DSG 3.1 (I)

I DON'T KNOW HOW TO INTERPRET SURFACE DENSITY FOR NON-FLAT DISTRIBUTIONS, IE A SPHERICAL ONE.

SDSG
3.1 (II)

START FROM: $\Sigma(R) = 2\int_R^\infty \frac{\rho(r)\, r\, dr}{\sqrt{r^2 - R^2}}$

$x = \frac{1}{R^2} \qquad y = \frac{1}{r^2} \rightarrow r = \frac{1}{\sqrt{y}} \rightarrow dr = -\frac{1}{2} y^{\left(-\frac{3}{2}\right)} dy$

$\hookrightarrow R^2 = x^{-1}; \quad R = x^{-\frac{1}{2}}$

REWRITE:

$$\Sigma\left(x^{-\frac{1}{2}}\right) = 2\int_{x^{-\frac{1}{2}}}^{0} \frac{\rho\left(y^{-\frac{1}{2}}\right)\left(-\frac{1}{2}\right) y^{-\frac{3}{2}}\, dy}{\sqrt{y^{-1} - x^{-1}}}$$

$$= \int_0^{x^{-\frac{1}{2}}} \frac{\rho(y)\, dy}{y^2 \sqrt{y^{-1} - x^{-1}}}$$

CHANGE VARIABLES AGAIN:

$x^{-\frac{1}{2}} \Rightarrow x'$

$$= \int_0^{x'} \frac{\rho(y)}{y^2} \frac{1}{\sqrt{y^{-1} - x'^2}} dy$$

NOT WHAT WE AIM FOR.

SDSG 3.2

a/ $R_0 = 10$ kpc $\quad A = -B = 15 \dfrac{km}{s \cdot kpc}$

(I)

$$A - B = \frac{v_0}{R_0} \Rightarrow v_0 = R_0(A-B)$$

$$= 10 \cdot (15 - -15) = \underline{300} \ \frac{km}{s}$$

HANDOUT 4 SLIDE 35: $v^2(R) = \dfrac{GM(R)}{R}$ FOR MOST Σs.

$$\Rightarrow M(R) = \frac{1}{G} R v^2(R)$$

$$= \frac{1}{6.67 \cdot 10^{-11}} \cdot 10 \cdot 10^3 \cdot 3.09 \cdot 10^{16} \cdot (300000)^2$$

$$\cong 4.2 \cdot 10^{41} \text{ kg}$$

$$\xrightarrow{:(2 \cdot 10^{30})} \sim \underline{2 \cdot 10^{11} M_\odot}$$

b/

1 GALACTIC YR: $\dfrac{2\pi R_0}{v_0} = \dfrac{2\pi \cancel{R_0}}{\cancel{R_0}(A-B)} = \dfrac{2\pi}{A-B}$

$$= \frac{2\pi}{15 - -15} = \frac{2}{30}\pi \ \frac{1 \text{ kpc}}{1 \text{ km}} \text{ sec} = \frac{2}{30}\pi \ \frac{3.09 \cdot 10^{16}}{1} \text{ sec}$$

$$= \underline{2.05 \cdot 10^{8} \text{ YRS}}$$

SUN IS ~4 BILLION YRS OLD SO ~20 GALACTIC YEAR OLD

SDSG
3.2(II)

c/
$$K^2 = -4B\Omega = -4B(A-B)$$
$$= -4(-15)(15 - -15) = 1.8\cdot 10^3 \frac{km^2}{s^2\,kpc^2}$$

$$\cancel{\frac{1\,kpc^2}{1\,km^2} = \frac{9.57\cdot 10^{38}}{10^6}}$$
$$\cancel{1.72\cdot 10^{36}\frac{1}{s^2}}$$
$$\cancel{\sqrt{\ } \rightarrow 1.31\cdot 10^{18}\frac{1}{s}}$$

$$1.8\cdot 10^3 \frac{km^2}{s^2\,kpc^2}$$
$$= 1.8\cdot 10^3\cdot\left(10^3\right)^2 \frac{m^2}{s^2\,kpc^2}$$
$$= 1.8\cdot 10^3\cdot\left(10^3\right)^2\cdot\left(3.09\cdot 10^{19}\right)^{-2} \frac{m^2}{s^2\,m^2}$$
$$= 1.88\cdot 10^{-3}\frac{1}{s^2} \longrightarrow 1.37\cdot 10^{-15}\frac{1}{s} = K$$
$$\Big\downarrow \cdot 60\cdot 60\cdot 24\cdot 365\cdot 2.05\cdot 10^8$$
$$8.86\frac{1}{GALYR} = K$$

$$\cancel{\frac{2\pi}{8.86}} =$$
$$\frac{8.86}{2\pi} \approx 1.4 \text{ RADIAL OSC. PER ORBIT}$$

SDSG 3.7. MAXWELLIAN DISTRIBUTION:

$$f(x,a) = \sqrt{\frac{2}{\pi}}\,\frac{1}{a^3}\,x^2 \exp\left(-\frac{x^2}{2a^2}\right)$$

