TOPICS II

1.

$$J_{\substack{EARTH\\SPIN}} = I\omega = \frac{2}{5} M_{EARTH} R_{EARTH}^2 \frac{2\pi}{P_{ROTATION}}$$

$$= \frac{2}{5} \cdot 6 \cdot 10^{24} \cdot \left(6.4 \cdot 10^6\right)^2 \frac{2\pi}{24 \cdot 60 \cdot 60}$$

$$= \underline{7.1 \cdot 10^{33}} \ kg\, m^2 \frac{1}{sec}$$

$$J_{\substack{MOON\\SPIN}} = I\omega = \frac{2}{5} M_{MOON} R_{MOON}^2 \frac{2\pi}{P_{\substack{ROTATION\\MOON}}}$$

$$= \frac{2}{5} \left(7.35 \cdot 10^{22}\right) \cdot \left(1740\right)^2 \cdot \frac{2\pi}{\underbrace{27.3}_{\text{LENGTH OF SIDEREAL MONTH IN DAYS}} \cdot 24 \cdot 60 \cdot 60}$$

$$= \underline{2.37 \cdot 10^{29}} \ kg\, m^2 \frac{1}{sec}$$

$$\frac{v^2}{R} = \frac{G M_{EARTH}}{R^2}$$

$$L_{ORBITAL} = M_{MOON}\, a_{EARTH-MOON}\, v_{MOON}$$

$$= M_{MOON}\, a \sqrt{\frac{G M_{EARTH}}{a}} = M_{MOON} \sqrt{G M_{EARTH}\, a}$$

$$= 7.35 \cdot 10^{22} \cdot \sqrt{G \cdot 6 \cdot 10^{24} \cdot 3.84 \cdot 10^{8}}$$

$$= 2.9 \cdot 10^{34} \ kg\, m^2 \frac{1}{sec}$$

i)

$$\frac{J_{\substack{EARTH\\SPIN}}}{L_{ORBITAL}} = \frac{7.1}{29} \simeq 0.24$$

$$\frac{J_{MOON}}{L_{ORBITAL}} = \frac{2.37}{2.9 \cdot 10^5} \simeq 8 \cdot 10^{-6} \quad \text{IE NEGLIGIBLE.}$$

WHEN SYSTEM BECOMES FULLY SYNCED:

$$\omega^{EARTH}_{SPIN} = \omega^{MOON}_{ORBITAL} \longrightarrow \Omega$$

E_ω

TO CONSERVE ANGULAR MOM:

$$I\omega + L_{ORBITAL} = \text{CONSTANT}$$

$$I\omega + M_M \sqrt{GM_E}\sqrt{a} = C$$

NOTING THAT:

$$\Omega^2 a = \frac{GM_{EARTH}}{a}$$

$$\sqrt{a} = \left(\frac{GM_{EARTH}}{\Omega^2}\right)^{\frac{1}{4}}$$

SUBSTITUTE THIS IN:

$$I\omega + M_M \sqrt{GM_E}\left(\frac{GM_{EARTH}}{\Omega^2}\right)^{\frac{1}{4}} = C$$

AT FULL SYNC, $\omega = \Omega = x$

$$I x + M_M \sqrt{G M_E} \left(G M_{EARTH}\right)^{\frac{1}{4}} x^{-\frac{1}{2}} = C$$

THE NUMBERS:

$$I = \frac{2}{5} M_{EARTH} R_{EARTH}^2 = \frac{2}{5} \cdot 6 \cdot 10^{24} \cdot \left(6.4 \cdot 10^6\right)^2 = 9.8 \cdot 10^{37} \ kg\, m^2$$

$$M_M \sqrt{G M_E} \left(G M_{EARTH}\right)^{\frac{1}{4}} = \cancel{5.9 \cdot 10^{34}} \ 6.6 \cdot 10^{33} \ kg\, m^2$$

$$C = 7.1 \cdot 10^{33} + 2.9 \cdot 10^{34} = 3.6 \cdot 10^{34}$$

REWRITE EQ.:

$$\cancel{9.8 \cdot 10^3 x + 5.9 \cdot 10 \ x^{-\frac{1}{2}} = 3.6}$$

$$9.8 \cdot 10^4 x + 6.6 \cdot x^{-\frac{1}{2}} = 36$$

$\boxed{\text{WHICH DOESN'T HAVE REAL SOLUTIONS.}}$ ⇒ STH IS WRONG.

