"FIND THE DENSITY" PART

$$S = \int d^3\underline{v}\, f$$

$$f = \frac{S_1}{(2\pi\sigma^2)^{\frac{3}{2}}} \exp\left(\frac{E}{\sigma^2}\right) = \underbrace{C}_{C} \exp\left(\frac{\Psi}{\sigma^2}\right) \exp\left(\frac{-\frac{1}{2}v^2}{\sigma^2}\right)$$

$$\Rightarrow S = \int_{v=-\infty}^{v=+\infty} C \exp\left(\frac{\Psi}{\sigma^2}\right) \exp\left(-\frac{v^2}{2\sigma^2}\right) dv = C \exp\left(\frac{\Psi}{\sigma^2}\right) \sqrt{2}\sqrt{\pi}\,\sigma$$

$$= \frac{S_1}{(2\pi)^{\frac{3}{2}}\sigma^{\frac{3}{2}}} \exp\left(\frac{\Psi}{\sigma^2}\right)\sqrt{2\pi}\,\sigma = \boxed{\frac{S_1}{2\pi\sigma^2} \exp\left(\frac{\Psi}{\sigma^2}\right)}$$

SO MY DENSITY INCREASES WITH
INCREASING POTENTIAL?
SEEMS WEIRD.

"POISSON EQUATION FOR THE SYSTEM" PART

$$\nabla^2 \Psi = 4\pi G S$$

IE
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Psi}{\partial r}\right) = 4\pi G \frac{S_1}{2\pi\sigma^2} \exp\left(\frac{\Psi}{\sigma^2}\right)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Psi}{\partial r}\right) = \frac{2GS_1}{\sigma^2} \exp\left(\frac{\Psi}{\sigma^2}\right)$$

"SHOW THAT" PART   USING POISSON & σ^2 VALUE GIVEN:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) = \frac{2 G S_1 m_0}{\mathscr{R} T} \exp\left(\frac{\psi}{\sigma^2}\right) \qquad / \cdot r^2 / \int$$

$$r^2 \frac{\partial \psi}{\partial r} = \int^r \frac{2 G S_1 m_0}{\mathscr{R} T} \exp\left(\frac{\psi}{\sigma^2}\right) \bcancel{} r'^2 \, dr \qquad / \int dV$$

$$\bcancel{(4\pi} r^2 \frac{\partial \psi}{\partial r} = \frac{4\pi G m_0}{\mathscr{R} T} \int_0^r \int_{\theta=0}^{\theta=\pi} \int_{\varphi=0}^{2\pi} \frac{S_1}{2\pi\sigma^2} \exp\left(\frac{\psi}{\sigma^2}\right) r'^2 \sin\theta \, dr \, d\theta \, d\varphi}_{M(r)}$$

$$r^2\frac{\partial \psi}{\partial r} = \frac{G m_0}{\mathscr{R} T} M(r)$$

WE'RE OK IF:

$$\frac{\partial \psi}{\partial r} = -\frac{1}{S}\frac{dS}{dr}$$

CHECK:

$$\frac{d\psi}{dr} = \frac{-1}{S}\frac{dS}{dr} = \frac{d}{dr}\left(-\ln S\right) \propto \frac{d}{dr}\left(-\frac{\psi}{\sigma^2}\right) \propto \frac{d\psi}{dr}$$

$$\Rightarrow \frac{\partial \psi}{\partial r} \propto -\frac{1}{S}\frac{dS}{dr} \Rightarrow r^2\left(-\frac{1}{S}\frac{dS}{dr}\right) \propto \frac{G m_0}{\mathscr{R} T} M(r)$$

$$\Rightarrow \frac{\mathscr{R} T}{M_0}\frac{dS}{dr} \propto -S\frac{G M(r)}{r^2}$$

$\Rightarrow$ SAME DENSITY STRUCTURE.

THIS IS THE EQUATION
GIVEN (APART FROM
THE PROPORTIONALITY
SIGN INSTEAD OF
EQUAL SIGN)