CS303A Homework 2

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Q1 Consider the following sentence:

[(Food ⇒ Party) ∨ (Drinks ⇒ Party)] ⇒ [(Food ∧ Drinks) ⇒ Party]

> a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.
>
> b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).
>
> c. Prove your answer to (a) using resolution.

a.

Let

$A = (Food \Rightarrow Party)$,

$B = (Drinks \Rightarrow Party)$,

$C = (Food \wedge Drinks)$

$D = [(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] = A \vee B$,

$E = [(Food \wedge Drinks) \Rightarrow Party] = (C \Rightarrow Party)$

Let 0 be False, and 1 be True.

Food	Party	Drinks	A	B	C	D	E
0	0	0	1	1	0	1	1
0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	1
1	0	0	0	1	0	1	1
1	0	1	0	0	1	0	0
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Because of $D \Rightarrow E$, which is also
$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$

So the sentence is true for all cases and it is valid.

b.

$$
\begin{aligned}
LHS &\equiv [(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \\
&\equiv (\neg Food \lor Party) \lor (\neg Drinks \lor Party) \\
&\equiv \neg Food \lor Party \lor \neg Drinks \lor Party \\
&\equiv \neg Food \lor Party \lor \neg Drinks \\
&\equiv \neg Food \lor \neg Drinks \lor Party
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
RHS &\equiv (Food \land Drinks) \Rightarrow Party \\
&\equiv \neg(Food \land Drinks) \lor Party \\
&\equiv \neg Food \lor \neg Drinks \lor Party
\end{aligned} \tag{2}
$$

The converted CNF of LHS is same as the RHS's, so it shows the original sentence is true, because for any P, there is $P \Rightarrow P$.

c.

Let shows its negation is unsatisfiable, which can prove the sentense is valid.

$$
\begin{aligned}
&\neg[(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \Rightarrow [(Food \land Drinks) \Rightarrow Party] \\
&\equiv [(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \land \neg[(Food \land Drinks) \Rightarrow Party] \\
&\equiv \neg Food \lor Party \lor \neg Drinks \lor Party \land Food \land Drinks \land \neg Party \\
&\equiv (\neg Food \lor Party \lor \neg Drinks) \land (Food \land Drinks \land \neg Party) \\
&\equiv (\neg(Food \land Drinks \land \neg Party) \land (Food \land Drinks \land \neg Party)) \\
&\equiv (\neg P \land P) \\
&\equiv False
\end{aligned} \tag{3}
$$

So the negation is always unsatisfiable.

Q.2

> This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions.
>
> For each of the logical expressions, state whether it
>
> (1) correctly expresses the English sentence;
>
> (2) is syntactically invalid and therefore meaningless;
>
> (3) is syntactically valid but does not express the meaning of the English sentence and tell why.

a. Paris and Marseilles are both in France

> (i) In(Paris ∧ Marseilles, France)
>
> (ii) In(Paris, France) ∧ In(Marseilles, France)
>
> (iii) In(Paris, France) ∨ In(Marseilles, France)

i	ii	iii
2	1	3

(i) : there is no place named both Paris and Marseilles, so it means $\emptyset \in France$, therefore meaningless.

(ii) : it means "Paris in France" and "Marseilles in France", which is correctly expressed.

(iii) : it means "Paris in France" or "Marseilles in France", which is also syntactically valid, but it doesn't express the full meaning of the English sentence.

b. There is a country that borders both Iraq and Pakistan

> (i) ∃c Country(c) ∧ Border(c, Iraq) ∧ Border(c, Pakistan)
>
> (ii) ∃c Country(c) ⇒ [Border(c, Iraq) ∧ Border(c, Pakistan)]
>
> (iii) [∃c Country(c)] ⇒ [Border(c, Iraq) ∧ Border(c, Pakistan)]
>
> (iv) ∃c Border(Country(c), Iraq ∧ Pakistan)

i	ii	iii	iv
1	3	2	2

(i) : It means "there is a country", "it borders Iraq" and "it borders Pakistan", which is correctly expressed.

(ii) : It means "there is a country implied that it borders both Iraq and Pakistan", which is also syntactically valid but if c is not a contry, there can be no country that borders both Iraq and Pakistan in this statement.

(iii) : Is syntactically invalid obviously because the second half of the sentence does not describe the scope of c, which expects a "∃".

(iv) : Is syntactically invalid obviously because "Country(c)" is not a valid region x for Border(x, y).

c. All countries that border Ecuador are in South America

> (i) ∀c Country(c) ∧ Border(c, Ecuador) ⇒ In(c, SouthAmerica)
>
> (ii) ∀c Country(c) ⇒ [Border(c, Ecuador) ⇒ In(c, SouthAmerica)]
>
> (iii) ∀c [Country(c) ⇒ Border(c, Ecuador)] ⇒ In(c, SouthAmerica)
>
> (iv) ∀c Country(c) ∧ Border(c, Ecuador) ∧ In(c, SouthAmerica)

i	ii	iii	iv
1	1	3	3

(i) : it means for all c, which c is a contry and borders Ecuador, can imply that c is in South America. So the sentence is correctly expressed.

