2.1 Given the following information:

$$S_x|\pm\rangle_x = \pm\frac{\hbar}{2}|\pm\rangle_x \qquad\qquad S_y|\pm\rangle_y = \pm\frac{\hbar}{2}|\pm\rangle_y$$

$$|\pm\rangle_x = \tfrac{1}{\sqrt{2}}[|+\rangle \pm |-\rangle] \qquad\qquad |\pm\rangle_y = \tfrac{1}{\sqrt{2}}[|+\rangle \pm i|-\rangle]$$

find the matrix representations of S_x and S_y in the S_z basis.

2.2 From the previous problem we know that the matrix representation of S_x in the S_z basis is

$$S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Diagonalize this matrix to find the eigenvalues and the eigenvectors of S_x.

2.5 Calculate the commutators of the spin-1/2 operators S_x, S_y, and S_z, thus verifying Eqs. (2.96).

2.9 For the state $|+\rangle$, calculate the expectation values and uncertainties for measurements of S_x, S_y and S_z in order to verify Eq. (2.108).

2.12 Diagonalize the S_x and S_y operators in the spin-1 case to find the eigenvalues and the eigenvectors of both operators.

2.18 A spin-1 particle is prepared in the state

$$|\psi\rangle = \tfrac{1}{\sqrt{14}}|1\rangle - \tfrac{3}{\sqrt{14}}|0\rangle + i\tfrac{2}{\sqrt{14}}|-1\rangle.$$

a) What are the possible results of a measurement of the spin component S_z, and with what probabilities would they occur?

b) Suppose that the S_z measurement on the particle yields the result $S_z = -\hbar$. Subsequent to that result a second measurement is performed to measure the spin component S_x. What are the possible results of that measurement, and with what probabilities would they occur?

c) Draw a schematic diagram depicting the successive measurements in parts (a) and (b).

2.22 A beam of spin-1/2 particles is sent through a series of three Stern-Gerlach analyzers, as shown in Fig. 2.15. The second Stern-Gerlach analyzer is aligned along the $\hat{\mathbf{n}}$ direction, which makes an angle θ in the x-z plane with respect to the z-axis.

a) Find the probability that particles transmitted through the first Stern-Gerlach analyzer are measured to have spin down at the third Stern-Gerlach analyzer?

b) How must the angle θ of the second Stern-Gerlach analyzer be oriented so as to maximize the probability that particles are measured to have spin down at the third Stern-Gerlach analyzer? What is this maximum fraction?

c) What is the probability that particles have spin down at the third Stern-Gerlach analyzer if the second Stern-Gerlach analyzer is removed from the experiment?

2.1 Find matrix representations of S_x and S_y in the S_z basis.

$\underline{\underline{S_x}}$

$$S_x \doteq \begin{pmatrix} \langle + | S_x | + \rangle & \langle + | S_x | - \rangle \\ \langle - | S_x | + \rangle & \langle - | S_x | - \rangle \end{pmatrix}$$

$\doteq S_x |+\rangle$ = matrix repr. of S_x acting on $|+\rangle$

$\doteq S_x |-\rangle$ = matrix repr. of S_x acting on $|-\rangle$

$$S_x |\pm\rangle_x = \pm \frac{\hbar}{2} |\pm\rangle_x \quad , \quad |\pm\rangle_x = \frac{1}{\sqrt{2}} \left(|+\rangle \pm |-\rangle \right)$$

$$\Leftrightarrow \quad S_x \frac{1}{\sqrt{2}} \left(|+\rangle \pm |-\rangle \right) = \pm \frac{\hbar}{2} \frac{1}{\sqrt{2}} \left(|+\rangle \pm |-\rangle \right)$$

$$\Leftrightarrow \quad S_x \left(|+\rangle \pm |-\rangle \right) = \pm \frac{\hbar}{2} \left(|+\rangle \pm |-\rangle \right)$$

Project out $|\pm\rangle$ components buy multiplying with $\langle \pm |$ from the left $\Rightarrow$

$$\begin{cases} \langle + | S_x | + \rangle \pm \langle + | S_x | - \rangle = \pm \frac{\hbar}{2} \big(\underbrace{\langle + | + \rangle}_{=1} \pm \underbrace{\langle + | - \rangle}_{=0} \big) \\ \langle - | S_x | + \rangle \pm \langle - | S_x | - \rangle = \pm \frac{\hbar}{2} \big(\underbrace{\langle - | + \rangle}_{=0} \pm \underbrace{\langle - | - \rangle}_{=1} \big) \end{cases}$$

