TOPICS 3

1/ FIRST PARAGRAPH

RADIATION PRESSURE: $P = \frac{L}{4\pi r^2 c}$

FORCE FROM RADIATION PRESSURE = FORCE ~~FROM~~ FROM GRAVITY.

THIS COMES FROM:

$P = \text{MOMENTUM FLUX IN UNIT TIME} = \frac{p}{At}$

$$= \frac{E_{TOTAL}^{\text{IN TIME } t}/c}{4\pi r^2 \cdot t} = \frac{L}{4\pi r^2 c}$$

$$\frac{L}{4\pi r^2 c}\pi a^2 = \frac{4}{3}\pi a^3 S \; M / r^2 \cdot G$$

$$\underline{a = \frac{3}{16}\frac{1}{\pi}\frac{L}{M}\frac{1}{GcS}}$$

AS WANTED.

SECOND PARAGRAPH.

$$\text{CIRCULAR ORBIT} \Rightarrow F_{NET} = \frac{v^2}{r} m$$

$$F_{NET} = F_{GRAV} - F_{RADIATION}$$

$$F_{NET} = G \frac{\frac{4}{3}\pi (2a)^3 \rho M}{r^2} - \frac{L}{4\pi r^2 c}(2a)^2 \pi = \frac{v^2}{r} \frac{4}{3}\pi (2a)^3 \rho$$

INTRODUCE CONSTANTS:

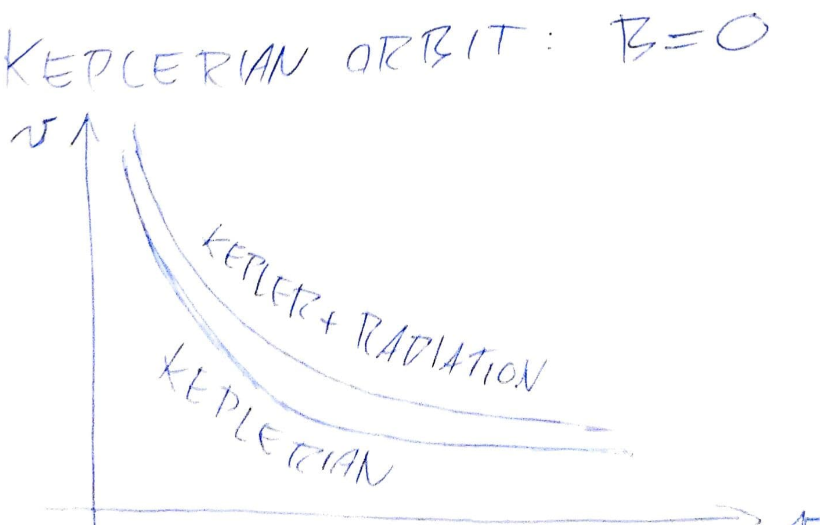
$$G \frac{4}{3}\pi (2a)^3 \rho M = A > 0 \qquad \frac{L}{4\pi \cancel{r^2} c}(2a)^3 \pi = B > 0$$

$$\frac{4}{3}\pi (2a)^3 \rho = C > 0$$

$$(A - B)\frac{1}{r^2} = \cancel{A} \frac{v^2}{r} C$$

$$v = \sqrt{\frac{1}{C}(A - B)\frac{1}{r}}$$

KEPLERIAN ORBIT: $B = 0$



LAST PARAGRAPH

STAR 1

$1M_\odot$

$L_1 \propto (1M_\odot)^3$

STAR 2

$2M_\odot$

$L_2 \propto (2M_\odot)^3 = 8M_\odot^3$

MASS OF ROCKY MATERIAL AROUND STAR 1.

$$= \int_{a_{\text{STAR 1 BLOWOUT}}}^{1\,km} \underbrace{a^{-3.5}\,da}_{dN(a)} \cdot \frac{4}{3}\pi a^3 \rho$$

(THIS INTEGRAL IS JUST:

NUMBER OF (THIS BIG OBJECTS) · MASS OF SUCH AN OBJECT SUMMED OVER ALL OBJECT SIZES.

