PII

2009Q10 (1)

$a^2=b^2=c^2=1$

$ab=c$

$abb=cb=a$

$$\boxed{\begin{matrix} 1 & a & b & c \\ a & 1 & c & b \\ b & c & 1 & a \\ c & b & a & 1 \end{matrix}}$$

V✓

✓

$acb=aca$

$ab=c$

$acbc=c$

$acb=1$

$ac=b^{-1}=b$

g_1, g_2 conjugate

as Abelian each elt in own conjugacy class.

$g_2g=gg_1=g_1g \Rightarrow g_1=g_2$

CONJUGACY CLASSES OF V: ✓

$\{1\}\ \{a\}\ \{b\}\ \{c\}$

explain why.

AN IRREDUCIBLE REP OF V:

~~$1=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $a=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ $b=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$ $c=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$~~

$\{1,a,b,c\} \to \{1,1,1,1\}$ ✓

NUMBER OF CONJUGACY CLASSES AND INEQUIVALENT IRREPS ARE EQUAL.

V HAS 4 INEQ. IRREP. ✓

REP1: $\{1,a,b,c\} \to \{1,1,1,1\}$ ✓

REP2: $\{1,a,b,c\} \to \{1,-1,1,-1\}$ ✓

REP3: $\{1,a,b,c\} \to \{1,1,-1,-1\}$ ✓

REP4: $\{1,a,b,c\} \to \{1,-1,-1,1\}$ ✓

$$\begin{matrix} D(1) & D(a) & D(b) & D(c) \\ D(a) & D(1) & D(c) & D(b) \\ D(b) & D(c) & D(1) & D(a) \\ D(c) & D(b) & D(a) & D(1) \end{matrix}$$

$\rightarrow$ THIS TABLE ISOMORPHIC TO V TABLE $\Rightarrow$ D IS A REP.

2009 P2 Q10 (II)

GROUP MULTIPLICATION TABLE OF V:

1	a	b	c
a	1	c	b
b	c	1	a
c	b	a	1

$ab = c = ba$

$abb = cb$

$a = cb$

✓ $ca = ccb = b$

VERY GOOD WORK

WELL DONE.

CONJUGACY CLASSES:

$g q_1 g^{-1} = q_2 \quad /\cdot g \quad g, q_1, q_2 \in V$

$g q_1 = q_2 g = g q_2$

↳ GROUP ABELIAN ✓

$\Rightarrow q_1 = q_2 \Rightarrow$ EVERY $g \in V$ IS ALONE ITS OWN CONJUGACY CLASS.

CONJUGACY CLASSES OF V:

$\{1\}, \{a\}, \{b\}, \{c\}$

surely $gqg^{-1} = q_2$ abelian $gg^{-1}q_1 = q_2$.

An irreducible representation, D, of group V is a set of matrices M(S) acting on vector space S, which S has no non-trivial proper subspaces under the action of matrices forming D. ✓

Number of conjugacy classes and the number of inequivalent irreducible representations of a finite group are equal.

✓

$\Rightarrow$ V HAS 4 INEQUIVALENT IRREP. THESE ARE:

So write ------

this is too involved a sentence

$\{1, a, b, c\} \to \{1, 1, 1, 1\} = d^{(1)}$

$\to \{1, -1, -1, 1\} = d^{(2)}$

$\to \{1, 1, -1, -1\} = d^{(3)}$

$\to \{1, -1, 1, -1\} = d^{(4)}$ good

PTO.

Also $|G| = \sum n_i^2$ n dim of irrep

MULTIPLICATION TABLE FOR MATRICES LISTED:

D(1)	D(a)	D(b)	D(c)
D(a)	D(1)	D(c)	D(b)
D(b)	D(c)	D(1)	D(a)
D(c)	D(b)	D(a)	D(1)

THIS TABLE IS ISOMORPHIC TO THE TABLE OF V PROVIDED ABOVE, SO D IS INDEED A REPRESENTATON OF V. ✓

CHARACTER TABLE:

	1	a	b	c
$d^{(1)}$	1	1	1	1
$d^{(2)}$	1	-1	-1	1
$d^{(3)}$	1	1	-1	-1
$d^{(4)}$	1	-1	1	-1

✓

DECOMPOSITION OF D:

$\{D(1)_{11}, D(a)_{11}, D(b)_{11}, D(c)_{11}\} = d^{(3)}$

$\{D(1)_{22}, D(a)_{22}, D(b)_{22}, D(c)_{22}\} = d^{(4)}$

$\{D(1)_{33}, D(a)_{33}, D(b)_{33}, D(c)_{33}\} = d^{(2)}$ ✓

EVERY VECTOR FORMED BY COLUMN OF CHARACTER TABLE ORTHOGONAL TO ANY COLUMN BELONGING TO DIFFERENT CONJUGACY CLASS (HERE: TO EVERY OTHER COLUMN)