

(a) $\ddot{y} + 2\dot{y} + 5y = f(t)$ $\quad | \cdot e^{-i\omega t} \quad | \int_{-\infty}^{\infty} dt$

$$\int_{-\infty}^{\infty} (\ddot{y} + 2\dot{y} + 5y) e^{-i\omega t} dt = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \tilde{f}(\omega)$$

$$\hookrightarrow \int_{-\infty}^{\infty} \ddot{y} e^{-i\omega t} dt = \left[\dot{y} e^{-i\omega t} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \dot{y} (-i\omega) e^{-i\omega t} dt =$$

BY THE ASSUMPTION THAT
THE RESPONSE OF THE SYSTEM $y(t)$
& SO THE DERIVATIVE $\dot{y}(t)$ DIES
AWAY AS $t \rightarrow \pm\infty$

$$= i\omega \left[\left[y e^{-i\omega t} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} y (-i\omega) e^{-i\omega t} dt \right] =$$

$$= (i\omega)^2 \int_{-\infty}^{\infty} y e^{-i\omega t} dt = -\omega^2 \tilde{y}$$

$$\hookrightarrow \int_{-\infty}^{\infty} 2\dot{y} e^{-i\omega t} dt = 2 \left[\left[y e^{-i\omega t} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} y (-i\omega) e^{-i\omega t} dt \right] =$$

$$= 2i\omega \tilde{y}$$

REWRITE EQUATION ABOVE:

$$-\omega^2 \tilde{y} + 2i\omega \tilde{y} + 5\tilde{y} = \tilde{f}(\omega)$$

$$\hookrightarrow \tilde{y} = \frac{-\tilde{f}(\omega)}{\omega^2 - 2i\omega - 5}$$

AS REQUIRED.

(b) (i) $\tilde{f}(\omega) = \frac{i}{\omega - 2i}$

$$y(t) = IF + [\tilde{y}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\tilde{f}(\omega)}{\omega^2 - 2i\omega - 5} e^{i\omega t} d\omega =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\frac{i}{\omega - 2i}}{\omega^2 - 2i\omega - 5} e^{i\omega t} d\omega$$

~~RES integrand~~
~~2 POLES~~
 - integrand

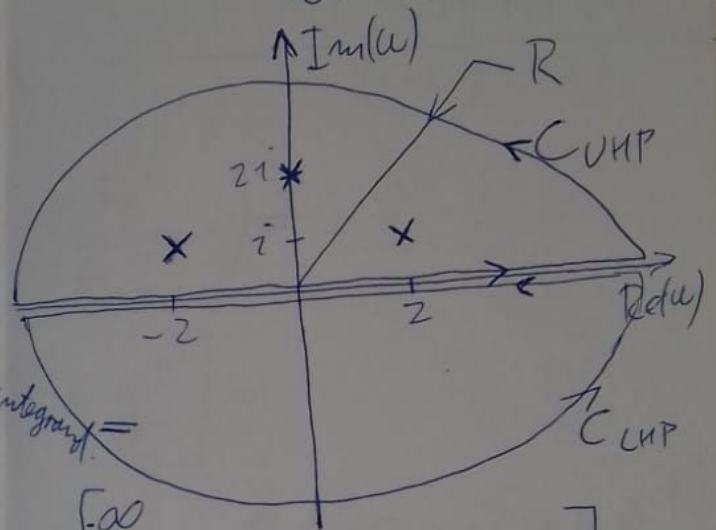
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$$\text{integrand} = \frac{-i \exp(i\omega t)}{(\omega^2 - 2i\omega - 5)(\omega - 2i)} =$$

$$\omega^2 - 2i\omega - 5 = (\omega - i)^2 - 4 = (\omega - i - 2)(\omega - i + 2)$$

$$= \frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)}$$

POLES: $\omega = 2 + i$
 $\omega = -2 + i$
 $\omega = 2i$



IF $t < 0$:

