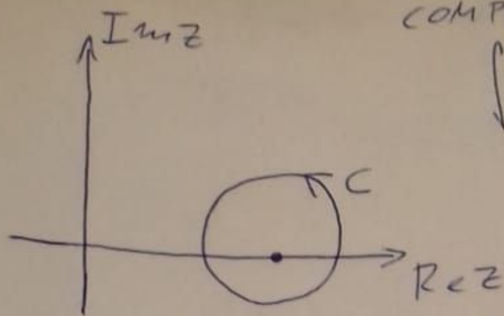


COMPUTE

$$\int_C \frac{e^{z^2}}{z-2}\,dz$$

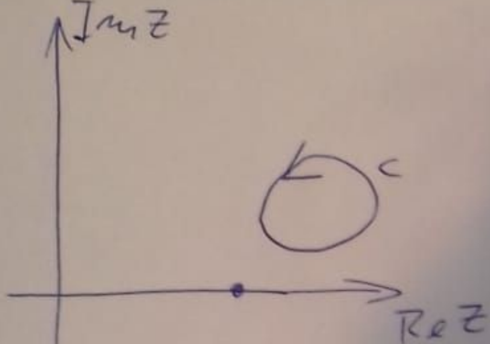
LET: $f(z)=e^{z^2}$

$z_0=2$ INSIDE C

BY CIF:

$$2\pi i\, f(z_0)=\int \frac{e^{z^2}}{z-2}\,dz =$$

$$= 2\pi i\, e^{2^2} = 2\pi i\, e^4$$



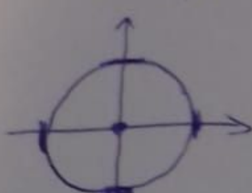
NO POLES INSIDE C FOR

$$f(z)=\frac{e^{z^2}}{z-2}$$

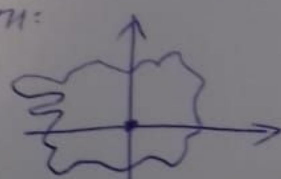
$$\Rightarrow \int_C \frac{e^{z^2}}{z-2}\,dz = 0$$

(BY RESIDUE THEOREM)

$$I=\int_C \frac{e^{2z}}{z^4}\,dz \qquad C: |z|=1$$



THE ANSWER IS THE SAME FOR THE PATH:



LET $f(z)=e^{2z}$

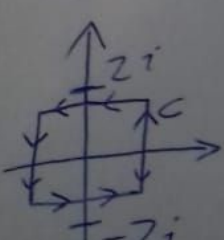
$z_0=0 \quad n=3$

BY CIFFD:

$$\int_C \frac{f(z)\,dz}{(z-z_0)^{n+1}} = \int_C \frac{e^{2z}\,dz}{(z-0)^4} = \frac{2\pi i}{3!} f'''(0) =$$

$$= 2\pi i\,(e^{2z})''' = \frac{8}{3}\pi i$$

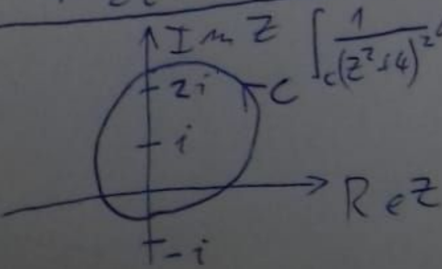
$$\int_C \frac{\cos z}{z(z^2+8)}\,dz$$



LET $f(z)=\dfrac{\cos z}{z^2+8}$

$$\int_C \frac{\cos z/(z^2+8)}{z}\,dz = \int_C \frac{f(z)}{z}\,dz = 2\pi i\,\frac{\cos 0}{0^2+8} = \frac{1}{4}\pi i$$

$$\int_C \frac{1}{(z^2+4)^2}\,dz$$



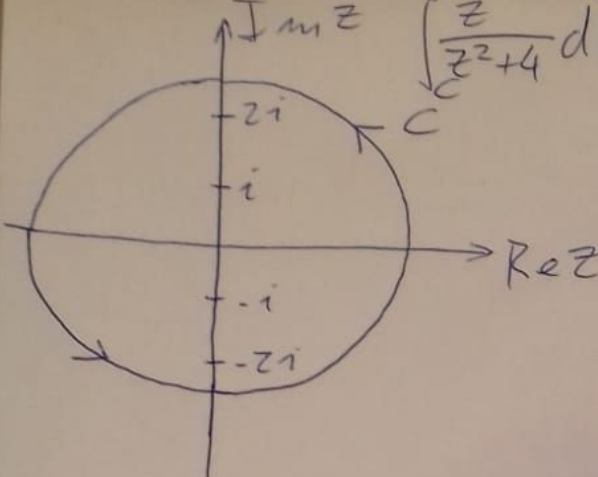
$$\frac{1}{(z^2+4)^2} = \frac{1}{(z-2i)^2(z+2i)^2}$$

LET $f(z)=\dfrac{1}{(z+2i)^2}$

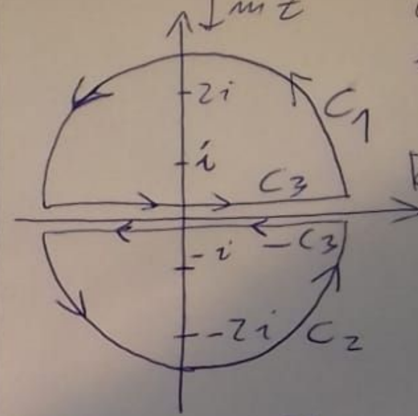
THIS IS ANALYTIC INSIDE C.

CIFFD:

$$\int_C \frac{1}{(z^2+4)^2}\,dz = \int_C \frac{f(z)\,dz}{(z-2i)^2} = 2\pi i\, f'(2i) = 2\pi i\left[\frac{-2}{(z+2i)^3}\right]_{z=2i} = \frac{-4\pi i}{-64i} = \frac{\pi}{16}$$



$$\int_C \frac{z}{z^2+4}\,dz$$



CANT FIND $f(z)$ WHICH IS ANALYTIC IN $C \Rightarrow$ SPLIT THE CURVE

$$\frac{z}{z^2+4} = \frac{z}{(z-2i)(z+2i)}$$

$$f_1(z) = \frac{z}{z+2i} \qquad f_2(z) = \frac{z}{z-2i}$$

so f_1 is analytic within C_1 & C_3

$$\frac{z}{z^2+4} = \frac{f_1(z)}{z-2i} = \frac{f_2(z)}{z+2i}$$

$$\int_C \frac{z\,dz}{z^2+4} = \int_{C_1+C_3-C_3+C_2} \frac{z\,dz}{z^2+4} =$$

$$= \int_{C_1+C_3} \frac{f_1(z)\,dz}{z-2i} + \int_{C_2-C_3} \frac{f_2(z)\,dz}{z+2i} =$$

f_1 IS ANALYTICAL W/IN C_1 & C_3 CURVES
f_2 IS SO W/IN $C_2 - C_3$
$\Rightarrow$ CAUCHY ~~H~~INTEGRAL FORMULA WORKS FOR BOTH

$$= 2\pi i\left[f_1(2i) + f_2(-2i)\right] =$$

$$= 2\pi i\left[\frac{1}{2} + \frac{1}{2}\right] = 2\pi i$$