4.5 Basic properties of the Fourier transform

Linearity:

$$g(x) = \alpha f(x) \quad \Leftrightarrow \quad \tilde{g}(k) = \alpha \tilde{f}(k) \tag{1}$$

$$h(x) = f(x) + g(x) \quad \Leftrightarrow \quad \tilde{h}(k) = \tilde{f}(k) + \tilde{g}(k) \tag{2}$$

Rescaling (for real α):

$$g(x) = f(\alpha x) \quad \Leftrightarrow \quad \tilde{g}(k) = \frac{1}{|\alpha|}\tilde{f}\left(\frac{k}{\alpha}\right) \tag{3}$$

Shift/exponential (for real α):

$$g(x) = f(x-\alpha) \quad \Leftrightarrow \quad \tilde{g}(k) = \mathrm{e}^{-\mathrm{i}k\alpha}\tilde{f}(k) \tag{4}$$

$$g(x) = \mathrm{e}^{\mathrm{i}\alpha x} f(x) \quad \Leftrightarrow \quad \tilde{g}(k) = \tilde{f}(k-\alpha) \tag{5}$$

Differentiation/multiplication:

$$g(x) = f'(x) \quad \Leftrightarrow \quad \tilde{g}(k) = \mathrm{i}k\tilde{f}(k) \tag{6}$$

$$g(x) = xf(x) \quad \Leftrightarrow \quad \tilde{g}(k) = \mathrm{i}\tilde{f}'(k) \tag{7}$$

Duality:

$$g(x) = \tilde{f}(x) \quad \Leftrightarrow \quad \tilde{g}(k) = 2\pi f(-k) \tag{8}$$

i.e. transforming twice returns (almost) the same function

Complex conjugation and parity inversion (for real x and k):

$$g(x) = [f(x)]^* \quad \Leftrightarrow \quad \tilde{g}(k) = [\tilde{f}(-k)]^* \tag{9}$$

Symmetry:

$$f(-x) = \pm f(x) \quad \Leftrightarrow \quad \tilde{f}(-k) = \pm\tilde{f}(k) \tag{10}$$