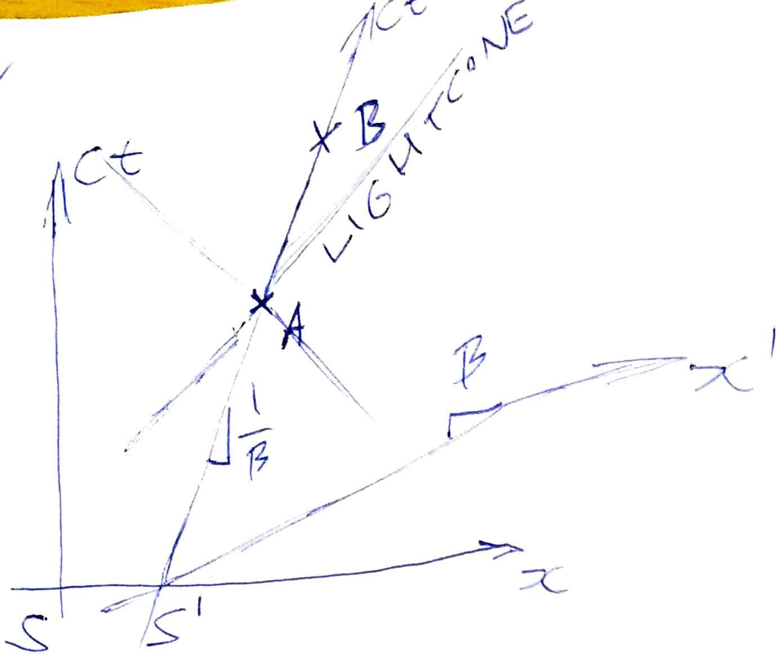
1. a/



A&B: 2 events timelike separated in S.

S': frame with standard configuration, with a Lorentz boost β.

Origin of S & S' might not coincide.

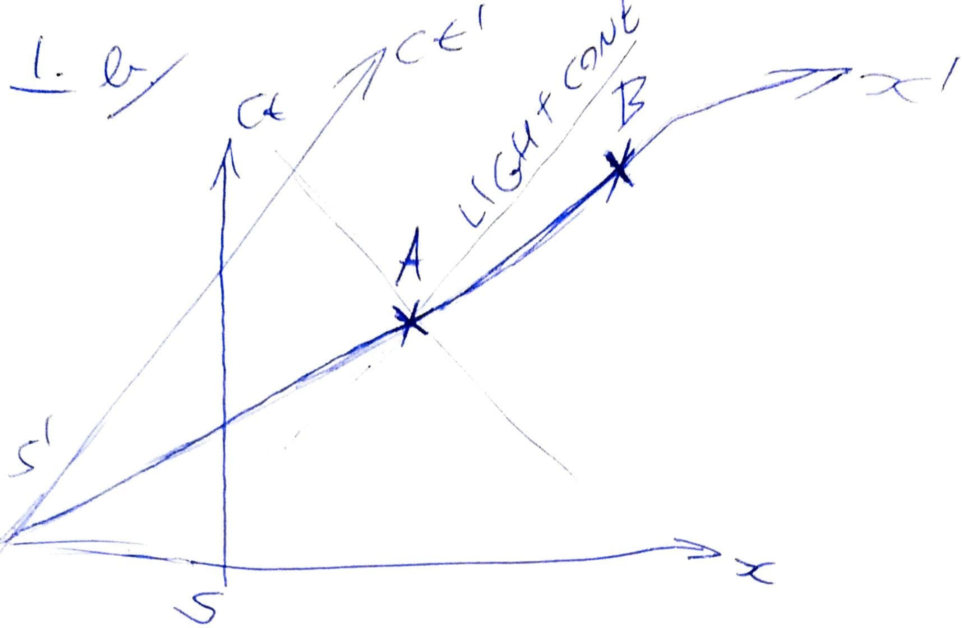
Slope of ct' axis when drawn in S: $\frac{1}{\beta}$

(SINCE $x = \beta ct$)

$-1 \leq \beta \leq 1 \Rightarrow 1 \leq \frac{1}{|\beta|} \Rightarrow$ IF A & B ARE TIMELIKE SEPARATED (IE THE LINE CONNECTING THEM WILL ALWAYS HAVE A SLOPE ≥ 1 IN S), WE CAN FIT A ct' AXIS ON THE TWO POINTS.

