CBE IN SPC:

$$\frac{\partial f}{\partial t} + \dot{r}\frac{\partial f}{\partial r} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\phi}\frac{\partial f}{\partial \phi} + \dot{v}_r\frac{\partial f}{\partial v_r} + \dot{v}_\theta\frac{\partial f}{\partial v_\theta} + \dot{v}_\phi\frac{\partial f}{\partial v_\phi} = 0$$

NO θ & ϕ DEPENDENCE $\Rightarrow \frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial \phi} = 0$

CONSIDER TERMS SEPARATELY:

$$\dot{r} = v_r$$

$$\dot{v}_r = \ddot{r} = a_r + r\dot{\theta}^2 + r\sin^2\theta\,\dot{\phi}^2$$

$$= -\frac{\partial \Phi}{\partial r} + \frac{1}{r}\left(v_\theta^2 + v_\phi^2\right)$$

$$\dot{v}_\theta = \frac{d}{dt}\left(r\dot{\theta}\right) = \dot{r}\dot{\theta} + r\ddot{\theta}$$

$$= a_\theta + \dot{r}\dot{\theta} + r\sin\theta\cos\theta\,\dot{\phi}^2$$

$$= -\frac{\partial \Phi}{\partial \theta}_{\to 0} - \frac{1}{r}\left(\dot{r} + r\dot{\theta} \mp (\sin\theta\,\dot{\phi}\,r)^2\cot\theta\right)$$

$$= -\frac{1}{r}\left(v_r v_\theta - v_\phi^2\cot\theta\right)$$

$$\dot{v}_\phi = \frac{d}{dt}\left(r\sin\theta\,\dot{\phi}\right) = \dot{r}(\sin\theta)\dot{\phi} + r(\cos\theta)\dot{\theta}\dot{\phi} + r(\sin\theta)\ddot{\phi}$$

$$= a_\phi - \dot{r}(\sin\theta)\dot{\phi} - r(\cos\theta)\dot{\theta}\dot{\phi}$$

$$= -\frac{\partial \Phi}{\partial \phi}_{\to 0} - \frac{1}{r}\left(r\sin\theta\,\dot{\phi}\,\dot{r} + r\dot{\theta}\,r\sin\theta\,\dot{\phi}\cot\theta\right)$$

$$= -\frac{1}{r}v_\phi\left(v_r + v_\theta\cot\theta\right)$$

COMBINING TERMS TOGETHER:

$$\frac{\partial f}{\partial t} + v_r\frac{\partial f}{\partial r} + \frac{1}{r}\left(v_\theta^2 + v_\phi^2\right)\frac{\partial f}{\partial v_r} - \frac{1}{r}\left(v_r v_\theta - v_\phi^2\cot\theta\right)\frac{\partial f}{\partial v_\theta} - \frac{1}{r}v_\phi\left(v_r + v_\theta\cot\theta\right)\frac{\partial f}{\partial v_\phi} - \frac{\partial \Phi}{\partial r}\frac{\partial f}{\partial v_r} = 0, \quad \text{AS REQUIRED.}$$