~~REASON: WE EXPECT x TO BE ON THE ORDER~~ $\cancel{\frac{2\pi}{27 \cdot 24 \cdot 60 \cdot 60} \sim 10^{-6}}$

~~IGNORE FIRST TERM IN EQUATION~~

$$\cancel{\frac{1}{x^{\frac{1}{2}}} = \frac{36}{6.6}}$$

$$|x| = 1.54 \cdot 10^{-3}$$

$$\frac{2\pi}{|x|} = 4 \cdot 10^3 \ sec \cong 68 \ HOURS$$

WHICH IS VERY OFF

$$E_{\substack{\text{EARTH}\\ \text{SPIN}}} = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} \cdot \frac{2}{5} \cdot 6 \cdot 10^{24} (6.4 \cdot 10^6)^2 \left(\frac{2\pi}{24 \cdot 60 \cdot 60}\right)^2 = 6.5 \cdot 10^{29} \text{ J}$$

$$E_{\substack{\text{ORBITAL}\\ \text{POTENTIAL}}} = -\frac{G M_{\text{MOON}} M_{\text{EARTH}}}{a}$$

$$= -\frac{G \cdot 7.35 \cdot 10^{22} \cdot 6 \cdot 10^{24}}{\cancel{6.4 \cdot 10^6}\ 3.84 \cdot 10^8}$$

$$\cancel{= -4.6 \cdot 10^{30} \text{ J}} = -7.66 \cdot 10^{28}$$

COULD'VE JUST USED VIRIAL THM.

$$E_{\substack{\text{ORBITAL}\\ \text{KINETIC}}} = \frac{1}{2} M_{\text{MOON}} v^2 = \frac{1}{2} M \left(\frac{2\pi a}{P}\right)^2$$

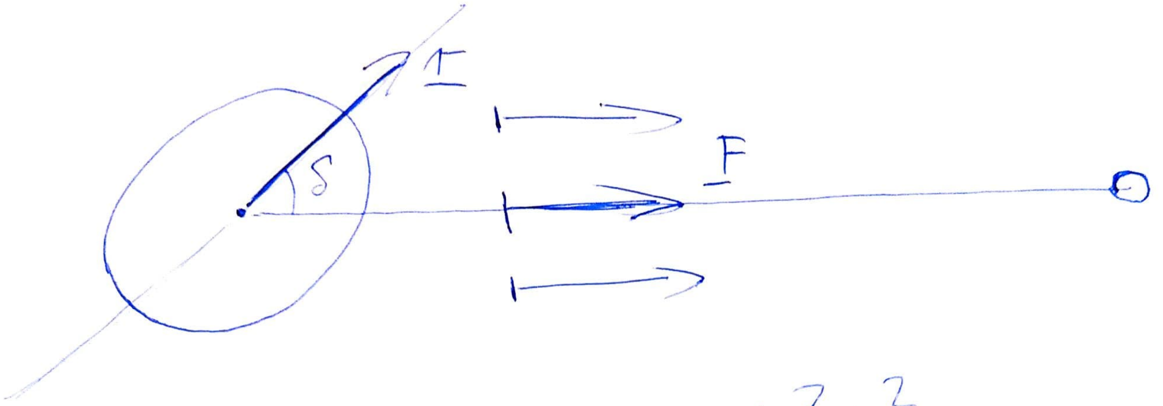
$$= \frac{1}{2}(7.35 \cdot 10^{22}) \cdot \left(\frac{2\pi \cdot 3.84 \cdot 10^8}{27 \cdot 24 \cdot 60^2}\right)^2 = 3.93 \cdot 10^{28}$$