(ii) : It can be converted to the (i), and it means for all c,which c is a contry implies that borders Ecuador, can imply that c is in South America. So the sentence is correctly expressed.

(iii) : It can be converted to "∀c [Country(c) ∧ ¬Border(c, Ecuador)] ∨In(c, SouthAmerica)" which means that for all c, c doesn't border Ecuador or is in South America which totally exists but doesn't express the origin meaning.

(iv) : It means that every country c borders Ecuador and also is in South America which totally exists but doesn't express the origin meaning.

d. No region in South America borders any region in Europe

> (i) ¬[∃c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)]
>
> (ii) ∀c,d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒ ¬Borders(c, d)]
>
> (iii) ¬∀c In(c, SouthAmerica) ⇒ ∃d In(d, Europe) ∧ ¬Borders(c, d)
>
> (iv) ∀c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ¬Borders(c, d)

i	ii	iii	iv
1	1	3	2

(i) : Using propositional resolution:

Let shows its negation is unsatisfiable, which can prove the sentense is valid:

"∃c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)" means that there is at least a tuple (c, d) where c in SouthAmerica, d in Europe and c borders d. The statement is unsatisfiable, so the origin statement is correctly expressed.

(ii) : It means for all c(a region) in SouthAmerica and d(a region) in Europe, c will not border d, which is correctly expressed.

(iii) : Using propositional resolution:

Let shows its negation is satisfiable, which can prove the sentense is unvalid:

"∃c ¬ In(c, SouthAmerica) ⇒ ∃d In(d, Europe) ∧ ¬Borders(c, d)" means that there is at least a region c not in SouthAmerica will implies that "there is at least a region d and region c doesn't border region d", which is abviously true but not expressed the origin meanings.

(iv) : "In(d, Europe) ⇒ ¬Borders(c, d)" can be converted to "¬In(d, Europe) ∨ ¬Borders(c, d)"

So it means that for all region in SouthAmerica, there will not a region d in Europe or there will not a region d borders c, which is syntactically invalid and therefore meaningless.

e. No two adjacent countries have the same map color

> (i) ∀x,y ¬Country(x) ∨ ¬Country(y) ∨ ¬Borders(x, y) ∨ ¬(MapColor(x)=MapColor(y)).
>
> (ii) ∀x,y (Country(x) ∧ Country(y) ∧ Borders(x, y)∧ ¬(x=y)) ⇒ ¬(MapColor(x)=MapColor(y)).
>
> (iii) ∀x,y Country(x) ∧ Country(y) ∧ Borders(x, y) ∧ ¬(MapColor(x)=MapColor(y)).
>
> (iv) ∀x,y (Country(x) ∧ Country(y) ∧ Borders(x, y)) ⇒ MapColor(x≠y).

i	ii	iii	iv
1	1	3	2

(i) : With the premise that x,y are countries, both "Borders(x, y)" and "(MapColor(x)=MapColor(y))" can not be true at the same time, otherwise the origin statement is false. So, it makes that no two adjacent countries have the same map color.

(ii) : For two region x and y, if x borders y, the LHS is True, and then the "¬MapColor(x)=MapColor(y)" must be also True. So there is no form like that "True ⇒ False", so it correctly expresses the English sentence.

(iii) : It means that for all two region x and y, x bothers y and they don't have the same map color which totally exists but doesn't express the origin meaning.

(iv) : It is obviously invalid because of "MapColor(x≠y)".

Q.3

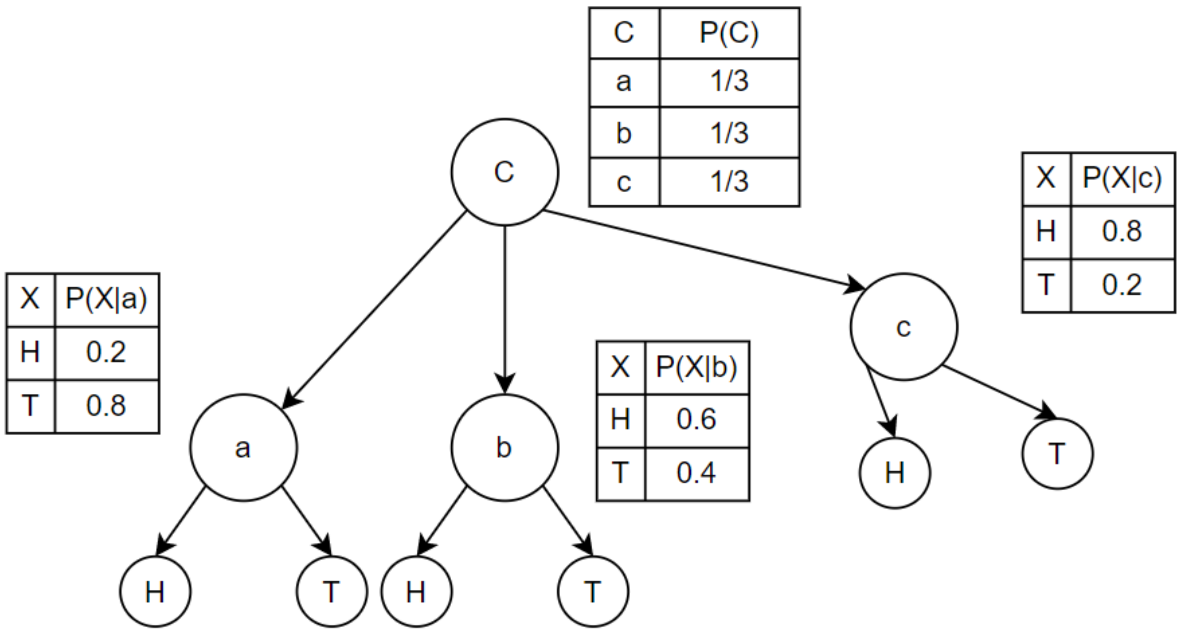
We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X1, X2, and X3

