SDSG 4.4(a)

CBE:

$$\frac{\partial f}{\partial t} + \underline{v}\cdot\nabla f - \nabla\Phi\cdot\frac{\partial f}{\partial \underline{v}} = 0$$

$$f(x,v) = f(E) = f\left(\frac{1}{2}v^2 + \Phi(x)\right) = f\left(\frac{1}{2}\sum_i v_i^2 + \Phi(x)\right)$$

$\frac{\partial f}{\partial t} = 0$ (IE NO EXPLICIT t DEPENDENCE)

$$\underline{v}\cdot\nabla f = v_i \frac{\partial}{\partial x_i} f = v_i \frac{\partial f}{\partial E}\frac{\partial E}{\partial x_i} = v_i \frac{\partial f}{\partial E}\frac{\partial \Phi}{\partial x_i}$$

$$\nabla\Phi\cdot\frac{\partial f}{\partial \underline{v}} = \frac{\partial}{\partial x_i}(\Phi)\cdot\frac{\partial}{\partial v_i}(f) = \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial E}\frac{\partial E}{\partial v_i} = v_i\frac{\partial f}{\partial E}\frac{\partial \Phi}{\partial x_i}$$

$$\Rightarrow \underline{v}\cdot\nabla f = \nabla\Phi\cdot\frac{\partial f}{\partial \underline{v}} \Rightarrow \text{CBE LHS} = 0$$

$$f(E) = \frac{\rho_0}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}v^2 - \Phi(x)\right)$$

$$\rho = \int_{\forall v} f\,dv = \frac{\rho_0}{\sqrt{2\pi}}\exp(-\Phi(x))\int_{-\infty}^{\infty}\exp\left(-\frac{1}{2}v^2\right)dv$$

$$= \frac{\rho_0}{\sqrt{2\pi}}\exp(-\Phi(x))\sqrt{2\pi} = \rho_0\exp(-\Phi(x))$$

POISSON EQ: $\nabla^2\Phi = 4\pi G\rho \Rightarrow \Phi'' = 4\pi G\rho_0\exp(-\Phi) = 4\pi G\rho_0 e^{-\Phi}$