2015P2Q10(I)

(a) R: ROTATION BY $\frac{2\pi}{6} = \frac{1}{3}\pi$ IN A DEFINED DIRECTION ✓ ✓

m_1: REFLECTION USING ONE OF THE SYMMETRY AXIS ✓ axes

$I = RR^{-1}$ $R = R$ $R^2 = R^2$ $R^3 = R^3$ $R^4 = R^4$ $R^5 = R^5$

$m_1 = m_1$ $m_2 = Rm_1$ $m_3 = R^2 m_1$ $m_4 = R^3 m_1$ $m_5 = R^4 m_1$

$m_6 = R^5 m_1$ ✓

(b) ORDER 2 SUBGROUPS: $SG_1 = \{I, R^3\}$ $SG_4 = \{I, m_3\}$

$SG_2 = \{I, m_1\}$ ✓ $SG_5 = \{I, m_4\}$

$SG_3 = \{I, m_2\}$ $SG_6 = \{I, m_5\}$

$SG_7 = \{I, m_6\}$ ✓

ORDER 3 SUBGROUPS: $SG8 = \{I, R^2, R^4\}$ ✓

YES, SG8 IS CYCLIC. are they not all cyclic? prime order grp has to be cyclic

(c) FOR THE ROTATION ELEMENTS OF THE GROUP, USE ROTATION MATRIX:

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$ but what is θ?? : θ IS ANGLE OF ROTATION

FOR m_1, USE:

$$m_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$ ✓

FOR $m_{n \neq 1}$, USE R AND m_1 TOGETHER, AS SHOWN IN THE ANSWER TO (a)

$$R = \begin{pmatrix} \cos\frac{1}{3}\pi & \sin\frac{1}{3}\pi \\ -\sin\frac{1}{3}\pi & \cos\frac{1}{3}\pi \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}, \quad m_2 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

2015P2Q10(II)

(d) $m_4^{-1} = m_4$

$$D(m_4) = \left[\begin{array}{cccccc|cccccc}
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
\hline
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0
\end{array}\right]$$

we have different cases + you have not explained this so I can not comment.

$$\left[D(m_4)\right]^n = \begin{cases} D(m_4) & \text{IF } n \text{ IS ODD} \\ I & \text{IF } n \text{ IS EVEN} \end{cases}$$

(e)

PART (d): $C_d = \{12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

this is not a trace.

PART (c): you just have to find the characters ie the traces.

eg $\mathrm{tr}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} = 0$

$\mathrm{tr}\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix} = 1.$ etc.