

III. 5

CONSIDER:

$$\frac{dH^2}{dt} = 2H \frac{dH}{dt}$$

WE ALSO HAVE:

$$\frac{dH^2}{dt} = \frac{d}{dt} \left(\frac{1}{3} \left(V(\phi) + \frac{1}{2} \dot{\phi}^2 \right) \right)$$

$$= \frac{1}{3} \frac{dV}{d\phi} \frac{d\phi}{dt} + \dot{\phi} \ddot{\phi}$$

$$= \frac{1}{3} \dot{\phi} \left(\underbrace{V' + \ddot{\phi}}_{\text{---}} \right)$$

$$= -H \dot{\phi}^2 \quad \boxed{-3H\dot{\phi}}$$

$$\Rightarrow 2H \frac{dH}{dt} = -H \dot{\phi}^2 \Rightarrow \underline{\frac{dH}{dt} = -\frac{1}{2} \dot{\phi}^2}$$

"HENCE DERIVE"

$$\frac{dH}{d\phi} = \frac{dH}{dt} \frac{dt}{d\phi} = \frac{\dot{H}}{\dot{\phi}}$$

SO WE HAVE:

$$H^{1/2} = \left(\frac{\dot{H}}{\dot{\phi}} \right)^2 = -\frac{1}{2} \dot{H}$$

SUBSTITUTE IN: (FOR $H^{1/2}$ & H^2)

$$H^{1/2} - \frac{3}{2} H^2 = -\frac{1}{2} \dot{H} - \cancel{\frac{3}{2}} \cancel{\frac{1}{2}} \left(V + \frac{1}{2} \dot{\phi}^2 \right)$$

$$= -\frac{1}{2} V - \frac{1}{2} \left(\dot{H} + \underbrace{\frac{1}{2} \dot{\phi}^2}_{0} \right)$$

$$= \underline{-\frac{1}{2} V}$$

"SHOW THAT"

$$H^1 \geq -\frac{3}{2} H^2 = -\frac{1}{2} V \quad /: H^2$$

$$\underbrace{\left(\frac{H^1}{H}\right)^2}_{L = \frac{1}{2}} - \frac{3}{2} = -\frac{1}{2} \frac{V}{H^2}$$

$$E_H - 3 = -\frac{V}{H^2} \quad /: -3 \cdot H^2$$

$$H^2 \left(1 - \frac{1}{3} E_H\right) = \frac{1}{3} V$$