4 eq., 4 unknowns: $\langle + | S_x | + \rangle$, $\langle + | S_x | - \rangle$, $\langle - | S_x | + \rangle$, $\langle - | S_x | - \rangle$

$$\Leftrightarrow \quad \begin{array}{cccc|c} ++ & +- & -+ & -- & \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & \hbar/2 \\ 1 & -1 & 0 & 0 & -\hbar/2 \\ 0 & 0 & 1 & 1 & \hbar/2 \\ 0 & 0 & 1 & -1 & \hbar/2 \end{array} \right) \begin{array}{c} \oplus \\ \\ \oplus \\ \\ \end{array} \sim$$

$$\sim \begin{array}{cccc|c} ++ & +- & -+ & -- & \\ \end{array}\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & \hbar/2 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \hbar/2 \\ 0 & 0 & 2 & 0 & \hbar \end{array}\right) \begin{array}{l} \overset{1}{\Rightarrow} \langle + | S_x | - \rangle = \frac{\hbar}{2} \\ \overset{2}{\Rightarrow} \langle + | S_x | + \rangle = 0 \\ \overset{3}{\Rightarrow} \langle - | S_x | - \rangle = 0 \\ \overset{4}{\Rightarrow} \langle - | S_x | + \rangle = \frac{\hbar}{2} \end{array}$$

$$\Rightarrow \boxed{S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

S_y

$$|\pm\rangle_y = \frac{1}{\sqrt{2}}\left(|+\rangle \pm i|-\rangle\right)$$

replace every $|-\rangle$ with $i|-\rangle$ in calculation for S_x

$\Rightarrow$ eq. for S_y:

$$\begin{array}{cccc|c} ++ & +- & -+ & -- & \\ \end{array}\left(\begin{array}{cccc|c} 1 & i & 0 & 0 & \hbar/2 \\ 1 & -i & 0 & 0 & -\hbar/2 \\ 0 & 0 & 1 & i & i\hbar/2 \\ 0 & 0 & 1 & -i & i\hbar/2 \end{array}\right) \oplus \sim \begin{array}{cccc|c} ++ & +- & -+ & -- & \\ \end{array}\left(\begin{array}{cccc|c} 1 & i & 0 & 0 & \hbar/2 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & i & i\hbar/2 \\ 0 & 0 & 2 & 0 & i\hbar \end{array}\right)$$

$\overset{2}{\Rightarrow} \langle + | S_y | + \rangle = 0 \qquad \overset{1}{\Rightarrow} i\langle + | S_y | - \rangle = \frac{\hbar}{2} \Leftrightarrow \langle + | S_y | - \rangle = -\frac{i\hbar}{2}$

$\overset{4}{\Rightarrow} \langle - | S_y | + \rangle = i\frac{\hbar}{2} \qquad \overset{3}{\Rightarrow} \langle - | S_y | - \rangle = 0$

$$\Rightarrow \boxed{S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$$

$\boxed{2.2}$

$$S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{eigenvalues, eigenvectors?}$$

Eigenvalue eq.:

$$S_x |\lambda\rangle_x = \lambda |\lambda\rangle_x \Leftrightarrow (S_x - \lambda I)|\lambda\rangle_x = 0$$

$$|\lambda\rangle_x \neq 0 \Rightarrow \det(S_x - \lambda I) = 0$$

$$\Leftrightarrow \begin{vmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - \left(\frac{\hbar}{2}\right)^2 = 0 \Leftrightarrow \boxed{\lambda = \pm \frac{\hbar}{2}}$$

$$\left(S_x - \left(\pm\frac{\hbar}{2}\right) I\right) \left|\pm\frac{\hbar}{2}\right\rangle_x = 0$$

$$\Leftrightarrow \left(\begin{array}{cc|c} \mp\frac{\hbar}{2} & \frac{\hbar}{2} & 0 \\ \frac{\hbar}{2} & \mp\frac{\hbar}{2} & 0 \end{array}\right) \pm \sim \left(\begin{array}{cc|c} \mp\frac{\hbar}{2} & \frac{\hbar}{2} & 0 \\ 0 & 0 & 0 \end{array}\right) \cdot\frac{1}{\hbar/2} \sim \left(\begin{array}{cc|c} \mp 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \left|\pm\frac{\hbar}{2}\right\rangle_x \doteq \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} \quad \text{or normalized:} \quad \boxed{\left|\pm\frac{\hbar}{2}\right\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}}$$

