1/

$$S = \frac{\partial}{\partial T}\left(k_B T \log Z_{IDEAL}\right)$$

SUB FOR Z_{IDEAL}

$$= \frac{\partial}{\partial T}\left(k_B T \log \frac{Z_1^N}{N!}\right)$$

SUB FOR $Z_1 \left(= \frac{V}{\lambda^3}\right)$

$$= \frac{\partial}{\partial T}\left(k_B T \log \frac{V^N}{N!\,\lambda^{3N}}\right)$$

SUB FOR $\lambda \left(= \sqrt{\frac{2\pi\hbar^2}{m k_B T}}\right)$

$$= \frac{\partial}{\partial T}\left(k_B T \log \frac{V^N}{N!\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}}\right)$$

$$= k_B \log \frac{V^N}{N!\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}} + k_B T \frac{\partial}{\partial T} \ln \frac{V^N}{N!\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}}$$

$$= k_B \log\left(\frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right) + k_B T \frac{\partial}{\partial T} \ln \frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}$$

STIRLING APPROX: $\log N! \simeq N \log N - N$

FIRST TERM:

$$k_B \log\left(\frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right) =$$

$$= k_B \log\left(V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right) - k_B \log N!$$

$$= N k_B \log\left(V\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\right) - k_B \log N!$$

$$= N k_B \log\left(\frac{V}{\lambda^3}\right) - k_B N \log N + N k_B$$

$$= N k_B \log\left(\frac{V}{\lambda^3 N}\right) + N k_B$$

SECOND TERM:

$$k_B T \frac{\partial}{\partial T} \ln \frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}} =$$

$$= k_B T \frac{\partial}{\partial T}\left[\ln V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}} - \ln N!\right]$$

N IS NOT DEPENDENT ON t, SO:

$$= k_B T \frac{\partial}{\partial T} \ln V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}$$

V ALSO t INDEPENDENT

$$= k_B T N \frac{\partial}{\partial T} \log\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

$$= k_B T N \frac{3}{2} \partial_T \log\left(\frac{m k_B T}{2\pi\hbar^2}\right)$$

$$\cancel{= k_B T N \frac{3}{2} \frac{2\pi\hbar^2}{m k_B T}}$$

$$= k_B T N \frac{3}{2} \partial_T \log T$$

$$= k_B T N \frac{3}{2} \frac{1}{T} = k_B N \frac{3}{2}$$

COLLECT TERMS:

$$S = N k_B \left[\log\left(\frac{V}{N\lambda^3}\right) + \frac{5}{2}\right]$$

IF N! IS NOT INCLUDED:

$$S = \frac{\partial}{\partial T}\left(k_B T \log \frac{V^N}{\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}}\right)$$

$$= \underbrace{k_B \log\left(V^N \cdot \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right)} + \underbrace{k_B T \frac{\partial}{\partial T} \ln V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}}$$

$$= N k_B \log\left(V\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\right)$$

$$= N k_B \log\left(\frac{V}{\lambda^3}\right)$$

$$\Rightarrow = k_B T \frac{\partial}{\partial T} T^{\frac{3N}{2}} = k_B T \frac{3}{2} N \frac{\partial}{\partial T} \ln T = \frac{3}{2} k_B N$$

COLLECT TERMS:

$$S_{\substack{\text{without} \\ N!\ \text{factor}}} = N k_B \log \frac{V}{\lambda^3} + \frac{3}{2} k_B N$$

$$= N k_B \log\left(\frac{V}{\lambda^3} + \frac{3}{2}\right)$$

NOTICE THAT WHILE:

$$S_{\substack{\text{with} \\ N!}}(2N, 2V) = 2 S_{\substack{\text{with} \\ N!}}(N, V)$$

HERE WE HAVE:

$$S_{\substack{\text{without} \\ N!}}(2N, 2V) \neq 2 S_{\substack{\text{without} \\ N!}}(N, V) \Rightarrow \text{NOT EXTENSIVE}$$

