

# TOPICS IN ASTROPHYSICS LECTURE I

for a quantity, characteristic timescale:

$$\text{timescale: } \tau = Q / |\dot{Q}|$$

$$\text{lengthscale: } l = Q / |Q|$$

$$\text{exponential form: } Q = Q_0 \exp\left(-\frac{t}{\tau}\right) \Rightarrow \tau = T$$

$$\text{power law: } Q = Q_0 \left(\frac{T}{t}\right)^{\alpha}$$

no characteristic time ~~length~~  
timescale for power law!

|| "self-similar":  
time length scales are themselves  
function of time / space.

straight line on log vs linear plot: exponential  
"it shows about some characteristic e-folding"  
(unsolved problems)

Dynamical timescale in presence of gravitating object

TRICK:  
MULTIPLICATION

$$\ddot{R} = -\frac{GM}{R^2} \Rightarrow \ddot{R} \dot{R} = -\frac{GM}{R^2}$$

$$M \bullet \overset{R(t)}{\longrightarrow} \\ R(0) = R_0 \quad t_{\text{freefall}}?$$

$$\text{i.e. } \frac{d}{dt} \left[ \frac{1}{2} \dot{R}^2 - \frac{GM}{R} \right] = 0$$

$$\frac{1}{2} \dot{R}^2 - \frac{GM}{R} = C$$

$$\frac{1}{2} \dot{R}^2 = \frac{GM}{R} + C$$

~~$$\frac{dR}{dt} = \sqrt{2GM} \left[ \frac{1}{R} + C \right]$$~~

$$\frac{dR}{dt} = \sqrt{2GM} \left[ \frac{1}{R} + C \right]$$

We want to fall inwards, let's put a  $\ominus$  sign in:

$$\frac{dR}{dt} = -\sqrt{2GM} \left( \frac{1}{R} - \frac{1}{R_0} \right)$$

C is rewritten so that  $\left. \frac{dR}{dt} \right|_{t=0} = 0$  as required by BC.

Unexpectedly bad integral.

Substitution:  $R = R_0 \sin^2 \theta$

$\theta: \frac{\pi}{2} \rightarrow 0$  (as we fall in,  $R \downarrow$ )

~~$$R = -\sqrt{2GM} \left( \frac{1}{R} - \frac{1}{R_0} \right)$$~~

$$t_{ff} = \int dt = \int \frac{1}{-\sqrt{2GM}} \frac{1}{\sqrt{\frac{1}{R} - \frac{1}{R_0}}} dR,$$

$$= -\frac{1}{\sqrt{2GM}} \int_{\theta=\frac{\pi}{2}}^0 \left( \frac{1}{R_0 \sin^2 \theta} - \frac{1}{R_0} \right)^{-\frac{1}{2}} R_0 \sin \theta \cos \theta d\theta$$

$$= -\frac{1}{\sqrt{2GM}} \sqrt{R_o} R_o 2 \int_0^{\frac{\pi}{2}} \left( \frac{1}{\sin^2 \theta} - 1 \right)^{\frac{1}{2}} \sin \theta \cos \theta d\theta$$

$\theta = \frac{\pi}{2}$     $\cot^2 \theta$

$$= -\sqrt{\frac{2R_o^3}{GM}} \int_0^{\frac{\pi}{2}} \tan^{\frac{1}{2}} \theta \sin \theta \cos \theta d\theta$$

$\theta = \frac{\pi}{2}$

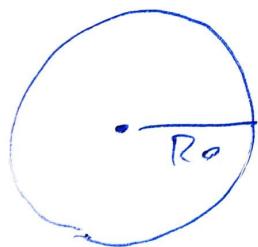
$$= -\sqrt{\frac{2R_o^3}{GM}} \int_0^{\frac{\pi}{2}} \cancel{\sin^{\frac{1}{2}} \theta} \sin^{\frac{1}{2}} \theta \cos \theta d\theta$$

$\theta = \frac{\pi}{2}$

$$= -\sqrt{\frac{2R_o^3}{GM}} \left(-\frac{\pi}{4}\right) = \frac{\pi}{2} \sqrt{\frac{R_o^3}{2GM}}$$

free-fall time.

5 Let's now have a uniform density on Earth.



$$M_{\text{enc}}(R_o) = M \quad M_{\text{enc}}(R) = \left(\frac{R}{R_o}\right)^3 M$$

$$\ddot{r} = -\frac{GM}{R^2} \left(\frac{R}{R_o}\right)^3 = -\frac{GM}{R_o^3} R$$

recognize as SHM.