2.5 Verify (2.96):

$$\begin{cases} [S_x, S_y] = i\hbar S_z \\ [S_y, S_z] = i\hbar S_x \\ [S_z, S_x] = i\hbar S_y \end{cases}$$

Commutator: $[A,B] = AB - BA$

$$[S_x, S_y] = S_x S_y - S_y S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{\hbar^2}{4}\left[\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\right] = \frac{\hbar^2}{4}\begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} =$$

$$= i\hbar \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \doteq i\hbar S_z$$ ok!

$$[S_y, S_z] = S_y S_z - S_z S_y \doteq \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right]$$

$$= \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\right] = \frac{\hbar^2}{4}\begin{pmatrix} 0 & 2i \\ 2i & 0 \end{pmatrix} = i\hbar\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\doteq i\hbar S_x$$ ok!

$$[S_z, S_x] = S_z S_x - S_x S_z \doteq \frac{\hbar^2}{4}\left[\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix} - \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\right]$$

$$= \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right] = \frac{\hbar^2}{4}\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = i\hbar\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\doteq i\hbar S_y$$

ok!

2.9 | Expectation value, uncertainties for state $|+\rangle$, measurements of S_x, S_y, S_z (eq. (2.108))

exp. value:

$$\langle S_x\rangle = \langle + | S_x | + \rangle \doteq (1 \;\; 0) \frac{\hbar}{2} \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{= \begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{\hbar}{2} \cdot (1 \;\; 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$\Leftrightarrow \boxed{\langle S_x \rangle = 0}$$

$$\langle S_y\rangle = \langle + | S_y | + \rangle \doteq (1 \;\; 0) \frac{\hbar}{2} \underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{= \begin{pmatrix} 0 \\ i \end{pmatrix}} = \frac{\hbar}{2} (1 \;\; 0) \begin{pmatrix} 0 \\ i \end{pmatrix} = 0$$

$$\Leftrightarrow \boxed{\langle S_y \rangle = 0}$$

$$\langle S_z\rangle = \langle + | S_z | + \rangle = (1 \;\; 0) \frac{\hbar}{2} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{= \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = \frac{\hbar}{2} (1 \;\; 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}$$

$$\Leftrightarrow \boxed{\langle S_z \rangle = \frac{\hbar}{2}}$$

uncertainties:

$$\Delta S_x^2 = \langle S_x^2 \rangle - \langle S_x \rangle^2 = \langle + | S_x^2 | + \rangle - \underbrace{\langle + | S_x | + \rangle}_{=0}^2$$

$$= \langle + | S_x^2 | + \rangle$$

$$\doteq (1 \;\; 0) \left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left(\frac{\hbar}{2}\right)^2 (1 \;\; 0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left(\frac{\hbar}{2}\right)^2$$

$$\Rightarrow \Delta S_x = \sqrt{\Delta S_x^2} = \frac{\hbar}{2} \quad \Leftrightarrow \quad \boxed{\Delta S_x = \frac{\hbar}{2}}$$

$$\Delta S_y^2 = \langle + | S_y^2 | + \rangle$$

$$\doteq (1 \;\; 0) \left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left(\frac{\hbar}{2}\right)^2 (1 \;\; 0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \left(\frac{\hbar}{2}\right)^2$$

$$\Rightarrow \Delta S_y = \sqrt{\Delta S_y^2} = \frac{\hbar}{2} \quad \Leftrightarrow \quad \boxed{\Delta S_y = \frac{\hbar}{2}}$$

$$\Delta S_z^2 = \langle S_z^2\rangle - \langle S_z\rangle^2 = \langle + | S_z^2 | + \rangle - \langle + | S_z | + \rangle^2$$

$$\doteq (1 \quad 0)\left(\frac{\hbar}{2}\right)^2 \underbrace{\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}_{=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}} \begin{pmatrix}1\\ 0\end{pmatrix} - \left(\frac{\hbar}{2}\right)^2 = \left(\frac{\hbar}{2}\right)^2 - \left(\frac{\hbar}{2}\right)^2 = 0$$

$$\Leftrightarrow \boxed{\Delta S_z = 0}$$

2.12) Find eigenvalues and eigenstates of spin-1

$$S_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

S_x:

