DSG 4.10 | I

START WITH $\overline{v_r^2}$:

$$\overline{v_r^2} = \int f \, v_r^2 \, d\text{VELOCITY SPACE}$$

$$= \int f_0(\mathcal{E}) \frac{1}{L} v_r^2 \, dV$$

USE: $L = \sqrt{v_\theta^2 + v_\phi^2}\, r$

$v_r = v \cos\alpha$

$dV = v^2 \sin\alpha \, dv \, d\alpha \, d\beta$

$\mathcal{E} = \Psi - \frac{1}{2}(v_r^2 + v_\theta^2 + v_\phi^2)$

$$= \int_{v=0}^{v=\infty} \int_{\alpha=0}^{\alpha=\pi} \int_{\beta=0}^{\beta=2\pi} f\left(\Psi - \frac{1}{2}(v_r^2 + v_\theta^2 + v_\phi^2)\right) \frac{v^2 \cos^2\alpha}{\sqrt{v_\theta^2 + v_\phi^2}\, r} v^2 \sin\alpha \, dv \, d\alpha \, d\beta$$

CONSIDER:

$$\sqrt{v_\theta^2 + v_\phi^2} = \sqrt{v^2 \sin^2\alpha \cos^2\beta + v^2 \sin^2\alpha \sin^2\beta}$$

$$= v \sin\alpha$$

REWRITE:

$$\overline{v_r^2} = \iiint f\left(\Psi - \frac{1}{2}v^2\right) \frac{v^2 \cos^2\alpha}{v \sin\alpha \, r} v^2 \sin\alpha \, dv \, d\alpha \, d\beta$$

$$= \iiint f\left(\Psi - \frac{1}{2}v^2\right) v^3 \cos^2\alpha \, dv \, d\alpha \, d\beta$$

NOTING THAT Ψ IS α & β INDEPENDENT,

REWRITE:

SDSG 4.10. $\boxed{\text{(II)}}$

$$= \underbrace{\int_{v=0}^{\infty} f\left(4-\frac{1}{2}v^2\right) v^3 dv}_{C} \int_{\cancel{\alpha}=0}^{\alpha=\pi} \int_{\beta=0}^{\beta=2\pi} \cos^2\alpha \, d\alpha \, d\beta$$

$$= C \, 2\pi \frac{\pi}{2} = \boxed{C\pi^2}$$

CONSIDER NOW $\overline{v_\Theta^2}$:

$$\overline{v_\Theta^2} = \int f \, v_\Theta^2 \, d\,\text{VELOCITYSPACE}$$

$$= \int_{v=0}^{\infty} \int_{\alpha=0}^{\pi} \int_{\beta=0}^{2\pi} f\left(4-\frac{1}{2}v^2\right) \overbrace{\frac{v^2 \sin^2\alpha \cos^2\beta}{\underbrace{v \sin\alpha}_{L}}}^{v_\Theta^2} v^2 \sin\alpha \, dv \, d\alpha \, d\beta$$

$$= \underbrace{\int_0^{\infty} f\left(4-\frac{1}{2}v^2\right) v^3 dv}_{C} \int_{\alpha=0}^{\pi} \sin^2\alpha \, d\alpha \int_{\beta=0}^{2\pi} \cos^2\beta \, d\beta = \boxed{C \frac{\pi}{2} \pi}$$

$\overline{v_\varphi^2}$:

$$\overline{v_\varphi^2} = \iiint f \frac{v^2 \sin^2\alpha \sin^2\beta}{v \sin\alpha} v^2 \sin\alpha \, dv \, d\alpha \, d\beta$$

$$= \underbrace{\int_0^{\infty} f\left(4-\frac{1}{2}v^2\right) v^3 dv}_{C} \int_0^{\pi} \sin^2\alpha \, d\alpha \int_0^{2\pi} \sin^2\beta \, d\beta = \boxed{C \frac{\pi}{2} \pi}$$

DSG
4.10 (III)

COMPARE RESULTS:

$$\frac{\overline{v_r^2}}{\overline{v_\theta^2}} = \frac{c\pi^2}{c\frac{\pi}{2}\pi} = 2$$

$$\frac{\overline{v_r^2}}{\overline{v_\phi^2}} = \frac{c\pi^2}{c\frac{\pi}{2}\pi} = 2$$

CONCLUDE:

$$\overline{v_\theta^2} = \overline{v_\phi^2} = \frac{1}{2}\overline{v_r^2}$$

AS REQUIRED.