IN S', A & B HAPPENS AT THE SAME SPATIAL LOCATION.

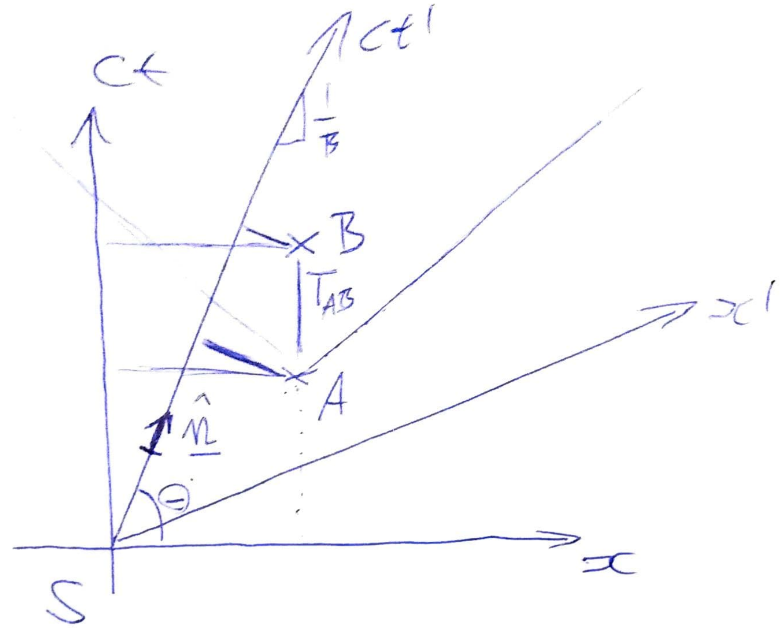
1\. b)



SIMILAR ARGUMENT AS IN a), BUT NOW FITTING x' AXIS TO THE TWO EVENTS.

spatially separated events ⇒ line connecting them have gradient < 1 in S ⇒ can fit x' on them which has gradient between -1 & 1. In S', the two events will be simultaneous.

2.a



All S' frames have a ct' axis with slope $\frac{1}{B}$ as drawn in S. Ie this slope must have a magnitude at least 1. Looking at the drawing, we can conclude that A will precede B in all S' frames.

A precedes B if:

$$\vec{AB} \cdot \hat{\underline{n}} > 0$$

(BOTH $\vec{AB}$ & $\hat{\underline{n}}$ IN S, AS DRAWN)

i.e

$$\begin{pmatrix} 0 \\ T_{AB} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} > 0$$

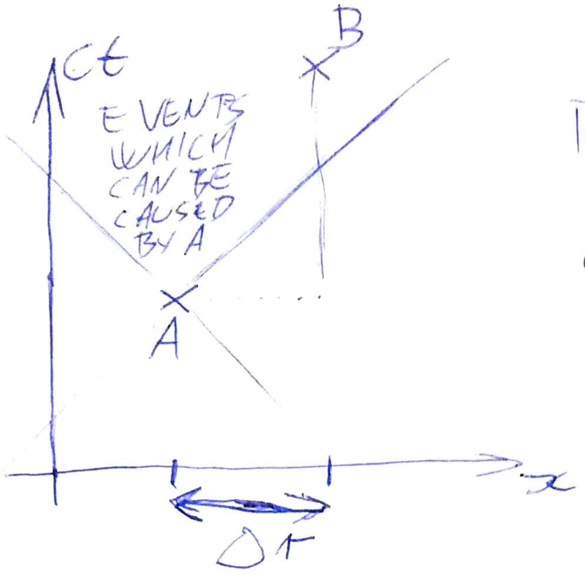
WITH $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

$T_{AB} > 0$.

$$T_{AB} \sin\theta > 0 \quad \text{FOR } \forall \ \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$$

WHICH IS TRUE, SO A ALWAYS PRECEDES B.

