1

$$S = \frac{\partial}{\partial T}\left(k_B T \log Z_{IDEAL}\right)$$

SUB FOR Z_{IDEAL}

$$= \frac{\partial}{\partial T}\left(k_B T \log \frac{Z_1^N}{N!}\right)$$

SUB FOR $Z_1 \left(= \frac{V}{\lambda^3}\right)$

$$= \frac{\partial}{\partial T}\left(k_B T \log \frac{V^N}{N!\,\lambda^{3N}}\right)$$

SUB FOR $\lambda \left(= \sqrt{\frac{2\pi\hbar^2}{m k_B T}}\right)$

$$= \frac{\partial}{\partial T}\left(k_B T \log \frac{V^N}{N!\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}}\right)$$

$$= k_B \log \frac{V^N}{N!\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}} + k_B T \frac{\partial}{\partial T} \ln \frac{V^N}{N!\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}}$$

$$= k_B \log\left(\frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right) + k_B T \frac{\partial}{\partial T} \ln \frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}$$

STIRLING APPROX: $\log N! \simeq N \log N - N$

FIRST TERM:

$$k_B \log\left(\frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right) =$$

$$= k_B \log\left(V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}\right) - k_B \log N!$$

$$= N k_B \log\left(V \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\right) - k_B \log N!$$

$$= N k_B \log\left(\frac{V}{\lambda^3}\right) - k_B N \log N + N k_B$$

$$= N k_B \log\left(\frac{V}{\lambda^3 N}\right) + N k_B$$

SECOND TERM:

$$k_B T \frac{\partial}{\partial T} \ln \frac{V^N}{N!}\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}} =$$

$$= k_B T \frac{\partial}{\partial T}\left[\ln V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}} - \ln N!\right]$$

N IS NOT DEPENDENT ON t, So:

$$= k_B T \frac{\partial}{\partial T} \ln V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}$$

V ALSO t INDEPENDENT

$$= k_B T N \frac{\partial}{\partial T} \log\left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

$$= k_B T N \frac{3}{2} \partial_T \log\left(\frac{m k_B T}{2\pi\hbar^2}\right)$$

$$\cancel{= k_B T N \frac{3}{2} \frac{2\pi\hbar^2}{m k_B T}}$$

$$= k_B T N \frac{3}{2} \partial_T \log T$$

$$= k_B T N \frac{3}{2} \frac{1}{T} = k_B N \frac{3}{2}$$

COLLECT TERMS:

$$S = N k_B \left[\log\left(\frac{V}{N\lambda^3}\right) + \frac{5}{2}\right]$$

IF $N!$ IS NOT INCLUDED:

$$S = \frac{\partial}{\partial T}\left(k_B T \log \frac{V^N}{\left(\frac{2\pi\hbar^2}{m k_B T}\right)^{\frac{3N}{2}}} \right)$$

$$= \underbrace{k_B \log\left(\frac{V^N}{\cancel{N!}} \cdot \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}} \right)} + \underbrace{k_B T \frac{\partial}{\partial T} \ln V^N \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3N}{2}}}$$

$$= N \cancel{k}k_B \log\left(V \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} \right)$$

$$= N k_B \log\left(\frac{V}{\lambda^3 \cancel{N}} \right)$$

$$\Rightarrow = k_B T \frac{\partial}{\partial T} T^{\frac{3N}{2}} = k_B T \frac{3}{2} N \frac{\partial}{\partial T} \ln T = \frac{3}{2} k_B N$$

COLLECT TERMS:

$$S_{\text{WITHOUT } N! \text{ FACTOR}} = N k_B \log \frac{V}{\lambda^3} + \frac{3}{2} k_B N$$

$$= N k_B \log\left(\frac{V}{\lambda^3} + \frac{3}{2} \right)$$

NOTICE THAT WHILE:

$$S_{\text{WITH } N!}(2N, 2V) = 2 S_{\text{WITH } N!}(N, V)$$

HERE WE HAVE:

$$S_{\text{WITHOUT } N!}(2N, 2V) \neq 2 S_{\text{WITHOUT } N!}(N, V) \Rightarrow \text{NOT EXTENSIVE}$$

