UNIVERZA V LJUBLJANI
FAKULTETA ZA MATEMATIKO IN FIZIKO
ODDELEK ZA FIZIKO

Uroš Grandovec

PREDNOSTI UPORABE REKURENTNIH NEVRONSKIH MREŽ PRI SIMULACIJI FERMI-PASTA-ULAM-TSINGOU SISTEMA

Magistrsko delo

Mentor: dr. Martin Horvat

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Kazalo

1 Uvod

uporaba:

2 FPUI sistem

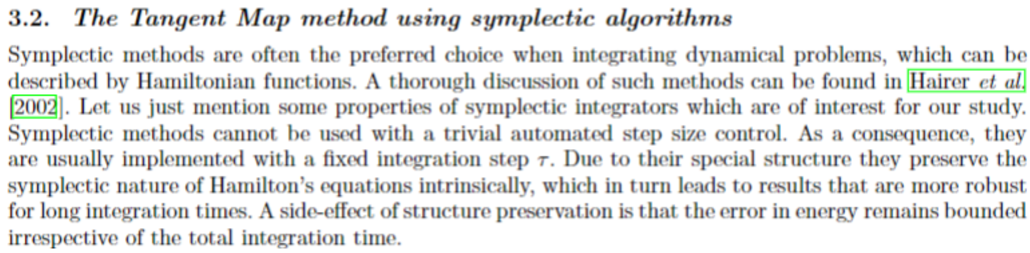
Obravnavali bomo sistem Fermi-Pasta-Ulam (FPU). Poznamo tudi obliki α-FPU ($\alpha \neq 0$, $\beta = 0$) in β-FPU ($\alpha = 0$, $\beta \neq 0$) sistema.

$$H = \sum_{i=0}^{N} \frac{p_i^2}{2} + \sum_{i=0}^{N} \left[\frac{1}{2}(q_{i+1} - q_i)^2 + \frac{\alpha}{3}(q_{i+1} - q_i)^3 + \frac{\beta}{4}(q_{i+1} - q_i)^4 \right] \tag{2.1}$$

Hamiltonove enačbe so potem

$$\begin{aligned}
\dot{q}_i &= \frac{\partial H}{\partial p_i} = p_i, \\
\dot{p}_i &= -\frac{\partial H}{\partial q_i} \\
&= (q_{j+1} - q_j)(\delta_j^i - \delta_{j+1}^i) + \alpha\,(q_{j+1} - q_j)^2(\delta_j^i - \delta_{j+1}^i) \\
&\quad + \beta\,(q_{j+1} - q_j)^3(\delta_j^i - \delta_{j+1}^i) \\
&= (q_{i+1} - q_i) - (q_i - q_{i-1}) + \alpha \left[(q_{i+1} - q_i)^2 - (q_i - q_{i-1})^2 \right] \\
&\quad + \beta \left[(q_{i+1} - q_i)^3 - (q_i - q_{i-1})^3 \right]
\end{aligned} \tag{2.2}$$

3 Rekurentne nevronske mreže



3.2. *The Tangent Map method using symplectic algorithms*

Symplectic methods are often the preferred choice when integrating dynamical problems, which can be described by Hamiltonian functions. A thorough discussion of such methods can be found in Hairer *et al.* [2002]. Let us just mention some properties of symplectic integrators which are of interest for our study. Symplectic methods cannot be used with a trivial automated step size control. As a consequence, they are usually implemented with a fixed integration step τ. Due to their special structure they preserve the symplectic nature of Hamilton's equations intrinsically, which in turn leads to results that are more robust for long integration times. A side-effect of structure preservation is that the error in energy remains bounded irrespective of the total integration time.

Slika 1: aa.