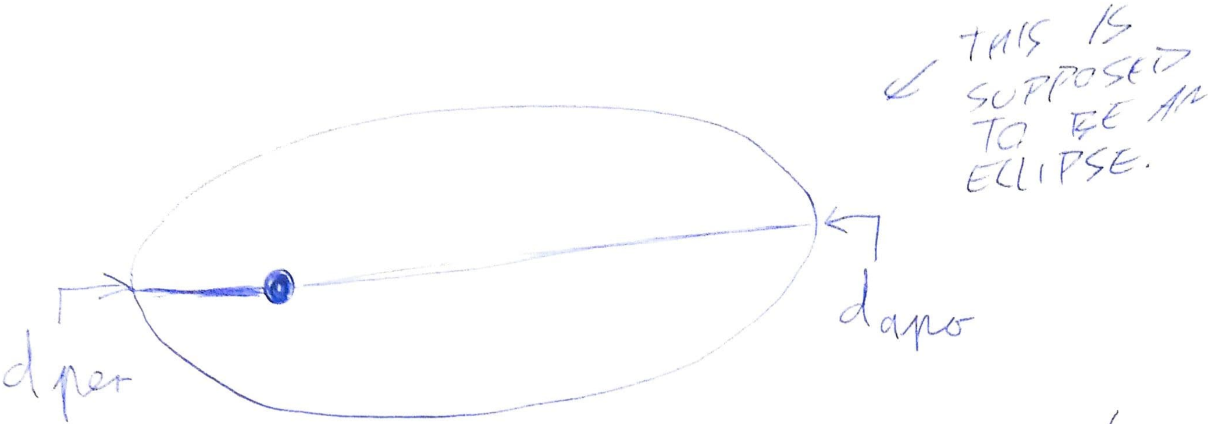
$$\propto \int_{a_{\text{STAR 1 BLOWOUT}}}^{1\,km} a^{-0.5}\,da \propto a^{\frac{1}{2}}\Big|_{\text{STAR 1 BLOWOUT}}^{1\,km}$$

MASS OF ROCKY MATERIAL AROUND STAR 2

$$= a^{\frac{1}{2}}\Big|_{\text{STAR 2 BLOWOUT}}^{1\,km}$$

$\Rightarrow$ RESULTING MASS IS DOMINATED BY UPPER SIZE, SO NO SIGNIFICANT DIFFERENCE IS EXPECTED.

2 / FIRST PARAGRAPH



$$T = 2\pi\sqrt{\frac{a^3}{GM}} \Rightarrow \frac{T^2}{4\pi^2} GM = a^3 \Rightarrow a = \left(\frac{GM}{4\pi^2} T^2\right)^{\frac{1}{3}}$$

$$d_{per} = a(1-e)$$

$$e = -\frac{d_{per}}{a} + 1$$

$$= -\frac{d_{PER}}{\left(\frac{GM}{4\pi^2} T^2\right)^{\frac{1}{3}}} + 1$$

$$= -\frac{10^4 \cdot 1.5 \cdot 10^{11}}{\left(\frac{G \cdot 2 \cdot 10^{30}}{4\pi^2}\left(2.6 \cdot 10^7 \cdot 365 \cdot 24 \cdot 60 \cdot 60\right)^2\right)^{\frac{1}{3}}} + 1$$

$$\simeq \underline{\underline{0.89}}$$

SECOND PARAGRAPH

ENERGY CONSERVATION:

$$\frac{1}{2}v^2 - \frac{\mu}{r} = C$$

ANGULAR MOM CONSERVATION.

$$d_p v_p = d_a v_a$$

$$\frac{1}{2}v_p^2 - \frac{\mu}{d_p} = \frac{1}{2}v_a^2 - \underbrace{\frac{\mu}{d_a}}_{\text{NEGLECT THIS TERM}}$$

USE: $d_p \ll d_a$

$$\frac{1}{2}v_p^2 - \frac{\mu}{d_p} \cong \frac{1}{2}v_a^2$$

SUB FOR v_a

$$\frac{1}{2}v_p^2 - \frac{\mu}{d_p} \cong \frac{1}{2}\underbrace{\left(\frac{d_p}{d_a}\right)^2}_{\ll 1 \Rightarrow \text{NEGLECT THIS TERM}} v_p^2$$

$$\frac{1}{2}v_p^2 - \frac{\mu}{d_p} \cong 0$$

REARRANGE, RENAME d_p TO r_p:

$$v_p \cong \left(\frac{2\mu}{r_p}\right)^{1/2}$$ AS WANTED.

