- Exercise 6.1

  - Show $\Phi = \delta_1 + \varepsilon_1 - \alpha_{11}$

$$L = \begin{bmatrix} 1 & & & \\ l_{21} & 1 & & \\ l_{31} & l_{32} & 1 & \\ & & \ddots & \\ l_{n1} & l_{n2} & \cdots & 1 \end{bmatrix} \quad D = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ & & & \ddots \\ & & & & d_n \end{bmatrix}$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{n1} \\ 0 & u_{22} & & & \\ 0 & 0 & u_{33} & & \\ & & & \ddots & \\ 0 & 0 & \cdots & & u_n \end{bmatrix} \quad E = \begin{bmatrix} \varepsilon_1 & & & \\ 0 & \varepsilon_2 & & \\ \vdots & & \ddots & \\ 0 & \cdots & & \varepsilon_n \end{bmatrix}$$

We know $U^T E L^T$ from above. (Don't want to draw it and take space).

$$A = LDL^T = L(DL^T) = LU^T U^{-T} D L^{-T} L^T = UEU^T$$

(E is diagonal matrix)

$$\Phi_1 = \delta_1 + \varepsilon_1 - \alpha_{11} = a_{11} - l_{21}^2 d_1 - u_{12}^2 \varepsilon_1 - \alpha_{11} =$$
$$= a_{11} - (l_{21}^2 + u_{12}^2)\Phi_1$$

we can write $\alpha_{11} = (L^T D L^T)_{11} = (l_{11}^2 d_1 + u_{11}^2 \varepsilon_1) = d_1 + \varepsilon_1$

Substitute!

$$\Phi_1 = a_{11} - (l_{21}^2 + u_{12}^2)(\delta_1 + \varepsilon_1 - d_1 - \varepsilon_1) = a_{11} - (l_{21}^2 + u_{12}^2)(\delta_1 - d_1)$$

~~Multiplying~~: This ~~proves~~ proves $\underline{\Phi_1 = \delta_1 + \varepsilon_1 - \alpha_{11}}$

~~$\Phi = \delta_1 + \varepsilon_1 - \alpha_{11}$~~

• What is cost of computing <u>one</u> twisted factorization given that you have already computed LDL^T & UEU^T factorizations?

Cost is dominated by computing $U = UE^{1/2}$ ~~$U\frac{1}{\sqrt{E}}L$~~ $U\left(\frac{1}{\sqrt{E}}\right)L$

3 parts:

1) ~~E~~ Multiplying U & L = $O(n^2)$ ~~#~~ floating point operations.

2) Computing $\frac{1}{\sqrt{E}}$: $O(n)$ (Computing square root of each diagonal element of E)

3) Multiplying $UE^{-1/2}L$: $O(n^2)$.

Total cost is therefore $O(n^2)$, of computing U.

$A = VDU^T$, requires 1 multiplication of matrix, diagonal matrix, & transpose. This is done in $O(n^2)$. Therefore overall cost of computing one twisted factorization is $O(n^2)$.

• Cost of computing <u>all</u> twisted factorizations...?

Cost of Φ_i involves computing LDL^T & UEU^T factorizations of A_i. These can be computed using previously computed factorizations of A_{i-1} & Φ_{i-1}.

Computing factorizations involves floating point operations:

1) Computing diagonal elements : $O(2)$
2) Computing subdiagonal elements : $O(2)$
3) Updating diagonal & subdiagonal elements Φ_{i-1} : $O(2)$
4) Cholesky factorization of tri. matrix with diagonal elements: $O(n)$.
5) Solving lower & upper triangular systems to get L & U: $O(n^2)$.

Total cost for <u>all</u> = $\underline{O(n^3)}$