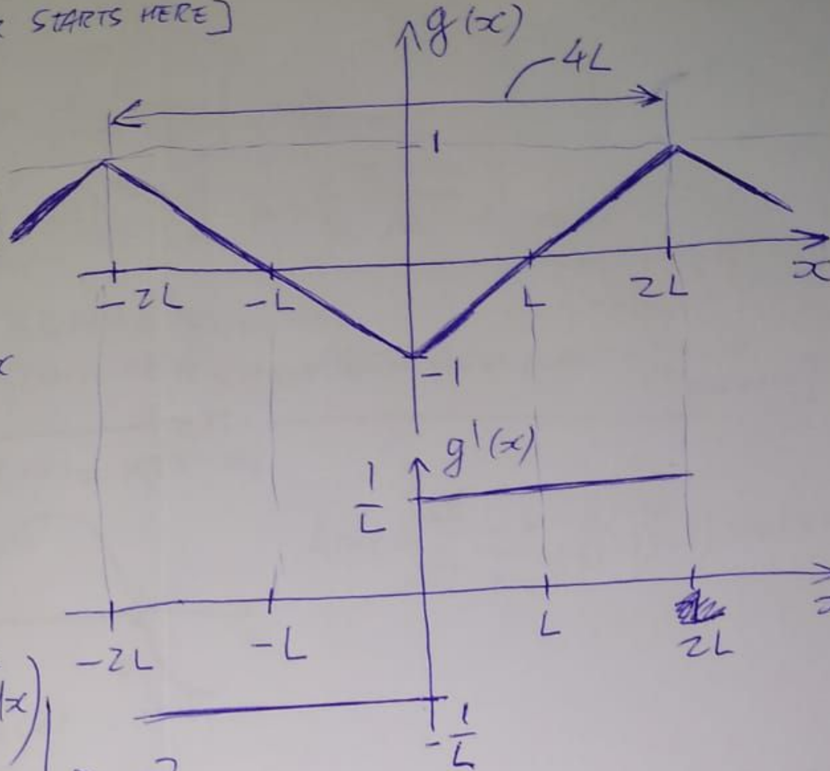
EXAMPLE SHEET 0 (I) [TIMER STARTS HERE]

4(b)



$$g(x)=\sum_{-\infty}^{\infty} c_n \exp\left(\frac{i\pi x}{L}n\right)$$

$$c_n=\frac{1}{P}\int_{-\frac{P}{2}}^{\frac{P}{2}} g\, e^{-\frac{ix\pi}{\left(\frac{P}{2}\right)}n}\,dx$$

$$=\frac{1}{4L}\int_{-2L}^{2L} g\, e^{-\frac{ix\pi}{2L}n}\,dx$$

$$=\frac{1}{4L}\left(\left[\;\right]-\int_{-2L}^{2L} g'\,\frac{-2L}{i\pi n}\, e^{-\frac{ix\pi}{2L}n}\,dx\right)$$

n^{-2} - CONTAINING TERMS WILL BE THE FIRST NONZERO TERMS.

$$=\frac{1}{\cancel{4L}_{2}}\,\frac{\cancel{2L}}{i\pi n}\int_{-2L}^{2L} \underbrace{g'}_{\text{ODD}}\left(\underbrace{\cos\left(-\frac{x\pi n}{2L}\right)}_{\text{EVEN}}+i\,\underbrace{\sin\left(-\frac{x\pi n}{2L}\right)}_{\text{ODD}}\right)dx$$

$$=\frac{1}{\cancel{2}}\,\frac{1}{\cancel{i}\pi n}\,\cancel{2}\int_{0}^{2L}\frac{1}{L}\,\cancel{i}\,\sin\left(-\frac{x\pi n}{2L}\right)dx$$

$$=\frac{1}{\pi n L}\int_{0}^{2L}\sin\left(-\frac{x\pi n}{2L}\right)dx=\frac{1}{\pi n \cancel{L}}\,\frac{-2\cancel{L}}{\pi n}\left[-\cos\left(-\frac{x\pi n}{2L}\right)\right]_{0}^{2L}$$

$$=\frac{-2}{\pi^2 n^2}\left(-\cos(\pi n)+1\right)=\frac{-2}{\pi^2 n^2}\left(-(-1)^n+1\right)$$

