Cosmology L I

Books

The Physics of Cosmology
- post grad level
- written by lecturer
- uploaded to moodle

Principles of Physical Cosmology

General Relativity: An Introduction for Physicists

Metric signature: $+ - - -$

$c = 1 \quad G = 1 \quad \hbar = 1$

Exams

Past papers offer best guide on what's examined

Planck length: $l_p = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}$ m

Planck mass: $m_p = \sqrt{\frac{\hbar c}{G}} \sim 10^{-8}$ kg

Planck time: $t_p = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-44}$ sec

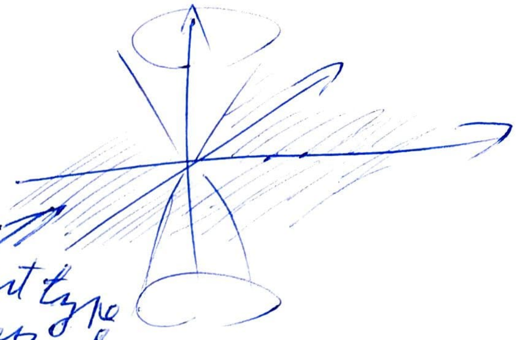
Natural units: universe is much bigger, heavier, older

Aim: why universe is so big, old.

Eddington suspected there's another constant in physics, which ~~is~~ sets the scale of universe.

What is science, what is not.

Does it make sense to talk about anything not in our past lightcone.



present type hypersurface.

Hubble Deep Field
Ultra

earlier time, higher redshift

with photons, cannot see beyond $z=1000$
it becomes hot, ionized, so optically thick

"technology horizons": limitations imposed by current telescopes, etc.

can study CMB for $z \sim 1000$
With neutrinos or grav. waves, can go even beyond that
Is something like multiverses science?
If we impose symmetries, explore models, then we're doing science, even if we predict not observable things.

Intrinsic & extrinsic curvature

cylinder

metric:

$ds^2 = r^2 d\theta^2 + dz^2$
separation of local points on cylinder

reexpress: $dx = r\,d\theta$

$ds^2 = dx^2 + dz^2$
returned expression into metric of flat sheet.

curvature is defined locally

The 3D embedding makes it looked like curved.

sphere

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

Riemann curvature tensor: tells us if metric is curved intrinsically.

Friedmann Robertson Walker metric

3-sphere embedded in 4-D Euclidean space

$$ds^2 = dx^2 + dy^2 + dz^2 + dw^2$$

3-sphere defined by:

$$x^2 + y^2 + z^2 + w^2 = a^2$$

Hence:

$$2x\,dx + 2y\,dy + 2z\,dz + 2w\,dw = 0$$

express dw, use euclidean metric to substitute:

$$ds^2 = dx^2 + dy^2 + dz^2 + \frac{(x\,dx + y\,dy + z\,dz)^2}{a^2 - (x^2 + y^2 + z^2)}$$

Transform to SPC:

$$x = r \sin\theta \sin\phi$$

$$y = r \sin\theta \cos\phi$$

$$z = r \cos\theta$$

$$ds^2 = \frac{dr^2}{1 - r^2/a^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$K = 1/a^2$ curvature is constant.

curvature tensor: $R_{ijkl} = K(g_{ik} g_{jl} - g_{il} g_{jk})$ (?)

Ricci tensor : $R_{jl} = g^{ik} R_{ijkl} = 2K g_{jl}$

Ricci scalar : $R = R^\sigma_\sigma = 6K$

$K>0$ $\quad K=0$ $\quad K<0$

closed $\quad$ flat $\quad$ open

$>180°$ $\quad 180°$ $\quad <180°$ triangles angles add

$$V = 2\int \sqrt{g}\, dr\, d\theta\, d\varphi = 2\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \frac{r^2 \sin\theta}{\sqrt{1-\frac{r^2}{a^2}}}\, dr\, d\theta\, d\varphi = 2\pi^2 a^3$$

TPOC 2.1

do it in 2D,
that'll explain why 2 is there.

full FRW:

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right]$$

Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

assume FRW, solve for univ. containing homogeneous isotropic perfect fluid:

energy momentum tensor

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu - P g^{\mu\nu}$$

matter content:

$$\rho(t) \quad P(t) \quad u^\mu = \frac{dx^\mu}{d\tau} \quad u^0 = u_0 = 1 \quad u^i = u_i = 0$$

$$ds^2 = c^2 dt^2 - R^2 \tilde{g}_{ij} dx^i dx^j$$

Have metric, energy momentum tensor.

$\Rightarrow$ ~~derive~~ derive Friedmann ~~eqn~~ equations

Appendix 2A of TPoC. $\leftarrow$ go through this (!)

get: two dynamical equations.

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \qquad \text{FR 1}$$

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \qquad \text{FR 2}$$

energy conservation requires:

$$\frac{d(\rho R^3)}{dR} = -3PR^2$$

non relativistic
non interacting

if $P=0$, this equation requires $\rho \propto R^{-3}$

if $\Lambda=0$, the second FR requires

$$\dot{R}^2 = \frac{8\pi G}{3}\rho_0 \frac{R_0^3}{R} - K$$

ie

$$\dot{R}^2 = \frac{A}{R} - K$$

while $A \equiv$ CONSTANT

if $K=0$, $\dot{R} \to 0$ as $R \to \infty$
initially expanding, expansion going to zero universe.

if $K<0$, $\dot{R} \to$ CONSTANT as $R \to \infty$

if $K>0$, $\dot{R}=0$ at max radius $R_m = A/K$
then contracts again.

$$K = \left[\frac{8\pi G}{3}\rho_0 - H_0^2\right] R_0^2$$

spatially flat universe critical density: $\rho_c = \frac{3H_0^2}{8\pi G}$

$\sim 10^{-26}$

matter density parameter: $\Omega_m = \frac{\rho_m}{\rho_c}$