2/

$$H = \frac{p^2}{2m} + \lambda q^4 \qquad -q \longrightarrow q$$

$$Z_1 = \frac{1}{2\pi\hbar}\int dq\, dp\, e^{-\beta\left(\frac{p^2}{2m} + \lambda q^4\right)}$$

$$= \frac{1}{2\pi\hbar}\underbrace{\int_{-\infty}^{\infty} dq\, e^{-\lambda\beta q^4}}\int_{-\infty}^{\infty} dp\, e^{-\beta\frac{p^2}{2m}}$$

FIRST TERM: $\displaystyle\int_{-\infty}^{\infty} dq\, e^{-\lambda\beta q^4} = \int_{-\infty}^{\infty} (\lambda\beta)^{-\frac{1}{4}} e^{-x^4}\, dx =$

LET $x = (\lambda\beta)^{\frac{1}{4}} q$

$dx = (\lambda\beta)^{\frac{1}{4}} dq$

$$= (\lambda\beta)^{-\frac{1}{4}}\int_{-\infty}^{\infty} e^{-x^4} dx$$

$$= (\lambda\beta)^{-\frac{1}{4}}\, 2\int_0^{\infty} e^{-x^4} dx$$

COMPUTER

$$= 2(\lambda\beta)^{-\frac{1}{4}}\,\Gamma\left(\frac{5}{4}\right)$$

SECOND TERM:

$$\int_{-\infty}^{\infty} dp\, e^{-\frac{\beta p^2}{2m}} =$$

LET: $y = +\sqrt{\frac{\beta}{2m}}\, p \Rightarrow dy = \sqrt{\frac{\beta}{2m}}\, dp$

$$= \int_{-\infty}^{\infty} dy\, e^{-y^2}\sqrt{\frac{2m}{\beta}} = \sqrt{\frac{2m\pi}{\beta}}$$

COLLECT TERMS:

$$Z_1 = \frac{1}{\cancel{2}\pi\hbar}\,\cancel{2}(\lambda\beta)^{-\frac{1}{4}}\,\Gamma\left(\frac{5}{4}\right)\sqrt{\frac{2m\pi}{\beta}} = \frac{1}{\pi\hbar}(\lambda\beta)^{-\frac{1}{4}}\sqrt{\frac{2m\pi}{\beta}}\,\Gamma\left(\frac{5}{4}\right)$$

$$= \frac{1}{\pi\hbar}(\lambda)^{-\frac{1}{4}}\cdot\sqrt{2m\pi}\,\beta^{-\frac{3}{4}}\,\Gamma\left(\frac{5}{4}\right) = \frac{1}{\pi\hbar}(\lambda)^{-\frac{1}{4}}\,\Gamma\left(\frac{5}{4}\right)\sqrt{2m\pi}\,k_B^{\frac{3}{4}}\,T^{\frac{3}{4}}$$

$$E = -\frac{\partial}{\partial \beta} \log Z = -\frac{\partial}{\partial \beta} \log Z_1^N = -N \frac{\partial}{\partial \beta} \log Z_1$$

$$= -N \frac{\partial}{\partial \beta} \log \text{BUNCH OF CONSTANTS} \cdot T^{\frac{3}{4}}$$

$$= -N \frac{\partial}{\partial \beta} \log T^{\frac{3}{4}}$$

USE:

$$\frac{\partial}{\partial \beta} = \frac{\partial T}{\partial \beta} \frac{\partial}{\partial T} = \frac{\partial T}{\partial \left(\frac{1}{k_B T}\right)} \frac{\partial}{\partial T} = \left(\frac{\partial \frac{1}{k_B T}}{\partial T} \right)^{-1} \frac{\partial}{\partial T}$$

$$= -k_B T^{+2} \frac{\partial}{\partial T}$$

$$E = -N(-k_B T^{+2}) \frac{\partial}{\partial T} \log T^{\frac{3}{4}}$$

$$= N k_B T^{+2} \cdot \frac{3}{4} \frac{\partial}{\partial T} \log T = N k_B T^{+2} \frac{1}{T} \cdot \frac{3}{4}$$

$$= \frac{3}{4} N k_B T$$

$$C_V = \left.\frac{\partial E}{\partial T}\right|_V = \left.\frac{\partial}{\partial T} \frac{3}{4} N k_B T\right|_V = \underline{\frac{3}{4} N k_B}$$

IF PARTICLES DISTINGUISHABILITY CHANGES, THEN NUMBER OF ALL POSSIBLE STATES CHANGES.