To get to 0: quarter of an oscillation

$$t_{\text{ff}} = \frac{1}{4} T_{\text{osc}} = \frac{1}{4} \frac{2\pi}{\omega} = \frac{1}{4} \frac{2\pi}{\left(\frac{GM}{R_o^3}\right)^{\frac{1}{2}}} = \frac{\pi}{2} \sqrt{\frac{R_o^3}{GM}}$$

Root  $\sqrt{2}$  more than prev: less mass enclosed, it smaller ~~less~~ acceleration.

Free fall velocity at  $R_0$  if particle is falling in from infinity

$$\frac{GMm}{r} = \frac{1}{2}mv^2$$

$$v_f(R_0) = \sqrt{\frac{2GM}{R_0}}$$

characteristic time scale:

$$\tau = \left( \frac{R_0}{\dot{r}} \right)_{r=R_0} = \frac{R_0}{v_f} = \sqrt{\frac{R_0}{2GM}}$$

### Sound propagation

ideal gas equation of state:  $PV = \cancel{N} N RT$

$\cancel{N} = 8.3 \frac{\text{atm} \cdot \text{K}}{\text{mol}}$   
number of moles

$$1/V$$

$$P = \frac{N}{V} RT$$

$$P = \frac{3}{\text{weight of a mole in kg}} RT$$

relative molecular weight  
 $\times 10^3, R \text{ kJ/kg}$

$$P = S \frac{1000 R}{\rho} T$$

$$L = \frac{R_*}{\rho} = 8300 \frac{J}{2gK}$$

## Alfvén waves

waves in magnetized media.  
transverse waves are made possible.

$$V_{\text{Alfvén}} \approx \sqrt{\frac{B^2}{S}}$$

## light crossing time

also energy transfer time for radiation  
(in optically thin media)

quasars: largely visible, though output comparable to entire galaxy  
lets take hours for no variability length scale  
assuming ~~all~~ we don't know about stuff, what can they let?

$$D + n \text{ hours} \Rightarrow c D + n \cdot 10^{12} \text{ m}$$

Can we put all stars in a galaxy this close?

$$R_{\text{gal}}^3 = N_* R_*^3 \quad \text{or} \quad R_{\text{gal}}^3 = N_* \frac{1}{2} R_*^3 \Rightarrow R_{\text{gal}} = N_*^{\frac{1}{3}} R_*$$


$$10^{10} \text{ stars}$$

this is already  
too big, it will  
stars are essentially  
"touching"  $\Rightarrow$  for not from  
stellar point

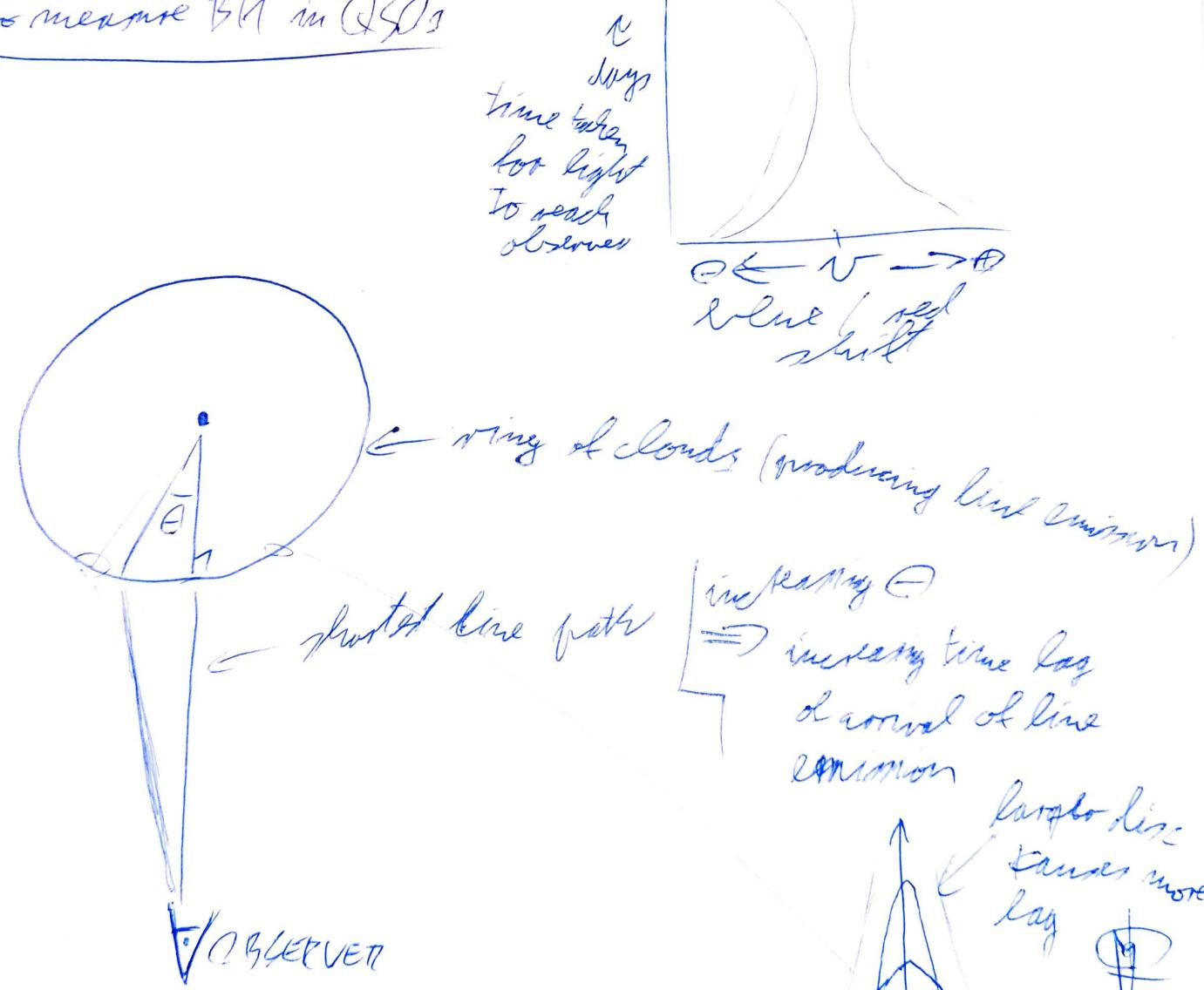
We also have limit on BH.