2

$$H = \frac{p^2}{2m} + \lambda q^4 \qquad -q \longrightarrow q$$

$$Z_1 = \frac{1}{2\pi\hbar}\int dq\, dp\, e^{-\beta\left(\frac{p^2}{2m} + \lambda q^4\right)}$$

$$= \frac{1}{2\pi\hbar} \underbrace{\int_{-\infty}^{\infty} dq\, e^{-\lambda\beta q^4}} \int_{-\infty}^{\infty} dp\, e^{-\beta\frac{p^2}{2m}}$$

FIRST TERM: $\int_{-\infty}^{\infty} dq\, e^{-\lambda\beta q^4} = \int_{-\infty}^{\infty} (\lambda\beta)^{-\frac{1}{4}} e^{-x^4}\, dx =$

LET $x = (\lambda\beta)^{\frac{1}{4}} q$ $\qquad = (\lambda\beta)^{-\frac{1}{4}} \int_{-\infty}^{\infty} e^{-x^4} dx$

$dx = (\lambda\beta)^{\frac{1}{4}} dq$

$$= (\lambda\beta)^{-\frac{1}{4}}\, 2 \int_0^{\infty} e^{-x^4} dx$$

COMPUTER

$$= 2(\lambda\beta)^{-\frac{1}{4}}\, \Gamma\left(\frac{5}{4}\right)$$

SECOND TERM:

$$\int_{-\infty}^{\infty} dp\, e^{-\beta\frac{p^2}{2m}} =$$

LET: $y = +\sqrt{\frac{\beta}{2m}}\, p \Rightarrow dy = \sqrt{\frac{\beta}{2m}}\, dp$

$$= \int_{-\infty}^{\infty} dy\, e^{-y^2} \sqrt{\frac{2m}{\beta}} = \sqrt{\frac{2m\pi}{\beta}}$$

COLLECT TERMS:

$$Z_1 = \frac{1}{\cancel{2}\pi\hbar}\, \cancel{2}(\lambda\beta)^{-\frac{1}{4}}\, \Gamma\left(\frac{5}{4}\right)\sqrt{\frac{2m\pi}{\beta}} = \frac{1}{\pi\hbar}(\lambda\beta)^{-\frac{1}{4}}\sqrt{\frac{2m\pi}{\beta}}\, \Gamma\left(\frac{5}{4}\right)$$

$$= \frac{1}{\pi\hbar}(\lambda)^{-\frac{1}{4}} \cdot \sqrt{2m\pi}\, \beta^{-\frac{3}{4}}\, \Gamma\left(\frac{5}{4}\right) = \frac{1}{\pi\hbar}(\lambda)^{-\frac{1}{4}}\Gamma\left(\frac{5}{4}\right)\sqrt{2m\pi}\, k_B^{\frac{3}{4}}\, T^{\frac{3}{4}}$$

$$E = -\frac{\partial}{\partial \beta} \log Z = -\frac{\partial}{\partial \beta} \log Z_1^N = -N \frac{\partial}{\partial \beta} \log Z_1$$

$$= -N \frac{\partial}{\partial \beta} \log \text{BUNCH OF CONSTANTS} \cdot T^{\frac{3}{4}}$$

$$= -N \frac{\partial}{\partial \beta} \log T^{\frac{3}{4}}$$

USE:

$$\frac{\partial}{\partial \beta} = \frac{\partial T}{\partial \beta} \frac{\partial}{\partial T} = \frac{\partial T}{\partial \left(\frac{1}{k_B T}\right)} \frac{\partial}{\partial T} = \left(\frac{\partial \frac{1}{k_B T}}{\partial T} \right)^{-1} \frac{\partial}{\partial T}$$

$$= -k_B T^{+2} \frac{\partial}{\partial T}$$

$$E = -N \left(-k_B T^{+2}\right) \frac{\partial}{\partial T} \log T^{\frac{3}{4}}$$

$$= N k_B T^{+2} \cdot \frac{3}{4} \frac{\partial}{\partial T} \log T = N k_B T^{+2} \frac{1}{T} \cdot \frac{3}{4}$$

$$= \frac{3}{4} N k_B T$$

$$C_V = \frac{\partial E}{\partial T}\bigg|_V = \frac{\partial}{\partial T} \frac{3}{4} N k_B T \bigg|_V = \underline{\frac{3}{4} N k_B}$$

IF PARTICLES DISTINGUISHABILITY CHANGES, THEN NUMBER OF ALL POSSIBLE STATES CHANGES.

