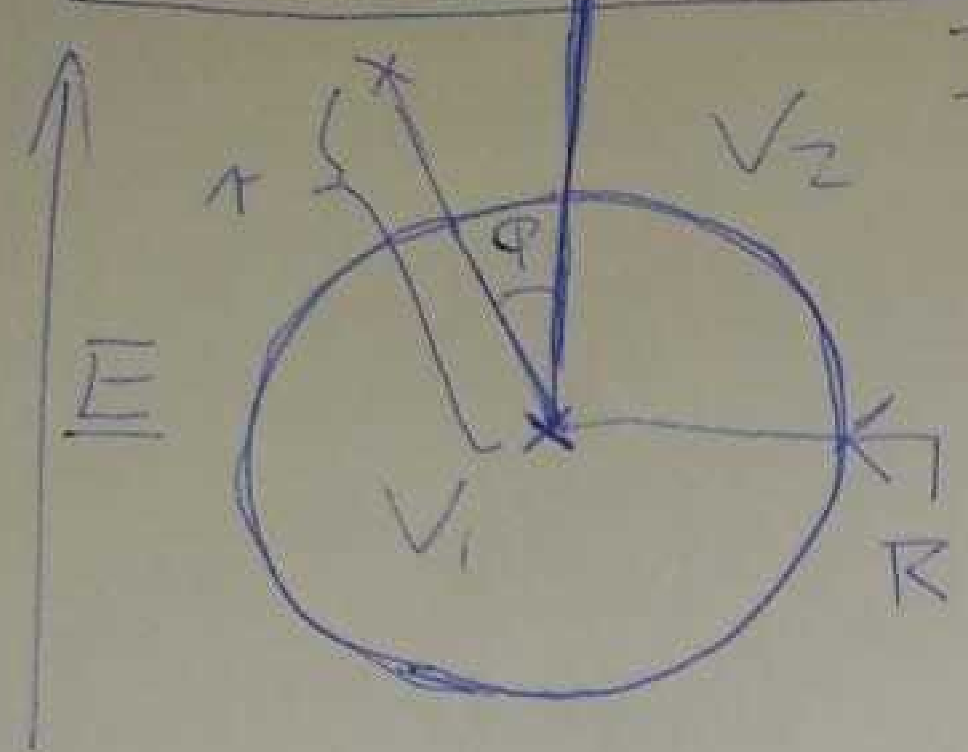
CYLINDER IN E FIELD



$D_\perp$ FIELD IS CONTINUOUS ACROSS BOUNDARY.

$$\varepsilon_1 \underbrace{\frac{\partial V_1}{\partial r}\Big|_{r=R}}_{E \text{ RADIAL INSIDE}} = \varepsilon_2 \underbrace{\frac{\partial V_2}{\partial r}\Big|_{r=R}}_{E \text{ RADIAL OUTSIDE}}$$

E FIELD IS EVERYWHERE FINITE $\Rightarrow$ CONTINUITY OF POTENTIAL.

$$V_1(R) = V_2(R)$$

FOR LARGE r, PRESENCE OF CYLINDER SHOULD BE NEGLIGIBLE:

$$\lim_{r \to \infty} V_2(r) = -E_0 r \cos\varphi$$

NO FREE CHARGE $\Rightarrow$ SOLVE LAPLACE

GENERAL LAPLACE SOL.:
$$V(r,\varphi) = A \ln r + K + \sum_{n=1}^{\infty} r^{n}\left(A_n \sin(n\varphi) + B_n \cos(n\varphi)\right) + \sum_{n=1}^{\infty} r^{-n}\left(C_n \sin(n\varphi) + D_n \cos(n\varphi)\right)$$

INSIDE CYLINDER: KILL $\ln$ & r^{-n} TERMS TO KEEP V FINITE AT $r=0$. SO:

$$V_1 = K_1 + \sum_{n=1}^{\infty} r^{n}\left(A_n \sin(n\varphi) + B_n \cos(n\varphi)\right)$$

OUTSIDE: KILL r^n TERMS & $\ln$ TERM TO KEEP THINGS FINITE EXCEPT $n=1$, TO SATISFY FAR FIELD CONDITION

$$V_2 = K_2 \underbrace{- E_0}_{B_1} r \cos\varphi + \sum_{n=1}^{\infty} r^{-n}\left(C_n \sin(n\varphi) + D_n \cos(n\varphi)\right)$$