$$E^{\text{ORBITAL}}_{\text{TOTAL}} \cong -3.9 \cdot 10^{28}$$

$$\frac{E_{\substack{\text{EARTH}\\ \text{SPIN}}}}{E^{\text{ORBITAL}}_{\text{TOTAL}}} = \frac{6.5 \cdot 10^{29}}{3.93 \cdot 10^{28}} \sim 16.5$$

$E^{\text{ORBITAL}}_{\text{TOTAL}}$ WILL NOT CHANGE TENFOLD, SO MOST E IS DISSIPATED.

TOPICS II
2



NOTES 2 PAGE 5: $E_{TIDAL} \sim \frac{G M_m^2 R_e^2}{R_{em}^6}$

TORQUE $=$ r $\times$ F

ENERGY IN "TORSION", IE ~~???~~ FROM TORQUE $=$ TORQUE $\cdot$ $\Delta\Theta$ — HOW MUCH EARTH IS TURNED BY TORQUE

$\propto$ TORQUE $\propto \underline{r} \times F \propto \sin\delta$

~~$G M_m^2 R_e^2 R_{em}^{-6} \propto$~~

$\Rightarrow$ TORQUE $\propto \sin\delta$

$\Rightarrow$ TORQUE $\propto$ ENERGY IN TORSION

$\Rightarrow$ TORQUE $\propto$ ENERGY IN TORSION $\cdot \sin\delta$

SOME CONSTANT

$\Rightarrow$ TORQUE $= f \frac{G M_m^2 R_e^2}{R_{em}^6} \sin\delta$

(I THINK THIS IS A VERY DODGY DERIVATION)

"ASSUMING THAT..."

$$\tau \propto \left(\frac{\text{TORQUE}}{\text{MOMENT OF INERTIA}}\right)^{-1}$$

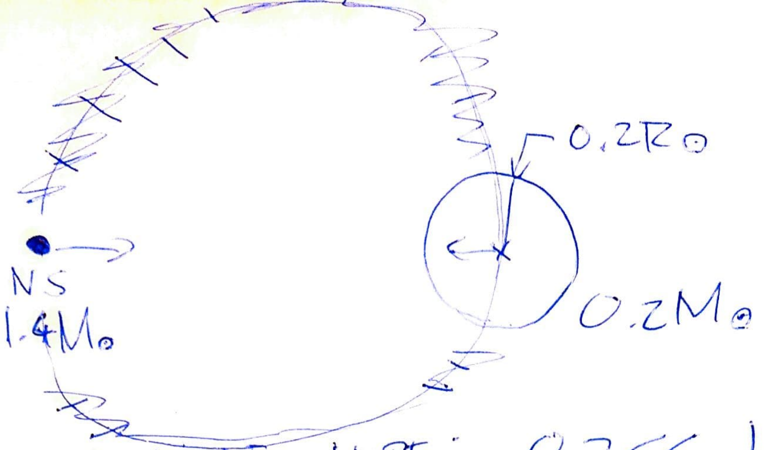
$$\frac{\tau_e}{\tau_m} \propto \left(\frac{\overbrace{M_m^2 R_e^5 \cancel{R_{em}^{-6}}}^{\propto \text{ TORQUE}} \Big/ \overbrace{M_e R_e^2}^{\propto I}}{M_e^2 R_m^5 \cancel{R_{em}^{-6}} \Big/ M_m R_m^2}\right)^{-1}$$

$$\propto \left(\frac{M_m^3 R_e^3}{M_e^3 R_m^3}\right)^{-1} \propto \left(\frac{M_m^3 M_e}{M_e^3 M_m}\right)^{-1} = \left(\frac{M_m}{M_e}\right)^{-2}$$

$$\propto \left(\frac{M_e}{M_m}\right)^{2} \gg 1 \Rightarrow$$ AGREES WITH OBSERVATION.

