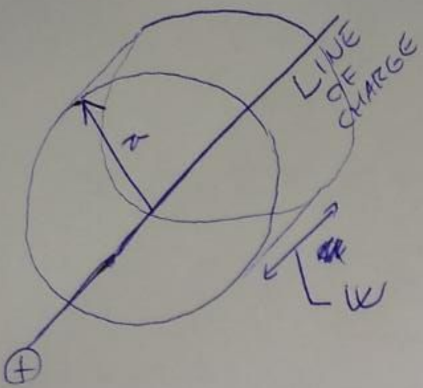


FIELD IS RADIAL
FIELD LINES ARE PERPENDICULAR TO LINE OF CHARGE.

CHARGE DENSITY: λ

$$\int \underline{E} \cdot \underline{ds} = \frac{Q}{\varepsilon_0} = \frac{W\lambda}{\varepsilon_0}$$

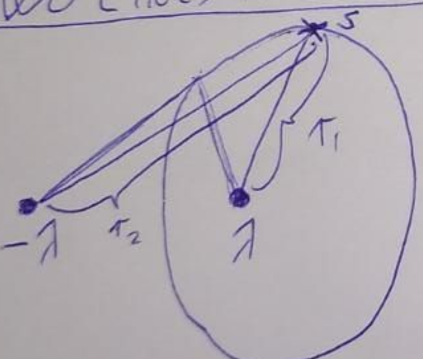
$$2\pi r \cancel{W} E = \cancel{W} \lambda \frac{1}{\varepsilon_0}$$

$$\cancel{E = 2\pi r \lambda \frac{1}{\varepsilon_0}}$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

$$V_{(r)} = -\int_{\infty}^{r} \underline{E}(r') \cdot \underline{dr'} = -\int_{\infty}^{r} \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} dr = -\frac{\lambda}{2\pi\varepsilon_0} \ln r$$

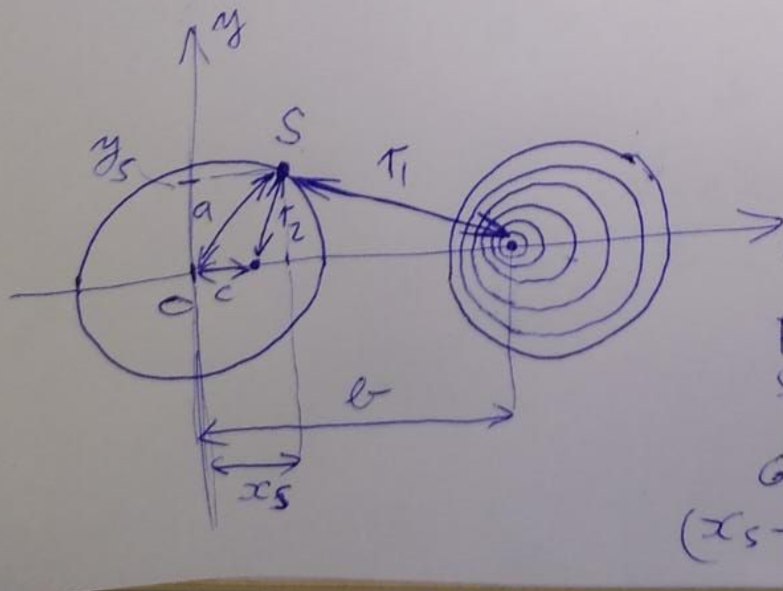
TWO LINES OF CHARGE



$$V_{@S} = -\frac{\lambda}{2\pi\varepsilon_0} \ln r_1 - -\frac{\lambda}{2\pi\varepsilon_0} \ln r_2 =$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left(\ln r_2 - \ln r_1\right) = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r_2}{r_1}$$

