$$\underline{\hat{n}} \cdot \underline{L} = \underline{\hat{n}}_z \hat{L}_z + \underline{\hat{n}}_y \hat{L}_y + \underline{\hat{n}}_x \hat{L}_x =$$

$$= \underline{\hat{n}}_z \hat{L}_z + \frac{1}{2}(\underline{\hat{n}}_x - i\underline{\hat{n}}_y)\hat{L}_+ + \frac{1}{2}(\underline{\hat{n}}_x + i\underline{\hat{n}}_y)\hat{L}_-$$

$$= \cos\theta \hat{L}_z + \frac{1}{2}\sin\theta e^{-i\varphi}\hat{L}_+ + \frac{1}{2}\sin\theta e^{i\varphi} L_-$$

$\underline{\hat{n}} = \underline{\hat{n}}_x$ IF $\theta = \frac{\pi}{2}, \varphi = 0$

$$\underline{\hat{n}}_x \cdot \underline{\hat{L}} = \hat{L}_x = \frac{1}{2}\hat{L}_+ + \frac{1}{2}\hat{L}_-$$

$$L_x|\psi\rangle = \ell|\psi\rangle$$

$L_+|1\,-1\rangle = \sqrt{2}|1\,0\rangle$

$L_+|1\,0\rangle = \sqrt{2}|1\,1\rangle$ $\quad$ $L_-|1\,0\rangle = \sqrt{2}|1\,-1\rangle$

$L_-|1\,1\rangle = \sqrt{2}|1\,0\rangle$

$$L_x(C_{-1}|1\,-1\rangle + C_0|1\,0\rangle + C_1|1\,1\rangle) = \qquad \alpha = \pm 1, 0$$

$$= (C_{-1}|1\,-1\rangle + C_0|1\,0\rangle + C_1|1\,1\rangle)\cdot\alpha$$

$$\frac{1}{2}(L_+ + L_-)(C_{-1}|1\,-1\rangle + C_0|1\,0\rangle + C_1|1\,1\rangle) =$$

$$= (C_{-1}|1\,-1\rangle + C_0|1\,0\rangle + C_1|1\,1\rangle)\cdot\alpha$$

$$= \frac{1}{2}\cancel{\hbar}\sqrt{2}(C_{-1}|1\,0\rangle + C_0|1\,1\rangle) +$$

$$+ \frac{1}{2}\cancel{\hbar}\sqrt{2}(C_0|1\,-1\rangle + C_1|1\,0\rangle) =$$

$$= \frac{1}{\sqrt{2}}(C_{-1} + C_1)|1\,0\rangle + \frac{1}{\sqrt{2}}C_0|1\,1\rangle + \frac{1}{\sqrt{2}}C_0|1\,-1\rangle$$

COMPARE COEFFICIENTS:

$\alpha=1$

$$\frac{1}{\sqrt{2}}(c_{-1}+c_1)=c_0$$

$$\frac{1}{\sqrt{2}}c_0=c_1$$

$$\frac{1}{\sqrt{2}}c_0=c_{-1}$$

$$|c_{-1}|^2+|c_0|^2+|c_1|^2=1$$

$$\left(\frac{1}{\sqrt{2}}c_0\right)^2+\left(\frac{1}{\sqrt{2}}c_1\right)^2+c_0^2=1$$

$$c_0^2=\frac{1}{2}$$

$$c_0=\pm\frac{1}{\sqrt{2}}$$

$$c_{\pm1}=\pm\frac{1}{2}$$

$$\frac{1}{\sqrt{2}}|10\rangle\pm\frac{1}{2}\left(|11\rangle+|1-1\rangle\right)$$

$$=\frac{1}{2}\left(\pm|11\rangle+|1-1\rangle+\sqrt{2}|10\rangle\right)$$

$\alpha=-1$

$$\frac{1}{\sqrt{2}}(c_{-1}+c_1)=-c_0$$

$$\frac{1}{\sqrt{2}}c_0=-c_1$$

$$\frac{1}{\sqrt{2}}c_0=-c_1$$

$\alpha=0$

$$\frac{1}{\sqrt{2}}(c_{-1}+c_1)=0$$

$$\frac{1}{\sqrt{2}}c_0=0$$

$$c_1=-c_{-1}$$

$$\frac{1}{2}\left(|11\rangle-|1-1\rangle\right)$$