[FROM WIKIPEDIA]

a/ $$\int_0^\infty f(v,\sigma)\, v\, dv = \sqrt{\frac{8}{\pi}}\cdot\sigma$$

b/ $$\int_0^\infty f(v,\sigma)\, v^2\, dv = 3\sigma^2 = \overline{v^2}$$

c/ THE $3\sigma^2$ IS ALREADY ONE-DIMENSIONAL. (SO I DON'T GET IT)

d/

e/ $$\frac{\int_{\sqrt{4\cdot\overline{v^2}}}^\infty f(v)\,dv}{\int_0^\infty f(v)\,dv} = 0.00738$$

DONE NUMERICALLY UNLESS WRITTEN OTHERWISE, LINK: BIT.LY/SDSG_3_7B

3.8

- Relaxation timescale:

[Mostly based on Binney & Tremaine]

If we assume a smooth matter distribution in a cluster, a star will have some trajectory which could be calculated. In reality, this smooth distribution is not a good approximation if we want to calculate orbits in the long term. The relaxation timescale expresses the time needed for the non-smoothness of the cluster's matter distribution to take significant effect on the individual stars' velocities.

- Crossing time:

The typical time a star needs to cross the cluster once.

- Comment briefly on why...

As N gets larger, the matter distribution approximation as smooth becomes better. Adding new stars to the system does increase its size, but apparently this is not as a big effect as the star has on smoothing the distribution [I don't think this is a good answer].

- Explain why it is reasonable...

Relaxation time can be thought of a unit of time in these situations. We can assume that every relaxation time, epsilon fraction of the stars are ejected. How much time does it take for all of the stars to be ejected? 1 / epsilon * relaxation time then. [Why not crossing time?]

DSG

3.8 LAST BIT

$M = 10^5 M_\odot$

$$N = \frac{M}{M_{STAR}} = \frac{10^5}{0.7} \cong 1.43 \cdot 10^5$$

$$\Rightarrow t_{e} \sim \left.\frac{N}{10 \ln N}\right|_{N=1.43\cdot 10^5} \cdot t_{CR} \Rightarrow t_e \sim 1200\, t_{CR}$$

$R = 5\,pc$

STAR SPEEDS ON ORDER OF MAG $\sim 100 \frac{km}{s}$

[THIS IS A GUESS]

$$\Rightarrow t_{CR} \sim \frac{2 \cdot 10 \cdot 3 \cdot 10^{13}}{100} = 6 \cdot 10^{12}\, s \sim \underline{2 \cdot 10^5\, YR}$$

$$\Rightarrow t_e \sim 1200 \cdot 2 \cdot 10^5 \sim \underline{2.3 \cdot 10^8\, YR}$$

~~Thus~~

$$\Rightarrow t_{eVAP} = \frac{2.3 \cdot 10^8}{7.4 \cdot 10^{-3}} = \underline{3 \cdot 10^{10}\, YR}$$

It takes longer for the cluster to evaporate than age of the universe. It is quite young though.

3.9 (I)

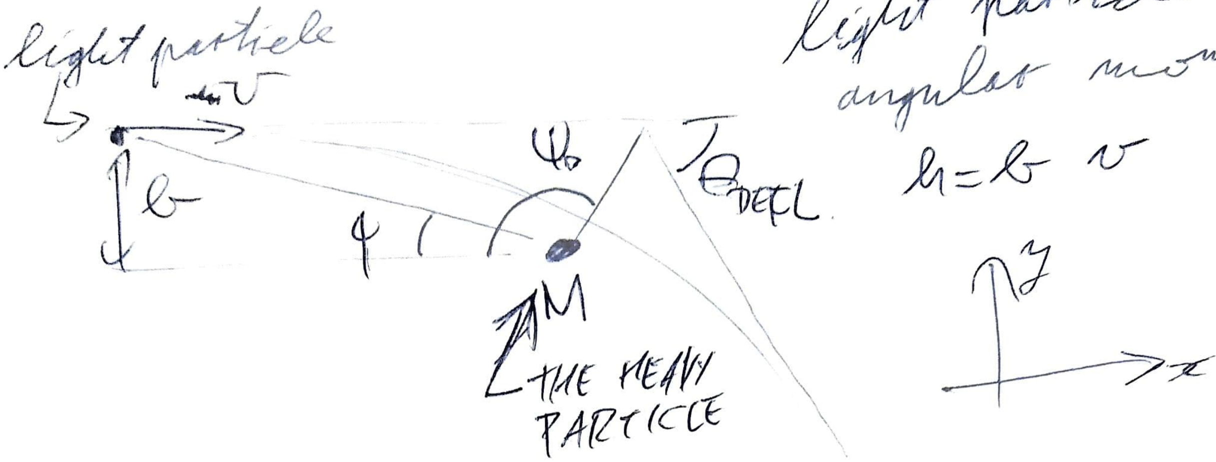
This phenomena is called gravitational focussing.
It arises because as the heavy particle moves in the stationary frame of light particles, ~~these~~ their orbit will be deflected.
Easiest to see this from frame of the heavy particle:

In this region, there'll be particles from all of this region. So ~~that~~ there'll be a density increase.