HENCE, ESTIMATE...

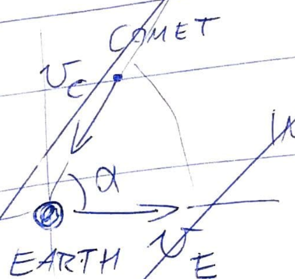
$$v_p \cong \left(\frac{2\mu}{r_p}\right)^{1/2}$$

$$\frac{v_{EARTH}^2}{r} = \frac{\mu}{r_p^2} \Rightarrow v_{EARTH} = \sqrt{\frac{\mu}{r_p}}$$

~~IF HEAD ON COLLISION WITH OPPOSITE DIRECTION~~

~~OF VELOCITIES:~~

~~MAX IMPACT VELOCITY:~~



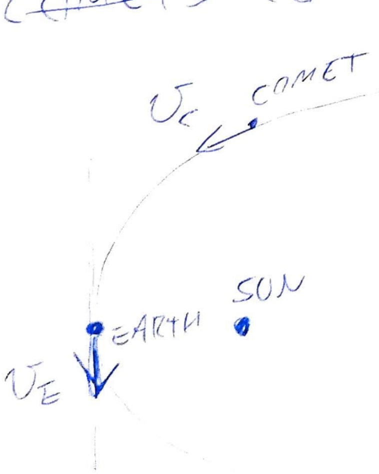
IMPACT VELOCITY:

$$v_E + v_C \cos\alpha$$

$$\text{AVERAGE QUANTITY} = \int \text{PROBABILITY DISTRIBUTION OF QUANTITY} \cdot \text{QUANTITY} \cdot dP$$

~~COMETS COME FROM ALL DIRECTIONS:~~

~~ASSUME UNIFORM~~



TAKE THIS PLANE. LOOK FROM SIDE WAYS:

$$\text{AVG QUANTITY} = \int \text{PROBAB DISTRIB OF QUANTITY} \cdot \text{QUANTITY } dP$$

PROBABILITY OF COMET COMING TO EARTH WITH ANGLE ~~α~~ BTWN ~~α~~ & $\alpha + d\alpha$ $= \dfrac{d\alpha}{2\pi}$

IMPACT VELOCITY $= v_E - v_C \cos\alpha$

$$\text{AVG IMPACT VELOCITY} = \int_0^{2\pi} \left(v_E - v_C \cos\alpha\right) \frac{d\alpha}{2\pi} = \underline{\underline{v_{EARTH}}}$$

$$\left(v_{EARTH} = \sqrt{\frac{\mu}{r}}\right)$$

NO DEPENDENCE ON COMET SPEED.

"ONE DISPLACED"...

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m \frac{\mu}{r}$$

THIS IS SPENT ON HEATING UP THE ATMOSPHERE & MAY BE LIFTING SOME PARTS OF IT?

I DON'T SEE HOW I COULD RELATE THIS TO ~~EARTH'S LOSING~~ MASS OF LOST ATMOSPHERE.

"THE DENSITY OF STARS"

$T_{NEMESIS} \approx 2.6 \cdot 10^7$ YRS

$\approx 8.2 \cdot 10^{14}$ SEC

$\cdot 20$

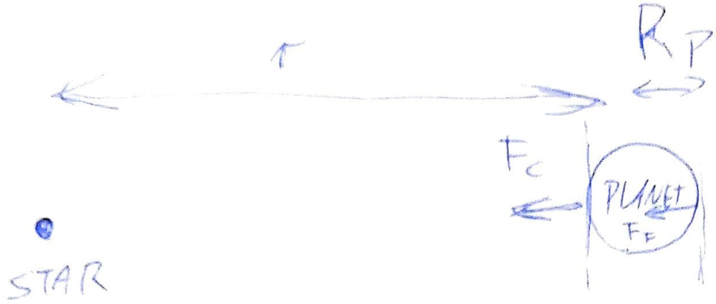
$1.6 \cdot 10^{16}$ km CLOSER

SOME STARS GET IN 1 ORBIT

≈ 500 pc

I DON'T SEE HOW TO CALCULATE FREQUENCY OF ENCOUNTERS BUT HYPOTHESIS DOES NOT SEEM PLAUSIBLE.

