

2.

- NUMBER OF STARS BORN PER UNIT TIME:  $B$
- NUMBER OF STARS BORN WITH MASS ON THE RANGE  $m \rightarrow m+dm$ :  $IMF(m)dm dt B$  (in  $dt$  time).
- NUMBER OF STARS BORN WITH MASS ON THE RANGE  $m \rightarrow m+dm$  BETWEEN 0 &  $T$ :

$$\begin{aligned} \text{TOTAL BORN} &= \int_0^T IMF(m)dm B dt \\ &= IMF(m)dm T B \end{aligned}$$

- WHICH SURVIVE TODAY? THOSE WHICH WERE BORN AFTER  $T - t_{ms}(m)$ . NUMBER OF STARS BORN WITH MASS ON THE RANGE  $m \rightarrow m+dm$  BETWEEN

$T - t_{ms}(m) \& T$ :

$$\begin{aligned} \text{BORN \& SURVIVING TO TODAY} &= \int_{T-t_{ms}(m)}^T IMF(m)dm B dt \\ &= IMF dm B t_{ms}(m) \end{aligned}$$

~~THIS WILL BE  $IMF(m)dm$~~   
~~PROPORTIONAL TO~~

- $PDF(m)dm \times \text{TOTAL NUMBER OF STARS}$  IS THE NUMBER OF STARS TODAY WITH MASS BTWN  $m$  &  $m+dm$ . IE WHAT WE HAVE ABOVE.

$$\Rightarrow \underline{PDF(m) \propto IMF(m) \cdot t_{ms}(m)}$$

IN THE LOWER LIMIT OF OUR INTEGRAL, WE'VE WRITTEN:

$$T - t_{ms}(m)$$

The highest  $t_{ms}$  is  $t_{ms}(m_{min})$ .

We're told:  ~~$t_{ms}$~~   $t_{ms}(m_{min}) < T$

$$\Rightarrow T - t_{ms} > 0 \quad \forall m$$

Which is quite good.

How will PDME change: it will not.  
(it has no time dependence ~~as~~  
as seen on the functional form)

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"Explain why"...

Total E in star  $\propto M$

time it takes to radiate it  $\propto t_{ms}(m)$

~~Being~~ on the main sequence means  
luminosity ~~is~~ is roughly constant.

$$\Rightarrow L = \frac{\text{energy radiated}}{\text{time it takes to radiate that energy}} \propto \frac{M}{t_{ms}(m)}$$

$$\underbrace{\text{TOTAL LUMINOSITY BETWEEN } m \text{ \& } m + dm \text{ IN PDME}}_{\downarrow \text{LET THIS BE}} = \underbrace{\text{LUMINOSITY OF EACH STAR WITH MASS } m}_{\text{LUMINOSITY OF EACH STAR WITH MASS } m} \times \underbrace{\text{NUMBER OF STARS w/ MASS } m}_{\text{NUMBER OF STARS w/ MASS } m}$$

LET THIS BE  $P(m)dm$  WHERE  $P(m)$  IS LUMINOSITY DENSITY.

$$\propto \frac{M}{t_{\text{ms}}(m)} \cdot \underbrace{\text{PDMF}(m) dm}_{\downarrow}$$

$$\propto \overbrace{\text{IMF}(m) t_{\text{ms}}(m)}^{\text{cancel}} dm \frac{M}{\cancel{t_{\text{ms}}(m)}}$$

$$\propto \text{IMF}(m) m dm$$

$\propto$  MASS OF STARS IN IMF BETWEEN  $m$  &  $m + dm$

AS REQUIRED.

~~P(m)~~

FROM ABOVE, GET LUMINOSITY DENSITY:

$$P(m) \propto \text{IMF}(m) m$$

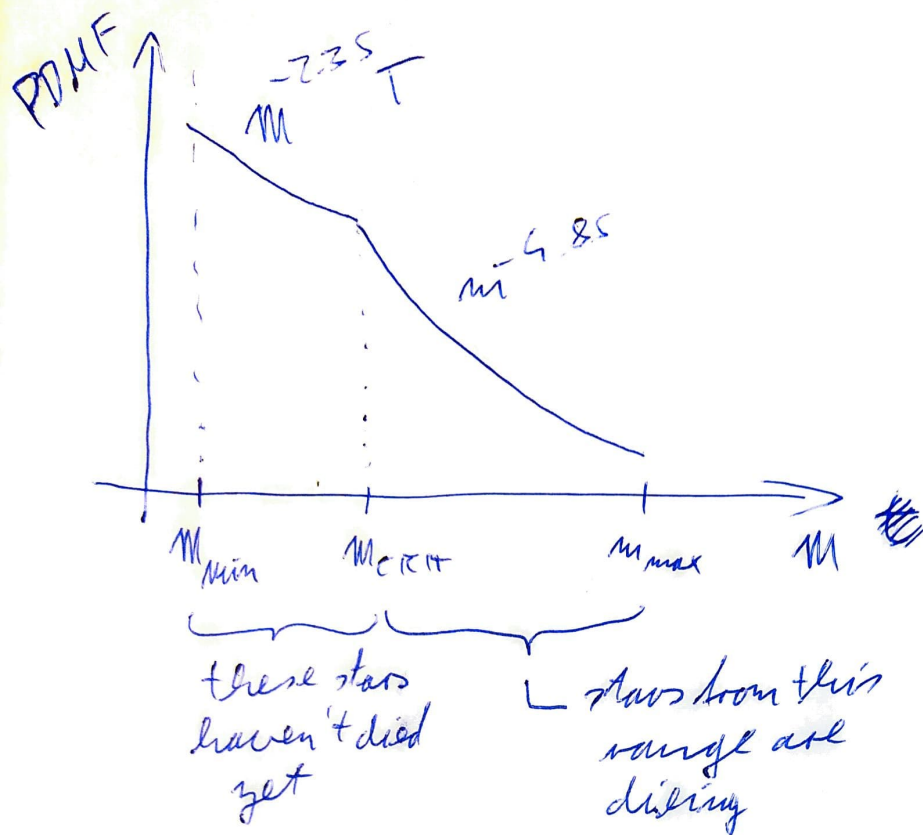
$$\text{IF } \text{IMF}(m) \propto m^{-2.35} \Rightarrow P(m) \propto m^{-1.35}$$

$\Rightarrow$  LOW MASS STARS DOMINATE TOTAL LIGHT DISTRIBUTION.

"Now consider"

$$\text{PDMF}(m) \propto \begin{cases} \text{IMF}(m) T & \text{IF } t_{\text{ms}}(m) > T \\ \text{IMF}(m) t_{\text{ms}} & \text{IF } t_{\text{ms}}(m) \leq T \end{cases}$$

$$\propto \begin{cases} \text{IMF}(m) T \\ \text{IMF}(m) m^{-2.5} \end{cases} \propto \begin{cases} m^{-2.35} T \\ m^{-2.35} m^{-2.5} \end{cases} \propto \begin{cases} m^{-2.35} T \\ m^{-4.85} \end{cases}$$



For a galaxy of  $T = 10^8$  yrs...

We have concluded earlier that if  $T > t_{ms}(M_{min})$ , low mass stars dominate the luminosity budget. If  $T < t_{ms}(M_{min})$ , this is even more true: ~~more~~ less low mass stars died, so they ~~don't~~ dominate over high mass stars which died anyway.