2.b)



Reading off from drawing:

$$c(t_B - t_A) \geq \Delta r$$

$$c\Delta t \geq \Delta r$$

$$\Delta t \geq \frac{\Delta r}{c}$$

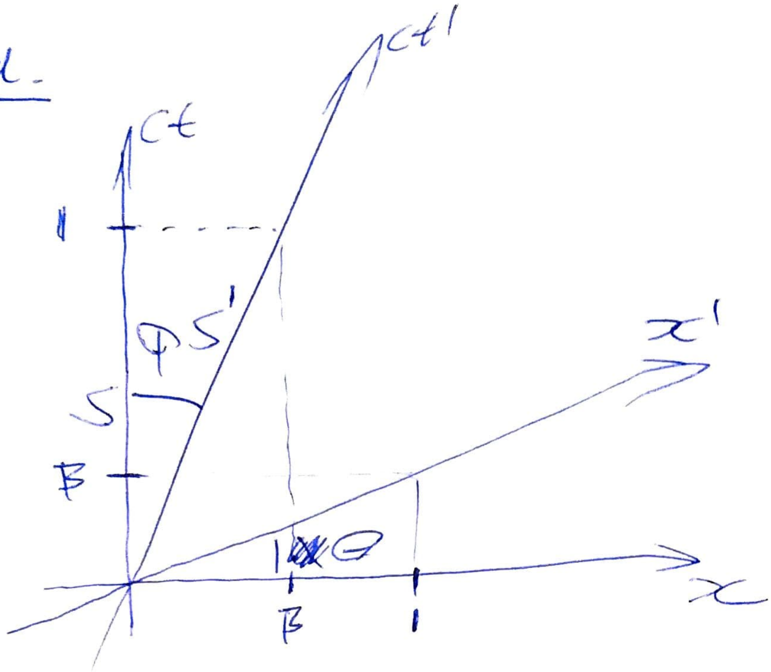
$\Downarrow$

There is enough time for light to go from A to B in S.

Δr is not getting any longer in any frames, ~~so~~ c stays constant, so there is surely enough time for light to go from A to B. Hence, they are timelike in all frames $\Rightarrow$ they are causally related.

(which is great, since causality better be Lorentz invariant.)

3a.



$ct' = \gamma(ct - \beta x)$

$t' = 0 \Rightarrow ct = \beta x \quad (I)$

$x' = \gamma(x - \beta ct)$

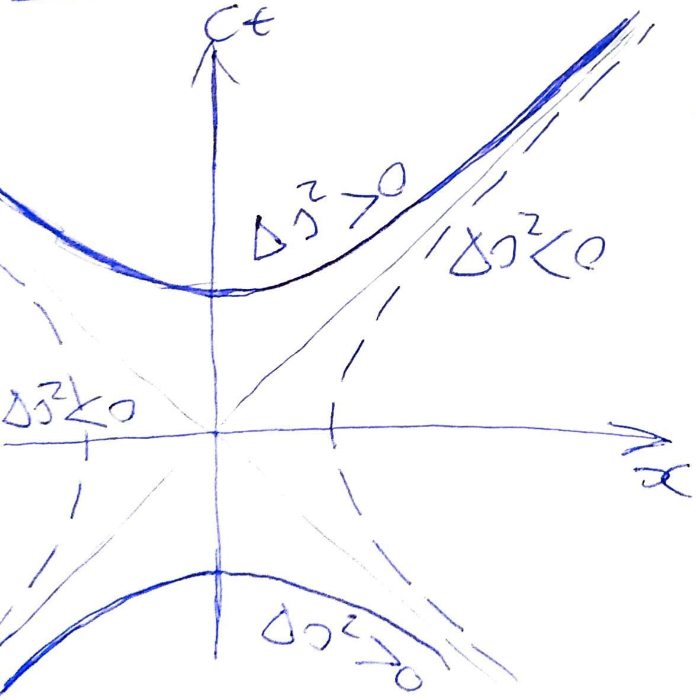
$x' = 0 \Rightarrow x = \beta ct$

$ct = \frac{x}{\beta}$

USING I: $\tan\theta = \frac{\beta}{1} \Rightarrow \theta = \arctan\beta = \arctan\left(\frac{v}{c}\right)$

USING II: $\tan\varphi = \frac{\beta}{1} \Rightarrow \varphi = \arctan\left(\frac{v}{c}\right)$

3b.



