Teste 19/18

1. $\int_0^{\frac{1}{4}} \frac{8}{2}\,dx + \int_{\frac{1}{4}}^{1} k\,dx = 1$

Base: $\frac{1}{4} - 0 = \frac{1}{4}$ Altura: $\frac{8}{2}$ Área: $\frac{1}{4}\cdot\frac{3}{2} = \frac{3}{8}$

Base: $1 - \frac{1}{4} = \frac{3}{4}$ Altura: $k$ Base: $k \cdot \frac{3}{2} = \frac{3k}{4}$

$\frac{3}{8} + \frac{3k}{4} = 1 \Leftrightarrow \frac{3}{8} + \frac{6k}{8} = \frac{8}{8} \Leftrightarrow 6k = 8-3 = 6k = 5 \Leftrightarrow k = \frac{5}{6}$ c.q.m

b)

$$\begin{cases} 0 & k < 0 \\ \frac{3k}{2} & 0 \le k < \frac{1}{4} \\ \frac{5k+1}{6} & \frac{1}{4} \le k \le 1 \\ 1 & k > 1 \end{cases}$$

Altura: $\frac{3}{2}$ Base: $k - 0 = k$ Área: $k \cdot \frac{3}{2} = \frac{3k}{2}$

Altura: $\frac{5}{6}$ Base: $k - \frac{1}{4}$ Área: $(k - \frac{1}{4})\frac{5}{6} = \frac{5}{6}k - \frac{5}{24}$

$\frac{3}{8} + \frac{5}{6}k - \frac{5}{24} = \frac{9}{24} + \frac{5k}{6} - \frac{5}{24} = \frac{5k}{6} + \frac{4}{24} = \frac{5k+1}{6}$

c)

1. Quantil

$\frac{3k}{2} = 0{,}25 \Leftrightarrow 3k = 0{,}50 \Leftrightarrow k = \frac{0{,}5}{3} \approx 0{,}1667$

2. Quantil

$\frac{5k+1}{6} = 0{,}5 \Leftrightarrow 5k+1 = 3 \Leftrightarrow \frac{5k}{5} = \frac{2}{5} = k = k = 0.4$

3. Quantil

$\frac{5k+1}{6} = 0.75 \Leftrightarrow 5k+1 = 4.5 \Leftrightarrow 5k = 3.5 \Leftrightarrow k = 0.7$

d) Dizemos X e Y são i.i.d, ambas têm a mesma distribuição

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

A: $P(X > \frac{1}{4}) = 1 - P(X \le \frac{1}{4}) = 1 - (\frac{3}{2}\cdot\frac{1}{4}) = 1 - \frac{3}{8} = \frac{4}{8}$ $A = \{x > \frac{1}{4}\}$

B: $P(Y < \frac{1}{4}) = \frac{3}{2}\cdot\frac{1}{4} = \frac{3}{8}$ $B = \{Y < \frac{1}{4}\}$

$\frac{4}{8} + \frac{3}{8} - (\frac{3}{8}\cdot\frac{4}{8}) = \frac{7}{8} - \frac{3}{16} = \frac{11}{16}$

ii)

$$\begin{cases} \frac{9}{4} & 0 \le x \le \frac{1}{2}, 0 \le y < \frac{1}{4} \\ \frac{5}{4} & 0 \le x < \frac{1}{2}, \frac{1}{4} \le y \le 1 \\ \frac{5}{4} & \frac{1}{2} \le x \le 1, 0 \le y < \frac{1}{4} \\ \frac{25}{36} & \frac{1}{4} < x \le 1, \frac{1}{4} < y < 1 \\ 0, & \text{caso contrário} \end{cases}$$

caso 1: $0 \le x \frac{1}{4}$ e $0 \le y < \frac{1}{4}$ $f_{X,Y}(x,y) = \frac{3}{2}\cdot\frac{3}{2} = \frac{9}{4}$

caso 2: $0 \le x < \frac{1}{2}$, $\frac{1}{4} \le y \le 1$ $f_{X,Y}(x,y) = \frac{3}{2}\cdot\frac{5}{6} = \frac{15}{12} = \frac{5}{4}$

caso 3: $\frac{1}{4} \le x \le 1$ e $0 \le y < \frac{1}{2}$ $f_{X,Y}(x,y) = \frac{5}{6}\cdot\frac{3}{2} = \frac{5}{4}$

caso 4: $\frac{1}{2} \le x \le 1$ e $\frac{1}{4} \le y < 1$ $f_{X,Y} = \frac{5}{6}\cdot\frac{5}{6} = \frac{25}{36}$

iii) ??

2. a) $P(Z \le 3) = \frac{1}{2}$ $P(Z \ge 4) = \frac{1}{4}$

$\frac{3-a}{a-b} = \frac{1}{2}$ $\frac{b-4}{b-a} = \frac{1}{4}$

$\begin{cases} \frac{3-a}{a-b} = \frac{1}{2} \\ \frac{b-4}{b-a} = \frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} 6-2a = b-a \\ 4b-16 = b-a \end{cases} \Leftrightarrow \begin{cases} b = 5 \\ a = 1 \end{cases}$

b) $\begin{cases} \frac{1}{5-1} & 1 \le x \le 5 \\ 0 & \text{c.c} \end{cases}$

$E[Z] = \int_1^5 z \cdot \frac{1}{4}\,dz = \frac{1}{4}\left[\frac{z^2}{2}\right]_1^5 = \frac{1}{4}\cdot(\frac{25}{2} - \frac{1}{2}) = \frac{1}{4}\cdot 12 = \frac{12}{4} = 3$

$E[Z^2] = \int_1^5 z^2 \cdot \frac{1}{4}\,dz = \frac{1}{4}\left[\frac{z^3}{3}\right]_1^5 = \frac{1}{4}\cdot(\frac{125}{3} - \frac{1}{3}) = \frac{1}{4}\cdot(\frac{124}{3}) = \frac{124}{12}$

$Var[Z] = \frac{124}{12} - 9 = \frac{4}{3}$

c) $P(Z \le 1{,}5) = \frac{1{,}5-1}{5-1} = \frac{0{,}5}{4} = 0{,}125$ $P(Z > 3) = \frac{5-3}{5-1} = \frac{2}{4} = 0{,}5$

resto $= 1 - (0{,}125 + 0{,}5) = 1 - (0{,}625) = 0{,}375$

$P_{multinomial} = \frac{10!}{1!\,8!\,1!}\cdot(0{,}125)^1\cdot(0{,}5)^8\cdot(0{,}375)^1$

d) $E[S_{100}] = 100 \times 3 = 300$ $Var[S_{100}] = 100 \times \frac{4}{3} = \frac{400}{3} \approx 133{,}33$ $P(S_{100} > 350)$

Desvio padrão $= \sqrt{\frac{400}{3}} = \frac{20}{\sqrt{3}} = 11{,}547$ $z = \frac{X - E}{\text{desvio pad}} = \frac{350 - 300}{11{,}547} = \frac{50}{11{,}547} = 4{,}33$

$P(z > 4{,}33) = 1 - P(z < 4{,}33) \approx 1 - 0{,}99999 \approx 0$