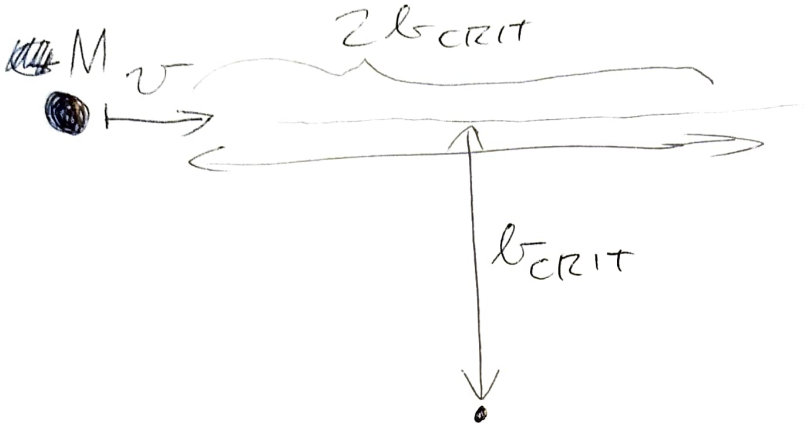
2018P3Q8

(ii)



IMPULSE APPROXIMATION:

PROXIMITY WITHIN TEST PARTICLE LASTS FOR: $\frac{2b_{CRIT}}{v}$

TEST PARTICLE ACCELERATES $\sim \frac{GM}{b_{CRIT}^2}$

Δv = TIME IN PROXIMITY × ACCELERATION

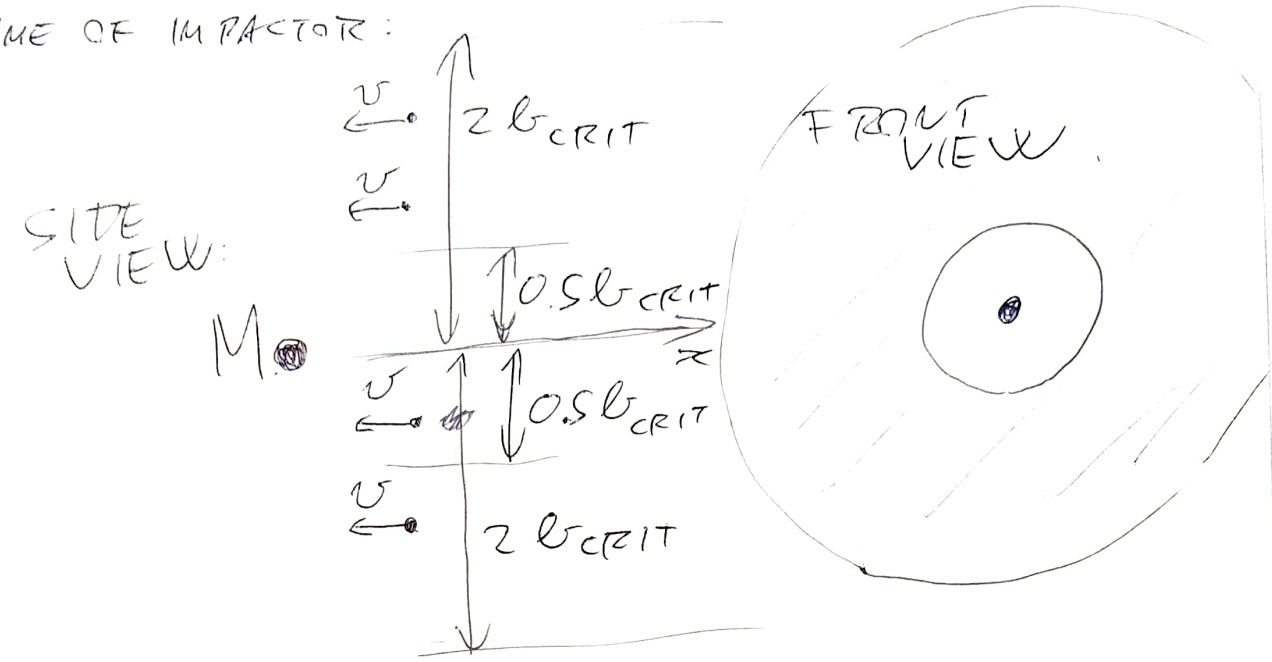
$$= \frac{2b_{CRIT}}{v} \frac{GM}{b_{CRIT}^2} = \frac{2GM}{v b_{CRIT}}$$

$\Delta v \sim v$

$$\Rightarrow b_{CRIT} = \frac{2GM}{v^2}$$

APPROX. EXPRESSION FOR DRAG FORCE.

FRAME OF IMPACTOR:



"DOMINATED BY IMPACT PARAMETER CLOSE TO b_{CRIT}":

APPROXIMATE BY CONSIDERING MATERIAL BTWN $0.5b_{CRIT}$ & $2b_{CRIT}$.