IE $Z_{NEW} = Z$ WHAT WE HAD PREVIOUSLY $\cdot\, \alpha$ FOR SAME α.

NOTICE THAT:

$$E = -\frac{\partial}{\partial \beta}\log(Z\alpha) = -\frac{\partial}{\partial \beta}\log Z + \cancelto{0}{\frac{\partial}{\partial \beta}\log\alpha}$$

$$= -\frac{\partial}{\partial \beta}\log Z$$

IE WE ARRIVE TO THE SAME E AS WE DID PREVIOUSLY, NO MATTER CONSTANT MULTIPLICATIVE FACTOR IN $\ln$.

$\Longrightarrow C_V$ IS NOT DEPENDENT ON DISTINGUISHABILITY.

3

FOR 1 PARTICLE:

$$Z_1 = \frac{1}{(2\pi\hbar)^3}\int d^3q\, d^3p\, e^{-\beta E(p)}$$

$$\int_{\text{ALL SPACE}} e^{-\beta|p|c} d^3p = \int e^{-\beta c p} p^2 \sin\theta\, dp\, d\theta\, d\phi = 4\pi\int_0^\infty e^{-\beta c p} p^2\, dp = 8\pi\frac{+1}{(\beta c)^3} \leftarrow \text{COMPUTER}$$

$$= \frac{1}{(2\pi\hbar)^3} V \left[\int e^{-\beta |p|^c} d^3p\right]$$

$$\cancel{= \frac{1}{(2\pi\hbar)^3} V \left(2\int_0^\infty e^{-\beta c p}\, dp\right)^3}$$

$$\cancel{= \frac{1}{(2\pi\hbar)^3} V\, 2^3 \left(\frac{1}{-\beta c}\left[e^{-\beta c p}\right]_0^\infty\right)^3}$$

$$= \frac{1}{(2\pi\hbar)^3} V \cdot 8\pi\left(\frac{1}{\beta c}\right)^3 = \frac{1}{(2\pi)^3} V \cdot 2^3\pi\left(\frac{k_B T}{\hbar c}\right)^3 = \frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$$

WE HAVE N PARTICLES BUT WE CANNOT DISTINGUISH THEM SO:

$$Z = \frac{1}{N!} Z_1^N = \frac{1}{N!}\left[\frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right]^N$$

AS WANTED.

(MY RELIANCE ON COMPUTERS TO DO INTEGRALS IS SOMEWHAT WORRYING)

$$dF = -S\,dT - p\,dV$$

$$p = -\left.\frac{\partial F}{\partial V}\right|_T$$

$$= -\frac{\partial}{\partial V}\left(-k_B T \log Z\right)\Big|_T$$

$$= \frac{\partial}{\partial V}\left(k_B T \log Z\right)\Big|_T$$

$$= k_B T \frac{\partial}{\partial V} \log Z\Big|_T$$

$$= k_B T \frac{\partial}{\partial V} \log \frac{1}{N!}\left[\frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right]^N$$

