2016P2Q5 (I) $\tilde{y}(\omega) = \dfrac{-\omega \tilde{f}(\omega)}{\omega^3 - i\omega^2 + 4\omega - 4i}$

(a)

$$f(t) = y'(t) \Longleftrightarrow \tilde{f}(\omega) = i\omega\, \tilde{y}(\omega)$$

$$(\omega^3 - i\omega^2 + 4\omega - 4i)\,\tilde{y}(\omega) = -\omega \tilde{f}(\omega)$$

$$\Updownarrow$$

$$(iy''' + iy'' + 4(-i)y' - 4iy) = if'$$

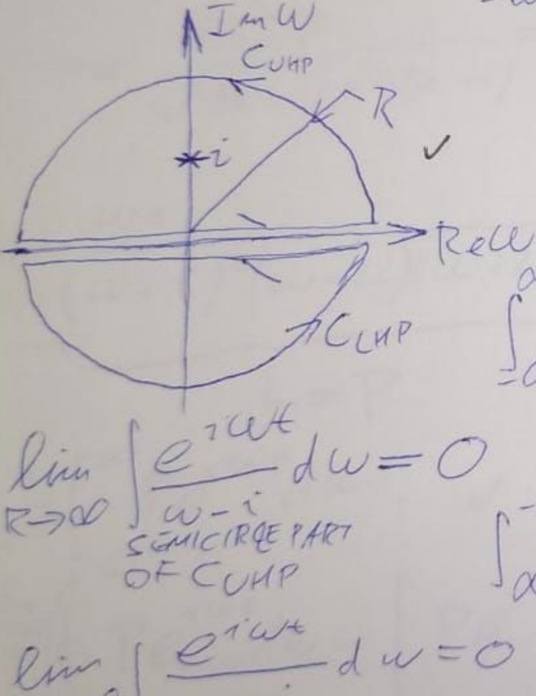
$$y''' + y'' - 4y' - 4y = f' \checkmark$$

$\leftarrow$ THIRD ORDER ODE

(b)

$$f(t) = \text{IFT}\left[\tilde{f}(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-i}{\omega - i} e^{i\omega t}\, d\omega =$$

POLE: $\omega = i$



$$= \frac{-i}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\omega - i} e^{i\omega t}\, d\omega \quad \checkmark$$

$$\int_{-\infty}^{\infty} \frac{1}{\omega - i} e^{i\omega t}\, d\omega = \lim_{R\to\infty}\left(\oint_{C_{UHP}} \frac{e^{i\omega t}}{\omega - i}\, d\omega - \int_{\substack{\text{SEMICIRCLE}\\ \text{PART OF } C_{UHP}}} \frac{e^{i\omega t}}{\omega - i}\, d\omega\right)$$

$$\lim_{R\to\infty}\int_{\substack{\text{SEMICIRCLE PART}\\ \text{OF } C_{UHP}}} \frac{e^{i\omega t}}{\omega - i}\, d\omega = 0 \quad \text{FOR } t > 0 \text{ BY JORDAN'S LEMMA.}$$

$$\int_{\infty}^{-\infty} \frac{1}{\omega - i} e^{i\omega t}\, d\omega = \lim_{R\to\infty}\left(\oint_{C_{LHP}} \frac{e^{i\omega t}}{\omega - i}\, d\omega - \int_{\substack{\text{SEMICIRCLE}\\ \text{PART OF } C_{LHP}}} \frac{e^{i\omega t}}{\omega - i}\, d\omega\right)$$

$$\lim_{R\to\infty}\int_{\substack{\text{SEMICIRCLE}\\ \text{PART OF}\\ C_{LHP}}} \frac{e^{i\omega t}}{\omega - i}\, d\omega = 0 \quad \text{FOR } t < 0 \text{ BY JORDAN'S LEMMA}$$

This would be clearer done bit by bit.

FOR $t > 0$:

$$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{\omega - i} e^{i\omega t}\, d\omega = \lim_{R\to\infty}\oint_{C_{UHP}} \frac{e^{i\omega t}}{\omega - i}\, d\omega = 2\pi i \left(\text{RES}\ \frac{e^{i\omega t}}{\omega - i}\right)\Bigg|_{\omega = i}$$

✓

FOR $t < 0$:

$$\int_{\infty}^{\infty} \frac{1}{\omega - i} e^{i\omega t}\, d\omega = \lim_{R\to\infty}\oint_{C_{LHP}} \frac{e^{i\omega t}}{\omega - i}\, d\omega = 0 \quad \text{(NO POLES IN } C_{LHP}\text{)}$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{\omega - i} e^{i\omega t}\, d\omega = 0$$

2016P2Q5(II)

$$2\pi i\left(\text{RES}\,\frac{e^{i\omega t}}{\omega-i}\right)\Bigg|_{\omega=i} = 2\pi i\, e^{i i t} = 2\pi i\, e^{-t}$$

$$f(t) = \frac{-i}{2\pi}\,2\pi i\, e^{-t} = e^{-t} \quad \text{FOR } t>0$$

$$f(t) = 0 \quad \text{FOR } t<0 \quad ✓$$

(c)