THEY LOSE ALL THEIR MOMENTUM PARALLEL WITH v TO M. (APPROXIMATE).

$$Mdv = -\rho \underbrace{\pi\left(2^2-0.5^2\right)b_{CRIT}^2}_{\text{AREA OF ANNULUS}} \cdot v\,dt \cdot S \cdot v$$

($\rho \sim 1$)

VOLUME SWIPED OUT BY ANNULUS IN TIME dt

MASS IN VOLUME

MOMENTUM OF MASS IN VOLUME

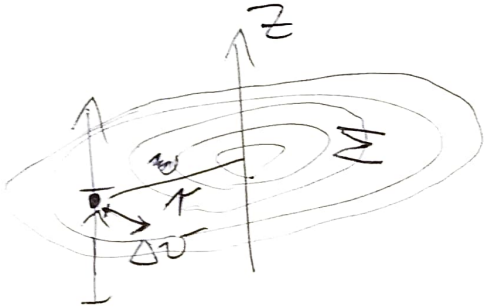
$$\Rightarrow M\frac{dv}{dt} \simeq -\rho_{crit}\, S v^2$$

$$\simeq -\left(\frac{GM}{v^2}\right)^2 S v^2$$

$$\Rightarrow \frac{dv}{dt} = -\frac{G^2 M S}{v^2}$$

(MINUS SIGN IS THERE TO INDICATE DIRECTION).

- "CONSIDER THE CASE"

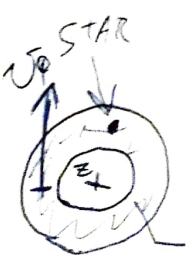


$L = \Delta v\, r$ = MASS OF DISC SIGNIFICANTLY EFFECTING THE STAR.

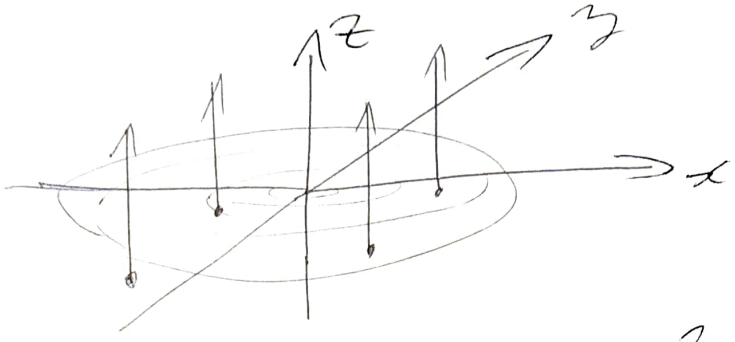
$$= \left(\frac{GM}{v^2}\right)^2 \Sigma$$

- VELOCITY CHANGE INDUCED PER UNIT MASS OF THIS MATERIAL · r

$$= \underbrace{\rho_{crit}^2 \Sigma}_{\text{MASS OF GAS HITTING STAR}} \cdot \underbrace{v_\phi}_{\text{MOMENTUM OVER UNIT MASS, IE SPEED OF GAS ENCOUNTERING STAR}} \cdot r = \underline{\left(\frac{GM}{v^2}\right)^2 \Sigma r v_\phi}$$



"NOW ASSUME"



ONE STAR ACQUIRES: $\Delta L \sim \left(\frac{GM}{v^2}\right)^2 \Sigma r v_\phi$

TOTAL ANGULAR MOMENTUM OF DISC INITIALLY:

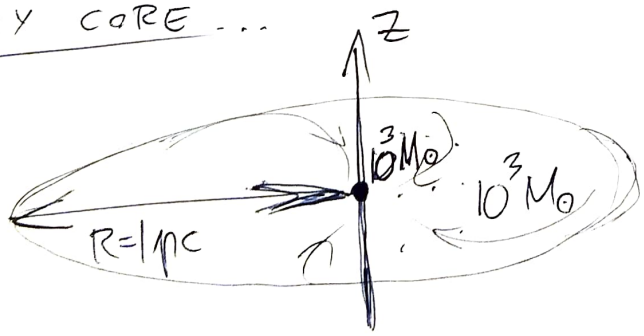
$$L = \int dL = \int_0^{R_{MAX}} 2\pi r (dr\,\Sigma) v_\phi \sim R_{MAX}^2 \Sigma v_\phi$$

USE: $r \sim R_{MAX}$, THEN

$$N = \frac{L}{\Delta L} = \frac{R_{MAX}^2 \Sigma v_\phi}{\left(\frac{GM}{v^2}\right)^2 \Sigma r v_\phi} = \frac{R_{MAX}}{\left(\frac{GM}{v^2}\right)^2} = \frac{R_{MAX}}{b_{crit}^2}$$

AND NOW WHAT?

GALAXY CORE ...



NUMBER OF STARS: HOW?

EVOLUTIONARY EFFECTS: H FIELDS BECOME DEPLETED?