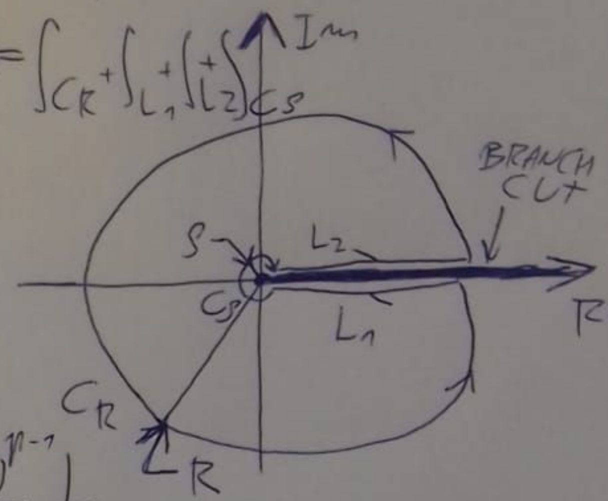
CALCULATION OF

$$I=\int_0^\infty \frac{x^{p-1}}{x^2+1}\,dx \qquad 0<p<2$$

$z^{p-1}=|z|^{p-1}e^{i(p-1)\theta}$, $0\leq\theta<\pi$, BRANCH CUT ALONG $\mathbb{R}^+$.

$$\oint \frac{z^{p-1}}{z^2+1}dz=2\pi i\left(\underset{z=+i}{\mathrm{RES}}\, f_+ + \underset{z=-i}{\mathrm{RES}}\, f_-\right)= \qquad \oint_\Gamma=\int_{C_R}+\int_{L_1}+\int_{L_2}+\int_{C_\delta}$$



$$=2\pi i\left(\frac{z^{p-1}}{z+i}\Big|_{z=i}+\frac{z^{p-1}}{z-i}\Big|_{z=-i}\right)=$$

$$=2\pi i\left(\frac{i^{p-1}}{2i}+\frac{(-i)^{p-1}}{-2i}\right)=2\pi i\left(\frac{i^{p-1}}{2i}-\frac{(-i)^{p-1}}{2i}\right)=$$

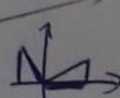
$$=\pi\left(i^{p-1}-(-i)^{p-1}\right)=\pi\left(i^{p-1}-(-1)^{p-1}(i)^{p-1}\right)=$$

$$=\pi\left(1-(-1)^{p-1}\right)(i)^{p-1}=\pi\left(1-(-1)^{p-1}\right)e^{i\frac{\pi}{2}(p-1)}=$$

$$=\pi\left(1-(e^{i\pi})^{p-1}\right)e^{i\pi(p-1)}e^{-i\frac{\pi}{2}(p-1)}=$$

$$=\pi\left(e^{-i\frac{\pi}{2}(p-1)}-e^{i\frac{\pi}{2}(p-1)}\right)e^{i\pi(p-1)}=2\pi i\sin\left(-\frac{\pi}{2}(p-1)\right)e^{i\pi(p-1)}=$$

$$=2\pi i e^{i\pi(p-1)}\sin\left(-p\frac{\pi}{2}+\frac{\pi}{2}\right)=2\pi i e^{i\pi(p-1)}\cos\left(-p\frac{\pi}{2}\right)=$$



$$=2\pi i\cos\left(\frac{\pi p}{2}\right)e^{i\pi(p-1)}$$

TAKE: $R\to\infty$, $\delta\to 0$; $\Rightarrow \int_{C_R}\to 0$, $\int_{C_\delta}\to 0 \Rightarrow \oint_\Gamma\frac{z^{p-1}}{z^2+1}dz=\int_0^\infty\frac{dx}{x^2+1}\left[x^{p-1}-x^{p-1}e^{2\pi i(p-1)}\right]=$

$$=\int_0^\infty\frac{x^{p-1}}{x^2+1}\left(1-e^{2\pi i(p-1)}\right)dx=2ie^{i\pi(p-1)}\sin(\pi p)\cdot I$$

USING:

$$\left(1-e^{2\pi i(p-1)}\right)=\left(e^{-\pi i(p-1)}-e^{\pi i(p-1)}\right)e^{\pi i(p-1)}=2i\sin\left(-\pi(p-1)\right)e^{\pi i(p-1)}=$$

$$=2i(-1)\sin(\pi p-\pi)e^{\pi i(p-1)}=2ie^{\pi i(p-1)}\sin(p\pi)$$

THEREFORE:

$$I = \frac{2\pi i \cos\left(\frac{\pi \mu}{2}\right) e^{i\pi(\mu-1)}}{2i\, e^{i\pi(\mu-1)} \sin(\pi\mu)} = \pi \cos\left(\frac{\pi\mu}{2}\right) \Big/ \sin(\pi\mu) =$$

$$= \pi \frac{\cancel{\cos\left(\frac{\pi\mu}{2}\right)}}{2\sin\left(\frac{\pi\mu}{2}\right)\cancel{\cos\left(\frac{\pi\mu}{2}\right)}} = \underline{\frac{\pi}{2\sin\left(\frac{\pi\mu}{2}\right)}}$$