

QUESTION  
TOPICS 3  
SHEET 1

$$M - M = 5 \log_{10} D - 5$$

$$M = -2.5 \log_{10} L + C$$

$$\Rightarrow M = 5 \log_{10} D - 5 - 2.5 \log_{10} L + C$$

$$= 5 \log_{10} D - 2.5 \log_{10} L + C$$

$D$  is between 39 & 41, but Kuiper belt is spread out quite more than 1 AU, and it is in the  $\log_{10}$  anyway, so we can say:

$$M \approx -2.5 \log_{10} L + C$$

Recall:

$$L = 4\pi R^2 \sigma T^4$$

$$\Rightarrow M \approx -2.5 \log_{10} (4\pi R^2 \sigma T^4) + C$$

$$\approx \boxed{-5 \log_{10} R + C}$$

~~$$\approx 10 \log_{10} T + C$$~~

Where we have assumed constant  $T$ , which is quite reasonable unless orbit is highly ~~eccentric~~ eccentric.

$$F(m) \propto \int_0^{N(m)} dN \propto \int_0^{R(m)} R^{-q} dR \propto \left[ R^{-q+1} \right]_0^{R(m)}$$

↑  
USING INFO  
IN QUESTION

$$M = -5 \log_{10} R + c \Rightarrow R = 10^{-\frac{1}{5}(m-c)}$$

SO WE END UP WITH  $10^{-\frac{1}{5}(m-c)} \propto 10^{-\frac{1}{5}(m-c)(-q+1)}$

$$F(m) \propto \left[ R^{-q+1} \right]_0^{\infty} \propto 10^{-\frac{1}{5}(m-c)(-q+1)}$$

$$\propto \frac{(c-m)(1-q)/5}{10}$$

IF  $c = m_{\min}$ , WE HAVE WHAT WE WANT.

(Right now I don't see why that'd be the case though)

"Observations show" ...

$$\log_{10} F(m) = 0.3m + c$$

$$\Rightarrow \underbrace{0.3m + c}_1 = \underbrace{(c_2 - m)(1-q)/5}_{= c_2(1-q)/5 + (q-1)/5 \cancel{m}}$$

$$= c_2(1-q)/5 + (q-1)/5 \cancel{m}$$

$$\Rightarrow 0.3 = \frac{q-1}{5} \Rightarrow q = 2.5$$

NOTE THAT

$$\frac{dN}{dR} \propto R^{-q} \Rightarrow N \propto R^{-q+1} \stackrel{\text{USING EQUATION ON PREVIOUS PAGE}}{\Rightarrow} N \propto R^{-1.5}$$

~~MASS  $\propto R^3$  (ASSUMING UNIFORM DENSITY)~~

TOTAL MASS OF OBJECTS BETWEEN  $R$  &  $R+dR$  = NUMBER OF OBJECTS BETWEEN  $R$  &  $R+dR$   $\times$  MASS OF THESE OBJECTS (MASS OF ONE)

$$\Downarrow \quad \frac{d}{dR} R^{1.5} \quad \frac{d}{dR} R^3$$

$$\text{Mass}(R) = R^{1.5}$$



MASS IS DOMINATED BY LARGE OBJECTS

We should go for shallow & wide.

"IF SKBS"

$$F(m) \propto 10^{-m(1-2.5)/5} = \propto 10^{+0.3m}$$

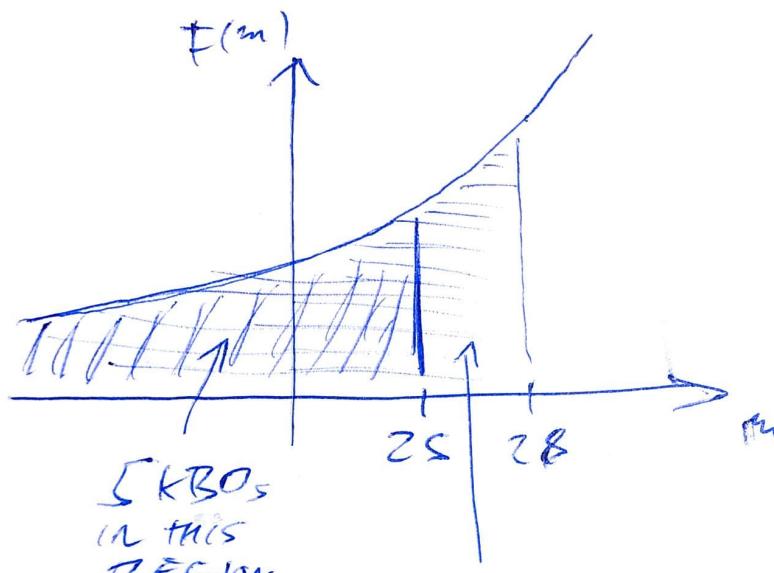
$$\frac{\int_{-\infty}^{25} 10^{+0.3m} dm}{\int_{-\infty}^{25} 10^{+0.3m} dm} = \frac{\int_{-\infty}^{25} 10^{+0.3m} dm + \int_{25}^{28} 10^{+0.3m} dm}{\int_{-\infty}^{25} 10^{+0.3m} dm}$$

$$= 1 + \frac{\int_{25}^{28} 10^{0.3m} dm}{\int_{-\infty}^{25} 10^{0.3m} dm}$$

JUST PUT THIS TO A CALCULATOR:

$$\approx 1 + 6.94 \approx 7.94$$

WHAT I'M DOING HERE:



How many in the  
≡ horizontally  
shaded region

$$\frac{X}{S} = \frac{\text{AREA OF HORIZONTALLY SHADeD REGION}}{\text{AREA OF VERTICALLY (ISa) SHADeD REGION}}$$

$$\approx 7.94$$

$\Rightarrow X \approx 40$  THIS MANY EXPECTED  
WITH  $m < 28$ .