COMPLEX FOURIER SERIES:

$$f(x)=\sum_{-\infty}^{\infty} c_n e^{\frac{in\pi x}{L}} \quad \text{WHERE} \quad c_n=\frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{in\pi x}{L}}dx$$

FOURIER SERIES WITH REAL COEFFICIENTS:

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+\sum_{n=1}^{\infty} b_n\sin\left(\frac{n\pi x}{L}\right)$$

$$a_0=\frac{1}{L}\int_{-L}^{L} f(x)dx$$

$$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

NOTE: $\int_{-L}^{L}\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx=\delta_{nm}L\left(=\delta_{nm}\frac{P}{2}\right)$

$$\int_{-L}^{L}e^{-\frac{in\pi x}{L}}e^{\frac{im\pi x}{L}}dx=\delta_{nm}\int_{-L}^{L}1\,dx=\delta_{nm}2L\left(=\delta_{nm}P\right)$$

THIS IS A WAY TO REMEMBER WHERE TO PUT $\frac{1}{L}$ & WHERE $\frac{1}{2L}$

COEFFS: REAL TO CMPLX, CMPLX TO REAL

$c_n=\frac{1}{2}(a_n-ib_n)$ FOR $n\geq 1$ WHICH MAKES SENSE BEC:

$$\frac{1}{2}(a_n-ib_n)=\frac{1}{2}\left(\frac{1}{L}\int_{-L}^{L}f(x)\cos\left(\frac{n\pi x}{L}\right)dx-i\frac{1}{L}\int_{-L}^{L}f(x)\sin\left(\frac{n\pi x}{L}\right)dx\right)$$

$$=\frac{1}{2L}\int_{-L}^{L}f(x)\left(\cos\frac{n\pi x}{L}dx-i\sin\left(\frac{n\pi x}{L}\right)dx\right)$$

$$=\frac{1}{2L}\int_{-L}^{L}f(x)\exp\left(-\frac{in\pi x}{L}\right)dx \text{ AS ABOVE}$$

$$C_{-n} = \frac{1}{2}(a_n + i b_n) \quad \text{FOR } n \geq 1$$

$$= \frac{1}{2}\left[\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx + i\frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx\right]$$

$$= \frac{1}{2L}\int_{-L}^{L} f(x)\left[\cos\left(\frac{n\pi x}{L}\right) + i\sin\left(\frac{n\pi x}{L}\right)\right]dx$$

$$= \frac{1}{2L}\int_{-L}^{L} f(x)\exp\left(\frac{in\pi x}{L}\right)dx \; (= C_{-n})$$

CONSISTENCY WITH EXPRESSION FOR C_n.

$$C_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\underbrace{(\exp 0)}_{\to 1}dx = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2}a_0$$

IF FOURIER SERIES IS IN FORM:

$$f(x) = \frac{1}{2}a_0 + \sum \dots$$

$= a_0$ IF FORM IS:

$$f(x) = a_0 + \sum \dots$$

$$a_n = C_n + C_{-n}$$

MAKES SENSE BEC:

$$C_n + C_{-n} = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{in\pi x}{L}}dx + \frac{1}{2L}\int_{-L}^{L} f(x)e^{\frac{in\pi x}{L}}dx$$

$$= \frac{1}{2L}\int_{-L}^{L} f(x)\left[e^{-\frac{in\pi x}{L}} + e^{\frac{in\pi x}{L}}\right]dx$$

$$= \frac{1}{2L}\int_{-L}^{L} f(x)\,2\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx = a_n$$

$b_n = i(c_n - c_{-n})$

MAKES SENSE BEC:

$$i(c_n - c_{-n}) = i\left(\frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{in\pi x}{L}}dx - \frac{1}{2L}\int_{-L}^{L} f(x)e^{\frac{in\pi x}{L}}dx\right)$$

$$= i\left(\frac{1}{2L}\int_{-L}^{L} f(x)\left[e^{-\frac{in\pi x}{L}} - e^{\frac{in\pi x}{L}}\right]dx\right)$$

$$= i\frac{1}{2L}\int_{-L}^{L} f(x)(-2i)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx = b_n$$