> a. Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

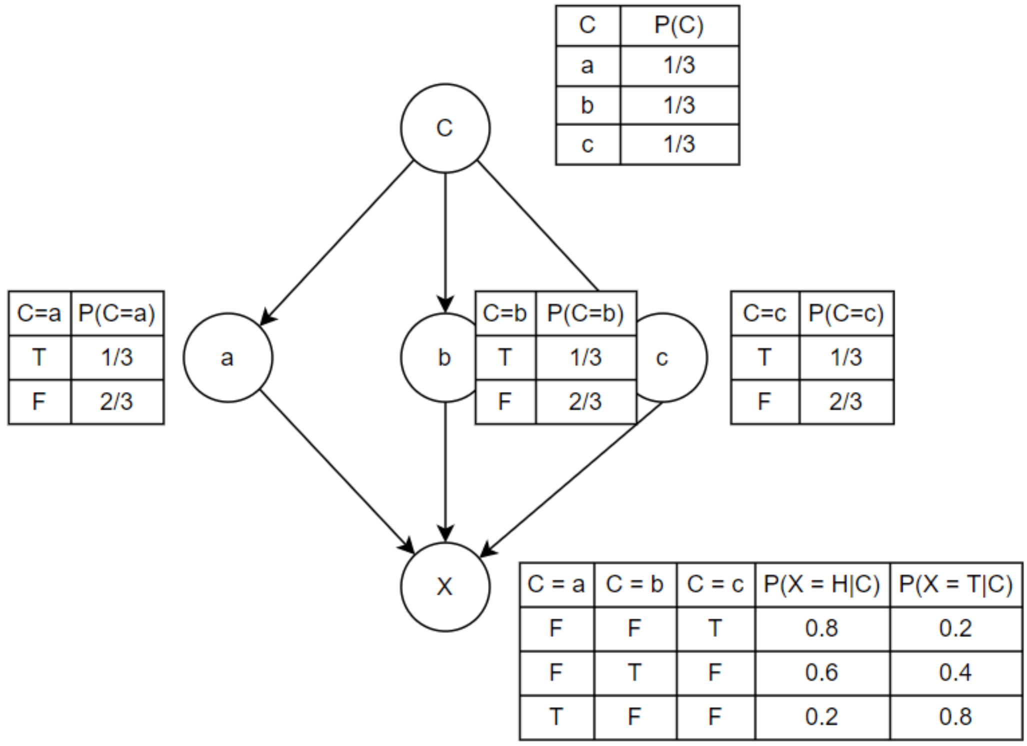
Let C be the operation of draw, X be the outcome. All X1, X2, and X3 can be expressed as X.

Let H be the coming up heads, and T be the not coming up heads.

We can draw Bayesian network with CPTs as following two figures.



What's different about the latter figure is that you take the flip of one of the three different biased coins and you represent the result with a variable X.



> b. Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

Ans: coin **b**.

Use Bayes' formula:

$$P(C|2H,1T) = \frac{P(2H,1T|C)P(C)}{P(2H,1T)}$$

Use Law of Total Probability:

$$P(2H,1T) = P(2H,1T|C=a)P(C=a) + P(2H,1T|C=b)P(C=b) + P(2H,1T|C=c)P(C=c)$$

We can calculate:

$$P(2H,1T|C=c)P(C=c) = C_3^1 0.8*0.8*0.2*\frac{1}{3}$$

$$P(2H,1T|C=b)P(C=b) = C_3^1 0.6*0.6*0.4*\frac{1}{3}$$

$$P(2H,1T|C=a)P(C=a) = C_3^1 0.2*0.2*0.8*\frac{1}{3}$$

So that:

$$P(C=c|2H,1T) = \frac{0.8*0.8*0.2}{0.8*0.8*0.2+0.6*0.6*0.4+0.2*0.2*0.8} = 0.4211$$

$$P(C=b|2H,1T) = \frac{0.6*0.6*0.4}{0.8*0.8*0.2+0.6*0.6*0.4+0.2*0.2*0.8} = 0.4737$$

$$P(C=a|2H,1T) = \frac{0.2*0.2*0.8}{0.8*0.8*0.2+0.6*0.6*0.4+0.2*0.2*0.8} = 0.1053$$

So the coin **b** is most likely to have been drawn from the bag.