III.1

FRACTION OF SPIRALS $= \dfrac{\text{NUMBER DENSITY OF SPIRALS}}{\text{TOTAL NUMBER DENSITY}}$

$$S = \int_{L_D}^{\infty} \varphi(L)\,dL$$

$\underbrace{L_D}$ THIS LUMINOSITY DEEP WE SEE.

IF $L_D < L_*$:

$$S = \int_{L_D}^{L_*} \varphi(L<L_*)\,dL + \int_{L_*}^{\infty} \varphi(L>L_*)\,dL$$

ELSE:

$$S = \int_{L_D}^{\infty} \varphi(L>L_*)\,dL$$

COMPUTE THESE INTEGRALS:

$$\int \varphi(L<L_*)\,dL = \int \frac{n_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} dL$$

$$= \frac{n_*}{L_*^{\alpha+1}} \frac{1}{\alpha+1} L^{\alpha+1}$$

$$\int \varphi(L>L_*)\,dL = \int \frac{n_*}{L_*} \exp\left(-\frac{L}{L_*}\right) dL = -n_* \exp\left(-\frac{L}{L_*}\right)$$

(i)

$L_{*E} > L_1 > L_{*S}$

IE POWER-LAW CONTRIB ONLY FOR ELLIPTICALS.

$$S_S = \int_{L_1}^{\infty} \phi(L > L_{*S})\, dL$$

$$= -n_{*S} \exp\left(-\frac{L}{L_{*S}}\right)\Big|_{L_1}^{\infty} = n_{*S} \exp\left(-\frac{L_1}{L_{*S}}\right)$$

$$S_E = \int_{L_1}^{\infty} \phi(L > L_{*E})\, dL$$

$$\cancel{= -n_{*E} \exp\left(-\frac{L}{L_{*E}}\right)\Big|_{L_1}^{L_{*E}} +}$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left[L^{\alpha_E+1}\right]_{L_1}^{L_{*E}} + \left[(-n_{*E}) \exp\left(-\frac{L}{L_{*E}}\right)\right]_{L_{*E}}^{\infty}$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_1^{\alpha_E+1}\right) +$$

$$+ n_{*E} \exp\left(-\frac{L_{*E}}{L_{*E}}\right)$$

$$= \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1} \left(L_{*E}^{\alpha_E+1} - L_1^{\alpha_E+1}\right) + n_{*E} \exp(-1)$$

PUTTING IN THE NUMBERS:

$$S_S = n_{*s} \exp\left(-\frac{5 \cdot 10^{11}}{10^{11}}\right) = n_{*s} e^{-5}$$

$$\cancel{S_E = \frac{4 n_{*s}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1}}$$

$$S_E = \frac{4 n_{*s}}{\left(10^{12}\right)^{-0.5+1}} \frac{1}{-0.5+1} \left(\left(10^{12}\right)^{-0.5+1} - \left(5 \cdot 10^{11}\right)^{-0.5+1} \right) + 4 n_{*s} e^{-1}$$

$$= 3.814\, n_{*s}$$

$$\text{FRACTION OF SPIRALS} = \frac{n_{*s} e^{-5}}{3.814\, n_{*s} + e^{-5} n_{*s}} \simeq \underline{\underline{1.77 \cdot 10^{-3}}}$$

(ii)

$$L_{*E} > L_{*S} > L_2$$

IE POWER-LAW & EXPONENTIAL TERM FOR BOTH SPIRALS & ELLIPTICALS. USING PREVIOUS RESULTS:

$$S_S = \frac{n_{*S}}{L_{*S}^{\alpha_S+1}} \frac{1}{\alpha_S+1}\left(L_{*S}^{\alpha_S+1} - L_2^{\alpha_S+1}\right) + n_{*S}\exp(-1)$$

$$S_E = \frac{n_{*E}}{L_{*E}^{\alpha_E+1}} \frac{1}{\alpha_E+1}\left(L_{*E}^{\alpha_E+1} - L_2^{\alpha_E+1}\right) + n_{*E}\exp(-1)$$

PUTTING IN THE NUMBERS:

$$S_S = \frac{n_{*S}}{\left(10^{11}\right)^{-1.5+1}} \frac{1}{-1.5+1}\left(\left(10^{11}\right)^{-1.5+1} - \left(10^{9}\right)^{-1.5+1}\right) +$$

$$+ n_{*S}\exp(-1)$$

$$= \frac{n_{*S}}{\left(10^{11}\right)^{-\frac{1}{2}}}(-2)\left(\left(10^{11}\right)^{-\frac{1}{2}} - \left(10^{9}\right)^{-\frac{1}{2}}\right) + n_S e^{-1}$$

$$= \underline{18.37\, n_{*S}}$$

$$S_E = \frac{4n_{s*}}{(10^{12})^{-0.5+1}} \frac{1}{-0.5+1} \left((10^{12})^{-0.5+1} - (10^{9})^{-0.5+1} \right) + 4n_{s*} e^{-1}$$

$$= 9.22\, n_{*s}$$

FRACTION OF SPIRALS $= \dfrac{18.37}{18.37 + 9.22} = \underline{\underline{0.67}}$

COMMENT ON RESULT:

THE DEEPER WE SEE THE MORE REALISTIC RESULTS WE GET.

IT IS IMPORTANT TO SEE DEEP ENOUGH TO OBSERVE THE POWER LAW PART OF THE DISTRIBUTION, WHICH IS THE DOMINATING CONTRIBUTOR TO NUMBER DENSITY.