2015P2Q6 (I) (a)

$x_i' = M_{ij} x_j$

$$\begin{pmatrix} \underline{e}_1' \\ \underline{e}_2' \\ \underline{e}_3' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \end{pmatrix}$$

WHERE: $M_{ij} = \underline{e}_i' \cdot \underline{e}_j$

(IS THIS A "RESTRICTION" THOUGH?)

if $\det M < 0$: M IS LEFT HANDED.

if $\det M > 0$: M IS RIGHT HANDED.

$T'_{ij} = M_{ip} M_{jq} T_{pq}$

(b)

$$T = \frac{1}{2}(T + T^T) + \frac{1}{2}(T - T^T) = S + A$$

$$M_{ip} M_{jq} S_{pq} = M_{ip} M_{jq} \frac{1}{2}\left(T_{pq} + (T^T)_{pq}\right) =$$

$$= M_{ip} M_{jq} \frac{1}{2}(T_{pq} + T_{qp}) = \frac{1}{2}(T'_{ij} + T'_{ji}) = \frac{1}{2}(T + T^T)'_{ij} =$$

$$= S'_{ij}$$

WE FIND: $M_{ip} M_{jq} S_{pq} = S'_{ij}$

$\Rightarrow$ S IS A SECOND ORDER TENSOR.

$$M_{ip} M_{jq} A_{pq} = M_{ip} M_{jq} \frac{1}{2}\left(T_{pq} - (T^T)_{pq}\right) =$$

$$= M_{ip} M_{jq} \frac{1}{2}(T_{pq} - T_{qp}) = \frac{1}{2}(T'_{ij} - T'_{ji}) = \frac{1}{2}(T - T^T)'_{ij} =$$

$$= A'_{ij}$$

$M_{ip} M_{jq} A_{pq} = A'_{ij} \Rightarrow$ A IS A SECOND ORDER TENSOR.

2015P2Q6 (II)

(c)

$$S=\frac{1}{2}(F+F^T)=\frac{1}{2}\left[\begin{pmatrix} x_1^2 & -x_1^2+x_1x_2-x_2^2 & x_1-x_2 \\ x_1^2+x_1x_2+x_2^2 & x_2^2 & -x_1-x_2 \\ -x_1+x_2 & x_1+x_2 & 3(x_1^2+x_2^2) \end{pmatrix}+\right.$$

$$\left.+\begin{pmatrix} x_1^2 & x_1^2+x_1x_2+x_2^2 & -x_1+x_2 \\ -x_1^2+x_1x_2-x_2^2 & x_2^2 & x_1+x_2 \\ x_1-x_2 & -x_1-x_2 & 3(x_1^2+x_2^2) \end{pmatrix}\right]=$$

$$=\begin{pmatrix} x_1^2 & x_1x_2 & 0 \\ x_1x_2 & x_2^2 & 0 \\ 0 & 0 & 3(x_1^2+x_2^2) \end{pmatrix}=S$$ (SYMMETRIC PART OF F THIS IS)

PRINCIPAL AXES & VALUES:

$$\underline{\underline{S}}\begin{pmatrix}0\\0\\1\end{pmatrix}=3(x_1^2+x_2^2)\begin{pmatrix}0\\0\\1\end{pmatrix}\Longrightarrow \underline{e}_1=\begin{pmatrix}0\\0\\1\end{pmatrix}\quad \lambda_1=3(x_1^2+x_2^2)$$

$$\det S=3(x_1^2+x_2^2)(x_1^2x_2^2-x_1x_2x_1x_2)=0$$

$$\operatorname{Tr} S=4(x_1^2+x_2^2)\Longrightarrow\sum_{i=1}^{3}\lambda_i=4(x_1^2+x_2^2)$$

$$\left.\begin{aligned}&\prod_{i=1}^{3}=0\\ &\lambda_1=3(x_1^2+x_2^2)\end{aligned}\right\}\Rightarrow\lambda_2=0\Rightarrow\lambda_3=x_1^2+x_2^2$$

$$Se_2=\begin{pmatrix}0\\0\\0\end{pmatrix}=\begin{pmatrix} x_1^2 & x_1x_2 & 0 \\ x_1x_2 & x_2^2 & 0 \\ 0 & 0 & 3(x_1^2+x_2^2) \end{pmatrix}\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\Rightarrow r=0$$

$$x_1^2p+x_1x_2q=0\quad /:x_1$$

$$x_1p+x_2q=0$$

$$e_2=\begin{pmatrix}-\frac{x_2}{x_1}\\1\\0\end{pmatrix}\Longleftarrow p=-\frac{x_2}{x_1}q$$

2015P2Q6(III)

$$S e_3 = \lambda_3 e_3$$

$$\begin{pmatrix} x_1^2 & x_1 x_2 & 0 \\ x_1 x_2 & x_2^2 & 0 \\ 0 & 0 & 3(x_1^2+x_2^2) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = (x_1^2+x_2^2) \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

$$3(x_1^2+x_2^2) r = (x_1^2+x_2^2) r$$
$$\Rightarrow r = 0$$

$$x_1^2 p + x_1 x_2 q = (x_1^2+x_2^2) p \qquad | - x_1^2 p$$

$$x_1 x_2 q = x_2^2 p$$

$$\frac{x_1}{x_2} q = p \longrightarrow e_3 = \begin{pmatrix} \frac{x_1}{x_2} \\ 1 \\ 0 \end{pmatrix}$$

