2010P2Q9(I)

CONJUGACY CLASS: SET OF ELEMENTS OF G WHICH ARE CONJUGATE TO EACH OTHER, IE.:

$$g_2 = g g_1 g^{-1}, \; g \in G$$ ✓

$g_1, g_2 \in$ OF THE SAME CONJUGACY CLASS.

proof by contradiction: of what?

where is the contradiction?

ASSUME $g_1, g_2 \in C_1$

$g_2, g_3 \in C_2$

$g_1 \notin C_2; \; g_3 \notin C_1$

$g_1, g_2 \in G;$

C_1, C_2 ARE CONJUGACY CLASSES

BY DEF. OF CONJ. CLASS:

$g_2 g = g g_1 \longrightarrow g_1 = g^{-1} g_2 g$

$g_2 g = g g_3$

~~$g g_1 = g g_3 \nexists g$~~

g_1 & g_3 NOT IN SAME CONJUGACY CLASS, SO

if pf by # then do 1st and likely use contrapositive. Look for #

$\nexists g$ S.T. $g g_1 = g_3 g$

This seems rather involved

try doing the same in 2 lines

$$\underbrace{g g^{-1} g_2 g}_{g_1} = g_3 g$$

$g_2 g = g_3 g$

THIS IS TRUE, SO:

$\exists g$ S.T. $g g_1 = g_3 g \Rightarrow$

$\Rightarrow$ assumption was wrong: g_1 & g_3 MUST BE IN SAME CONJUGACY CLASS $\Rightarrow$ NO OVERLAP BETWEEN DIFFERENT CONJUGACY CLASSES (IE THEY ARE DISJOINT) $\Rightarrow$ THEY ARE UNIQUE.

2010P2Q9(II) IF G IS ABELIAN: WLOG CHOOSE ELEMENT g_1.

$$g_1 g = g g_1 \quad \forall g \in G$$

$$g_1 = g g_1 g^{-1} \quad \forall g \in G$$

$g_1 \sim g_2$

$g^{-1} g_1 g = g_2$

$\therefore g_1 g^{-1} g = g_2$

$g_1 = g_2$.

$g g_1 g^{-1}$ ALWAYS GIVES g_1, NO MATTER CHOICE OF g, SO g_1 IS IN ITS CONJUGACY CLASS ALONE.

CENTRE Z

- $Ig = gI \quad \forall g \in G \Rightarrow I$ FORMS ITS OWN CONJUGACY CLASS ✓

$\Rightarrow \boxed{I \in Z}$ (*)

- $r, q \in Z$: IS rq IN Z? better.

~~$(rq)g = q(rg) = q$~~ ~~ASSOCIATIVITY~~ IN G

What are you assuming? is this closure??

$$rqg = r(qg) = r(gq) = (rg)q = (gr)q = g(rq)$$

ASSOC. IN G. $q \in Z \Rightarrow qg = gq \; \forall g$ ✓ ASSOC. $r \in Z$, $rg = gr \; \forall g$ ASSOC.

$$(rq)g = g(rq)$$

$\Rightarrow rq$ IS IN ITS OWN CONJUGACY CLASS

$\Rightarrow rq \in Z \Rightarrow Z$ IS CLOSED. ✓ (**)

- $g \in G, r \in Z$ $\Rightarrow g^{-1} \in G$

$gr = rg$

$g = rgr^{-1}$

$r^{-1} g = g r^{-1} \longrightarrow r^{-1} = g r^{-1} g^{-1} \Rightarrow r^{-1}$ FORMS ITS OWN CONJUGACY CLASS $\Rightarrow r^{-1} \in Z \Rightarrow$ if $r \in Z \Rightarrow r^{-1} \in Z$ (***)

$r = g^{-1} r g$

$\Rightarrow r^{-1} = g^{-1} r^{-1} g$

$= g^{-1} r^{-1} g$

2010 P2 Q9 (III) (*), (**), (***) $\Rightarrow$ Z IS A SUBGROUP

associativity..

• if $r \in Z \Rightarrow rg = gr \ \forall g \in G$

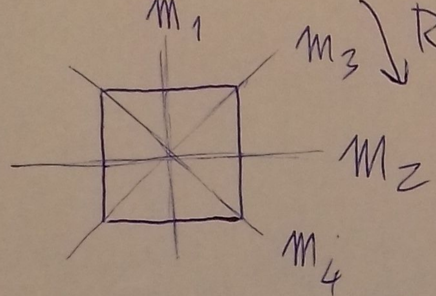
$Z \subseteq G$

$r t = t r \ \forall \ t \in Z$ $\forall$ not $\forall$

✓

$\Rightarrow Z$ IS ABELIAN SUBGROUP

• DESCRIBE THE GROUP D_4



m_1, m_2, m_3, m_4 : REFLECTION OF THE SQUARE ON THE RESPECTIVE AXIS DRAWN ON FIGURE ✓

R, R^2, R^3 : ROTATION BY 90°, 180°, 270° DEGREES RESPECTIVELY IN THE DIRECTION DRAWN. ✓✓

I : LEAVE THE SQUARE AS IT IS.

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓, $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ✓, $R^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $R^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ more usual

$m_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓, $m_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ✓, $m_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ✓, $m_4 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ✓

$m_3 \not\sim R$, BECAUSE:

$m_3 R \begin{smallmatrix} A & B \\ D & C \end{smallmatrix} = m_3 \begin{smallmatrix} D & A \\ C & B \end{smallmatrix} = \begin{smallmatrix} B & A \\ C & D \end{smallmatrix}$

~~$m_3 R \begin{smallmatrix} A & B \\ D & C \end{smallmatrix}$~~

REPRESENTS THE □

$\neq$ ✓

could use order of elements being different or do I and 6tg: $LIL^{-1} = r.s.r$ $I = r.$ #

This shows they do not commute not that they are not R conjugates

$R m_3 \begin{smallmatrix} A & B \\ C & D \end{smallmatrix} = R \begin{smallmatrix} D & B \\ C & A \end{smallmatrix} = \begin{smallmatrix} C & D \\ A & B \end{smallmatrix}$

OR: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ BECAUSE: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2010P2Q9(IV)

I	R	R^2	R^3	m_1	m_2	m_3	m_4
R	R^2	R^3	I	m_3	m_4	m_2	m_1
R^2	R^3	I	R	m_2	m_1	m_4	m_3
R^3	I	R	R^2	m_4	m_3	m_1	m_2
m_1	m_4	m_2	m_3	I	R^2	R	R^3
m_2	m_3	m_1	m_4	R^2	I	R^3	R
m_3	m_1	m_4	m_2	R	R^3	I	R^2
m_4	m_2	m_3	m_1	R^3	R	R^2	I

$I \in Z$: $gI = Ig$ ✓ $\forall g \in D_4$

$m_1 R \neq R m_1 \Rightarrow R \notin Z,\ m_1 \notin Z$

$R^2 g = g R^2 \ \forall g \in D_4$ (SEE GROUP TABLE ↑)

$\Rightarrow R^2 \in Z$

$R^3 m_2 \neq m_2 R^3 \Rightarrow R^3 \notin Z,\ m_2 \notin Z$

$m_3 R \neq R m_3 \Rightarrow m_3 \notin Z$

$m_4 R \neq R m_4 \Rightarrow m_4 \notin Z$

$Z = \{I, R^2\}$ ✓