2017 P1Q1(I)

(a) $\int_V (\nabla \cdot \underline{G})\, dV = \oint_S \underline{G} \cdot d\underline{S}$

WHERE V IS A CLOSED VOLUME BOUNDED BY CLOSED SURFACE S, $d\underline{S}$ IS OUTWARD POINTING NORMAL FROM THIS SURFACE.

LL

(b) $x^2+y^2=2z^2 \quad 0 \leq z < h$

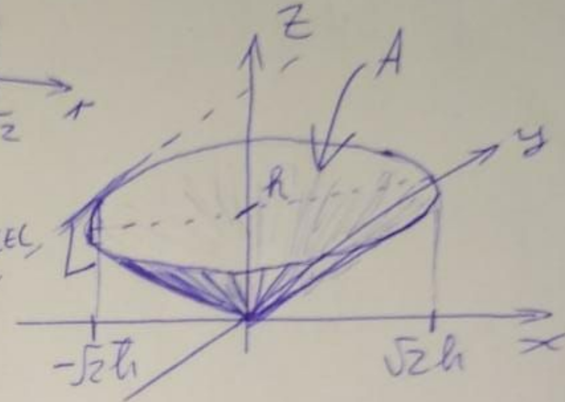
LET: $r^2=x^2+y^2$

$r^2=2z^2$

$z=\frac{1}{\sqrt{2}}r$

perspective

LINE PARALLEL W/ y AXIS



(c) LETS DEFINE A CLOSED CURVE S: $S = A \cup D$ WHERE D IS A DISC ~~LYING~~ LIEING PARALLEL ~~WITH~~ TO THE x & y AXES, AT HEIGHT h, CENTERED ON THE z AXIS

lying

$$\nabla \cdot \underline{G} = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \cdot \begin{pmatrix} x^3+2xy \\ y^3+\sin x \\ z \end{pmatrix} =$$

$$= 3x^2+2y+3y^2+1 = 3r^2+1+2y$$

$$\int_V (\nabla \cdot \underline{G})\, dV = \int_V 3r^2+1+\underbrace{2y}_{\text{ODD FUNCTION ON A SYMMETRIC INTEGRATION DOMAIN} \to \text{WE OMIT IT (WITHOUT CHANGING THE RESULT)}} dV = \int_V 3r^2+1\, dV =$$

$$= \int_{z=0}^{h} \int_{r=0}^{\sqrt{2}z} \int_{\theta=0}^{2\pi} (3r^2+1)\, r\, dr\, d\theta\, dz = 2\pi \int_{z=0}^{h} \int_{r=0}^{\sqrt{2}z} 3r^3+r\, dr\, dz =$$

$$= 2\pi \int_{z=0}^{h} 3\left[\frac{r^4}{4}\right]_0^{\sqrt{2}z} + \left[\frac{r^2}{2}\right]_0^{\sqrt{2}z} dz = 2\pi \int_{z=0}^{h} 3z^4+z^2\, dz = 2\pi\left(3\left[\frac{z^5}{5}\right]_0^h + \left[\frac{z^3}{3}\right]_0^h\right) =$$

$$= 2\pi\left(\frac{3}{5}h^5 + \frac{1}{3}h^3\right)$$

2017P1Q1(II)

$$\oint_S \underline{G}\cdot d\underline{S} \overset{\text{BY DIVERGENCE THM}}{=} \int_V (\nabla\cdot G)\,dV$$

$$= \underbrace{\int_D \underline{G}\cdot d\underline{S} + \int_A \underline{G}\cdot dS}_{\text{BY DEFINITION OF } S}$$

I think the underlying principle has been rather lost here. The geometry is too implicit.

REARRANGE:

$$\int_A \underline{G}\cdot d\underline{A} = \oint_S \underline{G}\cdot d\underline{S} - \int_D \underline{G}\cdot dS = \int_V (\nabla\cdot G)\,dV - \int_D \underline{G}\cdot d\underline{S}$$

LET'S FIND: $\int_D \underline{G}\cdot dS$

$$\int_D \underline{G}\cdot d\underline{S} = \int_D \begin{pmatrix} x^3+2xy \\ y^3+\sin x \\ z \end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} dS = \int_D z\,dS = \int_D h\,dS =$$

✓

$$= (\sqrt{2}h)^2\pi\, h = 2\pi h^3$$

RETURN to $\int_A \underline{G}\cdot d\underline{A}$:

$$\int_A \underline{G}\cdot d\underline{A} = 2\pi\left(\frac{3}{5}h^5 + \frac{1}{3}h^3\right) - 2\pi h^3 =$$

$$= 2\pi\left(\frac{3}{5}h^5 - \frac{2}{3}h^3\right)$$

✓

A BIT WEIRD BUT NOT ENTIRELY UNBELIEVABLE.

The dimensions in this question are odd.