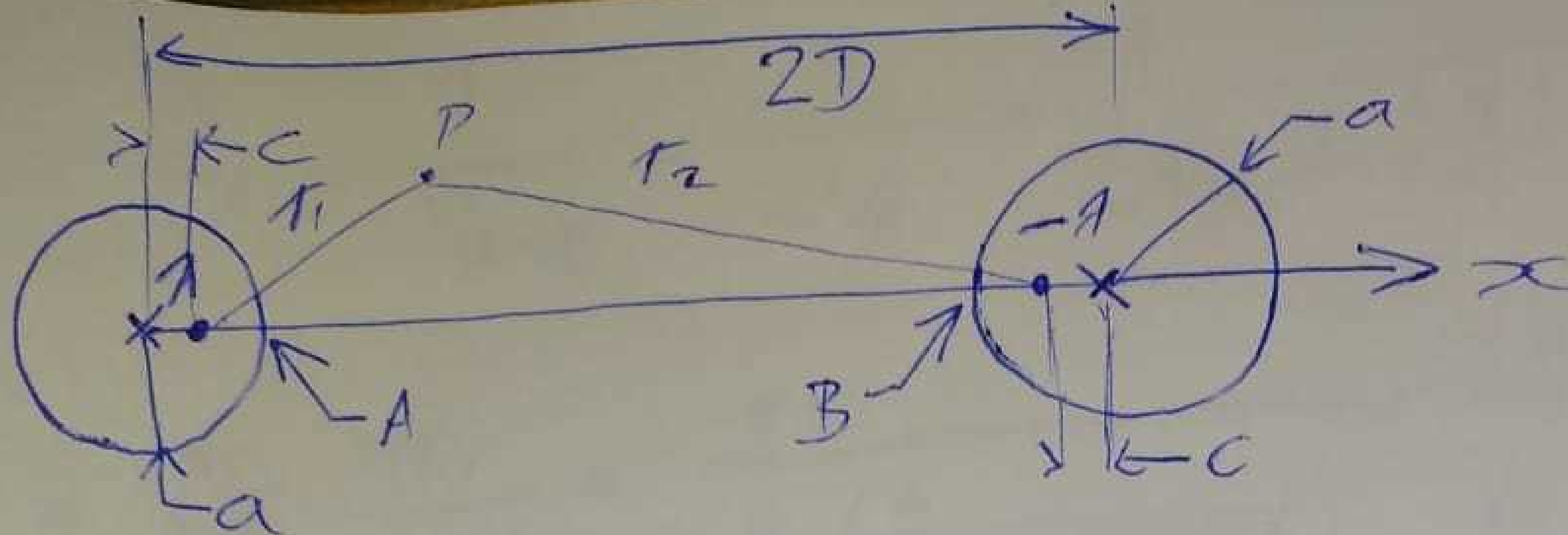

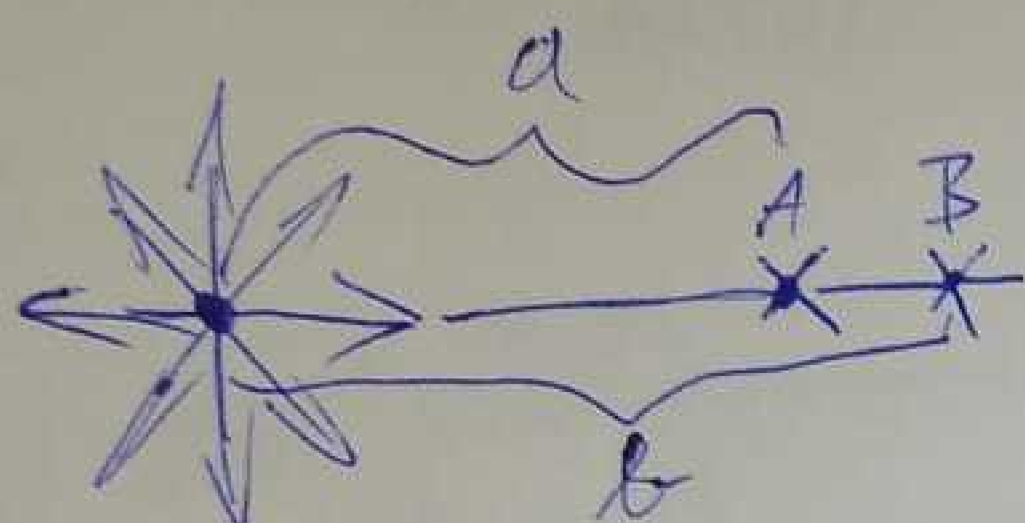

$$V_A - V_B = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left(\ln b + \ln\left(\frac{1}{a}\right)\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{1}{a}\right) + C$$

$$V @ P = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{1}{r_1}\right) + \frac{(-\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{1}{r_2}\right) + C$$

DON'T CARE ABOUT C:

$$= \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\frac{1}{r_1} - \ln\frac{1}{r_2}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1}$$

RECALL:

$$a^2 = cb$$

REWRITE:

$$a^2 = c(2D - c)$$

$$c^2 - 2Dc + a^2 = 0$$

$$c = \frac{2D \pm \sqrt{4D^2 - 4\cdot 1\cdot a^2}}{2} = D \pm \sqrt{D^2 - a^2}$$

TAKE SMALLER ROOT (OTHER IS OTHER IMAGE CHARGE)

$$c = D - \sqrt{D^2 - a^2}$$

AT A: $$\frac{r_2}{r_1} = \frac{(\overbrace{2D - c}^{b}) - a}{a - c} = \frac{b - a}{a - c} = \frac{\frac{a^2}{c} - a}{a - c} = \frac{a^2 - ac}{ac - c^2} = \frac{a(a-c)}{c(a-c)} = \frac{a}{c}$$

AT B: $$\frac{r_2}{r_1} = \frac{a - c}{b - (a - c) - c} = \frac{a - c}{b - a}$$

COMPARE THESE TWO

$$\hookrightarrow V_A = -V_B \Rightarrow V_A - V_B = 2V_A$$

$$C = \frac{Q}{V} = \frac{\lambda}{V_{A-B}} = \frac{\lambda}{2V_A} = \frac{\lambda}{2\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{c}\right)}$$

$$= \frac{\pi\varepsilon_0}{\ln\left(\frac{a}{c}\right)} = \frac{\pi\varepsilon_0}{\ln\left(\frac{a}{D-\sqrt{D^2-a^2}}\right)} = \frac{\pi\varepsilon_0}{\ln\frac{a}{D-\left(D\sqrt{1-\frac{a^2}{D^2}}\right)}}$$

$$\approx \frac{\pi\varepsilon_0}{\ln\frac{a}{D-D\left(1-\frac{1}{2}\frac{a^2}{D^2}\right)}} = \frac{\pi\varepsilon_0}{\ln\frac{a}{\frac{1}{2}\frac{a^2}{D}}} =$$

$$= \frac{\pi\varepsilon_0}{\ln\frac{2D}{a}}$$