$\Delta s^2 = (ct)^2 - x^2$

$ct = \sqrt{\Delta s^2 + x^2}$

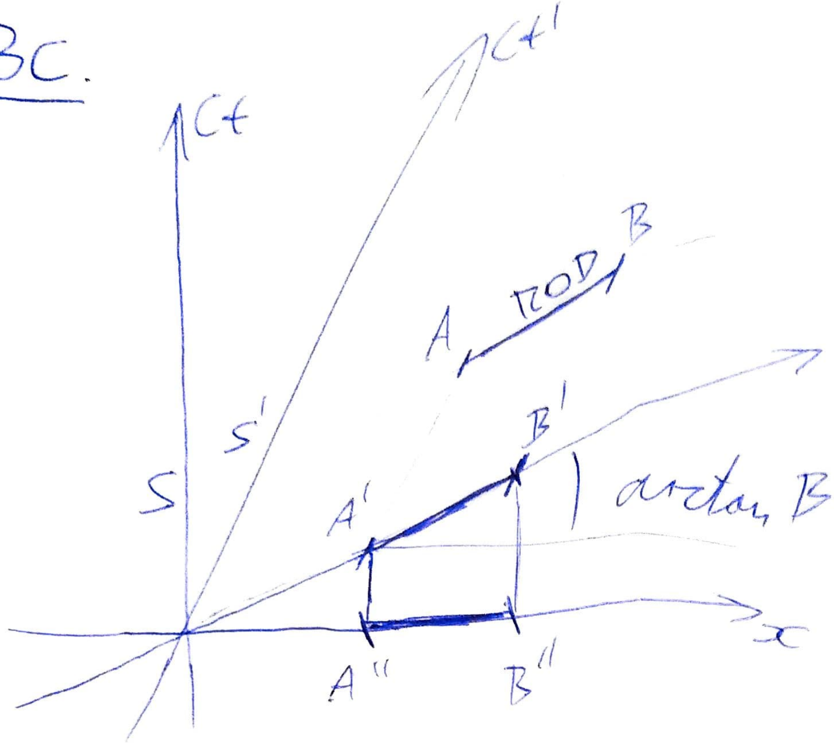
$\frac{d}{dx}(ct)\Big|_{x=0} = \frac{d}{dx}\Big|_{x=0} \sqrt{\Delta s^2 + x^2}$

$= \frac{1}{2}\left(\Delta s^2 + x^2\right)^{-\frac{1}{2}} \cdot 2x \Big|_{x=0} = \underline{\underline{0}}$

$x = \sqrt{\Delta s^2 - (ct)^2} \Rightarrow \frac{d}{d(ct)}\Big|_{ct=0} = \frac{1}{2}\left(\Delta s^2 - (ct)^2\right)^{-\frac{1}{2}} \cdot (-2)(ct)\Big|_{ct=0} = \underline{\underline{0}}$

$\Rightarrow$ tangents are parallel to axes.

3c.



length of rod in S' = AB = A'B'

in S: A''B''

$$\frac{A''B''}{AB} = \cos \arctan \beta \leq 1 \Rightarrow$$ CONTRACTION IN S.

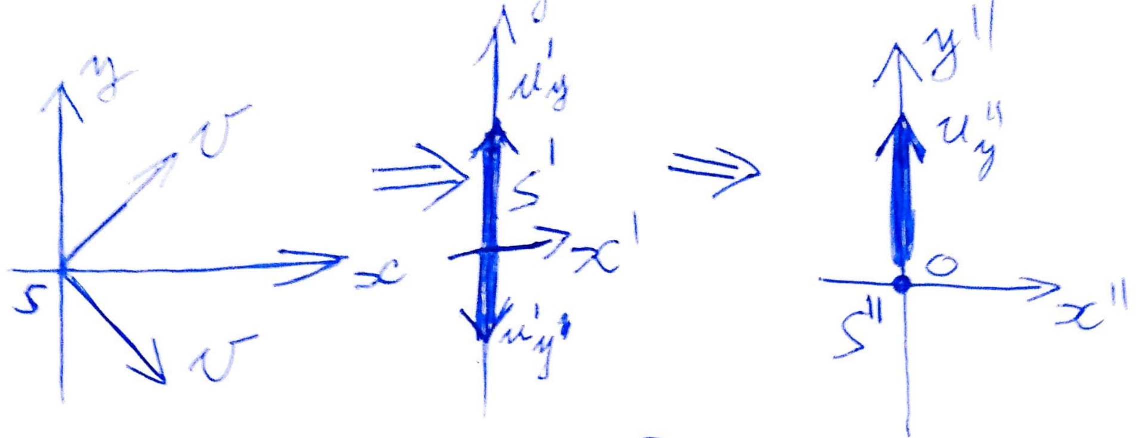
4

PLEASE SEE:

HTTPS://PHYSICS.STACKEXCHANGE.COM/A/588284/212053

(OR BIT.LY/LORENTZBOOST)

$\boxed{5}$



$$S \rightarrow \boxed{\text{BOOST ALONG (+)x AXIS, } \frac{v}{\sqrt{2}}} \rightarrow S' \rightarrow \boxed{\text{BOOST ALONG (−)y AXIS WITH SPEED } u'_y} \rightarrow S''$$

S-S' TRANSFORM:

$$u'_y = \frac{dy'}{dt'} = \cancel{\frac{dy}{\gamma_{\frac{v}{\sqrt{2}}}\left(dt - \frac{v\,dx}{c^2}\right)}} = \frac{u_y}{\gamma_{\frac{v}{\sqrt{2}}}\left(1 - \frac{u_x \frac{v}{\sqrt{2}}}{c^2}\right)} = \frac{u_y}{\gamma_{\frac{v}{\sqrt{2}}}\left(1 - \frac{v^2}{2c^2}\right)}$$

(LECTURE 3 SLIDES)

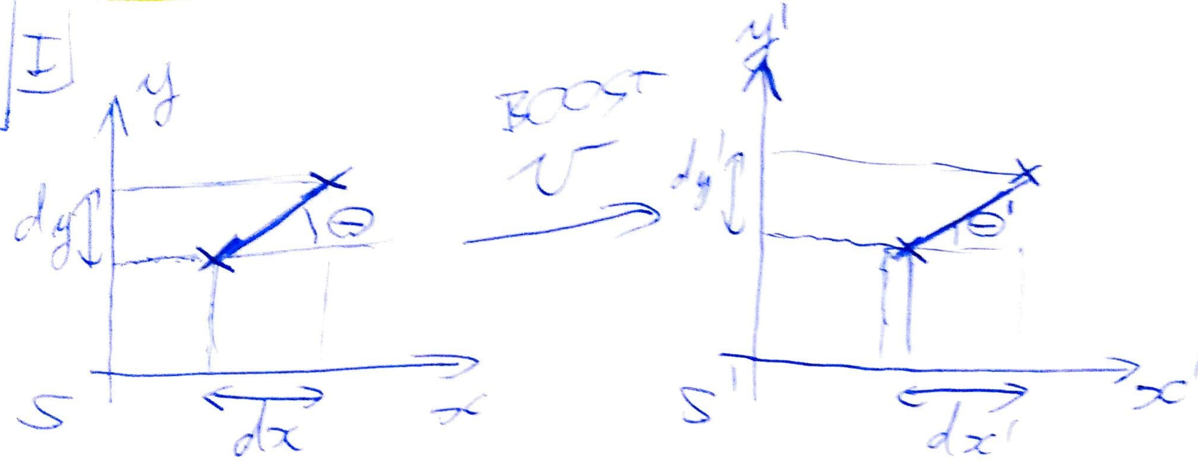
$$= \frac{\frac{v}{\sqrt{2}}}{\sqrt{1 - \frac{v^2}{2c^2}}} = \frac{v}{\sqrt{2 - \frac{v^2}{c^2}}}$$

S' S'' TRANSFORM:

$$u''_y = \frac{u'_y - (-u'_y)}{1 - \frac{u'_y(-u'_y)}{c^2}} = \frac{2u'_y}{1 + \frac{u'^2_y}{c^2}} = \frac{\frac{\sqrt{2}\,v}{\sqrt{1 - \frac{v^2}{2c^2}}}}{1 + \frac{v^2}{2c^2 - v^2}} =$$

$$= \left[\text{SOME ALGEBRA AT THIS POINT}\right] = v\sqrt{2 - \frac{v^2}{c^2}}$$