WE FOUND:

$$\lambda_1 = 3(x_1^2+x_2^2) \qquad \lambda_2 = 0 \qquad \lambda_3 = x_1^2 + x_2^2$$

$$e_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad e_2 = \begin{pmatrix} -\frac{x_2}{x_1} \\ 1 \\ 0 \end{pmatrix} \qquad e_3 = \begin{pmatrix} \frac{x_1}{x_2} \\ 1 \\ 0 \end{pmatrix}$$

GEOMETRICAL DESCRIPTION OF THE PRINCIPAL AXES:

e_1 HAS A CONSTANT COMPONENT IN THE x_3 DIRECTION AND HAS NO COMPONENT IN OTHER DIRECTIONS THROUGHOUT THE WHOLE SPACE.

$e_1 \cdot e_2 = e_1 \cdot e_3 = 0 \Rightarrow e_1$ IS PERPENDICULAR TO BOTH e_2 & e_3.

$e_2 \cdot e_3 = 0 \Rightarrow e_2$ IS ORTHOGONAL TO e_3 THROUGHOUT THE WHOLE SPACE. e_2 & e_3 HAVE A COMPONENT (CONSTANT LENGTH) IN THE x_3 DIRECTION.

~~THE COMPONENT OF e_2 & e_3 IN THE x_1 DIRECTION~~
~~MAGNITUDE OF THE~~
~~GOES TO ∞ AS WE APPROA~~
~~THE x~~ e_2 & e_3 HAVE A VARIABLE LENGTH COMPONENT IN THE x_1 DIRECTION, ITS LENGTH IS DEPENDENT ON WHERE WE ARE IN SPACE. SEE e_2 & e_3 DEFINITION ABOVE.

2015 P2 Q6 IV

TRANSFORMATION MATRIX M_{ij}?

$$\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}-\frac{x_2}{x_1}\\1\\0\end{pmatrix}\begin{pmatrix}\frac{x_1}{x_2}\\1\\0\end{pmatrix}\right) = \begin{pmatrix}M_{11} & M_{12} & M_{13}\\ M_{21} & M_{22} & M_{23}\\ M_{31} & M_{32} & M_{33}\end{pmatrix}\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right)$$

SO IS M JUST $\begin{pmatrix}0 & -\frac{x_2}{x_1} & \frac{x_1}{x_2}\\ 0 & 1 & 1\\ 1 & 0 & 0\end{pmatrix}$?

PROBABLY NOT. I AM LIKELY TO BE MISTAKEN HERE

(d) $F_{ij} = P\delta_{ij} + \hat{S}_{ij} + \hat{A}_{ij}$

$F_{ij} = S_{ij} + \hat{A}_{ij}$ $\qquad \hat{A}_{ij} = \frac{1}{2}(F - F^T)_{ij}$

$$\hat{S} = S - \frac{\mathrm{Tr}\, S}{3} I = \begin{pmatrix}x_1^2 & x_1x_2 & 0\\ x_1x_2 & x_2^2 & 0\\ 0 & 0 & 3(x_1^2+x_2^2)\end{pmatrix} - \frac{4(x_1^2+x_2^2)}{3} I =$$

$$= \begin{pmatrix}-\frac{1}{3}x_1^2 - \frac{4}{3}x_2^2 & x_1x_2 & 0\\ x_1x_2 & -\frac{4}{3}x_1^2 - \frac{1}{3}x_2^2 & 0\\ 0 & 0 & \frac{5}{3}(x_1^2+x_2^2)\end{pmatrix} = \hat{S}$$

$P = \frac{4(x_1^2+x_2^2)}{3}$ $\qquad \hat{A}_{ij} = \frac{1}{2}(F_{ij} - F_{ji}) = \begin{pmatrix}0 & -x_1^2 - x_2^2 & x_1 - x_2\\ x_1^2 + x_2^2 & 0 & -x_1 - x_2\\ -x_1 + x_2 & x_1 + x_2 & 0\end{pmatrix}$

CHECKING IF $\hat{S}$ HAS THE SAME PRINCIP. AXES AS FOUND PREVIOUSLY:

$\hat{S}\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{5}{3}(x_1^2+x_2^2)\begin{pmatrix}0\\0\\1\end{pmatrix}$ ✓

$$\hat{S}\begin{pmatrix}-\frac{x_2}{x_1}\\1\\0\end{pmatrix} = \begin{pmatrix}\left(-\frac{1}{3}x_1^2 - \frac{4}{3}x_2^2\right)\left(-\frac{x_2}{x_1}\right) + x_1x_2\\ x_1x_2\cdot\left(-\frac{x_2}{x_1}\right) + \left(-\frac{4}{3}x_1^2 - \frac{1}{3}x_2^2\right)\\ 0\end{pmatrix} \neq \begin{pmatrix}-\frac{x_1}{x_1}\\1\\0\end{pmatrix}$$

SO THE ANSWER IS NO (PROBABLY THE ANSWER IS YES AND IM WRONG SOMEWHERE)