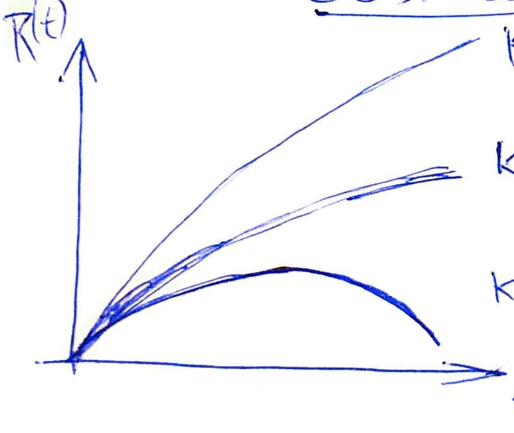
Cosmology L II



$K<0$ open $\Omega_m<1$

$K=0$ flat $\Omega_m=1$

$K>0$ closed $\Omega_m>1$

surprising: dynamical beh. arises & curvature link.

Studying behaviour tells us which cosmology is valid.

Can use Newtonian view as well
However, cannot give dynamical beh. & geometry

Exact sols in TPoC 2.2

FRE: $$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

CASES:

(a) $P=0$ (i.e. universe is filled with dust)

$\Lambda=0$ ~~i.e. flat~~

$K=0$ i.e. flat

$\rho \propto R^{-3}$ for dust, so: $\dot{R}^2 \propto R^{-1} \Rightarrow \boxed{R \propto t^{\frac{2}{3}}}$

Einstein-de Sitter cosmology

(b) $\Lambda=0$ $K=0$

$P=\frac{1}{3}\rho c^2$ (radiation dominated universe)

energy conservation:

$$\frac{d(\rho R^3)}{dR} = -3PR^2$$

$$\Rightarrow \rho \propto R^{-4} \Rightarrow \boxed{R \propto t^{\frac{1}{2}}}$$

↳ early times, curvature can be neglected then.

early time hot big bang model

(c) $P=0$ $K=0$ ΛCDM UNIVERSE

applicable to our universe late times (i.e. now)

$$\frac{R(t)}{R_0} = \left[\frac{\Omega_m}{2(1-\Omega_m)} \left(\cosh\left[\sqrt{3\Lambda}\, t\right] - 1 \right) \right]^{\frac{1}{3}}$$

$t \to \infty$ limit

$$R(t) \propto \exp \sqrt{\frac{\Lambda}{3}}\, t$$

from $R(t)$: $H(t) = \left(\frac{\Lambda}{3}\right)^{\frac{1}{2}}$

(?) $\hookrightarrow$ H is defd: $H(t) = \frac{\dot{R}}{R}$

$H =$ CONSTANT: de Sitter universe pivotal role in Inflationary Cosmology

from FRW:

$$3\frac{\ddot{R}}{R} = -4\pi G(\rho + 3P) + \Lambda$$

Λ dominated universe: $\ddot{R} > 0$: univ is accelerating.

~~thus~~ matter dominated universe: $\ddot{R} < 0$, can cause recollapse

even if $\Lambda = 0$, if $\rho + 3P$ negative, still can accelerate

if: $P < -\frac{1}{3}\rho c^2$ ie requiring negative pressure.

Problem: GR: local theory
does not determine global topology

i.e. a flat geometry w/ finite surface area: torus

main point:
we require more extensive physical model than GR.

Empirical issue if a topology applies.

Cosmological Redshift

FRW metric: $d\tau^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right]$

~~photons~~ photons travel along null paths:

2 observers: A & B
separated by coord dist. r_{AB}

A emit: t_1, B receive t_0

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_{AB}} \frac{dr}{\sqrt{1-kr^2}}$$

Physical dist: $R^2(t) \frac{dr^2}{1-kr^2}$

A emits another one @ $t_1 + \delta t_1$, received by B @ $t_0 + \delta t_0$
coord dist remains fixed: r_{AB} (it is comoving)
Arrive at:

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{R(t)} = \int_0^{r_{AB}} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_0} \frac{dt}{R(t)}$$

$$\Rightarrow \frac{\delta t_0}{R(t_0)} = \frac{\delta t_1}{R(t_1)}$$

expanding universe, so:
$R(t_0) > R(t_1)$ $\delta t_0 > \delta t_1$

light emitted ν_e freq. received lower freq by B:

(remark ab: ratio of freqs: inverse of ratio of time delays)

$$\frac{\nu_0}{\nu_e} = \frac{\delta t_1}{\delta t_0} = \frac{R(t_1)}{R(t_0)}$$

light redshifted to longer wavelengths:

$$\boxed{\underbrace{1+z}_{\text{this is the redshift.}} = \frac{\lambda_0}{\lambda_e} = \frac{R(t_0)}{R(t_1)}}$$

Now what: ~~look~~ at QSO emission lines, ie $L_y\,\alpha$

deduce redshift of quasar, ie $z\sim 7$

it tells us ratio of scale factors!

universe was factor of ~ 7 smaller back then.

~~Birkhoff's theorem~~

Birkhoff's theorem

metric inside empty cavity: Minkowski metric

$\Rightarrow$ we can apply Newton as long as cavity is small enough.

"Newtonian theory in Minkowski background"

Let $\underline{r}$ be physical coord, $\underline{x}$: comoving coord.

$\underline{r} = R(t)\,\underline{x}$

ie const scale, independent of time.

$\dot{\underline{r}} = \dot{R}(t)\,\underline{x} = \frac{\dot{R}}{R}\,\underline{r} = H(t)\,\underline{r}$

Neighbouring points will then receding from each other w/ speed proportional to distance.

Note that more gen:

$$\underline{v} = H\underline{r} + R(t)\dot{\underline{x}} = H\underline{r} + \underline{v}_p$$

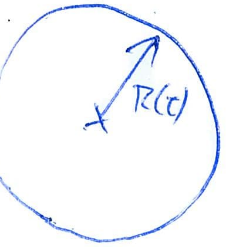
$\underline{v}_p$: peculiar velocity, ie departure from Hubble law

if homogeneous, $\underline{v}_p$ must be ~~zero~~.

It quantifies inhomogeneities in our univ.

Homog. isotrop. case

$$\nabla^2 \Phi = 4\pi G \rho$$



neglect everything outside

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2}$$

integrate that: E conserv.

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = \text{CONST.}$$

$$M = \frac{4}{3}\pi R^3 \rho \implies \rho \propto R^{-3}$$

FRIEDMANNS for $P=0$, $k=0$:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\rho \qquad \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho$$

This was Newtonian.

Point:

Newton is valid in GR scales: $R \ll ct$
(scales smaller than Hubble radius)

Full field eq. needed for scales $\sim ct$

Can use Newton grav. & eq. of motion because Minkowski background.

Newtonian perturbation is much simpler than GR.

Disadvantages:

- no link between dynamical evolution & spatial curvature
- no description of gravitational effects of pressure
- cannot describe ~~grav.~~ perturbations on scale $R \gtrsim ct$
- ~~cannot account~~
- redshift is caused by expansion of Univ., not Doppler-shift.

FRW:

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right]$$

typical galaxy: 10^6 density contrast
matter dominated
doesn't "participate" in the expansion

wavelength of a photon emitted by H: that defines a standard length

Expansion is relative to this standard length

$\Rightarrow$ redshift represents expansion of universe.