2016P2Q9

(a)

$H = \{I, h_1\}$ H is NORMAL $\Rightarrow g h_1 g^{-1} = h_x \quad \forall g \in G$

$h_x \in H$

(b)

HOMOMORPHISM:

A MAP $\Phi: G \to H$

WHICH SATISFIES

$\Phi(g_1 g_2) = \Phi(g_1)\Phi(g_2)$

✓

union complete conjugacy classes is normal subgp.

$g I g^{-1} = I$

$\Rightarrow g h_1 g^{-1} \neq I \Rightarrow g h_1 g^{-1} = h_1$

✓

$g h_1 = h_1 g$

ELEMENTS OF H COMMUTE W/ $\forall g \in G$

AND I COMMUTES W/ $\forall g \in G$

$\Rightarrow H \subseteq Z(G)$ ✓

✓

KERNEL OF HOMOMORPHISM:

$k_1, k_2, \ldots \in K,\ K \subseteq G,$

$\Phi(k) = I_H \quad \forall k \in K$

(THOSE ELEMENTS OF G ARE THE KERNEL WHICH MAP TO THE IDENTITY OF THE SECOND GROUP. image.

✓

(c) BY LAGRANGE'S THEOREM:

$\frac{|G|}{|H|} = n$ (H BEING PROPER SG.)

At least better to say $|G| = 21$. your logic is too skeletal.

POSSIBILITIES: $|H| = 3$

$|H| = 7$

why is this needed in general? separate Lagrange from this application of it to H.

ORDER OF EVERY ELEMENT OF GROUP DIVIDES ORDER OF GROUP.

3 & 7 ARE PRIMES. $\Rightarrow$ EVERY ELEMENT WHICH IS NOT THE IDENTITY HAS ORDER 3 OR 7 RESPECTIVELY

$\Rightarrow$ SUBGROUPS ARE CYCLIC

no. you need to say order of gp prime $\Rightarrow$ it is cyclic. (no proper subgps.).

so $|H| = 3$ or 7 cyclic.