WE ALWAYS SEE SAME SIDE OF MOON, MOON SEES ALL SIDES OF EARTH.

$\boxed{3}$



WITH A BIT OF HOPE: $0.2 \ll 1.4$

NOTES2 PAGE 4: $R_{ROCHE} = \left(\frac{M_2}{3M_1}\right)^{\frac{1}{3}} a$

$$= \left(\frac{0.2}{3 \cdot 1.4}\right)^{\frac{1}{3}} a$$

$$= 0.362a$$

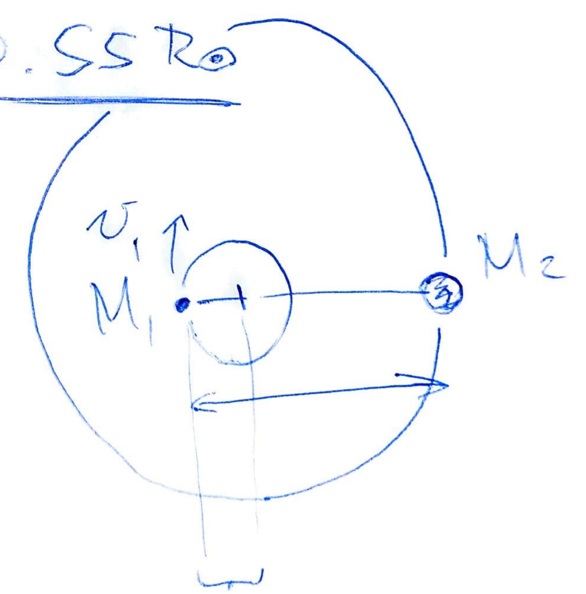
"FILLS ITS ROCHE LOBE" $\Rightarrow R_{ROCHE} = 0.2 R_\odot$

$$\Rightarrow a = \frac{0.2 R_\odot}{\cancel{R_{ROCHE}}\ 0.362} = \underline{0.55 R_\odot}$$

~~WHAT'S THE PERIOD?~~

$$\cancel{\frac{G M_1 M_2}{d^2} = \frac{v_1^2}{d} M_1}$$

$$\cancel{\frac{G M_2}{a^2} = v_1^2}$$



$$\frac{M_2}{M_1+M_2} a = \frac{0.2}{1.4+0.2} a$$

$$= 0.125a$$

WITHOUT LOSS OF GENERALITY, WE CAN SAY:

$$\frac{G \cancel{M_i} M_j}{a^{\cancel{2}}} = \frac{v_i^2}{\frac{M_j}{M_i+M_j}\cancel{a}} \cancel{M_i}$$

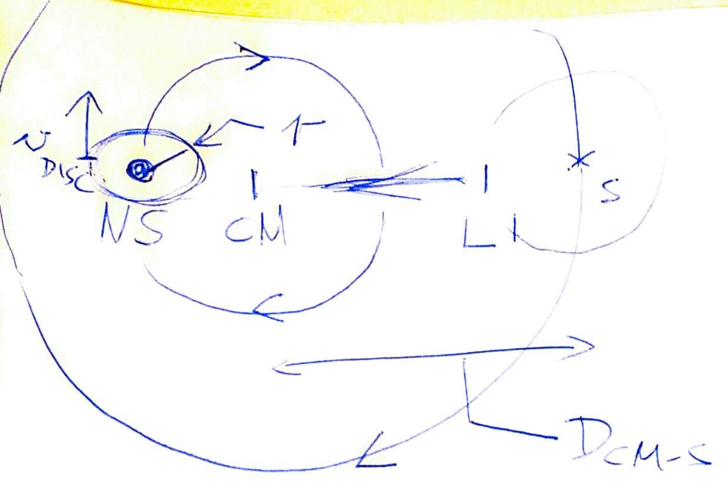
$$v_i^2 = G M_j^2 \frac{1}{M_i+M_j}\frac{1}{a}$$