$\boxed{6.\ \mathrm{I}}$



a/

$$\left.\begin{aligned} dx' &= \frac{dx}{\gamma_v} \\ dy' &= dy \end{aligned}\right] \Rightarrow \theta' = \arctan\left(\frac{dy'}{dx'}\right) = \arctan\left(\frac{dy}{dx}\gamma_v\right)$$

$$= \arctan\left((\tan\theta)\cdot\gamma_v\right)$$

$$\Rightarrow \theta = \arctan\left(\frac{\tan\theta'}{\gamma_v}\right)$$

b/

$$\cancel{u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}} \qquad \cancel{u_y' = \frac{u_y}{\gamma_v\left(1 - \frac{u_x v}{c^2}\right)}}$$

$$\cancel{\tan\theta' = \frac{u_y'}{u_x'} = \frac{\frac{u_y}{\gamma_v}}{u_x - v} = \frac{u_y}{\gamma(u_x - v)} = \frac{u'\sin\theta'}{\gamma_v(u'\cos\theta' - v)}}$$

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} \ ; \quad u_y = \frac{u_y'}{\gamma_v\left(1 + \frac{u_x' v}{c^2}\right)}$$

6.II

$$\tan\theta = \frac{u_y}{u_x} = \frac{u_y'}{\gamma_v(u_x'+v)} = \frac{u'\sin\theta'}{\gamma_v(u'\cos\theta'+v)}$$

IF $u'=c$:

$$\tan\theta = \frac{\sin\theta'}{\gamma_v(\cos\theta'+\beta_v)}$$

$$\theta = \arctan\frac{\sin\theta'}{\gamma_v(\cos\theta'+\beta_v)}$$

7

PLEASE SEE:

PS738.USER.SRCF.NET/RelativityES1Q7_JS.HTML

OR

BIT.LY/PS738MESON

8.

PLEASE SEE:

PS738.USER.SRCF.NET/ACCELERATINGSPACESHIP.HTML

OR

BIT.LY/ACCELERATINGSPACESHIP

RELATIVITY
ES I Q9

$$x^1 = x'^1 + x'^2$$

$$x^2 = x'^1 - x'^2$$

$$x^3 = 2x'^1 x'^2 + x'^3$$

$$dx^1 = dx'^1 + dx'^2$$

$$dx^2 = dx'^1 - dx'^2$$

$$dx^3 = 2x'^1 dx'^2 + 2x'^2 dx'^1 + dx'^3$$

$ds^2 =$

$$= (dx^1)^2 + (dx^2)^2 + (dx^3)^2 =$$

$$= (dx'^1 + dx'^2)^2 + (dx'^1 - dx'^2)^2 + (2x'^1 dx'^2 + 2x'^2 dx'^1 + dx'^3)^2$$

$$= 2(dx'^1)^2 + 2(dx'^2)^2 + \underbrace{(2x'^1 dx'^2 + 2x'^2 dx'^1 + dx'^3)^2}$$

$$\rightarrow 4(x'^1)^2 (dx'^2)^2 + 4(x'^2)^2 (dx'^1)^2 + (dx'^3)^2 +$$

$$+ 2x'^1 dx'^2 dx'^3 + 2x'^2 dx'^1 dx'^3 +$$

$$+ 4x'^1 x'^2 dx'^2 dx'^1$$

$$\Rightarrow g'_{ab} = \begin{pmatrix} 2 + 4(x'^1)^2 & 2x'^1 x'^2 & x'^2 \\ 2x'^1 x'^2 & 2 + 4(x'^1)^2 & x'^1 \\ x'^2 & x'^1 & 1 \end{pmatrix}$$

RELATIVITY
ESI Q9

g'_{ab} is not diagonal & also we have that columns of g'_{ab} are not orthogonal

$\Rightarrow$ coordinates aren't orthogonal.

