$\boxed{\text{III.1}}$

$$\text{FRACTION OF SPIRALS} = \frac{\text{NUMBER DENSITY OF SPIRALS}}{\text{TOTAL NUMBER DENSITY}}$$

$$S = \int_{\underbrace{L_D}_{\text{THIS LUMINOSITY DEEP WE SEE.}}}^{\infty} \varphi(L)\,dL$$

IF $L_D < L_*$:

$$S = \int_{L_D}^{L_*} \varphi(L < L_*)\,dL + \int_{L_*}^{\infty} \varphi(L > L_*)\,dL$$

ELSE:

$$S = \int_{L_D}^{\infty} \varphi(L > L_*)\,dL$$

COMPUTE THESE INTEGRALS:

$$\int \varphi(L < L_*)\,dL = \int \frac{n_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} dL$$

$$= \frac{n_*}{L_*^{\alpha+1}} \frac{1}{\alpha+1} L^{\alpha+1}$$

$$\int \varphi(L > L_*)\,dL = \int \frac{n_*}{L_*} \exp\left(-\frac{L}{L_*}\right) dL = -n_* \exp\left(-\frac{L}{L_*}\right)$$

(i)

$L_{*E} > L_1 > L_{*S}$

IE POWER-LAW CONTRIB ONLY FOR ELLIPTICALS.

$$S_S = \int_{L_1}^{\infty} \phi(L > L_{*S})\, dL$$

$$= -n_{*S} \exp\left(-\frac{L}{L_{*S}}\right)\Big|_{L_1}^{\infty} = n_{*S} \exp\left(-\frac{L_1}{L_{*S}}\right)$$

$$S_E = \int_{L_1}^{\infty} \phi(L > L_{*E})\, dL$$

$$\cancel{= -n_{*E} \exp\left(-\frac{L}{L_{*E}}\right)\Big|_{L_1}^{L_{*E}} +}$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left[L^{\alpha_E+1} \right]_{L_1}^{L_{*E}} + \left[(-n_{*E}) \exp\left(-\frac{L}{L_{*E}}\right)\right]_{L_{*E}}^{\infty}$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_1^{\alpha_E+1} \right) +$$

$$+ n_{*E} \exp\left(-\frac{L_{*E}}{L_{*E}}\right)$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_1^{\alpha_E+1} \right) + n_{*E} \exp(-1)$$

PUTTING IN THE NUMBERS:

$$S_S = n_{*S} \exp\left(-\frac{5\cdot 10^{11}}{10^{11}}\right) = n_{*S} e^{-5}$$

$$\cancel{S_E = \frac{4 n_{*S}}{\ \ } \frac{1}{\alpha_E+1} \ \alpha_E+1}$$

$$S_E = \frac{4 n_{*S}}{(10^{12})^{-0.5+1}} \frac{1}{-0.5+1} \left((10^{12})^{-0.5+1} - (5\cdot 10^{11})^{-0.5+1} \right) +$$

$$+ 4 n_{*S} e^{-1}$$

$$= 3.814\, n_{*S}$$

$$\text{FRACTION OF SPIRALS} = \frac{n_{*S} e^{-5}}{3.814\, n_{*S} + e^{-5} n_{*S}} \simeq \underline{1.77\cdot 10^{-3}}$$

(ii)

$$L_{*E} > L_{*S} > L_2$$

IE POWER-LAW & EXPONENTIAL TERM FOR BOTH SPIRALS & ELLIPTICALS. USING PREVIOUS RESULTS:

$$S_S = \frac{n_{*S}}{L_{*S}^{\alpha_S+1}} \frac{1}{\alpha_S+1}\left(L_{*S}^{\alpha_S+1} - L_2^{\alpha_S+1}\right) + n_{*S}\exp(-1)$$

$$S_E = \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1}\left(L_{*E}^{\alpha_E+1} - L_2^{\alpha_E+1}\right) + n_{*E}\exp(-1)$$

PUTTING IN THE NUMBERS:

$$S_S = \frac{n_{*S}}{(10^{11})^{-1.5+1}} \frac{1}{-1.5+1}\left((10^{11})^{-1.5+1} - (10^{9})^{-1.5+1}\right) +$$

$$+ n_{*S}\exp(-1)$$

$$= \frac{n_{*S}}{(10^{11})^{-\frac{1}{2}}}(-2)\left((10^{11})^{-\frac{1}{2}} - (10^{9})^{-\frac{1}{2}}\right) + n_S e^{-1}$$

$$= \underline{18.37\, n_{*S}}$$

$$S_E = \frac{4n_{s*}}{(10^{12})^{-0.5+1}} \frac{1}{-0.5+1} \left((10^{12})^{-0.5+1} - (10^9)^{-0.5+1} \right) + 4n_{s*} e^{-1}$$

$$= 9.22\, n_{*s}$$

$$\text{FRACTION OF SPIRALS} = \frac{18.37}{18.37+9.22} = \underline{0.67}$$

COMMENT ON RESULT:

THE DEEPER WE SEE THE MORE REALISTIC RESULTS WE GET.

