ \frac{2x-1}{3} & 1 \leq x \leq 2 \\ 1 & x>2 \end{cases}$$

calculo das rampas

Altura $= \frac{1}{3}$
base : $x - 0 = x$
Area $= x \cdot \frac{1}{3} = \frac{x}{3}$

Atura $= \frac{2}{3}$
base $= x-1$
Area $= (x-1)\frac{2}{3} = \frac{2x-2}{3}$

$$\frac{1}{3} + \frac{2x-2}{3} = \frac{2x-1}{3} =$$

c)

objetivo $(E[X]) = 7/6$ $\quad$ $var[X] = 11/36$

$$E[X] = \int_{-\infty}^{+\infty} x \cdot f(x)\,dx$$

$$\int_0^1 x \cdot \frac{1}{3}\,dx = \frac{1}{3}\int_0^1 x\,dx = \frac{1}{3}\left[\frac{x^2}{2}\right]_0^1 = \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}$$

$$\int_1^2 x \cdot \frac{2}{3}\,dx = \frac{2}{3}\int_1^2 x\,dx = \frac{2}{3}\left[\frac{x^2}{2}\right]_1^2 = \frac{2}{3}\cdot\left(\frac{4}{2} - \frac{1}{2}\right) = \frac{2}{3}\cdot\frac{3}{2} = \frac{6}{6}$$

$$\frac{1}{6} + \frac{6}{6} = \frac{7}{6}$$

$$E[x^2] = \int_{-\infty}^{+\infty} x^2 \cdot f(x)\, dx$$

$$\int_0^1 x^2 \cdot \frac{1}{3}\, dx = \frac{1}{3}\int_0^1 x^2\, dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

$$\int_1^2 x^2 \cdot \frac{2}{3}\, dx = \frac{2}{3}\int_1^2 x^2\, dx = \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2 = \frac{2}{3} \cdot \left(\frac{8}{3} - \frac{1}{3}\right) = \frac{2}{3} \cdot \frac{7}{3} = \frac{14}{9}$$

$$\frac{1}{9} + \frac{14}{9} = \frac{15}{9}$$

$$Var[X] = E[X^2] - (E[X])^2 = \frac{15}{9} - \left(\frac{7}{6}\right)^2 = \frac{15}{9} - \frac{49}{36} = \frac{60}{36} - \frac{49}{36} = \frac{11}{36}$$

d)

1 quantil

$$\frac{x}{3} = 0{,}25$$

$$x = 3 \times 0{,}25 = 0{,}75$$

2 quantil

$$\frac{2x-1}{3} = 0{,}5$$

$$2x - 1 = 1{,}5$$

$$2x = 2{,}5$$

$$x = 1{,}25$$

3 quantil

$$\frac{2x-1}{3} = 0{,}75$$

$$2x - 1 = 2{,}25$$

$$2x = 3{,}25$$

$$x = 1{,}625$$

e)

variavel x:

- $f(x) = 1/3$ se $0 \leq x < 1$
- $f(x) = 2/3$ se $1 \leq x \leq 2$
- Media $E[x] = 7/6$

var y:

- $f(y) = e^{-y}$ para $y \geq 0$
- Funçaõ Distribuiçaõ: $F_y(y) = P(Y \leq y) = 1 - e^{-y}$
- Probabilidade de sobrevivencia: $P(Y > y) = e^{-y}$
- Media $E[y] = 1/\lambda = 1$

saõ independentes: x e Y saõ independentes.

i) $$P(A \cup B) = P(A) + P(B) - [P(A) \times P(B)]$$

1. $P(A) = P(x \geq 4/3)$
- Base: $2 - \frac{4}{3} = \frac{6}{3} - \frac{4}{3} = \frac{2}{3}$
- Altura: $\frac{2}{3}$
- Area: $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

2. $P(B) = P(Y \geq 2)$
- $P(Y \geq y) = e^{-y}$
- $P(Y \geq 2) = e^{-2}$

$$= \frac{4}{9} + e^{-2} - \left(\frac{4}{9} \times e^{-2}\right)$$

$$= \frac{4}{9} + e^{-2}\left(1 - \frac{4}{9}\right)$$

$$= \frac{4}{9} + \frac{5}{9}e^{-2}$$

ii) como X e Y saõ independentes, a densidade conjunta é o produto das densidades individuais

$$f_{X,Y}(x,y) = f_X(x) \times f_Y(y)$$

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{3}e^{-y} & \text{se } 0 \leq x < 1 \text{ e } y \geq 0 \\ \frac{2}{3}e^{-y} & \text{se } 1 \leq x \leq 2 \text{ e } y \geq 0 \\ 0 & \text{caso contrario} \end{cases}$$

iii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = P(x < 1)$

$1 \times \frac{1}{3} = \frac{1}{3}$

$P(Y \leq X \cap X < 1)$

$$\int_0^1 \left(\int_0^x \frac{1}{3} e^{-y} \, dy \right) dx =$$

$$= \int_0^1 \frac{1}{3} \left[-e^{-y} \right]_0^x = \frac{1}{3} \left(-e^{-x} - (1) \right) = \int_0^1 \frac{1}{3} \left(1 - e^{-x} \right) dx$$

$$= \frac{1}{3} \int_0^1 (1 - e^{-x}) \, dx = \frac{1}{3} \left[x + e^{-x} \right]_0^1 = \frac{1}{3} \left((1 + e^{-1}) - (0 + 1) \right)$$

$$= \frac{1}{3} (1 + e^{-1} - 1) = \frac{1}{3} e^{-1}$$

$$\frac{\cancel{\frac{1}{3}} e^{-1}}{\cancel{\frac{1}{3}}} = e^{-1}$$

iv)

