2015P2Q5 (I)

(a) $\ddot{y}+2\dot{y}+5y=f(t) \quad / \cdot e^{-i\omega t} \quad / \int_{-\infty}^{\infty} dt$

$$\int_{-\infty}^{\infty}(\ddot{y}+2\dot{y}+5y)e^{-i\omega t}dt=\int_{-\infty}^{\infty}f(t)e^{-i\omega t}dt=\tilde{f}(\omega)$$

$$\hookrightarrow \int_{-\infty}^{\infty}\ddot{y}e^{-i\omega t}dt=\underbrace{\left[\dot{y}e^{-i\omega t}\right]_{-\infty}^{\infty}}_{\to 0}-\int_{-\infty}^{\infty}\dot{y}(-i\omega)e^{-i\omega t}dt=$$

BY THE ASSUMPTION THAT THE RESPONSE OF THE SYSTEM $y(t)$ & SO ~~ITS~~ DERIVATIVE $\dot{y}(t)$ DIES AWAY AS $t \to \pm\infty$

$$=i\omega\left[\left[ye^{-i\omega t}\right]_{-\infty}^{\infty}-\int_{-\infty}^{\infty}y(-i\omega)e^{-i\omega t}dt\right]=$$

$$=(i\omega)^2\int_{-\infty}^{\infty}ye^{-i\omega t}dt=-\omega^2\tilde{y}$$

$$\hookrightarrow \int_{-\infty}^{\infty}2\dot{y}e^{-i\omega t}dt=2\left[\underbrace{\left[ye^{-i\omega t}\right]}_{\to 0}-\int_{-\infty}^{\infty}y(-i\omega)e^{-i\omega t}dt\right]=$$

$$=2i\omega\tilde{y}$$

REWRITE EQUATION ABOVE:

$$-\omega^2\tilde{y}+2i\omega\tilde{y}+5\tilde{y}=\tilde{f}(\omega)$$

$$\hookrightarrow \tilde{y}=\frac{-\tilde{f}(\omega)}{\omega^2-2i\omega-5}$$

AS REQUIRED.

(b)(i) $\tilde{f}(\omega)=\frac{i}{\omega-2i}$

$$y(t)=\mathrm{IFT}[\tilde{y}(\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-\tilde{f}(\omega)}{\omega^2-2i\omega-5}e^{i\omega t}d\omega=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\underbrace{\frac{-\frac{i}{\omega-2i}}{\omega^2-2i\omega-5}e^{i\omega t}}_{\text{integrand}}d\omega \cancel{= \frac{2\pi i}{2\pi}\sum_{\text{POLES}}\text{RES integrand}}$$

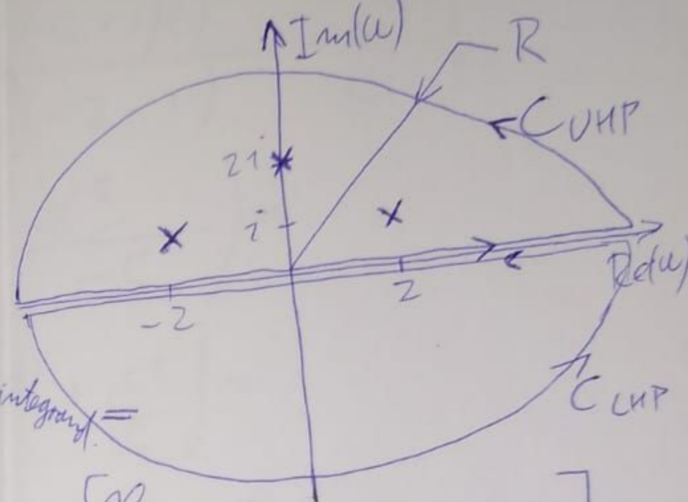
2015P2Q5(II)

integrand $= \dfrac{-i\exp(i\omega t)}{(\omega^2-2i\omega-5)(\omega-2i)} =$

$$\omega^2-2i\omega-5=(\omega-i)^2-4=((\omega-i)-2)((\omega-i)+2)$$

$$= \frac{-i\exp(i\omega t)}{((\omega-i)-2)((\omega-i+2))(\omega-2i)}$$

POLES: $\omega = 2+i$
$\omega = -2+i$
$\omega = 2i$



IF $t<0$:

