

TOPICS IN ASTROPHYSICS LECTURE I

for a quantity, characteristic

timescale: $\tau = Q / |\dot{Q}|$

lengthscale: $l = Q / |Q'|$

exponential form: $Q = Q_0 \exp(-\frac{t}{T}) \Rightarrow \tau = T$

power law: $Q = Q_0 (\frac{T}{t})$

no characteristic ~~time~~ ~~length~~
timescale for power law!

"self-similar":
time/length scales are themselves
function of time/space.

straight line on log vs linear plot: exponential
"it knows about some characteristic e-folding"
(unsolved problems)

• Dynamical timescale in presence of gravitating object

TRICK:

\vec{R} MULTIPLICATION

$$\ddot{\vec{R}} = -\frac{GM}{R^2} \Rightarrow \vec{R} \ddot{\vec{R}} = -\frac{GM \vec{R}}{R^2}$$

$M \bullet \xrightarrow{R(t)}$
 $R(0) = R_0$ $t_{\text{freefall?}}$

$$\text{i.e. } \frac{d}{dt} \left[\frac{1}{2} \dot{\vec{R}}^2 - \frac{GM}{R} \right] = 0$$

$$\frac{1}{2} \dot{R}^2 - \frac{GM}{R} = C$$

$$\frac{1}{2} \dot{R}^2 = \frac{GM}{R} + C$$

~~$$\frac{dR}{dt} = \sqrt{\frac{2GM}{R} + C}$$~~

$$\frac{dR}{dt} = \sqrt{2GM} \sqrt{\frac{1}{R} + C}$$

We want to fall inwards, let's put a \ominus sign in:

$$\frac{dR}{dt} = -\sqrt{2GM} \left(\sqrt{\frac{1}{R} - \frac{1}{R_0}} \right)$$

C is rewritten so that $\left. \frac{dR}{dt} \right|_{t=0} = 0$ as required by BC.

Unexpectedly bad integral.

Substitution: $R = R_0 \sin^2 \Theta$

$\Theta: \frac{\pi}{2} \rightarrow 0$ (as we fall in, R goes \downarrow)

~~$$R = -\sqrt{2GM} \sqrt{\frac{1}{R} - \frac{1}{R_0}}$$~~

$$t_{ff} = \int dt = \int \frac{1}{-\sqrt{2GM}} \frac{1}{\sqrt{\frac{1}{R} - \frac{1}{R_0}}} dR$$

$$= -\frac{1}{\sqrt{2GM}} \int_{\Theta=\frac{\pi}{2}}^0 \left(\frac{1}{R_0 \sin^2 \Theta} - \frac{1}{R_0} \right)^{-\frac{1}{2}} R_0 2 \sin \Theta \cos \Theta d\Theta$$

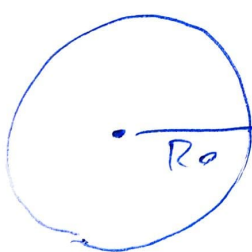
$$= -\frac{1}{\sqrt{2GM}} \sqrt{R_0} R_0 2 \int_{\theta=\frac{\pi}{2}}^0 \underbrace{\left(\frac{1}{\sin^2 \theta} - 1\right)}_{\cot^2 \theta}^{\frac{1}{2}} \sin \theta \cos \theta d\theta$$

$$= -\sqrt{\frac{2R_0^3}{GM}} \int_{\theta=\frac{\pi}{2}}^0 \tan \theta \sin \theta \cos \theta d\theta$$

$$= -\sqrt{\frac{2R_0^3}{GM}} \int_{\theta=\frac{\pi}{2}}^0 \sin^2 \theta d\theta$$

$$= -\sqrt{\frac{2R_0^3}{GM}} \left(-\frac{\pi}{4}\right) = \frac{\pi}{2} \sqrt{\frac{R_0^3}{2GM}} \text{ free-fall time.}$$

• Let's now have a uniform density on path.



$$M_{\text{enc}}(R_0) = M \quad M_{\text{enc}}(R) = \left(\frac{R}{R_0}\right)^3 M$$

$$\ddot{R} = -\frac{GM}{R^2} \left(\frac{R}{R_0}\right)^3 = -\frac{GM}{R_0^3} R$$

recognize as SHM.

To get to 0: quarter of an oscillation

$$t_{\text{ff}} = \frac{1}{4} T_{\text{osc}} = \frac{1}{4} \frac{2\pi}{\omega} = \frac{1}{4} \frac{2\pi}{\left(\frac{GM}{R_0^3}\right)^{\frac{1}{2}}} = \frac{\pi}{2} \sqrt{\frac{R_0^3}{GM}}$$

Root $\frac{1}{2}$ more than prev: less mass enclosed, if smaller ~~effect~~ ~~on~~ acceleration.

Free fall velocity at R_0 if particle is falling in from infinity

$$\frac{GMm}{r} = \frac{1}{2} m v^2$$

$$\Downarrow$$

$$v_{ff}(R_0) = \sqrt{\frac{2GM}{R_0}}$$

characteristic time scale:

$$\tau = \frac{R_0}{v_{ff}} = \frac{R_0}{\sqrt{\frac{2GM}{R_0}}} = \sqrt{\frac{R_0^3}{2GM}}$$

• Sound propagation

ideal gas equation of state: $pV = NRT$

$\frac{1}{V}$ number of moles $\rightarrow 8.3 \frac{\text{J}}{\text{mol K}}$

$$p = \frac{N}{V} RT$$

$$p = \frac{\text{weight of a mole in kg}}{\text{relative molecular weight}} RT$$