SPSG 3.9 (II)

[BASED ON HANDOUT]

CONSIDER THIS SETUP:



light particle angular momentum:

$h = b\,v$

Orbit eq. has solution:

$$\frac{1}{r} = C\cos(\varphi - \varphi_0) + \frac{GM}{h^2}$$

$$\frac{d}{dt}\left(\frac{1}{r}\right) = -r^{-2}\frac{dr}{dt} = -C\sin(\varphi-\varphi_0)\dot{\varphi}$$

$$\frac{dr}{dt} = C\underbrace{r^2}\sin(\varphi-\varphi_0)\underbrace{\dot{\varphi}}$$

AS $\varphi \to 0$, $\frac{dr}{dt} \to -v$

$$-v = C\,b\,v\sin(-\varphi_0)$$

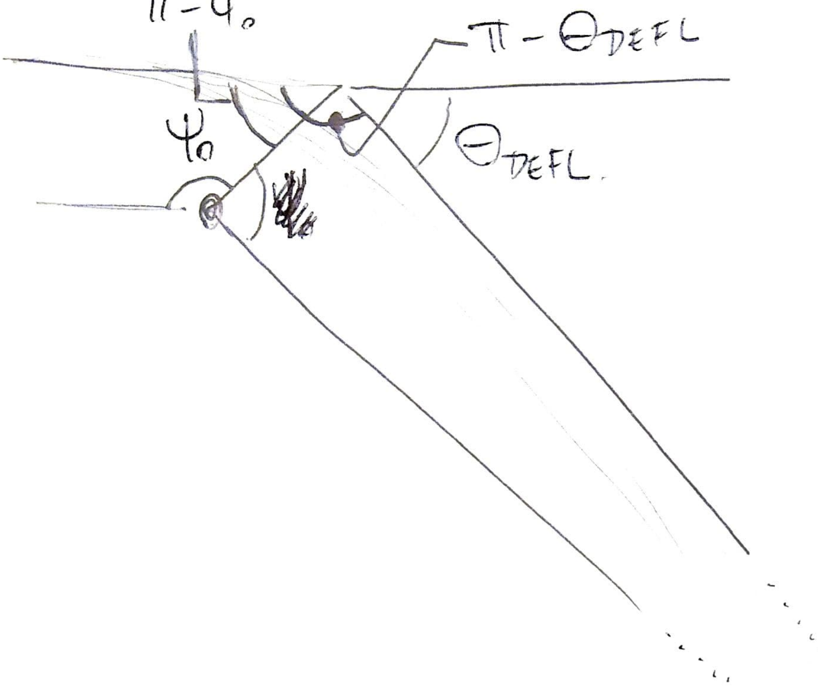
AS $r \to \infty$: (IE VERY EARLY IN TIME)

$$0 = C\cos\varphi_0 + \frac{GM}{b^2 v^2}$$

$$\tan\varphi_0 = -\frac{b v^2}{GM}$$

SDSG
3.9(III)

At a late stage:



$$(\pi - \Theta_{DEFL})\frac{1}{2} = \pi - \psi_0$$

$$\Downarrow$$

$$2\psi_0 - \pi = \Theta_{DEFL}.$$

$$\Downarrow$$

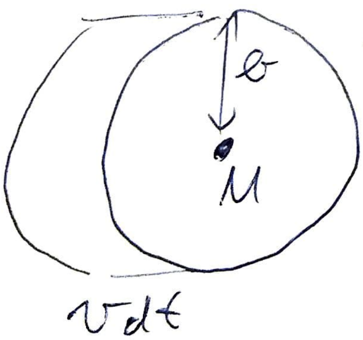
$$\tan\frac{\Theta_{DEFL}}{2} = -\frac{1}{\tan\psi_0}$$

COMBINING W/ PREV. PAGE:

$$\tan\frac{\Theta_{DEFL}}{2} = \frac{GM}{bv^2}$$

If a particle loses all of its x-directional momentum, dragging M (the big mass)

$$\tan\left(\frac{\frac{\pi}{2}}{2}\right) = 1 = \frac{GM}{bv^2} \Rightarrow b = \frac{GM}{v^2}$$



$v\,dt \cdot b^2\pi\cdot\rho$ ~~STATE~~ MASS OF PARTICLES INSIDE.
THEY LOSE THEIR x-DIRECTIONAL VELOCITY v.
MOMENTUM LOSS RATE THEN:

$$v^2 b^2 \pi\rho$$

$$\Rightarrow M\frac{dv}{dt} = -\pi\rho v^2\left(\frac{GM}{v^2}\right)^2 = -\pi\rho\frac{G^2M^2}{v^2}$$

DSG
3.9 IV

So the force is indeed proportional to $\frac{G^2 M^2 \rho}{V^2}$

on the massive particle, as desired.