Astrophysical Fluid Dynamics – Lecture I

what is fluid
where in astro
concept of fluid element
collisional / collisionless fluid

Eulerian vs Lagrangian framework

Kinematics descriptions:
streamlines
particle paths
streaklines

what is fluid.

fluid flows.
almost always gaseous in astro
fluid is treated as continuous medium
ie it is possessing well-defined macroscopic
properties, ie density velocity pressure

can come from kinetics of particles to fluid dynamics, but that's not what we're doing.

Sun
much of outer rings are convective
it can "ring"
observable normal modes
helioseismology: probing interior

Interstellar medium

Supernova remnants
shockwave going outwards.
evolution of shockwaves

Accreation discs
rotationally supported gas.
Spiralling inwards to e.g. BH

Giant Planets
much of it is fluid, weather patterns, etc

When fluid description work

fluid element

small enough so that macroscopic properties can be treated as constant

$$l_{region} \ll l_{scale} \sim \frac{q}{|\nabla q|}$$

large enough to contain large number of particles

$$n\, l^3_{region} \gg 1$$

$\underline{n}$ num of particles per unit volume

mean free path: distance travelled before direction of travel significantly changed, λ

collisional fluid: $l_{scale} \gg \lambda$

$$p = p(\rho, T)$$

collisionless fluid: $l_{scale} \ll \lambda$

velocity distrib of particles not determined locally

examples of collisionless fluids:

"stellar fluid" in galaxy

dark matter

intracluster medium of galaxy clusters

(transitional btw ~~c~~ collisional / less) intracluster medium

$n \sim 10^{-3} - 10^{-1}$ $\frac{\text{particle}}{\text{cm}^3}$

$T \sim 10^7 - 10^8$ K (full ionization)

$R \sim 1$ Mpc with 100 kpc core

$\lambda \approx 23$ kpc

Eulerian
~~Eulerian~~ vs Lagrangian framework

Eulerian
consider properties of the fluid as function of time in a frame of ref fixed in space.

Lagrangian
riding along w/ flow, how properties change

computational aspect
Eulerian $\Rightarrow$ grid code
Lagrangian $\Rightarrow$ smoothed particle codes

consider fluid element w/ quantity Q

moves from $\underline{r} \to \underline{r} + \delta \underline{r}$ in $t \to t + \delta t$

$$\frac{DQ}{Dt} = \lim_{\delta t \to 0} \frac{Q(\underline{r}+\delta\underline{r}, t+\delta t) - Q(\underline{r}, t)}{\delta t}$$

we are travelling w/ fluid element

We also have:

$$Q(\underline{r}+\delta\underline{r}, t+\delta t) = Q(\underline{r}, t) + \frac{\partial Q}{\partial t}\delta t + \delta\underline{r}\cdot\nabla Q + \text{higher order terms.}$$

We end up with:

$$\frac{DQ}{Dt} = \lim_{\delta t \to 0}\left[\frac{\partial Q}{\partial t} + \frac{\delta \underline{r}}{\delta t}\cdot\nabla Q + O(\delta t, |\delta \underline{r}|)\right]$$

$$\Rightarrow \frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \underline{u}\cdot\nabla Q$$

Lagrangian time derivative. Eulerian. gradient operator projected in direction of fluid flow

Eulerian can be zero without Lagrangian deriv being zero.

Kinematics

streamlines

curves that are instantaneously tangent to velocity vector

$$\frac{d\underline{r}}{ds} = \left(\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right)$$

from tangentiality:

$$\frac{d\underline{r}}{ds} \times \underline{u} = 0 \Rightarrow \text{unpack: } \frac{dx}{u_x} = \frac{dy}{u_y} = \frac{dz}{u_z}$$

particle paths

paths through space taken by individ fluid element

if not steady flow, ≠ streamlines

streaklines

locus of points of all fluid elements having passed through a point in the past

when steady flow, these 3 are the same.