$\mu \cdot 10^3, \text{ R.M.M.}$

$$P = \underbrace{S \frac{1000 R}{\nu}}_L T$$

$$L \equiv \frac{P_*}{\nu} = 8300 \frac{\text{J}}{\text{eV K}}$$

Alfven waves

waves in magnetized media.
transverse waves are made possible.

$$\nu_{\text{ALFVEN}} \sim \sqrt{\frac{B^2}{S}}$$

light crossing time

also energy transfer time for radiation
(in optically thin media)

quasars: highly variable, though output comparable to entire galaxies
lets take hours for variability length scale
assuming ~~we~~ we don't know about SMM, what can they tell?

$$0.1 \text{ hours} \Rightarrow c \delta t \sim 10^{12} \text{ m}$$

Can we put all stars in a galaxy this close?

$$R_{\text{gal}}^3 = N_* \underbrace{R_*^3}_{\text{star}} = N_* R_*^3 \Rightarrow R_{\text{gal}} = N_*^{1/3} R_*$$

$$\frac{1}{10^{10}} \frac{1}{10^8 \text{ m}}$$

this is already
too big, if all
stars are essentially
"touching" \Rightarrow ACR not from
stellar P. 2.1

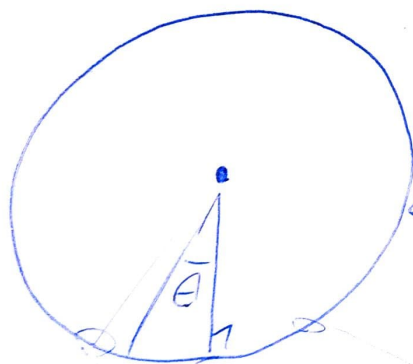
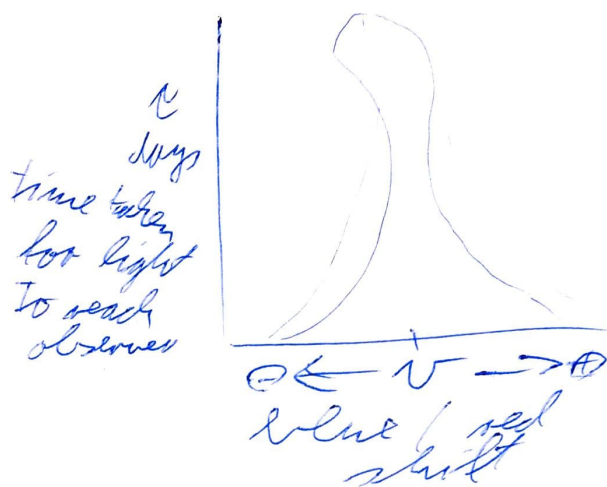
We also have limit on BH.

event horizon cannot be bigger than order 10^{12} m

$$\frac{GM_{\text{BH}}}{c^2} \leq 10^{12} \text{ m} \Rightarrow M_{\text{BH}} \leq 10^9 M_{\odot}$$

Variability data isn't violating black hole hypothesis.

"light echo" technique to measure BH in QSOs



← ring of clouds (producing line emission)

← shortest line path

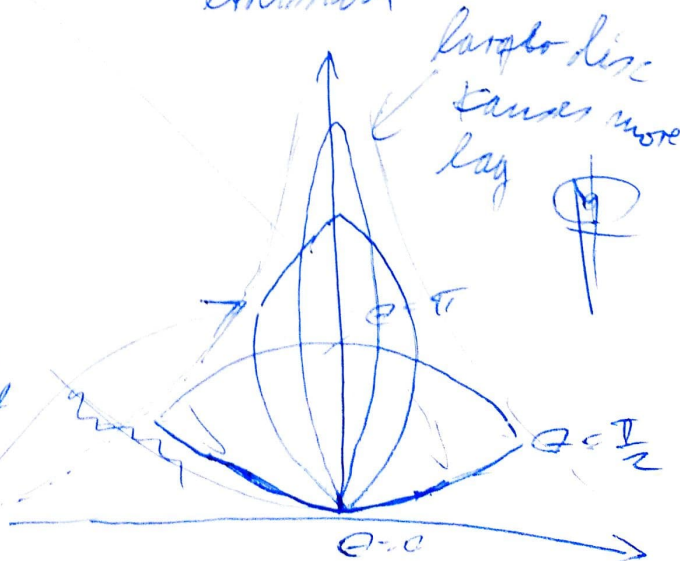
increasing θ
 \Rightarrow increasing time lag
 of arrival of line
 emission

✓ OBSERVER

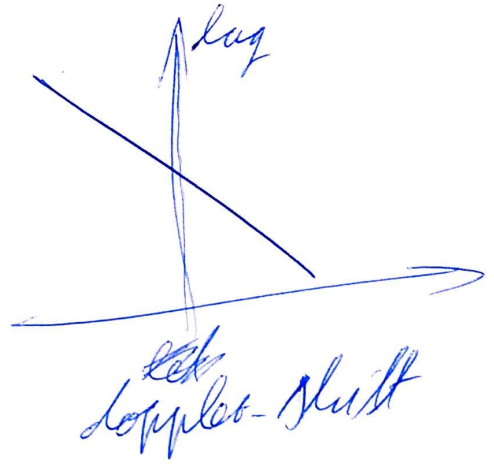
lets have rings of different size.

$$v = \sqrt{\frac{GM}{R}} \Rightarrow$$

smaller red & blueshift for larger discs



additional effect: ~~the~~ spherical infall:



add these two effects:



you are essentially measuring distance of clouds from BH.

Get v from doppler shift.

Use $v = \sqrt{\frac{GM}{R}}$, get mass of black hole.