eigenvalue equation: $\det(S_x - \lambda I) = 0$

$$\Leftrightarrow \begin{vmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\lambda & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\lambda \end{vmatrix} = 0 \quad \overset{r1}{\Leftrightarrow} \quad (-\lambda)\begin{vmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} \\ \frac{\hbar}{\sqrt{2}} & -\lambda \end{vmatrix} - \frac{\hbar}{\sqrt{2}}\begin{vmatrix} \frac{\hbar}{\sqrt{2}} & \frac{\hbar}{\sqrt{2}} \\ 0 & -\lambda \end{vmatrix} = 0$$

$$(-\lambda)\left(\lambda^2 - \left(\frac{\hbar}{\sqrt{2}}\right)^2\right) - \frac{\hbar}{\sqrt{2}}\left(\frac{\hbar}{\sqrt{2}}(-\lambda) - 0\right) = 0$$

$$\Leftrightarrow \lambda\left(\frac{\hbar^2}{2} - \lambda^2\right) + \frac{\hbar^2}{2}\lambda = 0 \quad \Leftrightarrow \quad \lambda\left(\hbar^2 - \lambda^2\right) = 0$$

$$\Leftrightarrow \boxed{\lambda = 0, \ \lambda = \pm\hbar}$$

eigen vectors:

$\lambda = +\hbar$

$$\begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} -\hbar & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\hbar & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\hbar \end{pmatrix} \end{array} \begin{array}{c} +\frac{1}{\sqrt{2}} \end{array} \sim \begin{pmatrix} -\hbar & \frac{\hbar}{\sqrt{2}} & 0 \\ 0 & -\frac{\hbar}{2} & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\hbar \end{pmatrix} \begin{array}{c} \sqrt{2} \end{array} \sim \begin{pmatrix} -\hbar & \frac{\hbar}{\sqrt{2}} & 0 \\ 0 & -\frac{\hbar}{2} & \frac{\hbar}{\sqrt{2}} \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{c} \frac{\sqrt{2}}{\hbar} \\ \frac{\sqrt{2}}{\hbar} \end{array}$$

$$\sim \begin{pmatrix} -\sqrt{2} & 1 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} b = \sqrt{2}\,c \\ b = \sqrt{2}\,a \end{cases} \Rightarrow |\hbar\rangle_x \doteq \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ \sqrt{2}\,a \\ a \end{pmatrix}$$

$$\Downarrow \quad a = c$$

$\langle \hbar | \hbar \rangle = 1 \iff |a|^2 (1 + 2 + 1) = |a|^2 \cdot 4 = 1$

$|a| = \frac{1}{2}$

$a = \frac{1}{2} \Rightarrow |\hbar\rangle_x \doteq \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$ or $\boxed{|\hbar\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle}$

Same procedure for the rest...

Answers:

$$\boxed{\begin{aligned} |-\hbar\rangle_x &= \frac{1}{2}|\hbar\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-\hbar\rangle \\ |0\rangle_x &= \frac{1}{\sqrt{2}}|\hbar\rangle - \frac{1}{\sqrt{2}}|-\hbar\rangle \end{aligned}}$$

for S_y:

$\boxed{\lambda = +\hbar, 0, -\hbar}$

$$\boxed{\begin{aligned} |\hbar\rangle_y &= \frac{1}{2}|+\hbar\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-\hbar\rangle \\ |0\rangle_y &= \frac{1}{\sqrt{2}}|\hbar\rangle + \frac{1}{\sqrt{2}}|-\hbar\rangle \\ |-\hbar\rangle_y &= \frac{1}{2}|\hbar\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-\hbar\rangle \end{aligned}}$$

2.18

$$|\psi\rangle = \frac{1}{\sqrt{14}}|1\rangle - \frac{3}{\sqrt{14}}|0\rangle + i\frac{2}{\sqrt{14}}|-1\rangle \qquad \text{(spin 1 particle)}$$

a) Results of S_z measurement : $+\hbar, 0, -\hbar$

$$P_0 = |\langle 0|\psi\rangle|^2 = |\langle 0|\frac{1}{\sqrt{14}}(|1\rangle - 3|0\rangle + i\cdot 2|-1\rangle)|^2$$

$$= |\frac{1}{\sqrt{14}}(-3)|^2 = \frac{9}{14}$$

$$P_1 = |\langle 1|\psi\rangle|^2 = |\langle 1|\frac{1}{\sqrt{14}}(|1\rangle + \ldots)|^2 = \frac{1}{14}$$

$$P_{-1} = 1 - P_0 - P_1 = \frac{4}{14}$$

b) $S_z : -\frac{\hbar}{2} \implies |\psi'\rangle = |-1\rangle$

$S_x : +\hbar, 0, -\hbar$ possible observations

Eigenvectors : $|1\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$

(2.113) $|0\rangle_x = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$

$|-1\rangle_x = \frac{1}{2}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$

$$P_{1x} = |{}_x\langle 1|\psi'\rangle|^2 = |\left(\frac{1}{2}\langle 1| + \frac{1}{\sqrt{2}}\langle 0| + \frac{1}{2}\langle -1|\right)|-1\rangle|^2$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

