2015 P2 Q8 (I)

a/ $gH = \{g, \boxed{\ } \, gh_1, gh_2 \dots\}$

what are the h_i? ✓

PROOF OF PARTITIONING:

ASSUME 1 ELEMENT IN COMMON BETWEEN g_1H & g_2H:

$g_1 h_1 = g_2 h_2 \rightarrow g_1 = g_2 h_2 h_1^{-1}$

$g_1 H = g_2 h_2 h_1^{-1} H = g_2 H_2$ ✓

↳ ASSUMPTION WAS WRONG

↳ NO ELEMENT IN COMMON, SO COSETS PARTITION GROUPS (G).

Is every element in a coset?

b/ $\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}$. x, y, z real.

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$x = y = z = 0$ ~~INVERSE~~ IDENTITY PRESENT (*)

$$\begin{pmatrix} 1 & x_1 & y_1 \\ 0 & 1 & z_1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & x_2 & y_2 \\ 0 & 1 & z_2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x_1+x_2 & y_1+x_1z_2+y_2 \\ 0 & 1 & z_1+z_2 \\ 0 & 0 & 1 \end{pmatrix}$$

SET IS CLOSED (**) ✓

$$\det\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} = 1 \Rightarrow \text{ALL MATRICES OF THE FORM } \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} \text{ ARE INVERTIBLE. } (***)$$

are the inverses of the CORRECT FORM?

(*)
(**) $\Rightarrow$ GROUP IS FORMED.
(***)

associativity? Too brief.

$\cancel{\begin{pmatrix} 1 & 0 & y_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & y_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$

$$\begin{pmatrix} 1 & 0 & y_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & x_2 & y_2 \\ 0 & 1 & z_2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x_2 & y_1+y_2 \\ 0 & 1 & z_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & x_2 & y_2 \\ 0 & 1 & z_2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & y_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x_2 & y_1+y_2 \\ 0 & 1 & z_2 \\ 0 & 0 & 1 \end{pmatrix}$$ ✓

$\underline{\underline{M}}\,\underline{\underline{M}}_{x=z=0} = \underline{\underline{M}}_{x=z=0}\underline{\underline{M}} \Rightarrow \underline{\underline{M}}_{x=z=0}$ COMMUTES W/ EVERY ELEMENT OF $G \Rightarrow \underline{\underline{M}}_{x=z=0}$ FORM THEIR OWN CONJUGACY CLASSES $\Rightarrow$ NORMAL SUBGROUP FORMED. ✓

2015P2Q8(ii)

$x, y, z = 0, 1, 2, 3$ ✓

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\det \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} = 1 \Rightarrow$ IDENTITY & INVERSES PRESENT.

$\hookrightarrow \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}$ ARE INVERTIBLE

need to show correct form not just g invertible.

$$M(x_1, y_1, z_1) \cdot M(x_2, y_2, z_2) = \begin{pmatrix} 1 & x_1 + x_2 & y_1 + x_1 z_2 + y_2 \\ 0 & 1 & z_1 + z_2 \\ 0 & 0 & 1 \end{pmatrix} \bmod 4$$

$0 \leq (x_1 + x_2) \bmod 4 \leq 3$

$0 \leq (y_1 + x_1 z_2 + y_2) \bmod 4 \leq 3$

$0 \leq (z_1 + z_2) \bmod 4 \leq 3$ SO GROUP IS CLOSED.

$\therefore$ IT IS INDEED A GROUP.

✓

ORDER: $4 \cdot 4 \cdot 4 = \underline{64}$

x, y, z ARE BOTH 0 OR 1 OR 2 OR 3 ✓

$x = z$ PART:

$$\begin{pmatrix} 1 & x_1 & y_1 \\ 0 & 1 & x_1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & x_2 & y_2 \\ 0 & 1 & x_2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x_1 + x_2 & y_1 + y_2 + x_1 x_2 \\ 0 & 1 & x_1 + x_2 \\ 0 & 0 & 1 \end{pmatrix} = R_1$$

$$\begin{pmatrix} 1 & x_2 & y_2 \\ 0 & 1 & x_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & x_1 & y_1 \\ 0 & 1 & x_1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x_1 + x_2 & y_1 + y_2 + x_1 x_2 \\ 0 & 1 & x_1 + x_2 \\ 0 & 0 & 1 \end{pmatrix} = R_2$$ ✓

$R_1 = R_2$ CLOSED INVERSES PRESENT IDENTITY PRESENT $\Rightarrow M_{x=z}$ FORM ABELIAN SG. ✓1. ✓

ORDER OF H: $4 \cdot 4 = \underline{16}$ ✓

$\frac{64}{16} = \underline{4}$ DISTINCT LEFT COSETS

Henry Lee