2008P2Q10

operation
$H = \{1, -1, i, -i\}$

$\times$	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

the order $1\ i\ -1\ -i$ $\cong C_4$.

make cyclic structure of gp + geometry clearer.

$S: H \to GL(2, \mathbb{R})$

$S(i) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$S(i)^2 = S(-1) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ✓

$S(-1)^2 = S(1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

$S(-i) = S(-1)S(i) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ✓

or do
$S(i)^2 = S(-1)$
$S(i)^3 = S(-i)$
$S(i)^4 = S(1)$.

$Q = \{\pm 1, \pm i, \pm j, \pm k\}$

$i^2 = j^2 = k^2 = ijk = -1$

$i^2 \to \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$j^2 \to \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$k^2 \to \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$ijk \to \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \cancel{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\Rightarrow$ GROUP OPERATIONS ARE PRESERVED, SO A REPRESENTATION IS GIVEN ?? like chart?

4×4 REP:

$i \to \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$

$j \to \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ ✓

$k \to \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

$\tilde{D}_3 = \tilde{D}(q_1)\tilde{D}(q_2) = S D(q_1) S^{-1} S D(q_2) S^{-1} =$

$= S D(q_1) D(q_2) S^{-1} =$

$= S D(q_3) S^{-1}, \ q_1, q_2, q_3 \in Q$ ✓

→ GROUP OPERATIONS ARE PRESERVED.

$Tr(S D(q) S^{-1}) = Tr(S^{-1} S D(q)) =$

$= Tr(D(q))$

USING: $Tr(ABC) = \cancel{\Sigma} A_{ij} B_{jk} C_{ki} =$ ✓

$= C_{ki} A_{ij} B_{jk} = Tr(CAB)$

→ SO CHARACTERS ARE PRESERVED.