

III.1)

$$\text{FRACTION OF SPIRALS} = \frac{\text{NUMBER DENSITY OF SPIRALS}}{\text{TOTAL NUMBER DENSITY}}$$

$$S = \int_{L_D}^{\infty} \Phi(L) dL$$

L_D
THIS LUMINOSITY DEEP WE SEE.

IF $L_D < L_*$:

$$S = \int_{L_D}^{L_*} \Phi(L < L_*) dL + \int_{L_*}^{\infty} \Phi(L > L_*) dL$$

ELSE:

$$S = \int_{L_D}^{\infty} \Phi(L > L_*) dL$$

L_D

COMPUTE THESE INTEGRALS:

$$\begin{aligned} \int \Phi(L < L_*) dL &= \int \frac{n_*}{L_*} \left(\frac{L}{L_*} \right)^\alpha dL \\ &= \frac{n_*}{L_*^{\alpha+1}} \frac{1}{\alpha+1} L^{\alpha+1} \end{aligned}$$

$$\int \Phi(L > L_*) dL = \int \frac{n_*}{L_*} \exp\left(-\frac{L}{L_*}\right) dL = -\frac{n_*}{L_*} \exp\left(-\frac{L}{L_*}\right)$$

(i)

$$L_{*E} > L_i > L_{*S}$$

IE POWER-LAW CONTRIB ONLY FOR ELLIPTICALS.

$$S_S = \int_{L_i}^{\infty} \phi(L > L_{*S}) dL$$

$$= -n_{*S} \exp\left(-\frac{L}{L_{*S}}\right) \Big|_{L_i}^{\infty} = n_{*S} \exp\left(-\frac{L_i}{L_{*S}}\right)$$

$$S_E = \int_{L_i}^{\infty} \phi(L > L_{*E}) dL$$

~~$$= -n_{*E} \exp\left(-\frac{L}{L_{*E}}\right) \Big|_{L_i}^{L_{*E}}$$~~

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left[L^{\alpha_E+1} \right]_{L_i}^{L_{*E}} + (-n_{*E}) \exp\left(-\frac{L}{L_{*E}}\right) \Big|_{L_i}^{\infty}$$

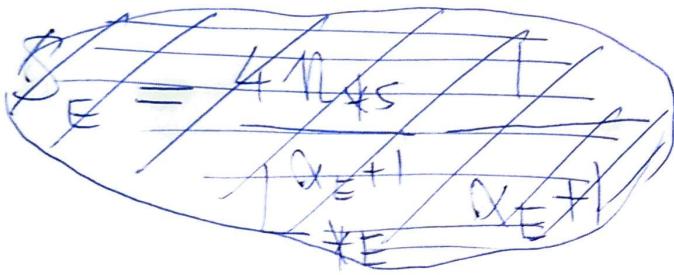
$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_i^{\alpha_E+1} \right) +$$

$$+ n_{*E} \exp\left(-\frac{L_{*E}}{L_{*E}}\right)$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_i^{\alpha_E+1} \right) + n_{*E} \exp(-1)$$

PUTTING IN THE NUMBERS:

$$S_S = n_{*s} e^{-s} \exp\left(-\frac{5 \cdot 10^{11}}{10^{11}}\right) = n_{*s} e^{-5}$$



$$S_E = \frac{4 n_{*s}}{\left(10^{12}\right)^{-0.5+1}} \frac{1}{-0.5+1} \left(\left(10^{12}\right)^{-0.5+1} - \left(5 \cdot 10^{11}\right)^{-0.5+1} \right) + 4 n_{*s} e^{-1}$$
$$= 3.814 n_{*s}$$

$$\text{FRACTION OF SPIRALS} = \frac{n_{*s} e^{-s}}{3.814 n_{*s} + e^{-s} n_{*s}} \approx \underline{\underline{1.77 \cdot 10^{-3}}}$$