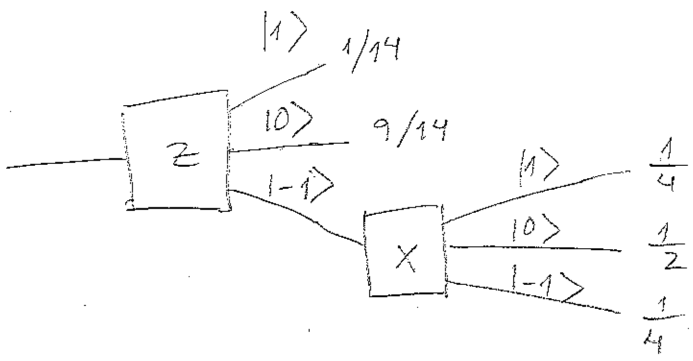
$$P_{0x} = |{}_x\langle 0|\psi'\rangle|^2 = |\left(\frac{1}{\sqrt{2}}\langle 1| - \frac{1}{\sqrt{2}}\langle -1|\right)|-1\rangle|^2$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$P_{-1x} = |{}_x\langle -1|\psi'\rangle|^2 = |\left(\frac{1}{2}\langle 1| - \frac{1}{\sqrt{2}}\langle 0| + \frac{1}{2}\langle -1|\right)|-1\rangle|^2$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

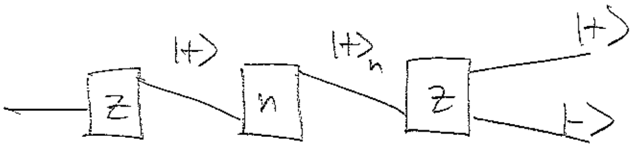
c.)



2.18.2

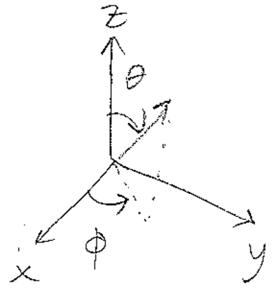
2.22

fig. 2.15 :



a) $P(+\to +n \to -) = P(+n|+)\cdot P(-|+n) = |{}_n\langle +|+\rangle|^2 \, |\langle -|+\rangle_n|^2$

Eigenvectors of S_n: (2.42)



$$\begin{cases} |+\rangle_n = \cos\left(\frac{\theta}{2}\right)|+\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|-\rangle \\ |-\rangle_n = \sin\left(\frac{\theta}{2}\right)|+\rangle - \cos\left(\frac{\theta}{2}\right)e^{i\phi}|-\rangle \end{cases}$$

$\phi = 0 \Rightarrow |+\rangle_n = \cos\left(\frac{\theta}{2}\right)|+\rangle + \sin\left(\frac{\theta}{2}\right)|-\rangle$

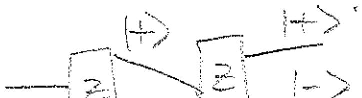
$$\Rightarrow P(+\to +n \to -) = \left|\cos\left(\frac{\theta}{2}\right)\right|^2 \cdot \left|\sin\left(\frac{\theta}{2}\right)\right|^2 = \left(\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\right)^2$$

$$= [\sin 2\alpha = 2\sin\alpha\cos\alpha] = \left(\frac{1}{2}\sin\theta\right)^2 = \underline{\underline{\frac{1}{4}\sin^2\theta}}$$

b) Maximize $P(+\to +n \to -)$ w.r.t. θ

$$\max_{\theta\in[0,\pi]} \frac{1}{4}\sin^2\theta = \underline{\underline{\frac{1}{4}}}, \text{ for } \boxed{\theta = \frac{\pi}{2}}$$

c) Remove middle analyzer:



$P(+\to -) = P(-|+) = |\langle -|+\rangle|^2 = \underline{\underline{0}}$