11.1

"HENCE SHOW THAT"

START WITH:

$$\left(\frac{\dot{T}}{T}\right)^2 = \frac{8}{3}\pi G \frac{aT^4}{c^2}$$

REARRANGE:

$$\frac{dT}{dt} = \pm\sqrt{\frac{8}{3}\pi G \frac{a}{c^2}}\, T^3$$

INTEGRATE UP:

$$\int \frac{dT}{T^3} = \int \pm\sqrt{\frac{8}{3}\pi G \frac{a}{c^2}}\, dt \qquad \text{/EVALUATE}$$

$$-\frac{1}{2}T^{-2} = \pm\sqrt{\frac{8}{3}\pi G \frac{a}{c^2}}\, t \qquad /\cdot -2$$

$$T^{-2} = \pm -\sqrt{\frac{32\pi G a}{3c^2}}\, t \qquad /\square^{-\frac{1}{2}}$$

$$T = \pm -\left(\frac{32\pi G a}{3c^2}\right)^{-\frac{1}{4}} t^{-\frac{1}{2}} \qquad \text{/REWRITE}$$

$$= +\left(\frac{3c^2}{32\pi G a}\right)^{\frac{1}{4}} t^{-\frac{1}{2}}$$

CHOOSE THIS SIGN TO MAKE SENSE PHYSICALLY.

11.2

CHEMICAL POTENTIAL:

- SERVES TO BREAK MATTER-ANTIMATTER SYMMETRY.

$$n = \frac{4\pi g_i}{h^3}\int_0^\infty \frac{p^2 \, dp}{e^{E(p)/kT} \pm 1}$$

$$= \frac{4\pi g_i}{h^3}\int_0^\infty \frac{p^2 \, dp}{e^{c\sqrt{p^2+m^2c^2}/kT} \pm 1}$$

SUBSTITUTE:

$$x = c\sqrt{p^2+m^2c^2}/(kT)$$

$$dx = c\,(p^2+m^2c^2)^{-\frac{1}{2}}\,p/(kT)\,dp$$

$$dp = \frac{kT}{c}(p^2+m^2c^2)^{\frac{1}{2}}/p\,dx$$

$$n = \frac{4\pi g_i}{h^3}\int_0^\infty \frac{p^{\cancel{2}}\,\frac{kT}{c}(p^2+m^2c^2)^{\frac{1}{2}}/\cancel{p}\,dx}{e^x \pm 1}$$

$$\cancel{= \frac{4\pi g_i}{h^3}\int_0^\infty p}$$

NOTICE THAT:

$$p \frac{kT}{c}\left(p^2+m^2c^2\right)^{\frac{1}{2}} = p \frac{kT}{c}\frac{kT}{c} x$$

WE HAD:

$$x = \frac{c}{kT}\sqrt{p^2+m^2c^2}$$

REARRANGE, AIMING FOR p:

$$\left(\frac{kT}{c} x\right)^2 = p^2+m^2c^2$$

$$p = \sqrt{\left(\frac{kT}{c} x\right)^2 - m^2c^2}$$

REWRITE NUMERATOR:

$$p \frac{kT}{c}\left(p^2+m^2c^2\right)^{\frac{1}{2}} = \sqrt{\left(\frac{kT}{c} x\right)^2 - m^2c^2}\left(\frac{kT}{c}\right)^2 x$$

$$n = \frac{4\pi g_i}{h^3}\int_0^\infty \frac{\sqrt{\left(\frac{kT}{c} x\right)^2 - m^2c^2}\left(\frac{kT}{c}\right)^2 x}{e^x \pm 1}\, dx$$

WHICH IS A BIT LESS HORRIBLE THAN WHAT WE'VE STARTED WITH BUT STILL NO STRAIGHTFORWARD WAY TO PROCEED.

ASSUMING $\mu << mc^2$, SHOW...

$$n = \frac{4\pi g_i}{h^3}\int_0^\infty \frac{p^2}{e^{\frac{c\sqrt{p^2+m^2c^2}-\mu}{kT}}\pm 1}\,dp$$

NON-REL LIMIT:

$$p^2 << m^2c^2$$

IE $$\sqrt{p^2+m^2c^2} \simeq mc + \cancelto{0}{\frac{d}{dp}\sqrt{p^2+m^2c^2}\Big|_{p=0}} p + \frac{1}{2}\frac{d^2}{dp^2}\sqrt{p^2+m^2c^2}\Big|_{p=0} p^2$$

$$\frac{d^2}{dp^2}\sqrt{p^2+m^2c^2}\Big|_{p=0}$$

$$= \frac{d}{dp}\left(p^2+m^2c^2\right)^{-\frac{1}{2}} p\Big|_{p=0}$$

$$= \cancelto{0}{\left(p^2+m^2c^2\right)^{-\frac{3}{2}} p\Big|_{p=0}} + \left(p^2+m^2c^2\right)^{-\frac{1}{2}}\Big|_{p=0}$$

$$= \frac{1}{mc}$$

$$\Rightarrow \sqrt{p^2+m^2c^2} = mc + \frac{1}{2}\frac{1}{mc}p^2$$

REWRITE INTEGRAL:

$$n = \frac{4\pi g_i}{h^3}\int_0^\infty \frac{p^2}{e^{(mc^2+\frac{1}{2}m p^2-\mu)/kT}\pm 1}\,dp$$

GIVEN $V \ll mc^2$, IT IS TEMPTING TO GET RID OF V, BUT IT IS IN THE FINAL RESULT SO WE SHOULDN'T DO THAT.