Se duas variaveis são independentes a sua convergencia e 0

$conv(X, Y) = 0$

Se duas variaveis são independentes $E[XY] = E[X] \cdot E[Y]$

$E[X] = 7/6$

$E[Y] = 1/\lambda = 1/1 \rightarrow 1$

$$E[X, Y] = \frac{7}{6} \times 1 = \frac{7}{6}$$

f) Sempre que aparecer Σ e valor aproximado é para usar o TLC

1. Media da Soma ($E[S_{50}]$)

$$E[S_{50}] = n \times E[x] = 50 \times \frac{7}{6} = \frac{350}{6} = 58,33$$

2. variancia da soma ($Var[S_{50}]$)

$$Var[S_{50}] = n \times Var[x] = 50 \times \frac{11}{36} = \frac{550}{36} \approx 15,28$$

Divisão padrão da Soma (σ_S)

$$\sigma_S = \sqrt{\frac{550}{36}} = \frac{\sqrt{550}}{6} \approx 3,91$$

$$Z = \frac{\text{Valor - Media}}{\text{Desvio Padrão}}$$

$$Z = \frac{60 - 58,33}{3,91} \approx 0,43$$

$$P(Z > 0,43) = 1 - P(Z \leq 0,43)$$

$$1 - 0,6664 = 0,3336$$

2. a)

1. Parte $\mu = 100$ $\sigma = \sqrt{100} = 10$

$P(X < 100) = P_{norm} = (100, 100, 10) = 0,5$

$P(X > 120) = 1 - P_{norm}(120, 100, 10) = 0,2275$

$P(100 < X < 120 = P_{norm}(120, 100, 10) - P_{norm}(100, 100, 10) = 0.4772$

2. Parte

Como diz no enunciado que:

- Pelo menos 2 dias fracos ($N_F \geq 2$)
- Pelo menos 3 dias fortes ($N_S \geq 3$)

Temos estas combinações possiveis

Cenario A: 2 dias fracos + 3 dias fortes = 5 . + Dia medio

Cenario B: 3 dias fracos + 3 dias fortes = 6

Cenario C: 2 dias fracos + 4 dias fortes = 6

3ª Parte (Multinomial)

$$P(A) = \frac{6!}{2!\,3!\,1!} \times (0,5)^2 \times (0,0228)^3 \times (0,4772)^1 \approx 0,000084$$

$$P(B) = \frac{6!}{3!\,3!} \times (0,5)^3 \cdot (0,0228)^3 = 0,000029$$

$$P(C) = \frac{6!}{2!\,4!} \times (0,5)^2 \cdot (0,0228)^4 = 0,000001$$

$$\text{total} = P(A) + P(B) + P(C) = 0,000114$$

b)

i) $D = A - 2B$

1. Media ($E[D]$)

$$E[D] = E[A-2B] = E[A] - 2E[B] = 100 - 2(80) = 100 - 160 = -60$$

Var ($Var[D]$)

$$Var[D] = E[D^2] - (E[D])^2$$

$$D^2 = A^2 - 4AB + 4B^2$$

$$E[D^2] = E[A^2] - 4E[A]E[B] + 4E[B^2]$$

$$Var(x) = E[x^2] - (E[x])^2$$

$$E[A^2] = Var(x) + (E[X])^2$$
$$= 100 + 10000 = 10.100$$

$$E[B^2] = Var(B) + (E[B])^2$$
$$= 90 + (80)^2$$
$$= 6.490$$

$$E[A] \times E[B] = 100 \times 80 = 8000$$

$$E[D^2] = \underbrace{E[A^2]}_{10100} - 4 \times \underbrace{E[A]\,E[B]}_{8000} + 4 \times \underbrace{E[B^2]}_{6490} = 4.060$$

$Var[D] = 4.060 - (-60)^2 = 4.060 - 3600$
$= 460$

ii) $P(A \geq 2B)$
$P(A - 2B \geq 0)$
$P(D \geq 0)$ $\sqrt{460} = 21,45$

$1 - Pnorm(0, -60, 21,45) = 0,0026$

3.

a) $F_N(e) = P(N \leq e) \Leftrightarrow$
$\Leftrightarrow F_N(e) = 1 - P(N > e)$
$P(N > e) = P(X_1 > e \text{ E } X_2 > e \text{ E} \ldots \text{ E } X_n > e)$
$P(N > e) = P(X_1 > e) \times P(X_2 > e) \times \ldots \times P(X_n > e)$
$P(N > e) = \prod_{i=1}^{n} P(X_i > e)$

$P(X_i > e) = 1 - F_{X_1}(e)$

$P(N > e) = \prod_{i=1}^{n} [1 - F_{X_i}(e)]$??

$F_N(e) = 1 - \prod_{i=1}^{n} [1 - F_{X_i}(e)]$

b)

??

4. ?!