EQUIPOTENTIAL SURFACES: $\frac{r_2}{r_1} = \text{CONSTANT}$



$$r_2^2 = (x_S - c)^2 + y_S^2$$

$$r_1^2 = (x_S - b)^2 + y_S^2$$

FOR EQUIPOTENTIAL SURFACE: $\frac{r_2^2}{r_1^2} = \ell$

GIVING:

$$(x_S - c)^2 + y_S^2 = \ell(x_S - b)^2 + \ell y_S^2$$

$$(x_s-c)^2+y_s^2=k(x_s-b)^2+ky_s^2$$

$$x_s^2-2x_sc+c^2+y_s^2=kx_s^2-2kx_sb+kb^2+ky_s^2$$

$\boxed{\text{IF: } kb=c}$ REMEMBER THIS SUBSTITUTION

$$x_s^2-2x_sc+c^2+y_s^2=kx_s^2-2x_sc+cb+ky_s^2$$

$$x_s^2+c^2+y_s^2=k(x_s^2+y_s^2)+cb$$

$$c^2=(k-1)(x_s^2+y_s^2)+cb$$

$$\frac{c^2-cb}{k-1}=x_s^2+y_s^2$$

$\downarrow$ USING: $cb=kbb=kb^2$

$$\frac{kb^2-c^2}{1-k}=x_s^2+y_s^2=r^2$$

IF EQUIPOTENTIAL SURFACE: RADIUS a CYLINDER

$$a^2=\frac{kb^2-c^2}{1-k}$$

~~AS $k=bc$~~ AS $kb=c$:

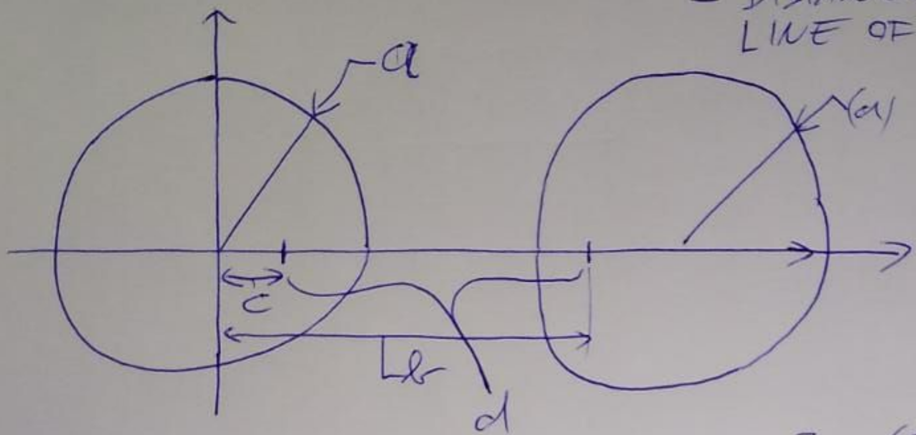
$$a^2=\frac{cb-c^2}{1-\frac{c}{b}}=\frac{(cb-c^2)b}{b-c}=$$

$$=\frac{(b-c)cb}{b-c}=cb=a^2$$

LETS SUPPOSE WE WANT:

CENTRE OF EQUIPOTENTIAL CYLINDER

- HAVING RADIUS a
- DISTANCE BETWEEN LINE OF CHARGES: $d = b - c$



OUR EQUATIONS: $bc = a^2$ (I)

$b - c = d$ (II)

WE WANT b & c, KNOWING d & a

$(I) \Rightarrow b = d + c$

$bc = a^2 \Rightarrow (d+c)c = a^2$

$c^2 + dc - a^2 = 0$

$$c = \frac{-d \pm \sqrt{d^2 - 4 \cdot 1 \cdot (-a^2)}}{2} = \frac{-d \pm \sqrt{d^2 + 4a^2}}{2}$$

$$b = d + c = d + \frac{-d}{2} \pm \frac{\sqrt{d^2 + 4a^2}}{2} = \frac{d}{2} \pm \frac{\sqrt{d^2+4a^2}}{2}$$

IF $\frac{1}{2}\sqrt{d^2 + 4a^2} = S$

THEN $c = -\frac{d}{2} \pm S$

$b = \frac{d}{2} \pm S$

$\sqrt{d^2 + 4a^2} > d \Rightarrow$ ONLY (+) OPTION IS PHYSICAL.

$c = -\frac{d}{2} + S$

$b = \frac{d}{2} + S$