~~$\lim_{R\to\infty}$ integrand~~

$\lim_{R\to\infty}$ INTEGRAND ON SEMICIRCLE PART OF C_{LHP} $=0$

$$\oint_{C_{LHP}} \frac{-i\exp(i\omega t)}{(\omega^2-2i\omega-5)(\omega-2i)}\,d\omega = 2\pi i \sum_{\text{POLES}} \text{RES integrand} =$$

$$=0$$

$$\to y(t)=\frac{1}{2\pi}\left[\int_{+\infty}^{-\infty}\text{integrand}\,d\omega+\int_{\substack{\text{SEMICIRCLE}\\\text{PART OF}\\ C_{LHP}}}\text{integrand}\,d\omega\right]=$$

GOES TO 0 EXPONENTIALLY (OR NEAR EXPONENTIALLY)
⇓
WHOLE INTEGRAL $=0$ WITH SEMICIRCLE PART OF LHP AS CONTOUR

$= 0$

$$=\frac{1}{2\pi}\oint_{C_{LHP}}\text{integrand}\,d\omega=0$$

$$\Downarrow$$

$$y(t<0)=0$$

IF $t>0$:

$$y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-i\exp(i\omega t)}{(\omega-i-2)(\omega-i+2)(\omega-2i)}=$$

$$=\frac{1}{2\pi}\lim_{R\to\infty}\int_{-R}^{R}\frac{-i\exp(i\omega t)\,d\omega}{(\omega-i-2)(\omega-i+2)(\omega-2i)}=$$

$$=\frac{1}{2\pi}\oint_{C_{UHP}}\frac{-i\exp(i\omega t)\,d\omega}{(\omega-i-2)(\omega-i+2)(\omega-2i)}-\int_{\substack{\text{SEMICIRCLE}\\\text{PART OF}\\ C_{UHP}}}\frac{1}{2\pi}\frac{-i\exp(i\omega t)\,d\omega}{(\omega-i-2)(\omega-i+2)(\omega-2i)}$$

INTEGRAND GOES TO 0 FASTER THAN ARCLENGTH OF SEMICIRCLE GOES TO ∞. $\Rightarrow$ INTEGRAL $=0$ AS $R\to\infty$

2015 P2 Q5 (III)

$$\Rightarrow y(t) = \frac{1}{2\pi} \oint_{C_{UHP}} \overbrace{\frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)}}^{\text{integrand}} d\omega = \frac{1}{\cancel{2\pi}} \cancel{2\pi} i \sum_{\text{POLES}} \text{RES}$$

$$i \sum_{\text{POLES}} \text{integrand} = \left.\frac{\exp(i\omega t)}{(\omega - i + 2)(\omega - 2i)}\right|_{\omega = 2+i} + \left.\frac{\exp(i\omega t)}{(\omega - i - 2)(\omega - 2i)}\right|_{\omega = -2+i} + \left.\frac{\exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)}\right|_{\omega = 2i} =$$

$$= \frac{\exp(i(2+i)t)}{4(2-i)} + \frac{\exp(i(-2+i)t)}{-4(-2-i)} + \frac{\exp(i(2i)t)}{(i-2)(i+2)} =$$

$$= e^{-t}\left(\frac{e^{2it}}{4(2-i)} + \frac{e^{-2it}}{4(2+i)} + \frac{e^{-t}}{-5}\right) =$$

$$= e^{-t}\left[\left(\frac{1}{4(2-i)} + \frac{1}{4(2+i)}\right) 2i \sin(2t) + \frac{e^{-t}}{5}\right] =$$

$$= e^{-t}\left[\frac{2+i+2-i}{4 - -1} 2i \sin(2t) + \frac{e^{-t}}{5}\right] = e^{-t} \frac{1}{5}\left(8i \sin(2t) + e^{-t}\right)$$

SO WE HAVE:

$$y(t) = \begin{cases} 0 & \text{FOR } t < 0 \\ e^{-t} \frac{1}{5}\left(8i \sin(2t) + e^{-t}\right) & \text{FOR } t > 0 \end{cases}$$

SHOULDN'T YOU GET A REAL ANSWER?

(b)(ii) FOR $t < 0$:

$$\ddot{y} + 2\dot{y} = f(t) \qquad \dot{y} = Y$$

$$\dot{Y} + 2Y = f(t)$$

~~f(t) IS A SOLUTION TO A FIRST ORDER~~

~~f(t) IS IN THE FORM: $f(t) = A \exp(i\beta t)$~~

$$y(t<0) = 0 \Rightarrow \dot{y}(t<0) = 0 \Rightarrow \ddot{y}(t<0) = 0$$

$$\Rightarrow f(t) = 0$$

~~SEEMS WRONG~~