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(1) EQUATION OF HYDROSTATIC EQUILIBRIUM:

$$\frac{1}{\rho} \nabla p = -\nabla \psi$$

"PERFECT STATIC GAS" \Rightarrow HYDROSTAT. EQM. EQUATION APPLIES

CONSTANT $T = 300K \Rightarrow$ ISOTHERMAL

$$pV = nRT \Rightarrow p = \frac{n}{V} RT$$

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$$= \frac{m}{M} RT \Rightarrow p = \frac{m}{M} \frac{1}{V} RT$$

$$= \frac{\rho}{M} RT = C \rho$$

[BUNCH OF CONSTANTS, INCLUDING T.

$$\frac{1}{\rho} \nabla p = \frac{1}{\rho} \nabla (C \rho) = C \frac{1}{\rho} \nabla \rho = \underline{g} = -g \hat{z}$$

"UNIFORM GRAVITATIONAL FIELD"

LET'S ASSUME THAT ATMOSPHERE IS FAIRLY THIN, NOT REQUIRING ∇ TO BE EVALUATED IN SPHERICAL POLARS (IE WE SAY EARTH IS FLAT, $\nabla = \frac{\partial}{\partial z} \hat{z}$).

$$C \frac{1}{\rho} \frac{\partial \rho}{\partial z} = -g$$

$$C \ln \rho = -gz + K$$

$$S = K \exp\left(-\frac{gz}{C}\right)$$

WE WANT: $S|_{z=0} = S_0 \Rightarrow K = S_0$

PREVIOUSLY WE HAD:

$$C = \frac{S}{M} RT$$

I THINK MORE COMMON NOTATION IS:

$$C = \frac{1}{N} R_* T$$

(NOTING THAT $1000N = M$
 $R_* = 1000R$)

WE END UP WITH:

$$\underline{S = S_0 \exp\left(-\frac{\rho g}{R_* T} z\right)}$$

Fluid approximation breaks down when we have order unity particles per unit volume (how big this unit volume should be, though?)

Let's say, when $S = 1 \text{ m}^{-3}$.

$$\exp\left(-\frac{30 \cdot 10}{8300 \cdot 300} z\right) = \frac{1}{3 \cdot 10^{25}} \Rightarrow z = 3.3 \cdot 10^5 \text{ m}$$

$\hookrightarrow \underline{330 \text{ km}}$

Constant grav. breaks down when

$$g_{\text{TRUE}} = 0.9 g_{\text{ASSUMED}}$$

(I'm just making an estimation here.)

$$\left(\frac{R}{R_0}\right)^2 = \frac{1}{0.9} \Rightarrow R = 1.05 R_0$$

$$= 6720 \text{ km}$$

$$\uparrow R_0 \approx 6400 \text{ km}$$

$$\text{Height} = R - R_0$$

$$= 6720 - 6400 \approx \underline{320 \text{ km}}$$

Similar number as we got previously.

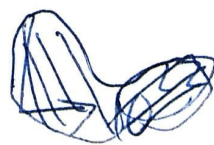
So similar in fact, that is probably has to do with us being lucky with the estimations, rather than physical reasons.

(ii)

$$\text{Ram pressure} = \rho u^2$$

$$u: \text{speed of Earth} = \frac{2\pi \cdot \text{sun-earth distance}}{365.24 \cdot 60 \cdot 60} = 3 \cdot 10^4 \frac{\text{m}}{\text{s}}$$

$$\rho u^2 \approx 10^5 \text{ Pa} \Rightarrow \rho = \frac{10^5}{u^2} \approx 10^{-4} \frac{\text{kg}}{\text{m}^3}$$



$$\underline{6 \cdot 10^{22} \text{ H per m}^3}$$