2.) **Note:** $m(X)$ and $n(X)$ return row and column dimensions of matrix (or vector) X, respectively, and "$\wedge$" is used to represent logical AND operator.

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), \quad L \rightarrow \begin{pmatrix} L_{TL} & 0 \\ L_{BL} & L_{BR} \end{pmatrix}, \quad U \rightarrow \begin{pmatrix} U_{TL} & U_{TR} \\ 0 & U_{BR} \end{pmatrix}, \quad \Delta A \rightarrow \begin{pmatrix} \Delta A_{TL} & \Delta A_{TR} \\ \Delta A_{BL} & \Delta A_{BR} \end{pmatrix}$$

- For the case of $n = 1$

$$A = LU$$

- ~~[illegible]~~

$$\begin{array}{c|c} \check{L}_{TL}\check{U}_{TL} = A_{TL} + \Delta A_{TL} & \check{L}_{TL}\check{U}_{TR} = A_{TR} + \Delta A_{TR} \\ \hline \check{L}_{BL}\check{U}_{TR} = A_{BL} + \Delta A_{BL} & \end{array}$$

with

$$\left|\left(\begin{array}{c|c} \Delta A_{TL} & \Delta A_{TR} \\ \hline \Delta A_{BL} & \end{array}\right)\right| \leq \gamma_k \left(\begin{array}{c|c} |\check{L}_{TL}||\check{U}_{TL}| & |\check{L}_{TL}||\check{U}_{TR}| \\ \hline |\check{L}_{BL}||\check{U}_{TL}| & \end{array}\right)$$

- For $n = K$

- Repartition

$$\left(\begin{array}{c|c} \Delta A_{TL} & \Delta A_{TR} \\ \hline \Delta A_{BL} & \Delta A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} \Delta A_{00} & \delta a_{01} & \Delta A_{02} \\ \hline \delta a_{10}^T & \delta\alpha_{11} & \delta a_{12}^T \\ \hline \Delta A_{20} & \delta a_{21} & \Delta A_{22} \end{array}\right)$$

$$\left|\left(\begin{array}{c|c|c} \Delta A_{00} & \delta a_{01} & \Delta A_{02} \\ \hline \delta a_{10}^T & & \\ \Delta A_{20} & & \end{array}\right)\right| \leq \gamma_k \left(\begin{array}{c|c|c} |\check{L}_{00}||\check{U}_{00}| & |\check{L}_{00}||\check{U}_{01}| & \check{L}_{00}\check{U}_{02} \\ \hline \check{l}_{10}^T \check{U}_{00} & & \\ \hline \check{L}_{20}\check{U}_{00} & & \end{array}\right)$$

$\longrightarrow$

Using Theorem's 3.12 R2-F, Theorem 5.1 R2-F, & Theorem 5.1 R4-F from reference 7 in the notes.

For

$v_{11} = \alpha_{11} - l_{10}^T u_{01}$

$u_{12}^T := a_{12}^T - l_{10}^T U_{02}$

$l_{21} := (a_{21} - L_{20} u_{01})/v_{11}$

$\check{v}_{11} + \delta\alpha_{11} = \alpha_{11} - l_{10}^T u_{01}$

$\wedge\ |\delta\alpha_{11}| \le \gamma_{k+1}(|l_{10}^T||u_{01}| + |\check{v}_{11}|)$ (3.12 R2-F)

$\check{u}_{12}^T + \delta a_{12}^T = a_{12}^T - l_{10}^T U_{02}$

$\wedge\ |\delta a_{12}^T| \le \gamma_{k+1}(|l_{10}^T||U_{02}| + |\check{u}_{12}^T|)$ (5.1 R2-F)

$\check{l}_{21} v_{11} + \delta a_{21} = a_{21} - L_{20} u_{01}$

$\wedge\ |\delta a_{21}| \le \gamma_{k+1}(|\check{L}_{20}||\check{u}_{01}| + |\check{l}_{21}||\check{v}_{11}|)$ (5.1 R4-F)

For $n = k+1$

$$m\left(\begin{array}{c|c} A_{00} & a_{01} \\ \hline a_{10}^T & \alpha_{11} \end{array}\right) = k+1$$

$$\left(\begin{array}{c|c|c} \check{L}_{00}\check{U}_{00} = A_{00} + \Delta A_{00} & \check{L}_{00}\check{u}_{01} = a_{01} + \delta a_{01} & \check{L}_{00}\check{U}_{02} = A_{02} + \Delta A_{02} \\ \hline \check{l}_{10}^T \check{U}_{00} = a_{10}^T + \delta a_{10}^T & \check{l}_{10}^T \check{u}_{01} + \check{v}_{11} = \alpha_{11} + \delta\alpha_{11} & \check{l}_{10}^T \check{U}_{02} + \check{u}_{12}^T = a_{12}^T + \delta a_{12}^T \\ \hline \check{L}_{20}\check{U}_{00} = A_{20} + \Delta A_{20} & \check{L}_{20}\check{u}_{01} + \check{l}_{21}\check{v}_{11} = a_{21} + \delta a_{21} & \end{array}\right)$$

$$\wedge \left|\left(\begin{array}{cc|c} \Delta A_{00} & \delta a_{01} & \Delta A_{02} \\ \hline \delta a_{10}^T & \delta\alpha_{11} & \delta a_{12}^T \\ \hline \Delta A_{20} & \delta a_{21} & \end{array}\right)\right| \le \gamma_k \left(\begin{array}{c|c|c} |\check{L}_{00}||\check{U}_{00}| & |\check{L}_{00}||\check{u}_{01}| & |\check{L}_{00}||\check{U}_{02}| \\ \hline |\check{l}_{10}^T||\check{U}_{00}| & |\check{l}_{10}^T||\check{u}_{01}| + |\check{v}_{11}| & |\check{l}_{10}^T||\check{U}_{02}| + |\check{u}_{12}^T| \\ \hline |\check{L}_{20}||\check{U}_{00}| & |\check{L}_{20}||\check{u}_{01}| + |\check{l}_{21}||\check{v}_{11}| & \end{array}\right)$$

Going forward

$$\left(\begin{array}{c|c} \Delta A_{TL} & \Delta A_{TR} \\ \hline \Delta A_{BL} & \Delta A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} \Delta A_{00} & \delta a_{01} & \Delta A_{02} \\ \hline \delta a_{10}^T & \delta\alpha_{11} & \delta a_{12}^T \\ \hline \Delta A_{20} & \delta a_{21} & \Delta A_{22} \end{array}\right)$$

This proves backward error analysis ~~aies~~ of bordered LU factorization alg. via induction.