2012P205 (I)

$$\hat{h}(k) = \hat{f}(k)\hat{g}(k) = \int_{-\infty}^{\infty} f(y) e^{-iky} dy \int_{-\infty}^{\infty} g(z) e^{-ikz} dz =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y) g(z) e^{-iky} e^{-ikz} dy\, dz =$$

$z = x - y \rightarrow dz = dx$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y) g(x-y) e^{-iky} e^{-ikx} e^{iky} dy\, dx =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y) g(x-y) dy\, e^{-ikx} dx =$$

$$= \int_{-\infty}^{\infty} h(x) e^{-ikx} dx \quad \checkmark$$

$$\rightarrow h(x) = \int_{-\infty}^{\infty} f(y) g(x-y) dy$$

$$h(x) = f(x) * f(x) = e^{-|x|} * e^{-|x|} = \int_{-\infty}^{\infty} e^{-|y|} e^{-|x-y|} dy =$$

$$= \begin{cases} \text{IF } x<0 \int_{-\infty}^{x} e^{y} e^{y-x} dy + \int_{x}^{0} e^{y} e^{-(y-x)} dy + \int_{0}^{\infty} e^{-y} e^{y-x} dy \\ \text{IF } x>0 \int_{-\infty}^{0} e^{y} e^{-(x-y)} dy + \int_{0}^{x} e^{-y} e^{-(x-y)} dy + \int_{x}^{\infty} e^{-y} e^{-(y-x)} dy \end{cases} =$$

$$= \begin{cases} x<0 \quad e^{-x}\left[\frac{1}{2}e^{2y}\right]_{-\infty}^{x} + e^{x}[y]_{x}^{0} + e^{+x}\left[-\frac{1}{2}e^{-2y}\right]_{0}^{\infty} = \\ x>0 \quad e^{-x}\left[+\frac{1}{2}e^{+2y}\right]_{-\infty}^{0} + e^{-x}[y]_{0}^{x} + e^{x}\left[-\frac{1}{2}e^{-2y}\right]_{x}^{\infty} = \end{cases}$$

2018 P2 Q5 (II)

$$x<0: \; e^{-x}\tfrac{1}{2}e^{2x} - e^{x}x + \tfrac{1}{2}e^{x} = (1-x)e^{x} \quad \text{FOR } x<0$$

$$x>0: \; e^{-x}\tfrac{1}{2} + e^{-x}x + e^{x}\tfrac{1}{2}e^{-2x} = (1+x)e^{-x} \quad \text{FOR } x>0$$

$$h(x) = f(x) * f(x) = \mathcal{F}^{+}\left[\tilde{h}(\xi)\right] = \mathcal{F}^{+}\left[\tilde{f}(\xi)^2\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\xi)^2 e^{i\xi x}\,d\xi$$

(BY CONVOLUTION THEOREM)

$$\tilde{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-i\xi x}\,dx = \int_{-\infty}^{\infty} e^{-|x|}e^{-i\xi x}\,dx = \int_{-\infty}^{0} e^{x}e^{-i\xi x}\,dx +$$

$$+ \int_{0}^{\infty} e^{-x}e^{-i\xi x}\,dx = \int_{-\infty}^{0} e^{(1-i\xi)x}\,dx + \int_{0}^{\infty} e^{-(1+i\xi)x}\,dx =$$

(layout.)

$$= \frac{1}{1-i\xi}\left[e^{(1-i\xi)x}\right]_{-\infty}^{0} + \frac{-1}{1+i\xi}\left[e^{-(1+i\xi)x}\right]_{0}^{\infty} = \frac{1}{1-i\xi} + \frac{-1}{1+i\xi} =$$

$$= \frac{1+i\xi}{(1-i\xi)(1+i\xi)} - \frac{1-i\xi}{(1-i\xi)(1+i\xi)} = \frac{2}{1+\xi^2}$$

PUT THIS BACK TO:

$$h(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overbrace{f(\xi)^2}^{\downarrow} e^{i\xi x}\,d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{4}{(1+\xi^2)^2} e^{i\xi x}\,d\xi =$$

$$= \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{e^{i\xi x}}{(1+\xi^2)^2}\,d\xi \quad \text{AS REQUIRED.}$$

LAYOUT IMPROVEMENT SHOULD BE:
- MORE SPACING BETWEEN DIFFERENT BITS?