$$dV = \sqrt{\det(g'_{ab})}\, dx'^1\, dx'^2\, dx'^3$$

10 | I

$x^2+y^2+z^2+q^2=a^2$

STAYING IN SURFACE REQUIRES:

$$0=2x\,dx+2y\,dy+2z\,dz+2q\,dq$$

$$dq=-\frac{x\,dx+y\,dy+z\,dz}{\sqrt{a^2-x^2-y^2-z^2}}$$

$$ds^2=dx^2+dy^2+dz^2+\frac{(x\,dx+y\,dy+z\,dz)^2}{a^2-x^2-y^2-z^2}$$

$x=r\cos\phi\sin\theta$

$y=r\sin\phi\sin\theta$

$z=r\cos\theta$

$$x\,dx+y\,dy+z\,dz=\frac{1}{2}d(x^2+y^2+z^2)=\frac{1}{2}d(r^2)=r\,dr$$

$\cancel{dx=\cos\phi\,d\rho+\rho(-\sin\phi)\,d\phi}$

$\cancel{dy=\sin\phi\,d\rho+\rho(\cos\phi)\,d\phi}$

10 | II

$$dx = \cos\varphi \sin\theta \, dr + r(-\sin\varphi)\sin\theta \, d\varphi + r\cos\varphi\cos\theta \, d\theta$$

$$dy = \sin\varphi \sin\theta \, dr + r(\cos\varphi)\sin\theta \, d\varphi + r\sin\varphi\cos\theta \, d\theta$$

$$dz = \cos\theta \, dr + r(-\sin\theta) \, d\theta$$

SPC IS ORTHOGONAL SO IN THE FOLLOWING SECTION I DISREGARD CROSS TERMS.

$$(dx)^2 = \cos^2\varphi \sin^2\theta (dr)^2 + r^2 \sin^2\varphi \sin^2\theta (d\varphi)^2 + r^2\cos^2\varphi\cos^2\theta (d\theta)^2$$

$$(dy)^2 = \sin^2\varphi \sin^2\theta (dr)^2 + r^2 (\cos\varphi)^2 \sin^2\theta (d\varphi)^2 + r^2 \sin^2\varphi \cos^2\theta (d\theta)^2$$

$$(dz)^2 = \cos^2\theta (dr)^2 + r^2 \sin^2\theta (d\theta)^2$$

COEFFICIENTS:

$$(dr)^2 : \cos^2\varphi \sin^2\theta + \sin^2\varphi \sin^2\theta + \cos^2\theta = \sin^2\theta + \cos^2\theta = 1$$

$$(d\varphi)^2 : r^2 \sin^2\varphi \sin^2\theta + r^2 \cos^2\varphi \sin^2\theta = r^2 \sin^2\theta$$

$$(d\theta)^2 : r^2\cos^2\varphi\cos^2\theta + r^2\sin^2\varphi\cos^2\theta + r^2\sin^2\theta = r^2$$

SO WE HAVE:

$$dx^2 + dy^2 + dz^2 = dr^2 + r^2\sin^2\theta (d\varphi)^2 + r^2 (d\theta)^2$$

$$d\sigma^2 = dr^2 + r^2\sin^2\theta (d\varphi)^2 + r^2 (d\theta)^2 + \frac{r^2 (dr)^2}{a^2 - r^2}$$

10 | III

$$= \left(1 + \frac{r^2}{a^2 - r^2}\right) dr^2 + r^2 \sin^2\theta (d\phi)^2 + r^2 (d\theta)^2$$

• LET $\left(\frac{r}{a}\right)^2 = \sin^2\chi$

THEN

$$\rightarrow \quad 1 + \frac{a^2 \sin^2\chi}{a^2 - a^2\sin^2\chi} = 1 + \tan^2\chi = \sec^2\chi$$

&

$$\rightarrow \quad r^2 \sin^2\theta (d\phi)^2 + r^2 (d\theta)^2 = a^2\left[\sin^2\chi \left((d\theta)^2 + \sin^2\theta (d\phi)^2\right)\right]$$

~~WE ALSO HAVE~~

BASED ON THE RESULT WE ARE SUPPOSED TO PROVE, WE HOPE:

$$a^2 (d\chi)^2 = \sec^2\chi (dr)^2$$

CHECK:

$$a^2 (d\chi)^2 = a^2 \left(d\left[\arcsin \frac{r}{a}\right]\right)^2$$

$$= a^2 \left(\frac{1}{\sqrt{1 - \left(\frac{r}{a}\right)^2}} \frac{1}{a} \, dr\right)^2$$

$$= \frac{1}{\sqrt{1 - \sin^2\chi}} (dr)^2 = \sec^2\chi (dr)^2$$

GOOD.

WE END UP WITH:

$$ds^2 = a^2\left[d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)\right]$$