IT IS IMPORTANT TO SEE DEEP ENOUGH TO OBSERVE THE POWER LAW PART OF THE DISTRIBUTION, WHICH IS THE DOMINATING CONTRIBUTOR TO NUMBER DENSITY.

III.2

$$x = \frac{1}{N} \sum_{i=1}^{N} \frac{V(i)}{V_{MAX}(i)}$$

MEAN:

$$\langle x \rangle = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{V(i)}{V_{MAX}(i)}$$

$$\cancel{= \int_0^1 P\left(\frac{V}{V_{MAX}}\right)}$$

$$= \int_0^1 PDF\left(\frac{V}{V_{MAX}}\right)\left(\frac{V}{V_{MAX}}\right) d\left(\frac{V}{V_{MAX}}\right)$$

VARIANCE:

$$VAR(x) = \langle x^2 \rangle - \langle x \rangle^2$$

$$\text{WHERE } \langle x^2 \rangle = \int_0^1 PDF\left(\frac{V}{V_{MAX}}\right)\left(\frac{V}{V_{MAX}}\right)^2 d\left(\frac{V}{V_{MAX}}\right)$$

WE AIM FOR: $PDF\left(\frac{V}{V_{MAX}}\right)$

PLAN TO GET THERE:

$$PDF(r) \Rightarrow PDF(v) \Rightarrow PDF\left(\frac{v}{v_{MAX}}\right)$$

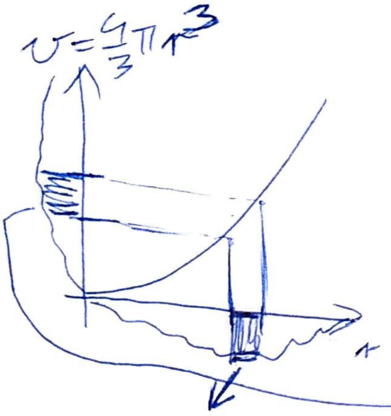
$$\text{NUMBER OF GALAXIES BTWN } r \,\&\, r+dr = 4\pi r^2 dr \underbrace{\rho}_{\text{NUMBER DENSITY OF GALAXIES}}$$

$$PDF(r)dr = \text{FRACTION OF GALAXIES BTWN } r \,\&\, r+dr = \frac{4\pi r^2 dr \rho}{\frac{4}{3} d_M^3 \pi \rho} =$$

$$= 3 \frac{r^2}{d^3_{MAX}} dr$$

USING THE NOTATION: $PDF_X(x)$

IS THE PROBABILITY DENSITY FUNCTION OF A SINGLE RANDOM VARIABLE (IN OUR CASE: PROPERTY X OF A GALAXY) AS A FUNCTION OF x.



$$PDF_R(r)\,dr = PDF_V(v(r))\,dv(r)$$

$$= PDF_V(v)\,d\left(\frac{4}{3}r^3\pi\right)$$

$$= PDF_V(v)\,4r^2\pi\,dr$$

$$\Rightarrow PDF_V(v) = PDF_R(r)/(4r^2\pi)$$

$$= \frac{3r^2/d_M^3}{4r^3\pi} = \frac{3}{4}\frac{1}{d_M^3}\frac{1}{\pi}$$

USING SAME MATH:

$$PDF_V(v)\,dv = PDF_{\frac{V}{V_{MAX}}}\left(\frac{v}{v_{MAX}}\right) d\left(\frac{v(v)}{v_{MAX}(v)}\right)$$

$$= PDF_{\frac{V}{V_{MAX}}}\left(\frac{v}{v_{MAX}}\right)\frac{1}{v_{MAX}}\,dv$$

$$\Rightarrow PDF_{\frac{V}{V_{MAX}}}\left(\frac{v}{v_{MAX}}\right) = v_{MAX}\,PDF_V(v) =$$

$$= \frac{4}{3}\pi d_M^3 \; \frac{3}{4}\frac{1}{d_M^3}\frac{1}{\pi} = 1$$

QUITE NICE.