event horizon cannot be bigger than order  $10^{12}$  m

$$\frac{GM_{BH}}{c^2} \leq 10^{12} \text{ m} \Rightarrow M_{BH} \leq 10^9 M_\odot$$

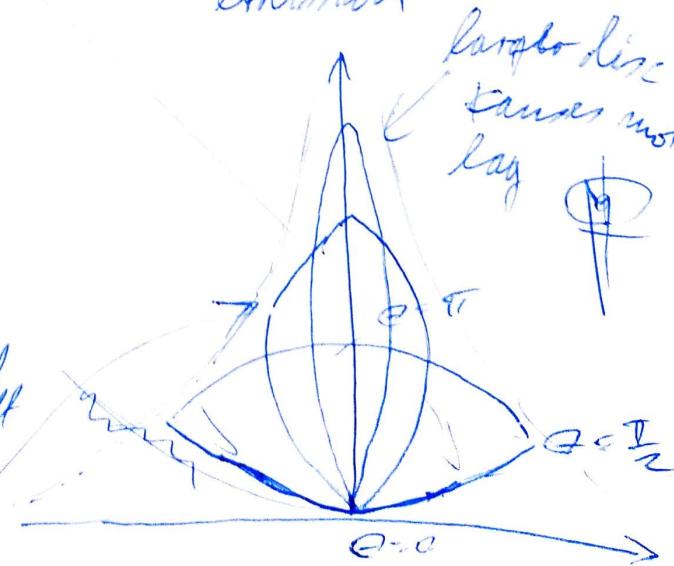
Novelty data isn't violating black hole hypothesis.

- "light echo" technique  
to measure BH in QSOs

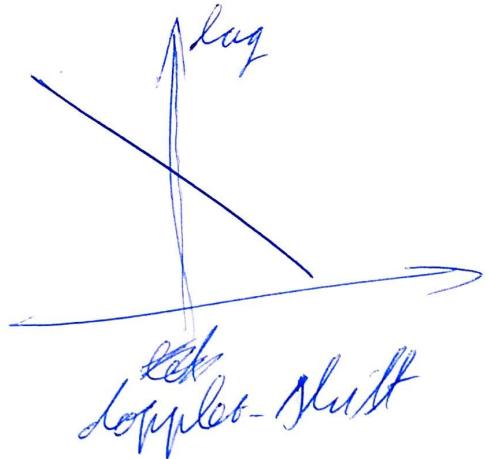


Let's have rings of different size.

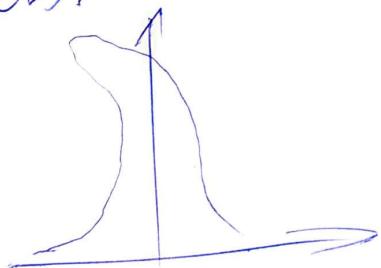
$$N = \sqrt{\frac{GM}{R}} \Rightarrow \begin{array}{l} \text{smaller red} \\ \text{blue shift} \\ \text{for larger} \\ \text{size} \end{array}$$



additional effect: spherical infall:



add these two effects:



You are essentially measuring distance of clouds from BH.

Get  $v$  from doppler shift.

Use  $v = \sqrt{\frac{GM}{R}}$ , get mass of black hole.