1.

a) d:

$$\lim_{e \to -\infty} F(e) = 0 \iff \lim_{e \to -\infty} d = 0 \Rightarrow d = 0$$

k:

1. Lado esquerdo sabemos que é 0
2. Lado direito: vale $1 - e^{-4(k-2)}$

$$0 = 1 - e^{-4(k-2)}$$
$$e^{-4(k-2)} = 1$$
$$-4(k-2) = 0$$
$$k - 2 = 0$$
$$k = 2$$

b)

Densidade para Distribuição → Integras
Distribuição para Densidade → Derivas

$$\begin{cases} 0' = 0 & \text{se } x < 2 \\ \left(1 - e^{-4(e-2)}\right)' = 0 - 4(x-2) = -4x + 8 = 4e^{-4(x-2)} & \text{se } x \geq 2 \end{cases}$$

c)

$$F_W(w) = P(W \leq w) \qquad P(2X - 4 \leq w)$$

isolar o x

$$P(2X \leq w + 4)$$
$$P\left(X \leq \frac{w+4}{2}\right)$$
$$P\left(X \leq \frac{w}{2} + 2\right)$$

$$F_W(e) = F_X\left(\frac{w}{2} + 2\right)$$

como sabemos que é maior que 2 então substituimos na parte positiva.

$$P_W(w) = 1 - e^{-4\left[\left(\frac{w}{2} + 2\right) - 2\right]} = 1 - e^{-2w}$$

$w = 2(2) - 4 = 0$ Logo w é definido $w \geq 0$

d)

i) $P(X>4) = 1 - P(X \leq 4) = 1 - (1 - e^{-4(4 \cdot 2)}) = 1 - 1 + e^{-8} = e^{-8}$

ii)

$P(X \leq 4.5 \mid X > 4) =$ $\qquad P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \dfrac{P(4 < X \leq 4.5)}{P(X>4)}$

$= \dfrac{F_X(4.5) - F_X(4)}{e^{-8}} = \dfrac{1 - e^{-10} - (1 - e^{-8})}{e^{-8}} = \dfrac{e^{-10} - e^{-8}}{e^{-8}} = \dfrac{e^{-8}}{e^{-8}} - \dfrac{e^{-10}}{e^{-8}} = 1 - e^{-2}$

$\simeq 0{,}865$

2.

a)

Lei Uniforme:

$P(c \leq Y \leq d) =$

$= \dfrac{\text{Tamanho do bocado}}{\text{tamanho total}}$

$= \dfrac{d-c}{b-a}$

$P(Y \leq 3) = \frac{1}{2}$ $\qquad P(Y \geq 4) = \frac{1}{4}$

$\dfrac{3-a}{b-a} = \dfrac{1}{2}$ $\qquad \dfrac{b-4}{b-a} = \dfrac{1}{4}$

$\begin{cases} \dfrac{3-a}{b-a} = \dfrac{1}{2} \\ \dfrac{b-4}{b-a} = \dfrac{1}{4} \end{cases} \Leftrightarrow \begin{cases} 6 - 2a = b - a \\ 4b - 16 = b - a \end{cases} \Leftrightarrow \begin{cases} -b - a = -6 \\ 3b + a = 16 \end{cases} \Leftrightarrow \begin{cases} b = 6 - a \\ 18 - 3a + a = 16 \end{cases}$

$\Leftrightarrow \begin{cases} \text{———} \\ \dfrac{-2a}{-2} = \dfrac{-2}{-2} \end{cases} \Leftrightarrow \begin{cases} b = 5 \\ a = 1 \end{cases}$ c.q.m

b

$\begin{cases} \frac{1}{4} & \text{se } 1 \leq y \leq 5 \\ 0 & \text{caso contrario} \end{cases}$

$E[Y] = \int_1^5 y \cdot \frac{1}{4} dy = \frac{1}{4} \int_1^5 y \, dy =$

$= \frac{1}{4} \left[\frac{y^2}{2}\right]_1^5 = \frac{1}{4}\left(\frac{25}{2} - \frac{1}{2}\right) = \frac{1}{4} \cdot \frac{24}{2} = \frac{24}{8} = 3$

$Var[Y] = E[Y^2] - (E[Y])^2 =$

$E[Y^2] = \int_1^5 y^2 \cdot \frac{1}{4} dy = \frac{1}{4} \int_1^5 y^2 dy = \frac{1}{4} \left[\frac{y^3}{3}\right]_1^5 = \frac{1}{4}\left(\frac{125}{3} - \frac{1}{3}\right) = \frac{1}{4} \cdot \frac{124}{3}$

$= \frac{124}{12}$

$= \frac{31}{3}$

$$\frac{31}{3} - 3^2 = \frac{31}{3} - 9 = \frac{31}{3} - \frac{27}{3} = \frac{4}{3}$$

e)

$P(Y<1,5) = 0.125$ $\quad$ $P(Y>3) = 0.5$

$$\text{resto} = 1-(0.125+0.5) = 0.375$$

$$P(\text{Multinomial}) = \frac{10!}{1!\,9!} \times 0.125^1 \times 0.5^9 \times 0.375^0$$

$$\approx 0.00244$$

d) Exercicio TLC

1. Calcular os Parâmetros da Soma (S_{100})

Media total $E[S_{100}]$

$$100 \times 3 = 300$$

Variancia Total ($Var[S_{100}]$)

$$100 \times \frac{4}{3} = \frac{400}{3} \approx 133{,}33$$

Desvio padrão

$$\sqrt{\frac{400}{3}} = \frac{20}{\sqrt{3}} \approx 11{,}547$$

2. Standardizar

$P(S_{100} > 325)$

$$z = \frac{\text{valor} - \text{Media}}{\text{Desvio padrao}}$$

$$z = \frac{325-30}{11{,}547}$$

$$z = \frac{25}{11{,}547} \approx 2{,}17$$

3. Parte final

$P(z > 2{,}17) =$

$= 1 - P(z < 2{,}17)$

$= 1 - 0{,}9850 = 0{,}0150$

e)

i)

$L = 5Y$

$E[L] = E[5Y] = 5 \times E[Y]$

$E[L] = 5 \times 3 = 15$

$Var[L] = Var(5Y) = 5^2 \times var(Y) = 25 \times \frac{4}{3} = \frac{100}{3} \approx 33.33$

ii) $\boxed{F_L(q_p) = P(L \leq q_p) = p}$

$P(5Y \leq q_p) = p =$

$= P(Y \leq \frac{q_p}{5}) = p$

$F_Y(\frac{q_p}{5}) = p$

$F_Y(y) = \frac{y-1}{5-1} = \frac{\frac{q_p}{5} - 1}{4} = p = \frac{q_p}{5} - 1 = 4p \Rightarrow q_p = 20p + 5$

iii)

$T = L_1 + L_2 + \dots + L_{10} = \sum_{i=1}^{10} L_i$

$E[T] = E[\sum_{i=1}^{10} L_i] = \sum_{i=1}^{10} E[L]$

$E[T] = 10 \times 15 = 150€$

$Var(T) = 10 \times \frac{100}{3} = \frac{1000}{3} \approx 333,33$

f) ??