$$P = \frac{2\pi a \frac{M_j}{M_i+M_j}}{v_i} = \frac{2\pi a}{\sqrt{G}} \frac{\frac{\cancel{M_j}}{M_i+M_j}\sqrt{M_i+M_j}\sqrt{a}}{\cancel{M_j}}$$

$$= \frac{2\pi a^{\frac{3}{2}}}{\sqrt{G}}\frac{1}{\sqrt{M_i+M_j}} = \frac{2\pi a^{3/2}}{\sqrt{G(M_i+M_j)}}$$

$$= \frac{2\pi (0.55 R_\odot)^{3/2}}{\sqrt{G(0.2+1.4)M_\odot}} = \cancel{\frac{2.45 \cdot 10^{16}\ \text{sec}}{(24 \cdot 60 \cdot 60)}}$$

$$= \underline{3.2 \cdot 10^{3}\ \text{SEC}}$$



ANGULAR MOMENTUM LOSS FROM SECONDARY:

$$\approx m_{STREAM} \cdot v_{SECONDARY} D_{CM-S}$$

$$\approx m_{STREAM} \underbrace{\sqrt{G} M_{NS} \sqrt{\frac{1}{M_S + M_{NS}}} \frac{1}{\sqrt{a}}}_{v_{SECONDARY}} \underbrace{\frac{M_{NS}}{M_{NS} + M_S} a}_{D_{CM-S}}$$

$$\approx m_{STREAM} \sqrt{G} M_{NS}^2 \sqrt{a} \frac{1}{(M_S + M_{NS})^{3/2}}$$

THIS GOES TO ANGULAR MOM OF DISC:

$$m_{STREAM} v_{DISC} r$$

NOTE: $\frac{v_{DISC}^2}{r} = \frac{GM_{NS}}{r^2} \Rightarrow v_{DISC} = \sqrt{\frac{GM_{NS}}{r}}$

EQUATE LOSS & GAIN:

$$\cancel{m_{STREAM}}\, v_{DISC}\, r = \cancel{m_{STREAM}} \sqrt{G}\, M_{NS}^{2} \sqrt{a}\, \frac{1}{\sqrt{(M_S+M_{NS})}^{\frac{3}{2}}}$$

$$\frac{\cancel{\sqrt{G}}\,\cancel{\sqrt{M_{NS}}}}{\sqrt{r}}\, r = \cancel{\sqrt{G}}\, M_{NS}^{\cancel{2}\,\frac{3}{2}} \sqrt{a}\, \frac{1}{\sqrt{(M_S+M_{NS})}^{\frac{3}{2}}}$$

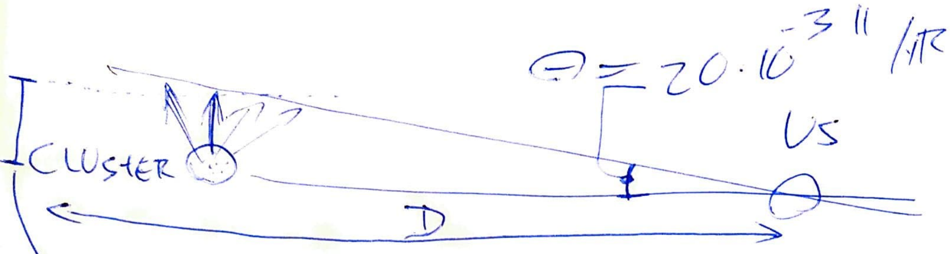
$$r = \left(\frac{M_{NS}}{M_S+M_{NS}}\right)^{3} a$$

$$r = \left(\frac{1.4}{1.4+0.2}\right)^{3} a$$

$$\underline{\underline{r \approx 0.67a}}$$

GIVEN THAT L1 IS $a-0.362a=0.638a$ AWAY FROM NS, THIS RESULT SEEMS TOO BIG.

