2013P1(i)

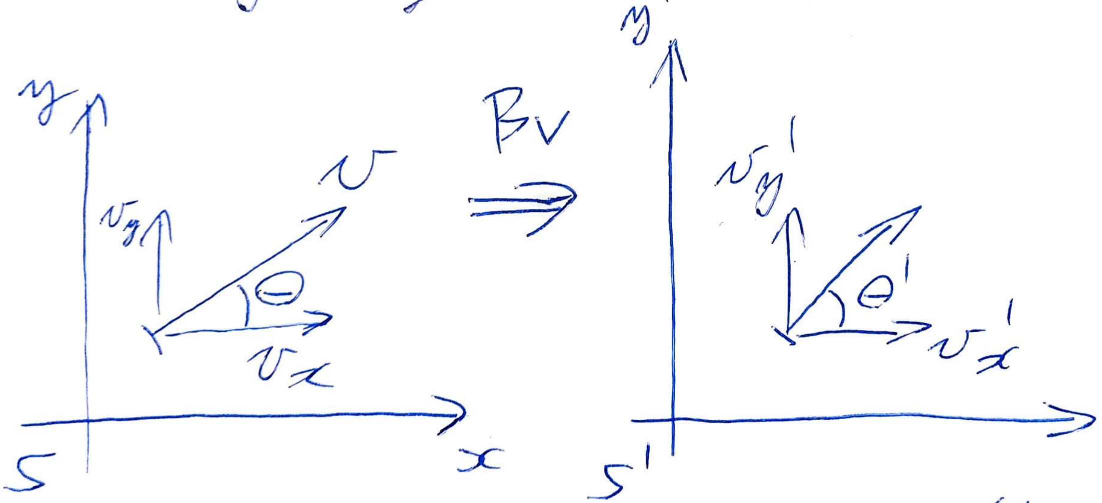
(i)

$$\begin{pmatrix} c\,dt' \\ dx' \\ dy' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 \\ -\beta\gamma & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\,dt \\ dx \\ dy \end{pmatrix}$$

$$\Rightarrow \quad c\,dt' = \gamma c\,dt - \beta\gamma\,dx$$
$$= \gamma(c\,dt - \beta\,dx)$$

$$\Rightarrow \quad dx' = -\beta\gamma c\,dt + \gamma\,dx = \gamma(dx - \beta c\,dt)$$

$$\Rightarrow \quad dy' = dy$$



$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

$$v_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - \beta c\,dt)}{\gamma(c\,dt - \beta\,dx)\frac{1}{c}}$$

$$= c \cdot \frac{\frac{dx}{dt} - \beta c}{c - \beta\frac{dx}{dt}} = \frac{v_x - \beta c}{c - \beta v_x} \cdot c$$

$$v_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma(c\,dt - \beta dx)\frac{1}{c}}$$

$$= \frac{dy/dt \qquad c}{\gamma\left(c - \beta\frac{dx}{dt}\right)} = \frac{v_y c}{\gamma(c - \beta v_x)}$$

$$\theta' = \text{ARCTAN}\,\frac{v_y}{v_x}$$

$$= \text{ARCTAN}\,\frac{\frac{v_y \cancel{c}}{\cancel{\gamma(c-\beta v_x)}}}{\frac{v_x - \beta c}{\cancel{c - \beta v_x}}\cancel{c}} = \text{ARCTAN}\,\frac{v_y}{\gamma(v_x - \beta c)}$$

$$= \text{ARCTAN}\,\frac{v\,\text{SIN}\,\theta}{\gamma(v\cos\theta - \beta c)}$$

$$= \text{ARCTAN}\,\frac{v\,\text{SIN}\,\theta}{\gamma_V(v\cos\theta - V)}$$

PHOTON CASE:

$$\theta'\Big|_{v=c} = \text{ARCTAN}\,\frac{c\,\text{SIN}\,\theta}{\gamma_V(c\cos\theta - V)}$$

IF $V \ll C$:

$$\gamma_V = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} = \left(1-\frac{V^2}{c^2}\right)^{-\frac{1}{2}} = 1+\frac{1}{2}\frac{V^2}{c^2}$$

$$\Theta'\Big|_{\substack{v=c \\ V\ll C}} = \text{ARCTAN} \frac{C \sin\Theta}{\left(1+\frac{1}{2}\frac{V^2}{c^2}\right)\left(C\cos\Theta - V\right)}$$

$$= \text{ARCTAN} \frac{C \sin\Theta}{C\cos\Theta - V + O\left(\frac{V^2}{c^2}\right)}$$

$$\simeq \text{ARCTAN} \frac{C \sin\Theta}{C\cos\Theta - V}$$