$\boxed{3}$



$$\frac{(F_C - F_F)}{m} = GM\left(\frac{1}{r^2} - \frac{1}{(r + 2R_P)^2}\right)$$

NOTE THAT:

$$(r + 2R_P)^{-2} = \left(r\left(1 + \frac{2R_P}{r}\right)\right)^{-2} = r^{-2}\left(1 + \frac{2R_P}{r}\right)^{-2}$$

$$\approx r^{-2}\left(1 - 4\frac{R_P}{r}\right)$$

SUBSTITUTE:

$$\frac{(F_C - F_F)}{m} = GM\left(\frac{1}{r^2} - \frac{1}{r^2}\left(1 - 4\frac{R_P}{r}\right)\right)$$

$$\simeq \frac{4GMR_P}{r^3}$$

$$F_C - F_F = F_T$$

$$\frac{F_T}{m} \approx \frac{4GMR_P}{r^3}$$

WHAT IS THE 4 DOING HERE?

LETS SAY FORCE NEEDED TO BREAK ROCK

$\approx$ AREA FACING STAR × STRENGTH

$$\approx \pi R_p^2 \sigma$$

$$F_T \sim \frac{GM_* R_p}{r^3} m = \pi R_p^2 \sigma$$

$$\frac{GM_* R_p}{r^3} \frac{4}{3} \cancel{\pi} r_p^3 \rho = \cancel{\pi} r_p^2 \sigma$$

$$\frac{GM_*}{r^3} \frac{4}{3} \rho \frac{1}{\sigma} = R_p^{-2}$$

$$R_p \approx \left(\frac{GM_*}{r^3} \frac{4}{3} \frac{\rho}{\sigma} \right)^{-\frac{1}{2}}$$

$$\cancel{\approx G \cdot 0.5 \cdot 6 \cdot 10}$$

$$\approx \left(\frac{G \cdot 0.5 \cdot 2 \cdot 10^{30}}{(0.2 \cdot \underbrace{7 \cdot 10^8}_{\text{SOLAR RADIUS}})^3} \frac{4}{3} \frac{3000}{100 \cdot 10^6} \right)^{-\frac{1}{2}} \approx \cancel{3 \cdot 10^5}$$

$$\approx 3.2 \cdot 10^4 \text{ m}$$

$$= R_{MAX}$$

FORCE PER UNIT MASS
ON SURFACE:

$$G \frac{\frac{4}{3}\pi R_{MAX}^3 \rho}{R_{MAX}^2} = \frac{4}{3}\pi G \rho R_{MAX}$$

(BUT IDK WHAT TO DO W/ THIS)

"IF THE LUMINOSITY OF THE WD"...

$$\text{POWER INCOMING} = \frac{\pi R_{MAX}^2}{4\pi r^2} L$$

$$= \frac{R_{MAX}^2}{4 r^2} L$$

$$= \frac{(3.2\cdot 10^4)^2}{4(7\cdot 10^8)^2} L \simeq 5\cdot 10^{-10} L \sim 1.6\cdot 10^{-8} L_\odot$$

$$L_\odot \approx 10^{26}\, W$$

$$\text{INCOMING POWER} \approx 10^{18}\, W$$

THE ROCK WON'T
WITHSTAND THIS
MUCH HEATING
FOR LONG.

ON DIMENSIONAL GROUNDS:

POWER IN · TIME TO SUBLIME = ΔH · MASS OF ROCK.

$$t = \Delta H \cdot \frac{\frac{4}{3}\pi R_{MAX}^3 \rho}{P}$$

$$= \Delta H \cdot \frac{\frac{4}{3}\pi (3.2 \cdot 10^4)^3 \, 3000}{10^{18}}$$

$$\approx 0.5 \Delta H$$

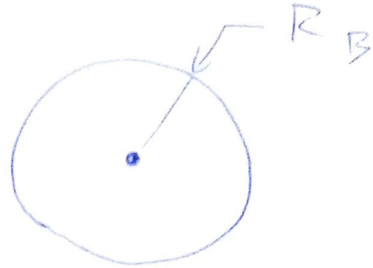
THIS NUMBER SEEMS SUSPICIOUSLY SMALL

"A ROCK ~~OF~~ RADIUS $R > R_{MAX}$"

PROBABLY HARDLY DOABLE IF GOT PREVIOUS ~~PART~~ WRONG SO I'LL GO TO THE NEXT PROBLEM.