(ii)

$$L_{*E} > L_{*S} > L_2$$

IE POWER-LAW & EXPONENTIAL TERM FOR BOTH SPIRALS & ELLIPTICALS. USING PREVIOUS RESULTS:

$$S_S = \frac{n_{*S}}{L_{*S}^{\alpha_S+1}} \frac{1}{\alpha_S+1} \left(L_{*S}^{\alpha_S+1} - L_2^{\alpha_S+1} \right) + n_{*S} \exp(-1)$$

$$S_E = \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_2^{\alpha_E+1} \right) + n_{*E} \exp(-1)$$

PUTTING IN THE NUMBERS:

$$S_S = \frac{n_{*S}}{(10^{11})^{-1.5+1}} \frac{1}{-1.5+1} \left((10^{11})^{-1.5+1} - (10^3)^{-1.5+1} \right) +$$

$$+ n_{*S} \exp(-1)$$

$$= \frac{n_{*S}}{(10^{11})^{-\frac{1}{2}}} (-2) \left((10^{11})^{\frac{1}{2}} - (10^3)^{\frac{1}{2}} \right) + n_S e^{-1}$$

$$= \frac{18.37 n_{*S}}{2}$$

$$S_E = \frac{4\pi s^*}{(10^{12})^{-0.5+1}} \frac{1}{-0.5+1} \left((10^{12})^{-0.5+1} - (10^9)^{-0.5+1} \right) + \frac{4\pi e^{-1}}{s^*}$$

$$= 9.22 \text{ M}_\odot$$

$$\text{FRACTION OF SPIRALS} = \frac{18.37}{18.37 + 9.22} = \underline{\underline{0.67}}$$

COMMENT ON RESULT:

THE DEEPER WE SEE THE MORE REALISTIC RESULTS WE GET.

IT IS IMPORTANT TO SEE DEEP ENOUGH TO OBSERVE THE POWER LAW PART OF THE DISTRIBUTION, WHICH IS THE DOMINATING CONTRIBUTOR TO NUMBER DENSITY.

III.2

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N \frac{V(i)}{V_{MAX}(i)}$$

MEAN:

$$\langle \bar{x} \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \frac{V(i)}{V_{MAX}(i)}$$



$$= \int_0^1 \text{PDF}\left(\frac{V}{V_{MAX}}\right) \left(\frac{V}{V_{MAX}}\right)^2 d\left(\frac{V}{V_{MAX}}\right)$$

VARIANCE:

$$\text{VAR}(\bar{x}) = \langle \bar{x}^2 \rangle - \langle \bar{x} \rangle^2$$

$$\text{WHERE } \langle \bar{x}^2 \rangle = \int_0^1 \text{PDF}\left(\frac{V}{V_{MAX}}\right) \left(\frac{V}{V_{MAX}}\right)^2 d\left(\frac{V}{V_{MAX}}\right)$$

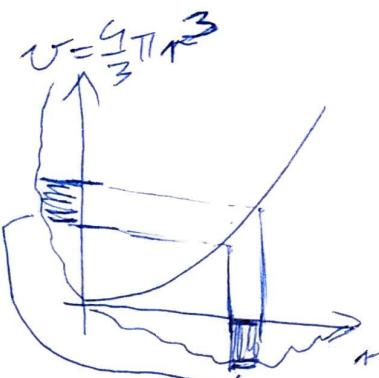
WE AIM TO GET: $\text{PDF}\left(\frac{V}{V_{MAX}}\right)$
 PLAN TO GET THERE:
 $\text{PDF}(r) \rightarrow \text{PDF}(v) \rightarrow \text{PDF}\left(\frac{v}{V_{MAX}}\right)$

NUMBER DENSITY
OF GALAXIES

$$\text{NUMBER OF GALAXIES}_{BETWEEN r \& r+dr} = 4\pi r^2 dr S$$

$$\text{PDF}(r) dr = \frac{\text{FRACTION OF GALAXIES}_{BETWEEN r \& r+dr}}{\text{NUMBER OF GALAXIES}_{BETWEEN r \& r+dr}} = \frac{4\pi r^2 dr S}{\frac{4}{3} d_M^3 \pi r^2} = 3 \frac{r^2}{d_M^3} dr$$