~~lim~~ ~~integrand~~
 $R \rightarrow \infty$ INTEGRAND ON SEMICIRCLE PART OF CLHP = 0
 $\lim_{R \rightarrow \infty}$ OF CLHP

$$\oint_{\text{CLHP}} \frac{-i \exp(i\omega t)}{(\omega^2 - 2i\omega - 5)(\omega - 2i)} d\omega = 2\pi i \sum_{\text{POLES}} \text{RES} = 0$$

$$\rightarrow y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{integrand} d\omega + \left[\text{SEMICIRCLE PART OF CLHP} \right] =$$

$$\frac{1}{2\pi} \oint_{\text{CLHP}} \text{integrand} d\omega = 0$$

~~y(t) = 0~~ $y(t < 0) = 0$

IF $t > 0$:

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)} d\omega =$$

$$= \frac{1}{2\pi} \lim_{R \rightarrow \infty} \int_{-R}^R \frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)} d\omega =$$

$$= \frac{1}{2\pi} \oint_{\text{CUHP}} \frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)} d\omega - \left[\frac{1}{2\pi} \int_{\text{SEMICIRCLE PART OF CUHP}} \frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)} d\omega \right]$$

SEMICIRCLE PART OF CUHP

INTEGRAND GOES TO 0 FASTER THAN ARC LENGTH OF SEMICIRCLE GOES TO $\infty \Rightarrow$ INTEGRAL = 0 AS $R \rightarrow \infty$

GOES TO 0 EXPONENTIALLY (OR NEAR EXPONENTIALLY)
 WHOLE INTEGRAL = 0 WITH SEMICIRCLE PART OF LHP AS CONTR

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$$\Rightarrow y(t) = \frac{1}{2\pi} \oint_{\text{UHP}} \frac{-i \exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)(\omega - 2i)} d\omega = \frac{1}{2\pi} 2\pi i \sum_{\text{poles}} \text{RES}_{\text{integrand}}$$

$$i \sum_{\text{poles}} \text{integrand} = \left. \frac{\exp(i\omega t)}{(\omega - i + 2)(\omega - 2i)} \right|_{\omega = 2+i} + \left. \frac{\exp(i\omega t)}{(\omega - i - 2)(\omega - 2i)} \right|_{\omega = -2+i} + \left. \frac{\exp(i\omega t)}{(\omega - i - 2)(\omega - i + 2)} \right|_{\omega = 2i}$$

$$= \frac{\exp(i(2+i)t)}{4(2-i)} + \frac{\exp(i(-2+i)t)}{-4(-2-i)} + \frac{\exp(i(2i)t)}{(i-2)(i+2)} =$$

$$= e^{-t} \left(\frac{e^{2it}}{4(2-i)} + \frac{e^{-2it}}{4(2+i)} + \frac{e^{-t}}{-5} \right) =$$

$$= e^{-t} \left[\left(\frac{1}{4(2-i)} + \frac{1}{4(2+i)} \right) 2i \sin(2t) + \frac{e^{-t}}{5} \right] =$$

$$= e^{-t} \left[\frac{2+i+2-i}{4-1} 2i \sin(2t) + \frac{e^{-t}}{5} \right] = e^{-t} \frac{1}{5} (8i \sin(2t) + e^{-t})$$

SO WE HAVE:

$$y(t) = \begin{cases} 0 & \text{for } t < 0 \\ e^{-t} \frac{1}{5} (8i \sin(2t) + e^{-t}) & \text{for } t > 0 \end{cases}$$

(b) (ii) For $t < 0$:

$$ij + 2j = f(t) \quad \dot{y} = Y$$

$$\dot{Y} + 2Y = f(t)$$

~~$f(t)$ IS A SOLUTION TO 1ST ORDER ODE~~

~~HOW $f(t)$ IS IN THE FORM: $f(t) = A \exp(i\beta t)$~~

$$y(t < 0) = 0 \Rightarrow \dot{y}(t < 0) = 0 \Rightarrow \ddot{y}(t < 0) = 0$$

$$\Rightarrow f(t) = 0$$

SEEMS WRONG.