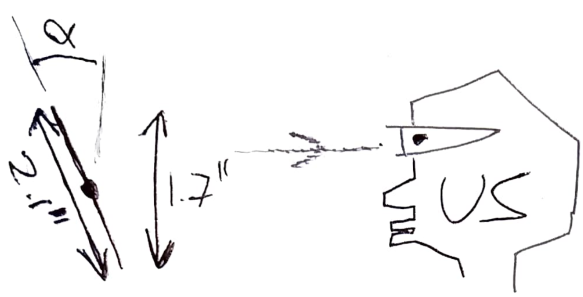
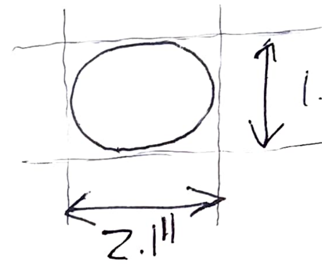
2017P4Q8

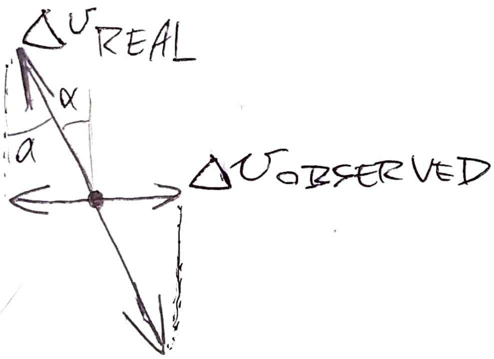
(i)

SIDE VIEW



FRONT VIEW

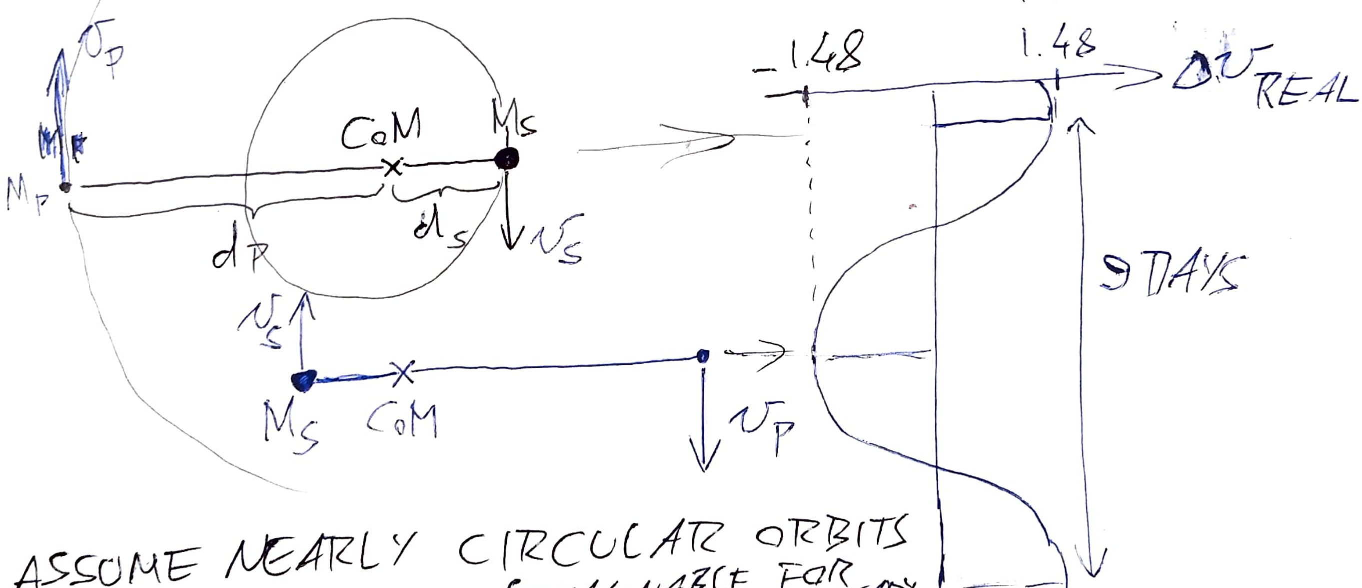




$$\Delta v_{REAL} = \frac{\Delta v_{OBSERVED}}{\cos\alpha}$$

$$= \frac{\Delta v_{OBSERVED}}{\cos\left(\arccos\frac{1.7}{2.1}\right)}$$

$$= 1.2 \cdot \frac{2.1}{1.7} = 1.48 \ \frac{km}{s}$$



ASSUME NEARLY CIRCULAR ORBITS (REASONABLE FOR PROTOPLANETARY DISCS)

$$2\pi d_s / v_s = P$$

$$d_s = \frac{P \cdot v_s}{2\pi} = \frac{9 \cdot 24 \cdot 60^2 \cdot 1.48 \cdot 10^3}{2\pi} = 1.83 \cdot 10^8 \, m$$

FROM DEFINITION OF C.M.:

$$M_p d_p = M_s d_s \Rightarrow M_p = M_s \frac{d_s}{d_p}$$

FORCE BALANCE:

$$\frac{v_p^2}{d_p} = G \frac{M_p M_s}{(d_p + d_s)^2}$$

$$v_p = d_p \cdot \omega = d_p \frac{2\pi}{P} \Rightarrow v_p^2 = 4\pi^2 \left(\frac{d_p}{P}\right)^2$$

REWRITE FORCE BALANCE EQUATION:

$$\frac{4\pi^2}{P^2} d_p = G \frac{M_s \frac{d_s}{d_p} M_s}{(d_p + d_s)^2}$$

$$\frac{4\pi^2}{P^2} d_p^2 (d_p + d_s)^2 = G M_s^2$$

$$\cancel{\frac{4\pi^2}{(24 \cdot 60^2)^2} d_p^2 (d_p + 1.83 \cdot 10^8)^2 = 6.67 \cdot 10^{-11} (1.99 \cdot 10^{30})^2}$$

$\cancel{d_p}$

$$\cancel{d_p^2 (d_p + 1.83 \cdot 10^8)}$$

$$\cancel{M_p = M_s \frac{d_s}{d_p} = 8}$$

~~SOLVE NUMERICALLY, OR SAY:~~ $\cancel{d_p \gg d_s}$, ~~THEN~~

$$\cancel{d_p^4 = G M_s^2 P^2 \frac{1}{4\pi^2}}$$

SOLVE NUMERICALLY OR SAY: $d_P \gg d_S$
(REASONABLE)

$$d_P^4 = \frac{G M_S^2 P^2}{4\pi^2}$$

$$d_P = \sqrt[4]{\frac{G M_S^2 P^2}{4\pi^2}} = 4.8 \cdot 10^{13}\, m$$

(WHICH IS CONVENIENTLY MUCH LARGER THAN $1.83 \cdot 10^{8}\, m$)

$$M_P = M_S \frac{d_S}{d_P} = 2 \cdot 10^{30} \cdot \frac{1.83 \cdot 10^{8}}{4.8 \cdot 10^{13}} = \underline{7.6 \cdot 10^{24}\, kg}$$

ORBITAL RADIUS $= \underline{4.8 \cdot 10^{13}}\, m$

Eccentricity deduction:

SINUSOIDAL VARIATION $\Rightarrow$ ORBIT PROBABLY HIGHLY CIRCULAR SO $e \ll 1$, $e \approx 0$.