$g(x)$ IS ZERO CENTERED & EVEN $\Rightarrow g(x)=\sum_{1}^{\infty} d_n \cos\left(\frac{\pi x n}{\frac{P}{2}}\right)$

$$=\sum_{1}^{\infty} a_n \cos\left(\frac{\pi x n}{2L}\right)$$

EXAMPLE SHEET 0 (II)

4(e)

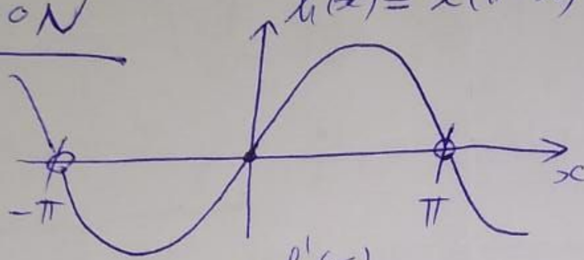
$$d_n = c_n + c_{-n} = +\frac{4}{\pi^2 n^2}\left((-1)^n - 1\right)$$

$$\text{so } g(x) = \sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2}\left((-1)^n - 1\right)\cos\left(\frac{\pi x n}{2L}\right)$$

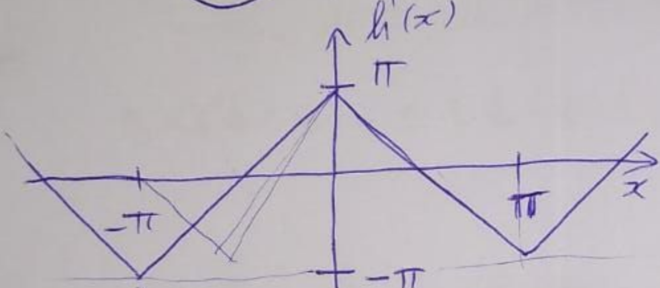
[33 MINS, INCLUDING COMPUTER CHECK & $\ominus$ SIGN SEARCH] [TIMER RESTARTS]

REFLECTION

$h(x) = x(\pi - x)$

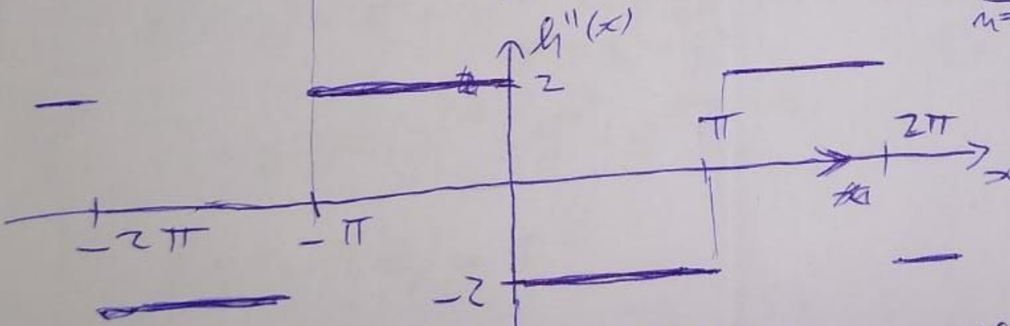


$$h(x) = \sum_{n=1}^{\infty} \frac{-4}{\pi n^3}\left((-1)^n - 1\right)\sin(nx)$$



$$h'(x) = \sum_{n=1}^{\infty} \pi \cdot \frac{-4}{\pi^2 n^2}\left((-1)^n - 1\right)\cos\left(\frac{\pi x n}{\pi}\right)$$

$$= \sum_{n=1}^{\infty} \frac{-4}{\pi n^2}\left((-1)^n - 1\right)\cos(nx)$$



GUESS:

$$h''(x) = \sum_{n=1}^{\infty} \frac{-4}{\pi n}\left((-1)^n - 1\right)\sin(nx)$$

YES, THIS IS TRUE (COMP. CHECK)

APART FROM $\ominus$, ITS $\oplus$

[24 MINS] [TIMER RESTARTS]