Cosmology L III Part I

CMB discovery: 1965
killed steady state proposals
proved hot big bang cosmology beyond any reasonable doubt

Penzias & Wilson
much bigger than expected noise
isotropic $\Rightarrow$ cosmic origin
Another group predicted $\sim 3K$ background radiation.
They couldn't measure the spectrum properly in 1965.
COBE satellite 1990: measured spectrum.
Found very nice black body radiation curve.
can estimate temperature from this
$T \simeq 2.7K$

Black body radiation:

$$\rho_\gamma c^2 = \boxed{\text{BUNCH OF STUFF}} \underbrace{\int_0^\infty \frac{x^3 dx}{e^x - 1}}_{\Gamma(4)\,\zeta(4) = \frac{\pi^4}{15}}$$

Riemann zeta function

early times were in thermal equilibrium then

$$\boxed{\rho_\gamma c^2 = a T_\gamma^4} \qquad a = \frac{8\pi^5 k^4}{15 h^3 c^3}$$

black body radiation exerts isotropic pressure:

$$P_\gamma = \frac{1}{3}\rho_\gamma c^2$$

Energy conservation: $\frac{d(\rho R^3)}{dR} = -3pR^2$

$\Longrightarrow \rho_\gamma \propto 1/R^4$ $\quad \rho_\gamma c^2 = aT_\gamma^4$

$T_\gamma \propto 1/R$

note that for dust, $\rho \propto 1/R^3$, unlike radiation which is considered here

$$\rho_\gamma(\nu,t)c^2 d\nu = \frac{1}{c^3}\frac{8\pi h\nu^3 d\nu}{\exp\left(\frac{h\nu}{kT_\gamma}\right)-1}$$

$T \propto 1/R$, $\nu \propto 1/R$ (redshift) $\Longrightarrow$ radiation maintains b.b. form as univ. expands.

number density of BB photons:

$$n_\gamma(\nu)d\nu = \frac{8\pi}{c^3}\frac{\nu^2 d\nu}{\exp\left(\frac{h\nu}{kT_\gamma}\right)-1}$$

integrate up:

$$n_\gamma = 16\pi\zeta(3)\left(\frac{kT_\gamma}{hc}\right)^3 = 410.6\left(\frac{T_\gamma}{2.726K}\right)^3 cm^{-3}$$

400 photons per cm^3

critical density is $\sim 1\, p^+/m^3$

$$\eta = \frac{n_b}{n_\gamma} = 2.75 \cdot 10^{-8}\,\Omega_b h^2$$

~~very~~ many more photons than baryons. The smallness of this is not yet well understood.

must have been an assymetry in matter-antimatter, so after $p^+ \bar{p}$ annihilation we have left with ~~[illegible]~~ p^+ a bit.

$$\Omega_\gamma(0) = \frac{\rho_\gamma}{\rho_c} = 2.4 \cdot 10^{-5} h^{-2}$$

└ hubble parameter

CMB plays negligible contrib to E density of univ. present day.

$$\rho_{matter} \propto R^{-3} \;\&\; \rho_\gamma \propto R^{-4}$$

$\Rightarrow$ E density of radiation dominates early times.

Furthermore: $T_\gamma \propto R^{-1} \Rightarrow$ early univ must have been hot & very dense.

~~it proof~~

i.e CMB proves hot bing bang cosmology

- ~~the~~ Issues after CMB discovery
  - is it BB $\rightarrow$ COBE
  - is it anysotropic?
    i.e different directions do have different temperatures or not

Discovery of Dipole Anisotropy

Dipole Anisotropy

if $v \ll c$:

$$T'(\theta') \approx T_0 \left[1 + \frac{v}{c}\cos\theta'\right]$$

i.e. motion of observer through the CMB induces dipole anisotropy.

Put radiometer to U2 spy plane, fly over Skila.

This was measured by Planck satellite as well.

Solar dipole, correct for motion of sun, motion compared to local group centre.

Anisotropy still present.

