

1.

a) A área total de um gráfico é 1

$$\int_0^1 \frac{1}{3} dx + \int_1^2 k dx = 1$$

$\text{Base: } 1 - 0 = 1$

$\text{Altura} = 1/3$

$\text{Área} = 1 \times \frac{1}{3}$



$\text{Base: } 2 - 1 = 1$

$\text{Altura} = k \quad \text{ou} \quad [ku]_1^2 = 2k - 1k = k$

$\text{Área} = k$

$$\frac{1}{3} + k = 1 \Leftrightarrow k = \frac{2}{3}$$

b)

$$\begin{cases} 0 & k < 0 \\ \frac{k}{3} & 0 \leq k < 1 \\ \frac{2k-1}{3} & 1 \leq k \leq 2 \\ 1 & k > 2 \end{cases}$$

Cálculo das rampas

$$\text{Altura} = \frac{1}{3}$$

base: $x - 0 = x$

$\text{Área} = x - \frac{1}{3} = \frac{x}{3}$

$\text{Altura} = \frac{2}{3}$

base: $x - 1$

$\text{Área} = (x-1) \frac{2}{3} = \frac{2x-2}{3}$

$$\frac{1}{3} + \frac{2x-2}{3} = \frac{2x-1}{3} =$$

c)

objetivo ($E[x]$) = $7/6$ var [x] = $17/36$

$$E[x] = \int_{-\infty}^{+\infty} x \cdot f(x) dx$$

$$\int_0^1 x \cdot \frac{1}{3} dx = \frac{1}{3} \int_0^1 x dx = \frac{1}{3} \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

$$\int_1^2 x \cdot \frac{2}{3} dx = \frac{2}{3} \int_1^2 x dx = \frac{2}{3} \left[\frac{x^2}{2} \right]_1^2 = \frac{2}{3} \cdot \left(\frac{4}{2} - \frac{1}{2} \right) = \frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6}$$

$$\frac{1}{6} + \frac{6}{6} = \frac{7}{6}$$

$$E[X^2] = \int_{-\infty}^{+\infty} x^2 \cdot f(x) dx$$

$$\int_0^1 x^2 \cdot \frac{1}{3} dx = \frac{1}{3} \int_0^1 x^2 dx = \frac{1}{3} \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

$$\int_1^2 x^2 \cdot \frac{2}{3} dx = \frac{2}{3} \int_1^2 x^2 dx = \frac{2}{3} \left[\frac{x^3}{3} \right]_1^2 = \frac{2}{3} \cdot \left(\frac{8}{3} - \frac{1}{3} \right) = \frac{2}{3} \cdot \frac{7}{3} = \frac{14}{9}$$

$$\frac{1}{9} + \frac{14}{9} = \frac{15}{9}$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 = \frac{15}{9} - \left(\frac{7}{6}\right)^2 = \frac{15}{9} - \frac{49}{36} = \frac{60}{36} - \frac{49}{36} = \frac{11}{36}$$

d)

1 quantil

$$\frac{k}{3} = 0,25$$

$$k = 3 \times 0,25 = 0,75$$

3 quantil

$$\frac{2k-1}{3} = 0,75$$

$$2k-1 = 2,25$$

2 quantil

$$2k = 3,25$$

$$\frac{2k-1}{3} = 0,5$$

$$k = 1,625$$

$$2k-1 = 1,5$$

$$2k = 2,5$$

$$k = 1,25$$

e)

Variável X:

- $f(x) = \frac{1}{3}$ se $0 \leq x < 1$
- $f(x) = \frac{2}{3}$ se $1 \leq x \leq 2$
- Média $E[x] = \frac{7}{6}$

Var Y:

- $f(y) = e^{-y}$ para $y \geq 0$
- Função Distribuição: $F_Y(y) = P(Y \leq y) = 1 - e^{-y}$
- Probabilidade de sobrevivência: $P(Y > y) = e^{-y}$
- Média $E[y] = 1/\lambda = 1$

São independentes: X, Y são independentes.

i) $P(A \cup B) = P(A) + P(B) - [P(A) \times P(B)]$

$$\begin{aligned} 1. P(A) &= P(X \geq 4/3) & 2. P(B) &= P(Y \geq 2) \\ \cdot \text{Base} &: 2 - \frac{4}{3} = \frac{6}{3} - \frac{4}{3} = \frac{2}{3} & \cdot P(Y \geq y) &= e^{-y} \\ \cdot \text{Altura} &: \frac{2}{3} & \cdot P(Y \geq 2) &= e^{-2} \\ \cdot \text{Área} &: \frac{2}{3} \times \frac{2}{3} = \frac{4}{9} \end{aligned}$$

$$\begin{aligned} &= \frac{4}{9} + e^{-2} - \left(\frac{4}{9} \times e^{-2} \right) \\ &= \frac{4}{9} + e^{-2} \left(1 - \frac{4}{9} \right) \\ &= \frac{4}{9} + \frac{5}{9}e^{-2} \end{aligned}$$

ii) Como X e Y são independentes, a densidade conjunta é o produto das densidades individuais

$$f_{X,Y}(x,y) = f_X(x) \times f_Y(y)$$

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{3}e^{-y} & \text{se } 0 \leq x < 1 \text{ e } y \geq 0 \\ \frac{2}{3}e^{-y} & \text{se } 1 \leq x \leq 2 \text{ e } y \geq 0 \\ 0 & \text{caso contrário} \end{cases}$$

$$iii) P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) : P(x < 1)$$

$$1 \times \frac{1}{3} = \frac{1}{3}$$

$$P(Y \leq X \cap X < 1)$$

$$\int_0^1 \left(\int_0^y \frac{1}{3} e^{-y} dy \right) dx =$$

$$= \int_0^1 \frac{1}{3} \left[-e^{-y} \right]_0^y = \frac{1}{3} (-e^{-y} - (-1)) = \int_0^1 \left(1 - e^{-x} \right) dx$$

$$= \frac{1}{3} \int_0^1 (1 - e^{-x}) dx = \frac{1}{3} \left[x + e^{-x} \right]_0^1 = \frac{1}{3} ((1 + e^{-1}) - (0 + 1)) \\ = \frac{1}{3} (1 + e^{-1} - 1) = \frac{1}{3} e^{-1}$$

$$\frac{\frac{1}{3} e^{-1}}{1} = e^{-1}$$

IV)