IE $Z_{NEW} = Z$ WHAT WE HAD PREVIOUSLY $\cdot\,\alpha$
FOR SAME α.

NOTICE THAT:

$$E = -\frac{\partial}{\partial\beta}\log(Z\alpha) = -\frac{\partial}{\partial\beta}\log Z + \cancelto{0}{\frac{\partial}{\partial\beta}\log\alpha}$$

$$= -\frac{\partial}{\partial\beta}\log Z$$

IE WE ARRIVE TO THE SAME E AS WE DID PREVIOUSLY, NO MATTER CONSTANT MULTIPLICATIVE FACTOR IN $\ln$.

$\Longrightarrow C_V$ IS NOT DEPENDENT ON DISTINGUISHABILITY.

3/

FOR 1 PARTICLE:

$$Z_1 = \frac{1}{(2\pi\hbar)^3}\int d^3q\, d^3p\, e^{-\beta E(p)}$$

$$\int_{\text{ALL SPACE}} e^{-\beta|p|c}\, d^3p = \int e^{-\beta c p}\, p^2 \sin\theta\, dp\, d\theta\, d\phi = 4\pi\int_0^\infty e^{-\beta c p}\, p^2\, dp = 8\pi\frac{+1}{(\beta c)^3} \quad \text{COMPUTER}$$

$$= \frac{1}{(2\pi\hbar)^3} V \int e^{-\beta|p|c}\, d^3p$$

$$\cancel{= \frac{1}{(2\pi\hbar)^3} V \left(2\int_0^\infty e^{-\beta c p}\, dp\right)^3}$$

$$\cancel{= \frac{1}{(2\pi\hbar)^3} V\, 2^3 \left(\frac{1}{-\beta c}\left[e^{-\beta c p}\right]_0^\infty\right)^3}$$

$$= \frac{1}{(2\pi\hbar)^3} V\cdot 8\pi\left(\frac{1}{\beta c}\right)^3 = \frac{1}{(2\pi)^3} V\cdot 2^3\pi\left(\frac{k_B T}{\hbar c}\right)^3 = \frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$$

WE HAVE N PARTICLES BUT WE CANNOT DISTINGUISH THEM SO:

$$Z = \frac{1}{N!} Z_1^N = \frac{1}{N!}\left[\frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right]^N$$

AS WANTED.

(MY RELIANCE ON COMPUTERS TO DO INTEGRALS IS ~~[illegible]~~ SOMEWHAT WORRYING)

$$dF = -S\,dT - p\,dV$$

$$p = -\frac{\partial F}{\partial V}\Big|_T$$

$$= -\frac{\partial}{\partial V}\left(-k_B T \log Z\right)\Big|_T$$

$$= \frac{\partial}{\partial V}\left(k_B T \log Z\right)\Big|_T$$

$$= k_B T \frac{\partial}{\partial V} \log Z\Big|_T$$

$$= k_B T \frac{\partial}{\partial V} \log \frac{1}{N!}\left[\frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right]^N$$