$$\cancel{= k_B T N \cdot V^{N-1} \frac{1}{N!}}$$

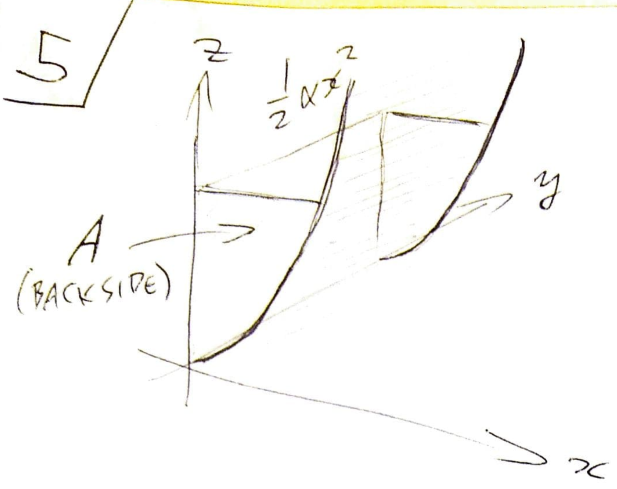
$$= k_B T \frac{\partial}{\partial V} \log V^N$$

$$= k_B T N \frac{\partial}{\partial V} \log V$$

$$= k_B T N \frac{1}{V} \implies pV = k_B T N$$

AS WANTED.

5



$$\underbrace{\text{NUMBER DENSITY OF ATOMS BETWEEN } x \,\&\, x+dx \text{ TIMES } dx}_{} = \underbrace{\text{PROBABILITY OF 1 ATOM BEING BETWEEN } x \,\&\, x+dx}_{} \times \underbrace{\underbrace{\text{TOTAL NUMBER OF ATOMS}}_{AN_0} \Big/ \underbrace{\text{AREA IN } zy \text{ PLANE}}_{A}}_{N_0}$$

$$P(x) = \frac{e^{-\beta E(x)}\,dx}{\int_0^\infty e^{-\beta E(x)}\,dx}$$

NORMALIZATION

$$= \exp\left(-\frac{1}{2}\alpha\beta x^2\right)\cdot \underbrace{\left(\int_0^\infty e^{-\frac{1}{2}\alpha\beta x^2}\,dx\right)^{-1}}_{}$$

$$\sqrt{\frac{\pi}{2\alpha\beta}}^{\,-1} = \sqrt{\frac{2\alpha\beta}{\pi}}$$

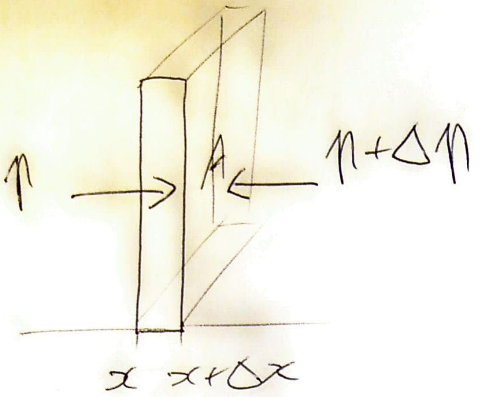
COLLECT TERMS:

$$S(x)\,dx = \exp\left(-\frac{1}{2}\alpha\beta x^2\right)\sqrt{\frac{2\alpha\beta}{\pi}}\,dx\,N_0$$

REWRITE

$$\Rightarrow S(x) = 2N_0\sqrt{\frac{\alpha\beta}{2\pi}}\,e^{-\alpha\beta x^2/2}$$

AS WANTED.



$$p(x)A = p(x+\Delta x)A \Rightarrow p = \text{CONSTANT}$$

HMMM, & NOW WHAT...

7

$$U(r) = \begin{cases} \infty & r < r_0/2 \\ 0 & \text{ELSE.} \end{cases}$$

$$\cancel{f(r) = e^{-\beta}}$$

$$f(r) = e^{-\beta U(r)} - 1$$

$$p = -\frac{\partial F}{\partial V} = \cancel{\frac{N k_B T}{V}}$$

$$F = F_{IDEAL} - N k_B T \underbrace{\log\left(1 + \frac{N}{2V}\int dr^2 f(r)\right)}_{\simeq \frac{N}{2V}\int dr^2 f(r)}$$

$$p = \underbrace{-\frac{\partial}{\partial V}\left(F_{IDEAL}\right.}_{\frac{N k_B T}{V}} \left. - N k_B T \frac{N}{2V}\int dr^2 f(r)\right)$$