USING THE NOTATION : $\text{PDF}_X(x)$



IS THE PROBABILITY DENSITY FUNCTION OF A SINGLE RANDOM VARIABLE (IN OUR CASE: PROPERTY X OF A GALAXY) AS A FUNCTION OF x .

$$\underbrace{\text{PDF}_R(r) dr}_{\text{PDF}_V(v) d\nu(v)}$$

$$= \text{PDF}_V(v) d\left(\frac{4}{3}r^3\pi\right)$$

$$= \text{PDF}_V(v) \cancel{4r^2\pi} dr$$

$$\Rightarrow \text{PDF}_V(v) = \text{PDF}_R(r) / (4r^2\pi)$$

$$= \frac{3r^2/dr}{4r^3\pi} = \frac{3}{4} \frac{1}{dr} \frac{1}{\pi}$$

USING SAME MATH:

$$\text{PDF}_V(v) d\nu = \text{PDF}_V\left(\frac{v}{v_{MAX}}\right) \lambda\left(\frac{v(v)}{v_{MAX}(v)}\right)$$

$$= \text{PDF}_V\left(\frac{v}{v_{MAX}}\right) \frac{1}{v_{MAX}} \cancel{\lambda(v)} dv$$

$$\Rightarrow \text{PDF}_V\left(\frac{v}{v_{MAX}}\right) = v_{MAX} \text{PDF}_V(v) =$$

$$= \frac{4}{3} \pi d_M^3 \cdot \frac{3}{4} \frac{1}{d_M^3} \frac{1}{\pi} = 1$$

QUITE NICE.

MAYBE THERE IS AN EASIER WAY TO GET HERE.

$$\langle x \rangle = \int_0^1 \underbrace{\text{PDF}\left(\frac{v}{v_{\max}}\right)}_{L_1} \left(\frac{v}{v_{\max}}\right) d\frac{v}{v_{\max}}$$

$$= \frac{1}{2} \left[\left(\frac{v}{v_{\max}} \right)^2 \right]_0^1 = \underline{\underline{\frac{1}{2}}}$$

$$\langle x^2 \rangle = \int_0^1 \underbrace{\text{PDF}\left(\frac{v}{v_{\max}}\right)}_{L_1} \left(\frac{v}{v_{\max}}\right)^2 d\frac{v}{v_{\max}}$$

$$= \left[\frac{1}{3} \left(\frac{v}{v_{\max}} \right)^3 \right]_0^1 = \frac{1}{3}$$

$$\text{VATC}(x) = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{12}}}$$

HOW CAN THIS BE TEST FOR EVOLUTION :

DEVIATIONS FROM ABOVE RESULTS INDICATE THAT GALAXY DISTRIBUTION IS NOT UNIFORM, AS WE ASSUMED.

WE CAN THEN OBTAIN A GALAXY

III.3

I DON'T KNOW WHAT IS AN "ACTION".

IF I ACCEPT THESE THINGS TO BE TRUE,
(I CANNOT DERIVE THEM):

~~XMAS~~

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial^k \phi \partial_k \phi + g_{\mu\nu} V(\phi)$$

$$S_\phi = T_{\phi\phi}$$

$$P_\phi = \frac{1}{3} T_{ii}$$

THEN I CAN OBTAIN:

$$S_\phi = \partial_\phi \phi \partial_\phi \phi - \frac{1}{2} g_{\phi\phi} \partial^k \phi \partial_k \phi + g_{\phi\phi} V(\phi)$$

$$\text{FLRW: } ds^2 = \frac{dr^2}{1 - \frac{r^2}{a^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 = c^2 dt^2$$

~~XMAS~~

$$g_{\phi\phi} = -c^2 = -1$$

$$S_\phi = \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} (\nabla \phi)^2 - V(\phi)$$

$$= \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 - V(\phi)$$

which is WRONG.

$$P_\phi = \frac{1}{3} T_{ii}$$

~~cancel~~

~~cancel~~

$$= \partial_i \phi \partial_i \phi - \frac{1}{2} g_{ii} \partial^k \phi \partial_k \phi + g_{ii} V(\phi)$$

GIVEN THAT I DIDN'T GET THE PREVIOUS
SIMPLER ONE, LET'S GIVE UP ON THIS PART.