$$\cancel{= k_B T N \cdot V^{N-1} \frac{1}{N!}}$$

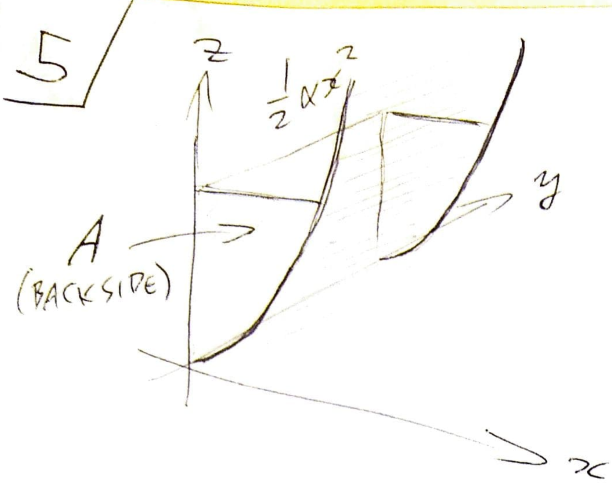
$$= k_B T \frac{\partial}{\partial V} \log V^N$$

$$= k_B T N \frac{\partial}{\partial V} \log V$$

$$= k_B T N \frac{1}{V} \Rightarrow \underline{pV = k_B T N}$$

AS WANTED.

5/



$$\underbrace{\text{NUMBER DENSITY OF ATOMS BETWEEN } x \,\&\, x+dx \text{ TIMES } dx}_{} = \underbrace{\text{PROBABILITY OF 1 ATOM BEING BETWEEN } x \,\&\, x+dx}_{} \times \underbrace{\underbrace{\text{TOTAL NUMBER OF ATOMS}}_{AN_0} \Big/ \underbrace{\text{AREA IN } zy \text{ PLANE}}_{A}}_{N_0}$$

$$P(x) = \frac{e^{-\beta E(x)}\,dx}{\int_0^\infty e^{-\beta E(x)}\,dx}$$

└ NORMALIZATION

$$= \exp\left(-\tfrac{1}{2}\alpha\beta x^2\right) \cdot \underbrace{\left(\int_0^\infty e^{-\frac{1}{2}\alpha\beta x^2}\,dx\right)^{-1}}_{\sqrt{\frac{\pi}{2\alpha\beta}}^{\,-1} = \sqrt{\frac{2\alpha\beta}{\pi}}}$$

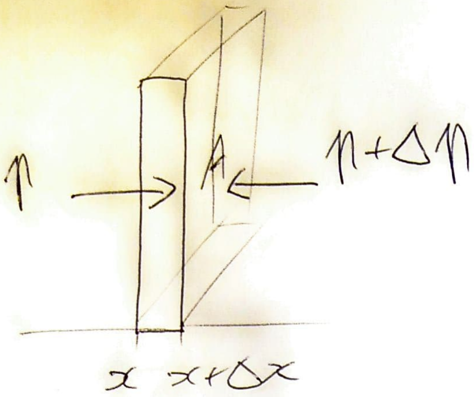
COLLECT TERMS:

$$\rho(x)\,dx = \exp\left(-\tfrac{1}{2}\alpha\beta x^2\right)\sqrt{\frac{2\alpha\beta}{\pi}}\,dx\,N_0$$

REWRITE

$$\Rightarrow \rho(x) = 2N_0\sqrt{\frac{\alpha\beta}{2\pi}}\,e^{-\alpha\beta x^2/2}$$

AS WANTED.



$$p(x)A = p(x+\Delta x)A \Rightarrow p = \text{CONSTANT}$$

HMMM, & NOW WHAT...

7/

$$U(r) = \begin{cases} \infty & r < r_0/2 \\ 0 & \text{ELSE.} \end{cases}$$

$$\cancel{f(r) = e^{-\beta}}$$

$$f(r) = e^{-\beta U(r)} - 1$$

$$p = -\frac{\partial F}{\partial V} = \cancel{\frac{N k_B T}{V}}$$

$$F = F_{IDEAL} - N k_B T \underbrace{\log\left(1 + \frac{N}{2V}\int dr^2 f(r)\right)}_{\simeq \frac{N}{2V}\int dr^2 f(r)}$$

$$p = \underbrace{-\frac{\partial}{\partial V}\left(F_{IDEAL}\right.}_{\frac{N k_B T}{V}} \left. - N k_B T \frac{N}{2V}\int dr^2 f(r)\right)$$