$\boxed{11.3}$

INTERACTION TIME $\propto$ SPACE TAKEN UP BY ONE NEUTRINO IN UNIT TIME $\times$ SPACE TAKEN UP BY ANOTHER NEUTRINO IN UNIT TIME / $\underbrace{\text{DISTANCE THEY NEED TO BE WITHIN TO INTERACT}}_{\propto \sqrt{\sigma}}$

I CANNOT GET UNITS MATCH UP.

$$\frac{3a}{8\pi} = 2.05 \cdot 10^7 \frac{1}{m^3 K^3}$$

SPACE TAKEN UP BY ONE NEUTRINO IN UNIT TIME $\propto \underbrace{(n_\nu)^{-1}}_{m^3K^3} \cdot$ SPEED OF THAT NEUTRINO

UNITS: $m^3K^3 \cdot \frac{m}{s} = m^4 K^3 sec^{-1}$

11.4

$$g_{EFF} = \sum_{BOSONS} g_i + \sum_{FERMIONS} \frac{7}{8} g_i$$

BOSONS: → PHOTON

$$g_{PHOTON} = 2$$

FERMIONS:

→ NEUTRINOS: N_ν TYPES

EACH: $g = 2$

→ e^- & e^+: EACH TWO SPIN STATES

→ NEUTRONS: $g = 1$ (NO SPIN)

→ PROTONS: $g = 2$ (TWO SPIN STATES)

COMBINE:

$$g_{EFF} = 2 + \frac{7}{8}\left(N_\nu \cdot 2 + 2 \cdot 2 + 1 + 2\right)$$

$$= 8.125 + \frac{7}{4} N_\nu$$

$$g_{eff}\Big|_{N_\nu = 3} \approx 1.34$$

$$g_{eff}\Big|_{N_\nu = 4} \approx 1.51$$

$N_\nu = 3$: $g_{eff} \propto T_F^6$ $\quad kT_F = 0.8\,MeV$

$N_\nu = 4$: $\underbrace{g_{eff} \propto T_F^6}$ $\quad \boxed{kT_F = 0.82\,MeV}$

THIS IS WHAT WE NEED TO SHOW

$$\Downarrow$$

$$T_F \propto g_{eff}^{\frac{1}{6}}$$

$$\frac{kT_F|_{N_\nu=4}}{kT_F|_{N_\nu=3}} = \frac{g_{eff}^{\frac{1}{6}}|_{N_\nu=4}}{g_{eff}^{\frac{1}{6}}|_{N_\nu=3}}$$

$$\Downarrow$$

$$T_F|_{N_\nu=4} = \left(\frac{g_{eff}|_{N_\nu=4}}{g_{eff}|_{N_\nu=3}}\right)^{\frac{1}{6}} T_F|_{N_\nu=3}$$

$$= \left(\frac{1.51}{1.34}\right)^{\frac{1}{6}} 0.8.. \simeq 0.816 \sim \underline{\underline{0.82}}\ MeV$$

GOOD

$$\left(\frac{N_n}{N_p}\right)_F = \frac{\cancel{AT^{3/2}}\exp\left(-\frac{m_n c^2}{kT}\right)}{\cancel{AT^{3/2}}\exp\left(-\frac{m_p c^2}{kT}\right)} = e^{-\frac{m_n c^2}{kT} + \frac{m_p c^2}{kT}}$$

$$= e^{\frac{c^2}{kT}(m_p - m_n)}$$

~~GOOGLE~~

WHERE kT is 0.8MeV & 0.82MeV FOR THE 2 CASES.

"AFTER FREEZE-OUT FREE NEUTRONS ARE DESTROYED" PART B

DON'T KNOW, THE $\frac{7}{43}$ TERM LOOKS PARTICULARY OUT OF THE BLUE.

"IF $t_3 = 300$" ...

$$\exp\left[-\frac{300}{877}\left(1+\frac{7}{43}\Delta N_\nu\right)^{-\frac{1}{2}}\right]$$

/ TAYLOR EXPAND $\left(1+\frac{7}{43}\Delta N_\nu\right)^{-\frac{1}{2}}$

$$\approx \exp\left[-\frac{300}{877}\left(1-\frac{1}{2}\frac{7}{43}\Delta N_\nu\right)\right]$$

/ EXPAND

$$\approx e^{-\frac{300}{877}} e^{\frac{7}{86}\Delta N_\nu}$$

/ USE: $e^x \approx 1+x$ when $|x| \ll 1$

$$\approx e^{-\frac{300}{877}}\left(1+\frac{7}{86}\Delta N_\nu\right)$$

EVALUATE

$$\approx 0.71 + 0.06\,\Delta N_{\nu}$$

WHICH DOES NOT SEEM TO BE RELATED TO WHAT WE WANT.

11.6

(i) $t_{ev} \sim \frac{10240\pi^2}{\hbar c^4} G^2 M^3$

REARRANGE:

$$\sqrt[3]{\frac{t_{ev}\,\hbar c^4}{10240\pi^2 G^2}} = M$$

EVALUATE:

$$M\Big|_{t_{ev}\sim t_0} = \sqrt[3]{\frac{13.7\cdot 10^9\cdot 365.25\cdot 60^2\,\hbar c^4}{10240\pi^2 G^2}} = 1.7\cdot 10^{11}\,kg$$

$$\sim 10^{-19} M_\odot$$

(ii)