TWO SPIN-HALF PARTICLES

NOTATION:

$$|\uparrow\rangle \otimes |\uparrow\rangle = |\uparrow\rangle|\uparrow\rangle = |\uparrow\uparrow\rangle$$
$$|\uparrow\rangle \otimes |\downarrow\rangle = |\uparrow\rangle|\downarrow\rangle = |\uparrow\downarrow\rangle$$
$$|\downarrow\rangle \otimes |\uparrow\rangle = |\downarrow\rangle|\uparrow\rangle = |\downarrow\uparrow\rangle$$
$$|\downarrow\rangle \otimes |\downarrow\rangle = |\downarrow\rangle|\downarrow\rangle = |\downarrow\downarrow\rangle$$

$$\vec{S}^{(AB)} = \vec{S}^{(A)} \otimes I^{(B)} + I^{(A)} \otimes \vec{S}^{(B)} = \vec{S}^{(A)} + \vec{S}^{(B)}$$

$$S_z^{(AB)}|\uparrow\uparrow\rangle = \left(S_z^{(A)} + S_z^{(B)}\right)|\uparrow\rangle \otimes |\uparrow\rangle =$$
$$= \left(S_z^{(A)}|\uparrow\rangle\right) \otimes |\uparrow\rangle + |\uparrow\rangle \otimes \left(S_z^{(B)}|\uparrow\rangle\right) =$$
$$= \left(\frac{\hbar}{2}|\uparrow\rangle\right) \otimes |\uparrow\rangle + |\uparrow\rangle \otimes \left(\frac{\hbar}{2}|\uparrow\rangle\right) =$$
$$= \left(\frac{\hbar}{2} + \frac{\hbar}{2}\right)|\uparrow\rangle \otimes |\uparrow\rangle = \hbar|\uparrow\uparrow\rangle$$

$$S_z^{(AB)}|\downarrow\downarrow\rangle = \left(S_z^{(A)} + S_z^{(B)}\right)|\downarrow\rangle \otimes |\downarrow\rangle =$$
$$= \left(S_z^{(A)}|\downarrow\rangle\right) \otimes |\downarrow\rangle + |\downarrow\rangle \otimes \left(S_z^{(B)}|\downarrow\rangle\right) =$$
$$= -\frac{\hbar}{2}|\downarrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \times \left(-\frac{\hbar}{2}|\downarrow\rangle\right) =$$
$$= \left(-\frac{\hbar}{2} - \frac{\hbar}{2}\right)|\downarrow\rangle \otimes |\downarrow\rangle = -\hbar|\downarrow\downarrow\rangle$$

$$S_z^{(AB)}|\uparrow\downarrow\rangle = \left(S_z^{(A)} + S_z^{(B)}\right)|\uparrow\rangle \otimes |\downarrow\rangle =$$
$$= \left(S_z^{(A)}|\uparrow\rangle\right) \otimes |\downarrow\rangle + |\uparrow\rangle \otimes \left(S_z^{(B)}|\downarrow\rangle\right) =$$
$$= \left(\frac{\hbar}{2}|\uparrow\rangle\right) \otimes |\downarrow\rangle + |\uparrow\rangle \otimes \left(-\frac{\hbar}{2}|\downarrow\rangle\right) =$$
$$= \left(\frac{\hbar}{2} - \frac{\hbar}{2}\right)\left(|\uparrow\rangle \otimes |\downarrow\rangle\right) = 0$$

THESE COMBINATION OF SPINS ABOVE ARE EIGENSTATES OF S_z, BUT NOT ALL OF THEM ARE EIGENSTATES OF S^2.

$$\vec{S}^2 = \left(\vec{S}^{(AB)}\right)^2 = \left(\vec{S}^{(A)} \otimes I^{(B)} + I^{(A)} \otimes \vec{S}^{(B)}\right)^2 =$$
$$= \left(\vec{S}^{(A)}\right)^2 \otimes I^{(B)} + 2\vec{S}^{(A)} \otimes \vec{S}^{(B)} + I^{(A)} \otimes \left(\vec{S}^{(B)}\right)^2$$

$$\left(\vec{S}^{(A)}\right)^2 = \frac{\hbar^2}{4}\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2\right) = \frac{3}{4}\hbar^2 I$$

REWRITE RESULT FOR $\vec{S}^2$:

$$\vec{S}^2 = \frac{3}{4}\hbar^2 I^{(A)} \otimes I^{(B)} + 2\vec{S}^{(A)} \otimes \vec{S}^{(B)} + \frac{3}{4}\hbar^2 I^{(A)} \otimes I^{(B)} =$$

$$= \frac{\hbar^2}{4}\left(6 I \otimes I + 2\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right)\right)$$

$$\vec{S}^2 |\uparrow\uparrow\rangle = \frac{\hbar^2}{4}\left(6 I \otimes I + 2\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right)\right) \ldots$$

$$\ldots |\uparrow\rangle \otimes |\uparrow\rangle = \frac{\hbar^2}{4}\left(6|\uparrow\uparrow\rangle + 2|\downarrow\downarrow\rangle + 2\left(\frac{1}{2i} + \frac{1}{2i}\right)^2 |\downarrow\downarrow\rangle + 2|\uparrow\uparrow\rangle\right) =$$

$$= \frac{\hbar^2}{4} 8 |\uparrow\uparrow\rangle = 2\hbar^2 |\uparrow\uparrow\rangle = \hbar^2 1(1+1)|\uparrow\uparrow\rangle$$

$$\vec{S}^2 |\downarrow\downarrow = 2\hbar^2 |\downarrow\downarrow\rangle$$

$$\vec{S}^2 |\uparrow\downarrow\rangle = \vec{S}^2 |\downarrow\uparrow\rangle = \hbar^2 (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$\vec{S}^2 \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) = 2\hbar^2 \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$ $\frac{1}{\sqrt{2}}$ FOR NORMALIZ.

$$\vec{S}^2 \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = 0$$

TRIPLET STATES TO SPIN QUANTUM NUMBER: $s = 1$

MAGNETIC SPIN QUANTUM NUM: $m_s = -1, 0, 1$

$$|1, -1\rangle = |\downarrow\downarrow\rangle$$

$$|1, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|1 + 1\rangle = |\uparrow\uparrow\rangle$$

SINGLET STATE: FOR $s = 0$, $m_s = 0$:

$$|0, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$