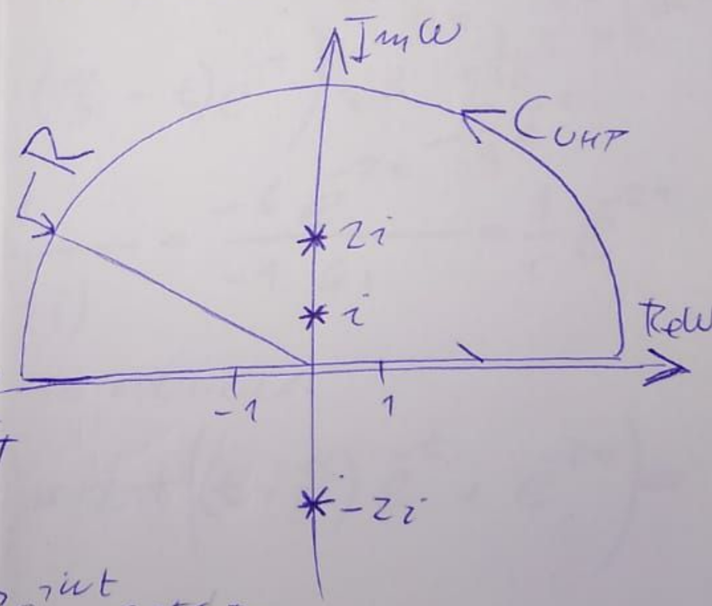
$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{y}(\omega)e^{i\omega t}\,d\omega = \begin{cases} \text{FOR } t<0 & \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} 0\, e^{i\omega t}\,d\omega = 0 \\ \text{FOR } t>0 & \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\underbrace{\frac{-\omega\frac{-i}{\omega-i}}{\omega^3-i\omega^2+4\omega-4i}}\, e^{i\omega t}\,d\omega \end{cases}$$

$$= \frac{\omega\frac{i}{\omega-i}}{\omega^2(\omega-i)+4(\omega-i)} = \frac{\omega i}{\omega^2(\omega-i)^2+4(\omega-i)^2} = \frac{\omega i}{(\omega-i)^2(\omega^2+4)} ✓ =$$

$$= \underbrace{\frac{\omega i}{(\omega-i)^2(\omega-2i)(\omega+2i)}}_{L=P} \Rightarrow$$

SIMPLE POLES AT: $\omega = \pm 2i$

DOUBLE POLE AT: $\omega = i$



$$\frac{1}{2\pi}\int_{-\infty}^{\infty} P e^{i\omega t}\,d\omega = ✓$$

$$\stackrel{\lim_{R\to\infty}}{=}\left(\oint_{C_{UHP}} P e^{i\omega t}\,d\omega - \int_{\substack{\text{SEMICIRCLE}\\\text{PART OF}\\ C_{UHP}}} P e^{i\omega t}\,d\omega\right)\cdot\frac{1}{2\pi}$$

$\lim_{R\to\infty}\int_{\substack{\text{SEMICIRCLE}\\\text{PART OF}\\ C_{UHP}}} P e^{i\omega t}\,d\omega = 0$ BECAUSE $Pe^{i\omega t}$ GOES TO 0 FASTER IN THE UPPER HALF PLANE W/ $z>0$ THAN THE ARCLENGTH OF THE SEMICIRCLE GOES TO ∞ AS $R\to\infty$. too vague

SO WE HAVE: $\displaystyle\int_{-\infty}^{\infty} P e^{i\omega t}\,d\omega = \oint_{C_{UHP}} P e^{i\omega t}\,d\omega = 2\pi i \sum_{\substack{\text{POLES}\\\text{IN } C_{UHP}}} \text{RES}\left(P e^{i\omega t}\right)$ BY CAUCHY'S THEOREM

2016P2Q5(III) • CALCULATE THE RESIDUES

$$\underset{\omega=i}{RES}\left(Pe^{i\omega t}\right)=\lim_{\omega\to i}\frac{d}{d\omega}\left[\cancel{(\omega-i)^2}\frac{\omega i e^{i\omega t}}{\cancel{(\omega-i)^2}(\omega-2i)(\omega+2i)}\right]=$$

$$=\lim_{\omega\to i}\frac{d}{d\omega}\left[\frac{\omega i e^{i\omega t}}{\omega^2+4}\right]=i\lim_{\omega\to i}\frac{(\omega^2+4)(\omega e^{i\omega t})'-}{}$$

$$\frac{-\omega e^{i\omega t}(\omega^2+4)'}{(\omega^2+4)^2}=i\lim_{\omega\to i}\frac{(\omega^2+4)(\omega(it)e^{i\omega t}+e^{i\omega t})-}{}$$

$$\frac{-\omega e^{i\omega t}2\omega}{(\omega^2+4)^2}=i\lim_{\omega\to i}\left[(\omega(it)e^{i\omega t}+e^{i\omega t})-\frac{2\omega^2e^{i\omega t}}{(\omega^2+4)^2}\right]=$$