$$\cancel{= \frac{N k_B T}{V}\left(1 + \frac{\partial}{\partial V}\right.}$$

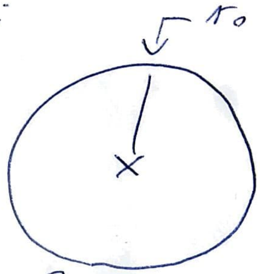
$$p = \frac{N k_B T}{V} - \frac{N k_B T N}{2V^2}\int dr^2 f(r)$$

$$p = \frac{N k_B T}{V}\left(1 - \frac{N}{2V}\int dr^2 f(r)\right)$$

$$\frac{pV}{N k_B T} = 1 - \frac{N}{2V}\underbrace{\int dr^2 f(r)}$$

FOCUS ON THIS BIT

CONSIDER GAS IN THIS AREA:



$V = r_0^2 \pi$

$$\cancel{\int dr^2 f(r) = \int_{r=0}^{r=r_0}\int_{\varphi=0}^{\varphi=2\pi} f(r)\, r\, dr\, d\varphi}$$

$$\cancel{= \int_0^{r_0}\int_0^{2\pi} e^{-\beta U}}$$

$$\int_0^{\infty} f(r)\, dr^2 = \int_0^{\frac{r_0}{2}} f(r)\, dr^2 + \int_{\frac{r_0}{2}}^{\infty} f(r)\, dr^2$$

$$= \int_0^{\frac{r_0}{2}} -1\, dr^2 + \underbrace{\int_{\frac{r_0}{2}}^{\infty} 0\, dr^2}_{0}$$

$$= \left(\frac{r_0}{2}\right)^2 \pi \cdot (-1)$$

ARRIVE TO:

$$\frac{pV}{N k_B T} = 1 - \frac{N}{2V}\left(\frac{r_0}{2}\right)^2 \pi (-1)$$

IE

$$pV = N k_B T \left(1 + \frac{N}{2V}\left(\frac{r_0}{2}\right)^2\right)$$

8

$$U(r) = \frac{\alpha}{r^n} \qquad n > 3 \qquad \alpha > 0$$

$$\frac{pV}{N k_B T} = 1 - \frac{N}{2V} \underbrace{\int d^3r\, f(r)}_{\text{THIS IS THE SECOND VIRIAL COEFF.}}$$

$$\cancel{= 1 - \frac{N}{V}\left(\frac{a}{k_B T} - b\right)}$$

SECOND VIRIAL COEFF

$$= \int d^3r \left(e^{-\beta U(r)} - 1\right)$$

$$= \int dr^3 \left(e^{-\beta \alpha r^{-n}} - 1\right)$$

$$= \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \left(e^{-\beta \alpha r^{-n}} - 1\right) r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$= 4\pi \int_{r=0}^{\infty} \left(e^{-\beta \alpha r^{-n}} - 1\right) r^2\, dr$$

& I SHOULD SOMEHOW EVALUATE THIS BUT IDK HOW.

9

$d=2$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right)^2 (n_1^2 + n_2^2) = \frac{2\pi^2\hbar^2}{mL^2}(n_1^2 + n_2^2)$$

$$\sum_{\vec{n}} \int d^2 n = \frac{V}{(2\pi)^2}\int d^2 k = \frac{V}{(2\pi)^2}\int k\, d\phi\, dk$$

$$= \frac{\pi V}{(2\pi)^2}\int_0^\infty k\, dk$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow dE = \frac{\hbar^2 k}{m}\, dk$$

$$\frac{\pi V}{(2\pi)^2}\int k\, dk = \frac{V}{4\pi}\int \frac{m}{\hbar^2 \cancel{k}}\, dE\, \cancel{k} = \frac{V}{4\pi}\int \frac{m}{\hbar^2}\, dE$$

$$\Rightarrow g(E) = \frac{V}{4\pi}\frac{m}{\hbar^2}$$

CONSTANT, AS HINT SUGGESTS

$d=1$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{2\pi\hbar^2}{mL^2}(n^2)$$

$$\sum_{\vec{n}} \approx \int d^1 n = \frac{V}{2\pi}\int d^1 k = \frac{V}{2\pi}\int dk = \int \frac{V}{2\pi} \overbrace{\frac{m}{\hbar^2 k}\, dE}^{dk \text{ FROM } E = \frac{\hbar^2 k^2}{2m}}$$