"SHOW FURTHER" ...

EQUATION OF STATE:

$$\omega = \frac{P_\phi}{S_\phi} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi) - \frac{1}{6} (\nabla \phi)^2}{\frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{1}{2} (\nabla \phi)^2} \quad \downarrow \quad \begin{matrix} \phi \text{ IS CONST.} \\ \text{SPATIALLY} \end{matrix}$$

$$= \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)}$$

III.4

$$V(\phi) = V_0 \exp(-\gamma\phi)$$

EQ. OF MOTION:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\ddot{\phi} + 3H\dot{\phi} + (-\gamma)V_0 \exp(-\gamma\phi) = 0$$

$$\ddot{\phi} + 3 \frac{R}{R} \dot{R} \dot{\phi} - \gamma V_0 e^{-\gamma\phi} = 0$$

CHECK IF PROVIDED

SOLUTION IS RIGHT:

~~$$\ddot{\phi} = \frac{2}{\pi} t^{-1}$$~~

$$\dot{\phi}(t) = \frac{2}{\pi} t^{-1}$$

$$\ddot{\phi}(t) = -\frac{2}{\pi} t^{-2}$$

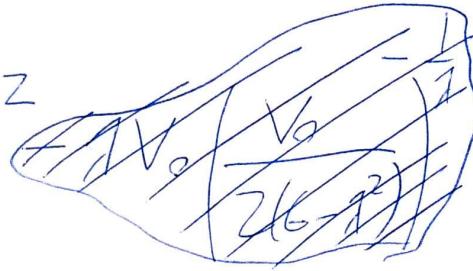
~~$$R(t) = R_0 \left(\frac{t}{t_0} \right)^2$$~~

$$\dot{R}(t) = R_0 \left(\frac{1}{t_0} \right)^{\frac{2}{\pi^2}} \frac{2}{\pi^2} t^{\frac{2}{\pi^2}-1}$$

$$\frac{\dot{R}}{R} \dot{\phi} = \frac{R_0 \left(\frac{1}{t_0} \right)^{\frac{2}{\pi^2}} \frac{2}{\pi^2} t^{\frac{2}{\pi^2}-1}}{R_0 \left(\frac{t}{t_0} \right)^{\frac{2}{\pi^2}}} \quad \frac{2}{\pi} t^{-1} = \frac{4}{\pi^3} t^{-2}$$

$$\ddot{\phi} + \frac{3\bar{I}}{R} \dot{\phi} - \pi V_0 e^{-\lambda\phi} =$$

$$= -\frac{2}{\lambda} t^{-2} + 3 \frac{4}{\lambda^3} t^{-2}$$



$$- \pi V_0 \exp \left(- \sqrt{\frac{2}{\lambda}} \ln \left[\dots \right]^{\frac{1}{2}} \frac{t}{\lambda^2 t} \right)$$

$$= -\frac{2}{\lambda} t^{-2} + \cancel{\frac{12}{\lambda^3} t^{-2}} - \pi V_0 \left[\dots \right]^{\frac{1}{2}} \frac{t}{\lambda^2 t}^{-2}$$

$$= -\frac{2}{\lambda} t^{-2} + \frac{12}{\lambda^3} t^{-2} - \pi V_0 \left[\dots \right]^{-1} \frac{1}{\lambda^4} t^{-2}$$

$$= -\frac{2}{\lambda} t^{-2} + \cancel{\frac{12}{\lambda^3} t^{-2}} - \cancel{\pi V_0} \frac{\sqrt{2(\phi-\lambda^2)}}{V_0} \frac{1}{\lambda^3} t^{-2}$$

$$= -\frac{2}{\lambda} t^{-2} + \frac{2}{\lambda^3} t^{-2} = 0 \quad \text{indeed.}$$

IDK HOW TO SOLVE
THE EQUATION PROPERLY
THOUGH.