Questions arising:

CMB defines preferred reference frame. SR violation?

no, preferred ref frame in FRW metric. (3)

What causes this motion?

inhomogeneity of matter around us

COSMOLOGY L III P II

de Sitter space

recall FRW: max ly sym space
characterized by 1 number k.
3D

de Sitter space:
max. symmetric 4D spacetime.

curvature tensor

$$R_{\kappa\nu\rho\sigma} = K(g_{\nu\sigma}g_{\kappa\rho} - g_{\rho\nu}g_{\kappa\sigma})$$

Ricci tensor

$$R_{\nu\sigma} = g^{\kappa\rho}R_{\kappa\nu\rho\sigma} = K(N-1)g_{\nu\sigma}$$

Ricci scalar

$$R = KN(N-1)$$

$N = 4$ for 4D.

Rewrite Einstein field equation:

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{\sigma}_{\sigma}\right)$$

for empty universe:

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

de Sitter geometry satisfies:

$$R_{\mu\nu} = 3Kg_{\mu\nu}$$

$\Longrightarrow$ de Sitter geometry is sol of EFE if there is no matter $T_{\mu\nu} = 0$ & $\Lambda \neq 0$

$$\Rightarrow K = \frac{1}{3}\Lambda$$

i.e curvature of spacetime is related to cosm. const.

geometry of de Sitter space?
consider constant curvature hyperboloid embedded in 2+1 dimensional Minkowski space.

$$-z^2+x^2+y^2=H^{-2} \quad \text{(eq of hyperboloid)}$$

similar to FRW: there we embedded sphere

metric:
$$ds^2=dz^2-dx^2-dy^2$$

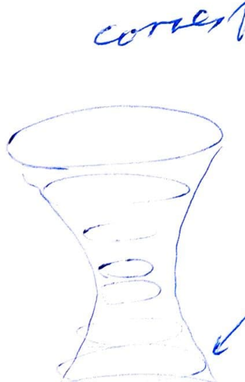
eliminate dz:
$$ds^2=\frac{(x\,dx+y\,dx)^2}{x^2+y^2-H^{-2}}-dx^2-dy^2$$

make coordinate transform:
$$x=H^{-1}\cosh(Ht)\begin{vmatrix}\cos\chi\\ \sin\chi\end{vmatrix}$$
$$y=$$

metric reduces to:
$$ds^2=dt^2-\frac{1}{H^2}\cosh^2(Ht)\,d\chi^2$$

recognize form as FRW with 1 spatial coordinate

corresponds to closed universe



entire surface covered by:
$$-\infty<t<\infty,$$
$$0\leq\chi\leq 2\pi$$

This universe has minimum size reached at $t=0$.

lets make another coord. transform.

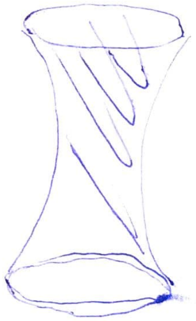
[ALGEBRA]

$z = z(t, \xi)$
$y = y(t, \xi)$
$x = x(t, \xi)$

arrive to:

$$ds^2 = dt^2 - H^{-2} e^{(2Ht)} \underbrace{d\xi^2}_{\text{spatially flat. just a line}}$$

spatially flat universe
expanding exponentially



"time-sliced" hyperboloid in a different way

Now we are equipped to
do embedding in 5D minkowski space.

loads of algebra.

FRW metric with closed spatial sections:

$$ds^2 = dt^2 - H^{-2} \cosh^2(Ht)\left[d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

analogue to contracting - expanding universe.
closed spatial sections

can choose
time coord.,
change time
slicing: $ds^2 = dt^2 - R(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2)$

analogous to top of this page.

$R \propto \exp(Ht)$ $\quad H = \sqrt{\frac{\Lambda}{3}}$ $\quad k = H^2$

KEY POINTS

if we can timeslice resulting in

closed open flat sections

what does it mean to say that our Universe is approaching the de Sitter solution with flat spatial geometry?

keep in mind:

de Sitter sol is a vacuum solution

$\Longrightarrow$ no unique way of timeslicing

(why?)

existence of matter

fixes timeslicing.

no arbitrary choice,

matter distrib defines natural timeslicing

de Sitter sol is a max. ly symmetric spacetime (ie 4D)

with curvature K.

Think of it as a thing in 4D.

~~[illegible]~~ de Sitter is important for early & late time universe.