TOPICS SHEET I | Q1

$N(L)\,dL$: Number of objects in the sample with luminosity between L & $L+dL$. Note that this refers to the whole sample, not per ~~unit~~ unit volume.

We can get this number by: Counting the objects with luminosity between L & $L+dL$ over whole space. We count only those which we are able to see.

i.e.:

$$N(L)dL = \int \begin{matrix}\text{VOLUME DENSITY} \\ \text{OF STARS BTWN } L \& L+dL\end{matrix} \; d\,\begin{matrix}\text{VISIBLE}\\ \text{SPACE}\end{matrix}$$

We can observe "down to a certain apparent magnitude", i.e. a star is visible if $\frac{L}{r^2} > C$ for some constant C.

ASSUMPTION I: spherical symmetry of star distribution centered on us. (i.e. ignore θ & ϕ dep. of space)

$$N(L)dL \propto \int_0^{\substack{\text{DEEPEST}\\ \text{WE CAN SEE}}} \begin{pmatrix}\text{VOLUME DENSITY}\\ \text{OF STARS BTWN}\\ L \& L+dL\end{pmatrix} \underbrace{r^2 dr}_{\text{VOLUME ELEMENT}}$$

(INTEGRAL OVER θ & ϕ IE $\sin\theta \, d\theta \, d\phi$ WOULD ONLY GIVE A ~~CONSTANT TERM SO WE DON'T~~ MULTIPLICATIVE CONSTANT TERM SO WE DON'T CARE)

~~DEEPEST WE CAN SEE A~~

DEEPEST WE CAN SEE A STAR WITH LUMINOSITY L: $\frac{L}{r^2} > c \Rightarrow r \leq \sqrt{\frac{L}{c}}$

VOLUME DENSITY OF STARS BTWN L & L+dL $= n(L)dL$ (THE PROBLEM TOLD US SO)

SUBSTITUTING IN:

$$N(L)dL \propto \int_0^{\sqrt{\frac{L}{c}}} n(L)dL \; r^2 dr$$

/ MOVE OUT r-INDEP TERMS

$$\propto n(L)dL \int_0^{\sqrt{\frac{L}{c}}} r^2 dr$$

/ INTEGRATE

$$\propto n(L)dL \left. \frac{1}{3} r^3 \right|_0^{\sqrt{\frac{L}{c}}}$$

/ KILL MULTIPLICATIVE CONSTANTS

$$\propto n(L)dL \, L^{\frac{3}{2}}$$

$\Downarrow$ /: dL

$$N(L) \propto n(L) L^{\frac{3}{2}}$$

$\Downarrow$

$$n(L) \propto N(L) L^{-\frac{3}{2}}$$

AS REQUIRED.