4/

$R_B = \frac{GM}{2c_1^2}$



INCOMING FLUID
SPEED AT R_B: C_1

MASS ACCRETING PER UNIT TIME

$$\equiv \underbrace{C_1}_{\text{RATE}} \underbrace{4\pi R_B^2}_{\text{AREA}} \rho$$

$$= C_1 4\pi \left(\frac{GM}{2C_1^2}\right)^2 \rho = \cancel{\frac{GM\rho}{2C_1}}\ \pi \frac{G^2M^2}{C_1^3}\rho$$

CHECK DIMENSIONS:

$$\frac{m}{s} \cdot m^2 \cdot \frac{kg}{m^3} = \frac{kg}{s} \Rightarrow \text{OK.}$$

$$\cancel{M(t) = M \frac{1}{G} t}$$

$$\frac{dM}{dt} = \pi \frac{G^2M^2}{C_1^3}\rho = \frac{\pi G^2 \rho}{C_1^3} M^2$$

$\cancel{M}$

$$\frac{dM}{M^2} = \frac{\pi G^2 S}{C_s^3}\, dt$$

$$-M^{-1} = \frac{\pi G^2 S}{C_s^3}\, t + C_0$$

WE WANT: $-M(t)^{-1} = -M_0^{-1}$ AT $t=0$

$$\Rightarrow C_0 = -M_0^{-1}$$

$$-M^{-1} = \underbrace{\frac{\pi G^2 S}{C_s^3}}\, t - \frac{1}{M_0}$$

CHECK DIMENSIONS: $\left(\frac{m^3}{kg\, s^2}\right)^2 \frac{kg}{m^3} \left(\frac{m}{s}\right)^{-3} s$

$\cancel{\frac{m^3}{kg\, s^2} \frac{kg}{m^3} \left(\frac{s}{m}\right)^3 s}$

$$= \frac{m^6\, m^{-3}\, m^{-3}\, s^4}{kg\ s^4} = \frac{1}{kg}$$

$\Rightarrow$ GOOD

$$M = \frac{1}{-\frac{\pi G^2 S}{C_s^3}\, t + \frac{1}{M_0}}$$

SO I'LL GET TO ∞ MASS IN VERY FINITE TIME $\Rightarrow$ THIS IS LIKELY WRONG.

OH, THE PROBLEM ALSO SAYS THIS CANNOT HOLD INDEFINITELY, SO THERE'S HOPE.

5.

ANGULAR MOMENTUM OF DISC: $M a v$

$$\frac{v^2}{a} = \frac{GM}{a^2} \Rightarrow v = \sqrt{\frac{GM}{a}}$$

$$L = M_{ANNULUS}\, a \sqrt{\frac{GM_*}{a}}$$

THIS TURNS INTO ROTATING THE PLANET.

$$L = I\omega = \frac{2}{5} M_{PLANET} R_{PLANET}^2 \frac{2\pi}{P_{DAY}}$$

$M_{PLANET} \approx M_{ANNULUS}$ BY END OF ACCRETION.

$$\Rightarrow a\sqrt{\frac{GM_*}{a}} = \frac{2}{5} R_p^2 \frac{2\pi}{P_{DAY}}$$

$$\frac{5}{2}\sqrt{\mu a}\frac{1}{R_p^2}\frac{1}{2\pi} = P_{DAY}^{-1}$$

$$P_{DAY} = \frac{4\pi}{5}\frac{R_p^2}{\sqrt{\mu a}} \neq \frac{2}{2}\frac{R_p^2}{\sqrt{\mu a}}$$

SO I DID SOMETHING WRONG.

USING GIVEN FORMULA

$$P_{DAY} = \frac{2}{3} \frac{(10 \cdot 6400 \cdot 10^3)^2}{(G\, 2 \cdot 10^{30} \cdot 150\, 10^9)^{1/2}}$$

$$\neq \cancel{\ }$$

$$= 0.6 \text{ SEC} \qquad (?)$$

IF THIS IS CORRECT, PLANET ISN'T STABLE