2014P2Q10(I)

(i) Representation D of group G is a set of matrices D which form a group under multiplication and preserves group operations in G. ie a homomorphism.

Faithful representation is when the group formed by elements of D is isomorphic to G.

D_l & D_m are equivalent representations if they can be transformed by similarity transformation, that is $SD_lS^{-1}=D_m$, for single S, for all D_l, D_m, while preserving group structure. then??

The character of a representation is the set of traces of its matrices.

(ii) $C_4=\{I,a,a^2,a^3\}$

	I	a	a^2	a^3
I	I	a	a^2	a^3
a	a	a^2	a^3	I
a^2	a^2	a^3	I	a
a^3	a^3	I	a	a^2

(iii) $D_2=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$

$\begin{vmatrix}-\lambda&1\\-1&-\lambda\end{vmatrix}=0$

$\lambda^2+1=0$

$\left.\begin{array}{l}\lambda_1=i\\\lambda_2=-i\end{array}\right]\Rightarrow SD_2S^{-1}=\begin{pmatrix}i&0\\0&-i\end{pmatrix}$

You have to explain what you are doing

THE REPRESENTATIONS ARE INEQUIVALENT $\Leftarrow E_2 \neq E_4$

$D_4=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$

$\begin{vmatrix}-\omega&-1\\1&-\omega\end{vmatrix}=0\Rightarrow \omega_1=i$, $\omega_2=-i$

$\Rightarrow SD_4S^{-1}=\begin{pmatrix}i&0\\0&-i\end{pmatrix}$

just look at the traces.

2014 P2 Q10(II)

$$\begin{pmatrix}1&0&0\\0&0&b\\0&c&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&0&b\\0&c&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&bc&0\\0&0&bc\end{pmatrix}$$

(iv) $T(a)=\begin{pmatrix}1&0&0\\0&0&b\\0&c&0\end{pmatrix}$ $\quad T(a^2)=\begin{pmatrix}1&0&0\\0&bc&0\\0&0&bc\end{pmatrix}$ ✓ $\quad T(\overset{a^4.}{\underline{a^3}})=\begin{pmatrix}1&0&0\\0&b^2c^2&0\\0&0&b^2c^2\end{pmatrix}$

$$T(a)T(a^2)=\begin{pmatrix}1&0&0\\0&0&b\\0&c&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&bc&0\\0&0&bc\end{pmatrix}=\begin{pmatrix}1&0&0\\0&0&b^2c\\0&bc^2&0\end{pmatrix}=T(a^3)$$

$$\Downarrow$$

$$bc^2=b^2c=b^2c^2=0$$

$$\Uparrow$$

$$T(a^2)T(a)=\begin{pmatrix}1&0&0\\0&bc&0\\0&0&bc\end{pmatrix}\begin{pmatrix}1&0&0\\0&0&b\\0&c&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&0&b^2c\\0&bc^2&0\end{pmatrix}=T(a^3)$$

$$T(a^2)T(a^3)=\begin{pmatrix}1&0&0\\0&bc&0\\0&0&bc\end{pmatrix}\begin{pmatrix}1&0&0\\0&b^2c^2&0\\0&0&b^2c^2\end{pmatrix}=\begin{pmatrix}1&0&0\\0&b^3c^3&0\\0&0&b^3c^3\end{pmatrix}=T(a^3)T(a^2)=T(a)$$

$$\Downarrow$$

$$b^3c^3=b^3c^3=$$
$$=b=c=0$$

SEEMS WRONG.

$T(a^2)=\begin{pmatrix}1&&\\&bc&\\&&bc\end{pmatrix}$

$T(a^3)=$ this $\times\begin{pmatrix}1&&\\&&b\\&c&\end{pmatrix}$

$T(a^4)=\begin{pmatrix}1&&\\&b^2c^2&\\&&b^2c^2\end{pmatrix}$.

Faithful rep $\quad bc=-1 \quad$ ie $b=1, c=-1$ or $b=-1, c=1$.

Unfaithful rep $bc=1 \quad b, c=1, -1$.

orthogonality $\Rightarrow |b|=|c|=1$.

Have just been listening
to Clive Oppenheimer (Earth Sci)
on the radio. All about volcanoes.