2013P1 (ii)
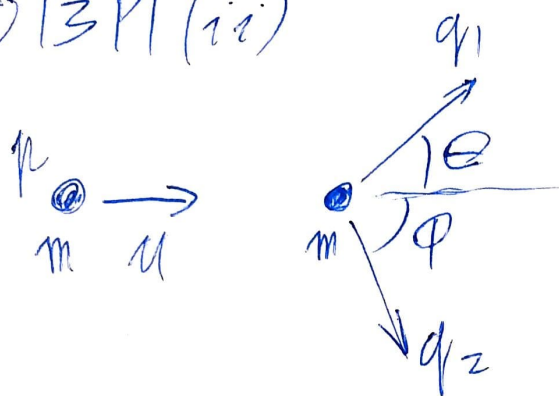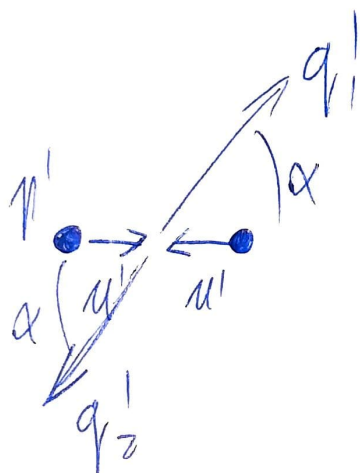

LAB'S FRAME

LORENTZ BOOST u

ZERO MOMENTUM FRAME

$$\gamma_u mc^2 - mc^2 = KE = T_o$$

$$T_o = (\gamma_u - 1)mc^2$$

ZMF: WHERE $p'^{TOTAL}_x = 0$ INITIALLY

INITIALLY, IN LAB'S FRAME:

$$p_{TOTAL} = \begin{pmatrix} E_{TOTAL}/c \\ p_{x\,TOTAL} \\ p_{y\,TOTAL} \end{pmatrix}$$

ZMF:

$$p'_{TOTAL} = \begin{pmatrix} \gamma & -\beta\gamma & 0 \\ -\beta\gamma & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_{TOTAL}/c \\ p_{x\,TOTAL} \\ p_{y\,TOTAL} \end{pmatrix} = \begin{pmatrix} \cdots \\ \frac{E_{TOT}}{c}(-\beta\gamma) + \gamma p^{TOT}_x \\ \cdots \end{pmatrix}$$

$$\frac{E_{TOT}}{c}(-\beta \gamma) + \gamma p_x^{TOT} = 0$$

$$\frac{\gamma_u mc^2 + mc^2}{c}(-\beta \gamma) + \gamma \underbrace{p_x^{TOT}}_{\downarrow} = 0$$

$$(\gamma_u + 1)\, mc\, (-\beta) + \overbrace{\gamma m u}^{\gamma_u} = 0$$

$$B = \frac{\gamma_u u}{(\gamma_u + 1)\, c}$$

$$u' = \frac{\gamma_u}{\gamma_u + 1}\, u$$

(SPEED OF PARTICLE (WHICH WAS STATIONARY IN LAB) IN ZMF = SPEED OF ZMF IN LAB'S FRAME, BECAUSE TO GO BACK FROM ZMF TO LAB WE NEED INVERSE BOOST LAB→ZMF OR JUST INVERSE BOOST BY SPEED OF CONCERNED PARTICLE TO MAKE IT STATIONARY ⟹ ZMF SPEED IN LAB'S FRAME = SPEED OF PARTICLE IN ZMF)

ZMF: ENERGY IS CONSERVED
       MOMENTUM IS CONSERVED.

$$q_1''^\mu = \left(\gamma_{u'}\, mc,\; \gamma_{u'}\, m\, u'\cos\alpha,\; \gamma_{u'}\, m\, u'\sin\alpha,\; 0 \right)$$

$$q_1^\mu = \text{INVERSE LORENTZ BOOST } q_1''^\mu$$

$$= \begin{pmatrix} \gamma_{u'} & \gamma_{u'}\beta_{u'} & & \\ \gamma\beta_{u'} & \gamma_{u'} & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \gamma_{u'}\, m \begin{pmatrix} c \\ u'\cos\alpha \\ u'\sin\alpha \\ 0 \end{pmatrix}$$

$$= \gamma_{u'}\, m \left(\gamma_{u'} c + \gamma_{u'}\beta_{u'}\, u'\cos\alpha, \right.$$

$$\gamma_{u'}\beta_{u'}\, c + \gamma_{u'}\, u'\cos\alpha,$$

$$u'\sin\alpha,$$

$$\left. 0 \right)$$

x COMPONENT:

$$\gamma_{u'}\, m \left(\gamma_{u'}\beta_{u'}\, c + \gamma_{u'}\, u'\cos\alpha \right) =$$

$$= \gamma_{u'}^2\, m \left(u' + u'\cos\alpha \right) = \gamma_{u'}^2\, m\, u' \left(1 + \cos\alpha \right)$$

MOMENTUM IN LAB'S FRAME =

$$\sqrt{q^{\mu\,2}_{1\,x} + q^{\mu\,2}_{1\,y}} =$$

$$= \partial^2_{\mu'} m \mu' \sqrt{\left(1 + \cos\alpha\right)^2 + \sin^2\alpha}$$

$$= \partial^2_{\mu'} m \mu' \sqrt{\cos^2\alpha + 2\cos\alpha + 1 + \sin^2\alpha}$$

$$= \gamma^2_{\mu'} m \mu' \sqrt{2\cos\alpha + 2}$$

$$= \partial^2_{\mu'} m \mu' \sqrt{2} \sqrt{\cos\alpha + 1}$$

ANGLE:

$$\Theta = ARCTAN \frac{\mu' \sin\alpha}{\mu'\left(1 + \cos\alpha\right)}$$

$$= ARCTAN \frac{\sin\alpha}{1 + \cos\alpha}$$

$$= ARCTAN \frac{2 \sin\frac{\alpha}{2} \cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = ARCTAN\, TAN\,\frac{\alpha}{2} = \frac{\alpha}{2}$$

SANITY CHECK: $\ominus = \frac{\alpha}{2}$

THIS DOES NOT RECOVER NEWTONIAN DYNAMICS AT SMALL SPEEDS SO THIS IS WRONG.