

(i)

EQUATION OF HYDROSTATIC EQUILIBRIUM:

$$\frac{1}{\rho} \nabla p = -\nabla \psi$$

"PERFECT STATIC GAS"  $\Rightarrow$  HYDROSTAT. EQM. EQUATION APPLIES

CONSTANT  $T = 300K \Rightarrow$  ISOTHERMAL

~~$pV = nRT \Rightarrow p = \frac{n}{V} RT$~~

$$pV = nRT$$

$$= \frac{m}{M} RT \Rightarrow p = \frac{m}{M} \frac{1}{V} RT$$

$$= \frac{S}{M} RT = CS$$

[BUNCH OF CONSTANTS,  
INCLUDING T.]

$$\frac{1}{\rho} \nabla p = \frac{1}{\rho} \nabla (CS) = C \frac{1}{S} \nabla S = \underline{\underline{q}} = -g \underline{\underline{\hat{z}}}$$

"UNIFORM GRAVITATIONAL FIELD"

LET'S ASSUME THAT ATMOSPHERE IS FAIRLY THIN, NOT REQUIRING  $\nabla S$  TO BE EVALUATED IN SPHERICAL POLARS (IE WE SAY EARTH IS FLAT,  $\nabla = \frac{\partial}{\partial z} \hat{z}$ ).

$$\cancel{C} \frac{1}{S} \frac{\partial}{\partial z} S = -g \cancel{F}$$

$$C \ln S = -gz + K$$

$$S = K \exp\left(-\frac{g z}{C}\right)$$

WE WANT:  $S|_{z=0} = S_0 \Rightarrow K = S_0$

PREVIOUSLY WE HAD:

$$C = \frac{S}{M} RT$$

I THINK MORE COMMON NOTATION IS:

$$C = \frac{1}{N} R_* T \quad (\text{NOTING THAT } 1000N=M \\ R_* = 1000R)$$

WE END UP WITH:

$$\underline{S = S_0 \exp\left(-\frac{Ng}{R_* T} z\right)}$$

Fluid approximation breaks down  
when we have order unity particles per  
unit volume (how big this unit volume  
should be, though?)

Let's say, when  $S = 1 \text{ m}^{-3}$ .

$$\underline{\exp\left(-\frac{30 \cdot 10}{8300 \cdot 300} z\right) = \frac{1}{3 \cdot 10^{25}}} \Rightarrow z = 3.3 \cdot 10^5 \text{ m} \\ \underline{\rightarrow 330 \text{ km}}$$

Constant grav. breaks down when

$$g_{\text{TRUE}} = \text{the } 0.9 \text{ g assumed}$$

(I'm just making an estimation here.)

$$\left(\frac{R}{R_0}\right)^2 = \frac{1}{0.9} \Rightarrow R = 1.05 R_0 \\ = 6720 \text{ km} \\ \boxed{R_0 = 6400 \text{ km}}$$

$$\text{Height} = R - R_0$$

$$= 6720 - 6400 \approx \underline{320 \text{ km}}$$

Similar number as we got previously.

So similar in fact, that is probably has to do with us being lucky with the estimations, rather than physical reasons.

(ii)

$$\text{Ram pressure} = 3u^2$$

$$u: \text{speed of Earth} = \frac{2\pi \cdot \text{sun-earth distance}}{365.24 \cdot 60 \cdot 60} = 3 \cdot 10^4 \frac{\text{m}}{\text{s}}$$

$$3u^2 \approx 10^5 \text{ Pa} \Rightarrow S = \frac{10^5}{u^2} \approx 10^{-4} \frac{\text{kg}}{\text{m}^3}$$



$$G \cdot 10^{22} \frac{\text{N}}{\text{kg}^2 \cdot \text{m}^3}$$