USE: $\frac{1}{k} = \sqrt{\frac{\hbar^2}{2mE}}$

$$= \int \frac{V}{2\pi}\frac{m}{\hbar^2}\sqrt{\frac{\hbar^2}{2mE}}\, dE = \frac{V\sqrt{m}}{2\pi\,\hbar}\frac{1}{\sqrt{2}}\int E^{-\frac{1}{2}} dE$$

$$= \cancel{-\frac{1}{2}\frac{V}{2\pi}\frac{\sqrt{m}}{\hbar}} \Rightarrow g(E) = \frac{V}{2\pi\hbar}\sqrt{\frac{m}{2}}\, E^{-\frac{1}{2}}$$

DECREASES, AS EXPECTED.

11

$$\text{AVERAGE NUMBER OF PHOTONS} = \int \text{PROBABILITY OF HAVING THIS MANY PHOTONS} \times \text{THIS MANY PHOTONS} \; d\,\text{ALL POSSIBLE NUMBER OF PHOTONS}$$

$$= \int \text{NUMBER OF PHOTONS AT A GIVEN ENERGY LEVEL} \; d\,\text{ALL ENERGY LEVELS}$$

$$\cancel{= \int \frac{V}{\pi^2 \hbar^3 c^3} E^2 \, dE}$$

~~THEN IDK WHAT.~~

$$\text{MEAN PHOTON ENERGY} = \int \text{NUMBER OF PHOTONS AT A GIVEN ENERGY LEVEL} \cdot \text{ENERGY OF THAT LEVEL} \; d\,\text{ALL ENERGY LEVELS}$$

~~MOST LIKELY~~

$$\frac{d\,\text{NUMBER OF PHOTONS AT A GIVEN ENERGY LEVEL}}{d\,\text{ENERGY LEVELS}} = 0$$

MOST LIKELY ENERGY OF A PHOTON

$$E(\omega)d\omega \propto \frac{\omega^3}{e^{-\beta\hbar\omega}-1}d\omega \propto N\underbrace{\hbar\omega}_{\text{energy of 1 photon}}d\omega$$

$$\Rightarrow N(\omega) \propto \frac{\omega^2}{e^{-\beta\hbar\omega}-1} \Rightarrow N(E) \propto \frac{\omega^2}{e^{-\beta\hbar\omega}-1}$$

$$\langle N \rangle = \int \frac{\omega^2}{e^{-\beta\hbar\omega}-1}d\omega \propto \int \frac{\omega^2}{e^{-\omega}-1}d\omega$$

THIS ISN'T EVEN CONVERGENT.

12/

BY EXPERIENCE: YES WE CAN.

CHECK:

$$L = \varepsilon 4\pi R^2 = 4\pi R^2 \sigma T^4$$

E FLUX ON EARTH PER UNIT AREA:

$$E = 4\pi R^2 \sigma T^4 \left(\frac{r}{R}\right)^2 = 4\pi r^2 \sigma T^4$$

IF EARTH IS ALSO BB-LIKE: (?)

$$4\pi r^2 \sigma T_{SUN}^4 = \sigma T_{EARTH}^4$$

$$T_{EARTH} = (4\pi)^{\frac{1}{4}} r^{\frac{1}{2}} T_{SUN}$$

$$= 1.9 \cdot (1.5 \cdot 10^8)^{\frac{1}{2}} \cdot 6000$$

$$=$$

WAIT, THIS IS WAY TOO HIGH.