CONTINUITY OF V AT $r=R$:

$$K_1 + \sum_{n=1}^{\infty} R^{n}\left(A_n \sin(n\varphi) + B_n \cos(n\varphi)\right) = K_2 - E_0 r \cos\varphi + \sum_{n=1}^{\infty} R^{-n}\left(C_n \sin(n\varphi) + D_n \cos(n\varphi)\right)$$

UNIQUENESS OF SERIES:

CONSTANT: $K_1 = K_2$ (WE DON'T CARE TBH)

$$A_n R^n = \frac{C_n}{R^n}$$

SIN TERMS

$n \neq 1$ COS TERMS:

$$B_n R^n = \frac{D_n}{R^n} \quad \text{FOR } n \neq 1$$

$$R B_1 = -E_0 R + \frac{D_1}{R}$$

$n=1$ COS TERMS

$D_\perp$ CONTINUITY CONDITIONS, AT $r = R$

$$\varepsilon_1 = \varepsilon_0 \varepsilon_r \qquad \varepsilon_2 = \varepsilon_0$$

$$\varepsilon_r \sum_{n=1}^{\infty} n R^{n-1} \left(A_n \sin(n\varphi) + B_n \cos(n\varphi)\right) =$$

$$= -E_0 \cos\varphi + \sum_{n=1}^{\infty} (-n) R^{-n-1} \left(C_n \sin(n\varphi) + D_n \cos(n\varphi)\right)$$

EQUATING SIN TERMS:

$$\varepsilon_r n R^{n-1} A_n = -n R^{-n-1} C_n$$

$n \neq 1$ COS TERMS:

$$\varepsilon_r n R^{n-1} B_n = -n R^{-n-1} D_n$$

$n = 1$ COS TERMS:

$$\varepsilon_r B_1 = -E_0 - D_1 R^{-2}$$

COMPARE THESE WITH EQUATIONS FROM CONTINUITY OF V, $\varepsilon_r \geqslant 1$:

$$A_n = C_n = 0$$

FOR $n \neq 1$:

$$B_n = D_n = 0$$

SURVIVING EQS:

$$R B_1 = -E_0 R + \frac{D_1}{R} \qquad \varepsilon_r B_1 = -E_0 - D_1 R^{-2}$$

$$B_1 = -E_0 + D_1 R^{-2} \qquad \cancel{D_1 = \varepsilon_r B_1 + E_0}$$

$$D_1 R^{-2} = -E_0 - \varepsilon_r B_1$$

$$B_1 = -E_0 + -E_0 - \varepsilon_r B_1$$

$$(\varepsilon_r + 1) B_1 = -2E_0$$

$$B_1 = -\frac{2E_0}{\varepsilon_r + 1}$$

$$D_1 = -E_0 R^2 - \varepsilon_r B_1 R^2$$

$$= -E_0 R^2 - \varepsilon_r \frac{-2E_0}{\varepsilon_r + 1} R^2$$

$$= -E_0 R^2 \left(1 + \varepsilon_r \frac{-2}{\varepsilon_r + 1}\right)$$

$$= -E_0 R^2 \left(\frac{\varepsilon_r + 1}{\varepsilon_r + 1} + \frac{-2\varepsilon_r}{\varepsilon_r + 1}\right)$$

$$= -E_0 R^2 \left(\frac{1 - \varepsilon_r}{1 + \varepsilon_r}\right)$$

$$= \frac{\varepsilon_r - 1}{\varepsilon_r + 1} E_0 R^2 = D_1$$

GET V_1 (POT. INSIDE)

$$V_1 = r\,\frac{-2E_0}{\varepsilon_r+1}\cos\varphi = -\frac{2E_0}{\varepsilon_r+1}\,r\cos\varphi \quad \left(= \frac{-2E_0}{\varepsilon_r+1}\,\xi \right)$$

ξ BEING THE COORD AXIS PARALLEL W/ FIELD.

& OUTSIDE:

$$V_2 = r^{-1}\,\frac{\varepsilon_r-1}{\varepsilon_r+1}\,E_0 R^2\cos\varphi - E_0 r\cos\varphi$$

$$= \frac{\varepsilon_r-1}{\varepsilon_r+1}\,\frac{E_0 R^2}{r}\cos\varphi - E_0 r\cos\varphi$$

$$\underline{E}_{IN} = -\nabla V_1 = -\frac{dV_1}{d\xi}\,\hat{\underline{\xi}} = \frac{2E_0}{\varepsilon_r+1}\,\hat{\underline{\xi}}$$

$\Rightarrow$ UNIFORM FIELD INSIDE.