2014P2Q5(I)

(i) $\ddot{u} + 2\gamma\dot{u} + \gamma^2 u = c^2 u''$

$\downarrow$ FT_x USING: $f(x) = g'(x) \Leftrightarrow \tilde{f}(k) = ik\tilde{g}(k)$

$$\ddot{\tilde{u}} + 2\gamma\dot{\tilde{u}} + \gamma^2\tilde{u} = -k^2c^2\tilde{u}$$

(ii) $\tilde{u}(k,t) = \exp(rt)$

$$(r^2 + 2\gamma r + \gamma^2)\exp(rt) = -k^2c^2\exp(rt)$$

$$r^2 + 2\gamma r + \gamma^2 + k^2c^2 = 0$$

$$r = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\cdot 1\cdot(\gamma^2 + k^2c^2)}}{2} = -\gamma \pm \sqrt{-(kc)^2} =$$

$$= -\gamma \pm ikc$$

SO WE HAVE THE GENERAL SOLUTION TO $\tilde{u}$:

$$\tilde{u}(k,t) = \tilde{A}(k)e^{(-\gamma + ikc)t} + \tilde{B}(k)e^{(-\gamma - ikc)t}$$

$$u(x,t) = IFT_x[\tilde{u}(k,t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\gamma t}\left[\tilde{A}(k)e^{+ikct} + \tilde{B}(k)e^{-ikct}\right]\cdot e^{ikx}dk =$$

$$= \frac{1}{2\pi}e^{-\gamma t}\int_{-\infty}^{\infty}\tilde{A}(k)e^{ik(x+ct)} + \tilde{B}(k)e^{ik(x-ct)}dk =$$

$$= e^{-\gamma t}\left(A(x+ct) + B(x-ct)\right)$$

(iii) [WILL RETURN TO THIS WHEN I'M ABLE TO SOLVE THIS, NOW I TRIED BUT DID NOT SUCCEED]