Se duas variáveis são independentes a sua covariância é 0

$$\text{cov}(x, y) = 0$$

Se duas variáveis são independentes $E[xy] = E[x] \cdot E[y]$

$$E[x] = \frac{1}{6}$$

$$E[y] = 1/\lambda = 1/1 \rightarrow 1$$

$$E[xy] = \frac{1}{6} \cdot 1 = \frac{1}{6}$$

f) Sempre que aparecer Σ , valor aproximado é para usar o TLC

1. Média da Soma ($E[S_{50}]$)

$$E[S_{50}] = n \times E[x] = 50 \times \frac{7}{6} = \frac{350}{6} = 58,33$$

2. Variância da Soma ($Var[S_{50}]$)

$$Var[S_{50}] = n \times Var[x] = 50 \times \frac{11}{36} = \frac{550}{36} \approx 15,28$$

Divisão padrão da Soma (σ_s)

$$\sigma_s = \sqrt{\frac{550}{36}} = \sqrt{\frac{550}{6}} \approx 3,91$$

$$z = \frac{\text{Valor} - \text{Média}}{\text{Desvio Padrão}}$$

$$z = \frac{60 - 58,33}{3,91} \approx 0,43$$

$$P(z > 0,43) \approx 1 - P(z \leq 0,43)$$

$$1 - 0,6664 = 0,3336$$

2. a)

1. Parte $\mu = 100 \quad \sigma = \sqrt{100} = 10$

$$P(x < 100) = P_{\text{norm}}(100, 100, 10) = 0,5$$

$$P(x > 120) = 1 - P_{\text{norm}}(120, 100, 10) = 0,2275$$

$$P(100 < x < 120) = P_{\text{norm}}(120, 100, 10) - P_{\text{norm}}(100, 100, 10) = 0,4772$$

2. Parte

Como diz no enunciado que:

- Pelo menos 2 dias fracos ($N_F \geq 2$)
- Pelo menos 3 dias fortes ($N_S \geq 3$)

Temos estas combinações possíveis

Cenario A: 2 dias fracos + 3 dias fortes = 5 . + Dia medio

Cenario B: 3 dias fracos + 3 dias fortes = 6

Cenario C: 2 dias fracos + 4 dias fortes = 6

3 = Parte (Multinomial)

$$P(A) = \frac{6!}{2!3!1!} \times (0,5)^2 \times (0,0228)^3 \times (0,4772)^1 \approx 0,0000089$$

$$P(B) = \frac{6!}{3!3!} \times (0,5)^3 \times (0,0228)^3 = 0,000029$$

$$P(C) = \frac{6!}{2!4!} \times (0,5)^2 \times (0,0228)^4 = 0,000001$$

$$\text{Total} = P(A) + P(B) + P(C) = 0,000114$$

b)

$$\text{ii } D = A - 2B$$

1. Média ($E[D]$)

$$E[D] = E[A - 2B] = E[A] - 2E[B] = 100 - 2(80) = 100 - 160 = -60$$

Var ($\text{Var}[D]$)

$$\text{Var}[D] = E[D^2] - (E[D])^2$$

$$D^2 = A^2 - 4AB + 4B^2$$

$$E[D^2] = E[A^2] - 4E[A]E[B] + 4E[B^2]$$

$$\text{Var}(x) = E[x^2] - (E[x])^2$$

$$E[A^2] = \text{Var}(A) + (E[A])^2$$

$$= 100 - 10000 = 10 \cdot 100$$

$$E[B^2] = \text{Var}(B) + (E[B])^2$$

$$= 90 - (80)^2$$

$$= 6 \cdot 490$$

$$E[D^2] = \overbrace{E[A^2]}^{10000} - 4 \times E[A]E[B] + \overbrace{4 \times E[B^2]}^{6490} = 4.060$$

8000

6490

$$E[A] \times E[B] = 100 \times 80 = 8000$$

$$\text{Var}[D] = 4.060 - (-60)^2 = 4.060 - 3600$$

$$= 460$$

ii) $P(A \geq 2B)$
 $P(A - 2B \geq 0)$
 $P(D \geq 0)$

$$\sqrt{460} = 21,45$$

$$1 \cdot P_{\text{norm}}(0, -60, 21,45) = 0,0026$$

3.

a) $F_N(e) = P(N \leq e) \Leftrightarrow$

$$\Leftrightarrow f_N(e) = 1 - P(N > e)$$

$$P(N > e) = P(x_1 > e \wedge x_2 > e \wedge \dots \wedge x_n > e)$$

$$P(N > e) = P(x_1 > e) \times P(x_2 > e) \times \dots \times P(x_n > e)$$

$$P(N > e) = \prod_{i=1}^n P(x_i > e)$$

$$P(x_i > e) = 1 - F_{x_i}(e)$$

$$P(N > e) = \prod_{i=1}^n [1 - F_{x_i}(e)] \quad ??$$

$$\bar{F}_N(e) = 1 - \prod_{i=1}^n [1 - F_{x_i}(e)]$$

??

4. ??