MAYBE THERE IS AN EASIER WAY TO GET HERE.

$$\langle x \rangle = \int_0^1 \underbrace{PDF\left(\frac{V}{V_{MAX}}\right)}_{1} \left(\frac{V}{V_{MAX}}\right) d\frac{V}{V_{MAX}}$$

$$= \frac{1}{2}\left[\left(\frac{V}{V_{MAX}}\right)^2\right]_0^1 = \frac{1}{2}$$

$$\langle x^2 \rangle = \int_0^1 \underbrace{PDF\left(\frac{V}{V_{MAX}}\right)}_{1} \left(\frac{V}{V_{MAX}}\right)^2 d\frac{V}{V_{MAX}}$$

$$= \left[\frac{1}{3}\left(\frac{V}{V_{MAX}}\right)^3\right]_0^1 = \frac{1}{3}$$

$$VAR(x) = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

HOW CAN THIS BE TEST FOR EVOLUTION :

DEVIATIONS FROM ABOVE RESULTS INDICATE THAT GALAXY DISTRIBUTION IS NOT UNIFORM, AS WE ASSUMED.

~~WE CAN THEN OBTAIN A GALAXY~~

III.4

$$V(\phi) = V_0 \exp(-\lambda\phi)$$

EQ. OF MOTION:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\ddot{\phi} + 3H\dot{\phi} + (-\lambda)V_0 \exp(-\lambda\phi) = 0$$

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} - \lambda V_0 e^{-\lambda\phi} = 0$$

CHECK IF PROVIDED SOLUTION IS RIGHT:

$$\cancel{\dot{\phi} = \frac{2}{\lambda}\left(\frac{V_0}{2(6-\lambda^2)}\right)^{-\frac{1}{2}} t^{-2}}$$

$$\dot{\phi}(t) = \frac{2}{\lambda} t^{-1}$$

$$\ddot{\phi}(t) = -\frac{2}{\lambda} t^{-2}$$

$$\cancel{R(t) = R_0\left(\frac{t}{t_0}\right)^{\frac{2}{\lambda^2}}}$$

$$\dot{R}(t) = R_0\left(\frac{1}{t_0}\right)^{\frac{2}{\lambda^2}} \frac{2}{\lambda^2} t^{\frac{2}{\lambda^2}-1}$$

$$\frac{\dot{R}}{R}\dot{\phi} = \frac{\cancel{R_0}\cancel{\left(\frac{1}{t_0}\right)^{\frac{2}{\lambda^2}}} \frac{2}{\lambda^2} t^{\cancel{\frac{2}{\lambda^2}}-1}}{\cancel{R_0}\cancel{\left(\frac{t}{t_0}\right)^{\frac{2}{\lambda^2}}}} \frac{2}{\lambda} t^{-1} = \frac{4}{\lambda^3} t^{-2}$$

$$\ddot{\varphi} + \frac{3\dot{R}}{R}\dot{\varphi} - \lambda V_0 e^{-\lambda\varphi} =$$

$$= -\frac{2}{\lambda}t^{-2} + 3\frac{4}{\lambda^3}t^{-2} \cancel{-\lambda V_0\left(\frac{V_0}{2(6-\lambda^2)}\right)^{-\frac{1}{\lambda}}}$$

$$-\lambda V_0 \exp\left(-\cancel{\lambda}\frac{2}{\cancel{\lambda}}\ln\left[[\dots]^{\frac{1}{2}}\lambda^2 t\right]\right)$$

$$= -\frac{2}{\lambda}t^{-2} + \frac{12}{\lambda^3}t^{-2} - \lambda V_0\left[[\dots]^{\frac{1}{2}}\lambda^2 t\right]^{-2}$$

$$= -\frac{2}{\lambda}t^{-2} + \frac{12}{\lambda^3}t^{-2} - \lambda V_0[\dots]^{-1}\lambda^{-4}t^{-2}$$

$$= -\frac{2}{\lambda}t^{-2} + \frac{\cancel{12}}{\lambda^3}t^{-2} - \cancel{V_0}\frac{2(\cancel{6}-\lambda^2)}{\cancel{V_0}}\lambda^{-3}t^{-2}$$

$$= -\frac{2}{\lambda}t^{-2} + \frac{2\lambda^2}{\lambda^3}t^{-2} = 0$$ INDEED.

IDK HOW TO SOLVE THE EQUATION PROPERLY THOUGH.