$$=i\left(i(it)e^{-t}+e^{-t}-\frac{2(i)^2e^{-t}}{3^2}\right)=$$

I lack a 3 here

PROBABLY SOME ALGEBRA ERROR KERRY THINKS:

$$=i\left((1-t)e^{-t}+\frac{2}{9}e^{-t}\right)=i\left(\left(\frac{11}{9}-t\right)e^{-t}\right)$$

$$\frac{1}{2}e^{-2t}+\frac{1}{18}e^{2t}+\frac{1}{3}te^{-t}-\frac{5}{9}e^{-t}$$

$$\underset{\omega=2i}{RES}\left(Pe^{i\omega t}\right)=\frac{(2i)i e^{-2t}}{(2i-i)^2(2i+2i)}=\frac{-4e^{-2t}}{-1\,4i}=\frac{1}{i}e^{-2t}$$

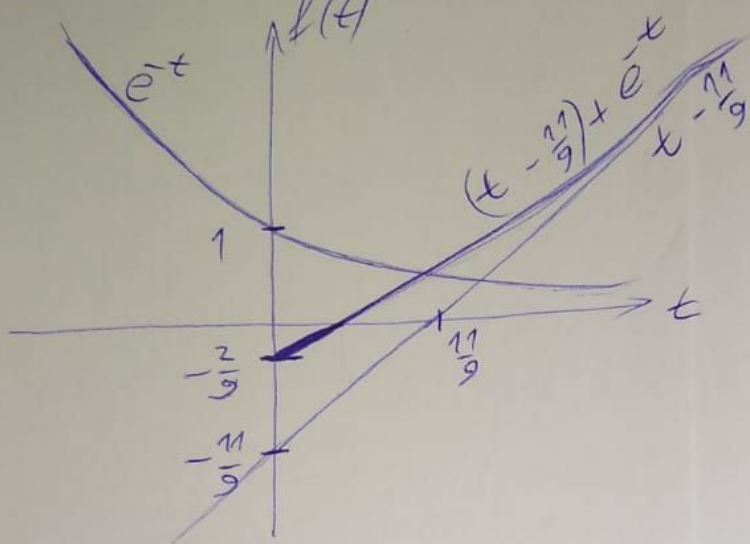
• PUT THE RESIDUES BACK TO FORMULA:

$$2\pi i\left(i\left(\frac{11}{9}-t\right)e^{-t}+\frac{1}{i}e^{-2t}\right)=2\pi\left(\left(t-\frac{11}{9}\right)e^{-t}+e^{-2t}\right)=$$

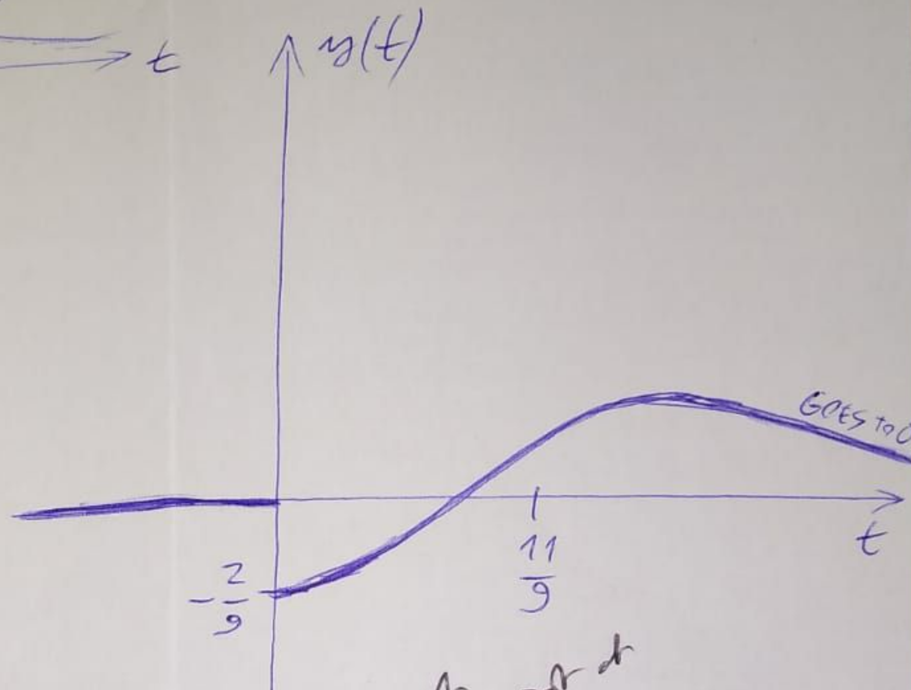
$$=2\pi e^{-t}\left(\left(t-\frac{11}{9}\right)+e^{-t}\right)$$

$$y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Pe^{i\omega t}d\omega=e^{-t}\left(\left(t-\frac{11}{9}\right)+e^{-t}\right)\quad \text{FOR } t>0$$

2016 P2 Q5 IV)



$$y(t) = \begin{cases} 0 & \text{for } t < 0 \\ e^{-t}\left(\left(t - \frac{11}{9}\right) + e^{-t}\right) & \text{for } t > 0 \end{cases}$$



This is not what I got at all
See me