GIVEN:

$$S_x|\pm\rangle_x = \pm\frac{\hbar}{2}|\pm\rangle_x \qquad S_y|\pm\rangle_y = \pm\frac{\hbar}{2}|\pm\rangle_y$$

$$|\pm\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle \pm |-\rangle\right)$$

$$|\pm\rangle_y = \frac{1}{\sqrt{2}}\left(|+\rangle \pm i|-\rangle\right)$$

FIND MATRIX REP OF S_x & S_y IN S_z BASIS

$$S_x = \begin{pmatrix} \langle +|S_x|+\rangle & \langle +|S_x|-\rangle \\ \underbrace{\langle -|S_x|+\rangle}_{S_x|+\rangle} & \underbrace{\langle -|S_x|-\rangle}_{S_x|-\rangle} \end{pmatrix}$$

$$S_x|\pm\rangle_x = \pm\frac{\hbar}{2}|\pm\rangle_x \qquad |\pm\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle \pm |-\rangle\right)$$

$$S_x \frac{1}{\sqrt{2}}\left(|+\rangle \pm |-\rangle\right) = \pm\frac{\hbar}{2}\frac{1}{\sqrt{2}}\left(|+\rangle \pm |-\rangle\right)$$

$$S_x\left(|+\rangle \pm |-\rangle\right) = \pm\frac{\hbar}{2}\left(|+\rangle \pm |-\rangle\right)$$

I A — ⊕ B — ⊖

$\langle +|\cdot$

$$\langle +|S_x|\left(|+\rangle \pm |-\rangle\right) = \langle +|S_x|+\rangle \pm \langle +|S_x|-\rangle =$$

$$= \pm\frac{\hbar}{2}\Big(\underbrace{\langle +|+\rangle}_{1} \pm \underbrace{\langle +|-\rangle}_{=0}\Big) = \pm\frac{\hbar}{2}$$

$\langle -|\cdot$

$$\langle -|S_x|\left(|+\rangle \pm |-\rangle\right) =$$

$$= \langle -|S_x|+\rangle \pm \langle -|S_x|-\rangle =$$

II A — ⊕ B — ⊖

$$= \pm\frac{\hbar}{2}\Big(\underbrace{\langle -|+\rangle}_{0} \pm \underbrace{\langle -|-\rangle}_{1}\Big) = \frac{\hbar}{2}$$

4 UNKNOWNS: $\langle +|S_x|+\rangle$

$\langle +|S_x|-\rangle$

$\langle -|S_x|+\rangle$

$\langle -|S_x|-\rangle$

$$\begin{array}{c} \mathrm{I}_A \rightarrow \\ \mathrm{I}_B \rightarrow \\ \mathrm{II}_A \rightarrow \\ \mathrm{II}_B \rightarrow \end{array}\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & \frac{\hbar}{2} \\ 1 & -1 & 0 & 0 & -\frac{\hbar}{2} \\ 0 & 0 & 1 & 1 & \frac{\hbar}{2} \\ 0 & 0 & 1 & -1 & \frac{\hbar}{2} \end{array}\right) =$$

$$= \begin{array}{c} \begin{array}{cccc} ++ & +- & -+ & -- \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & \hbar/2 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \hbar/2 \\ 0 & 0 & 2 & 0 & \hbar \end{array}\right) \end{array}$$

$\langle +|S_x|-\rangle = \frac{\hbar}{2}$

$\rightarrow \langle +|S_x|+\rangle = 0$

$\rightarrow \langle -|S_x|-\rangle = 0$

$\rightarrow \langle -|S_x|+\rangle = \frac{\hbar}{2}$

$$\Longrightarrow \boxed{S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

S_y PART.

$$|\pm\rangle_y = \frac{1}{\sqrt{2}}\left(|+\rangle \pm i|-\rangle\right)$$

REPLACE $|-\rangle$ WITH $i|-\rangle$ IN S_x CALC.

$$\left(\begin{array}{cccc|c} 1 & i & 0 & 0 & \frac{\hbar}{2} \\ 1 & -i & 0 & 0 & -\frac{\hbar}{2} \\ 0 & 0 & 1 & i & i\frac{\hbar}{2} \\ 0 & 0 & 1 & -i & i\frac{\hbar}{2} \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & i & 0 & 0 & \hbar/2 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & i & i\hbar/2 \\ 0 & 0 & 2 & 0 & i\hbar \end{array}\right)$$

$\rightarrow i\langle +|S_y|-\rangle = \frac{\hbar}{2} \Rightarrow \langle +|S_y|-\rangle = -i\frac{\hbar}{2}$ $(/\cdot i)$

$\rightarrow \langle +|S_y|+\rangle = 0$

$\langle -|S_y|-\rangle = 0$

$\rightarrow \langle -|S_y|+\rangle = i\frac{\hbar}{2}$

$$\Longrightarrow \boxed{S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$$