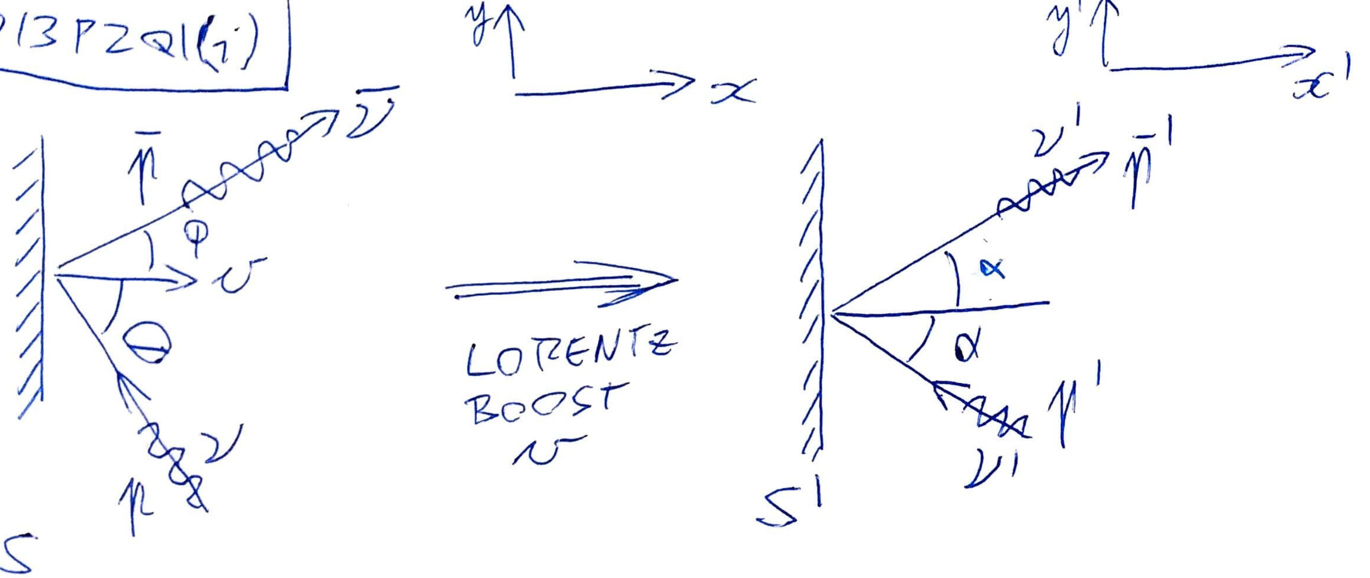
2013P2Q1(i)



$$p = \frac{h\nu}{c}\left(1, -\cos\theta, \sin\theta, 0\right)$$

$$\bar{p} = \frac{h\bar{\nu}}{c}\left(1, \cos\varphi, \sin\varphi, 0\right)$$

$=$ LORENTZ BOOST TO S' · REFLECT USING CLASSICAL RESULT · INVERSE BOOST BACK TO S · p

$$= \begin{pmatrix} \gamma & -\gamma\beta & \\ -\gamma\beta & \gamma & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & \\ \gamma\beta & \gamma & \\ & & 1 \end{pmatrix} \frac{h\nu}{c}\begin{pmatrix} 1 \\ -\cos\theta \\ \sin\theta \end{pmatrix}$$

REFLECT
x COMPONENT
KEEP OTHERS

$$= \begin{pmatrix} \gamma & \gamma\beta & \\ -\gamma\beta & -\gamma & \\ & & 1 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & \\ \gamma\beta & \gamma & \\ & & 1 \end{pmatrix} \frac{h\nu}{c}\begin{pmatrix} 1 \\ -\cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \gamma^2+\gamma^2\beta^2 & \gamma^2\beta+\gamma^2\beta & 0 \\ -\gamma^2\beta-\gamma^2\beta & -\gamma^2\beta^2-\gamma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{h\nu}{c} \begin{pmatrix} 1 \\ -\cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \gamma^2(1+\beta^2) & 2\gamma^2\beta & \\ -2\gamma^2\beta & -\gamma^2(1+\beta^2) & \\ & & 1 \end{pmatrix} \frac{h\nu}{c} \begin{pmatrix} 1 \\ -\cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \gamma^2(1+\beta^2)-2\gamma^2\beta\cos\theta \\ -2\gamma^2\beta+\gamma^2(1+\beta^2)\cos\theta \\ \sin\theta \end{pmatrix} \frac{h\nu}{c}$$

$$= \begin{pmatrix} 1 \\ \cos\varphi \\ \sin\varphi \end{pmatrix} \frac{h\bar{\nu}}{c} \Rightarrow \underline{\bar{\nu} = \nu\left[\gamma^2(1+\beta^2)-2\gamma^2\beta\cos\theta\right]}$$

$$\Rightarrow \cos\varphi = \frac{\nu}{\bar{\nu}}\left(-2\gamma^2\beta+\gamma^2(1+\beta^2)\cos\theta\right)$$

$$= \underline{\frac{-2\beta+(1+\beta^2)\cos\theta}{-2\beta\cos\theta+(1+\beta^2)}}$$