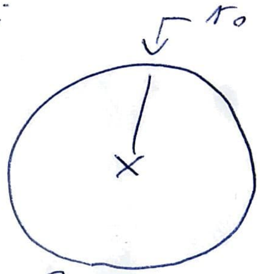
$$\cancel{= \frac{N k_B T}{V}\left(1 + \frac{\partial}{\partial V}\right.}$$

$$p = \frac{N k_B T}{V} - \frac{N k_B T\, N}{2V^2}\int dr^2 f(r)$$

$$p = \frac{N k_B T}{V}\left(1 - \frac{N}{2V}\int dr^2 f(r)\right)$$

$$\frac{pV}{N k_B T} = 1 - \frac{N}{2V}\underbrace{\int dr^2 f(r)}_{\text{FOCUS ON THIS BIT}}$$

CONSIDER GAS IN THIS AREA:



$V = r_0^2 \pi$

$$\cancel{\int dr^2 f(r) = \int_{r=0}^{r=r_0}\int_{\varphi=0}^{\varphi=2\pi} f(r)\, r\, dr\, d\varphi}$$

SPACE

$$\cancel{= \int_0^{r_0}\int_0^{2\pi} e^{-\beta U}}$$

$$\int_0^{\infty} f(r)\, dr^2 = \int_0^{\frac{r_0}{2}} f(r)\, dr^2 + \int_{\frac{r_0}{2}}^{\infty} f(r)\, dr^2$$

$$= \int_0^{\frac{r_0}{2}} -1\, dr^2 + \underbrace{\int_{\frac{r_0}{2}}^{\infty} 0\, dr^2}_{0}$$

$$= \left(\frac{r_0}{2}\right)^2 \pi \cdot (-1)$$

ARRIVE TO:

$$\frac{pV}{N k_B T} = 1 - \frac{N}{2V}\left(\frac{r_0}{2}\right)^2 \pi(-1)$$

IE

$$pV = N k_B T \left(1 + \frac{N}{2V}\left(\frac{r_0}{2}\right)^2\right)$$

8

$$U(r) = \frac{\alpha}{r^n} \qquad n > 3 \qquad \alpha > 0$$

$$\frac{pV}{Nk_BT} = 1 - \frac{N}{2V} \underbrace{\int d^3r\, f(r)}$$

THIS IS THE SECOND VIRIAL COEFF.

$$\cancel{= 1 - \frac{N}{V}\left(\frac{a}{k_BT} - b\right)}$$

SECOND VIRIAL COEFF

$$= \int d^3r \left(e^{-\beta U(r)} - 1\right)$$

$$= \int dr^3 \left(e^{-\beta \alpha r^{-n}} - 1\right)$$

$$= \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \left(e^{-\beta\alpha r^{-n}} - 1\right) r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= 4\pi \int_{r=0}^{\infty} \left(e^{-\beta\alpha r^{-n}} - 1\right) r^2 \, dr$$

& I SHOULD SOMEHOW EVALUATE THIS BUT IDK HOW.

9

$d=2$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right)^2 (n_1^2 + n_2^2) = \frac{2\pi^2\hbar^2}{mL^2}(n_1^2 + n_2^2)$$

$$\sum_{\vec{n}} \int d^2 n = \frac{V}{(2\pi)^2}\int d^2 k = \frac{V}{(2\pi)^2}\int k\, d\varphi\, dk$$

$$= \frac{\pi V}{(2\pi)^2}\int_0^\infty k\, dk$$

$$E = \frac{\hbar^2 k^2}{2m} \Longrightarrow dE = \frac{\hbar^2 k}{m} dk$$

$$\frac{\pi V}{(2\pi)^2}\int_0 k\, dk = \frac{V}{4\pi}\int \frac{m}{\hbar^2 \cancel{k}} dE\, \cancel{k} = \frac{V}{4\pi}\int \frac{m}{\hbar^2} dE$$

$$\Longrightarrow g(E) = \frac{V}{4\pi}\frac{m}{\hbar^2}$$

CONSTANT, AS HINT SUGGESTS