$\boxed{5}$



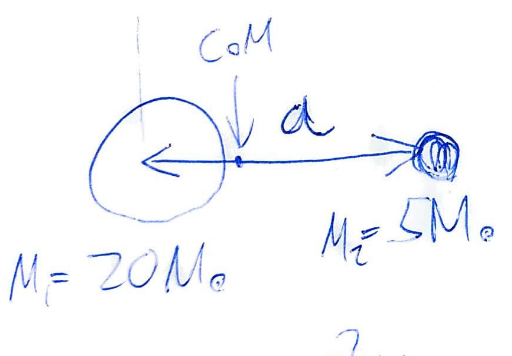
$$20 \cdot 10^{-3}{}'' = 5.55 \cdot 10^{-6}{}^\circ$$

$$d = D\Theta = 500 \cdot 5.55 \cdot 10^{-6} \cdot \frac{\pi}{180} = \underline{4.84 \cdot 10^{-5}} \frac{PC}{YR}$$

$$= 4.75 \cdot 10^{9} \frac{m}{yr}$$

THIS IS LOWER LIMIT.

REDSHIFT MEASUREMENTS OF CLUSTER & STAR WOULD MAKE THIS MORE ACCURATE, BY ALLOWING US TO CALCULATE PERPENDICULAR-TO-SKY VELOCITY COMPONENT.



$$\frac{GM_1M_2}{a^2} = \frac{v_1^2 M_1}{d(M_1, CoM)} = \frac{v_2^2 M_2}{d(M_2, CoM)}$$

$$d(M_i, CoM) = \frac{M_j}{M_i + M_j} a$$

$$\frac{GM_1M_2}{a^2} = \frac{v_1^2 M_1}{\frac{M_2}{M_1+M_2}a} = \frac{v_2^2 M_2}{\frac{M_1}{M_1+M_2}a}$$

$$= v_1^2 \frac{M_1}{M_2}(M_1+M_2)\frac{1}{a} = v_2^2 \frac{M_2}{M_1}(M_1+M_2)\frac{1}{a}$$

$$\Downarrow \qquad\qquad \Downarrow$$

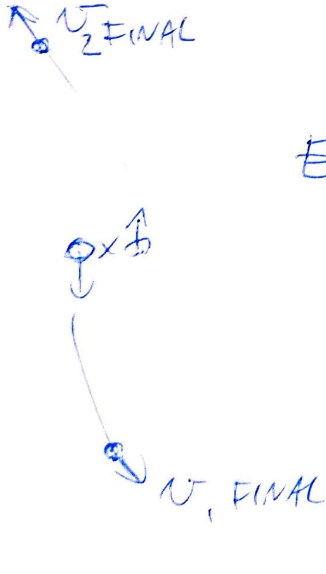
$$v_1^2 = \frac{GM_2^2}{M_1+M_2}\frac{1}{a} = \frac{G}{a}M_\odot \qquad \Big| \; v_2^2 = \frac{GM_1^2}{M_1+M_2}\frac{1}{a} = \frac{G}{a}16M_\odot$$

$M_1 = 20M_\odot$
$M_2 = 5M_\odot$

$M_1 = 20M_\odot$
$M_2 = 5M_\odot$

AFTER EXPLOSION

$M_1' = 1.4M_\odot$ $\qquad$ $M_2 = 5M_\odot$



~~ENERGY CONSERVATION~~

$$\cancel{\frac{1}{2}M_1'v_1^2 + \frac{1}{2}M_2v_2^2 = \frac{1}{2}M_1'v}$$

ENERGY CONSERVATION:

$$KE + PE\Big|_{EXPLOSION} = KE + PE\Big|_{LATER}$$

$$PE\Big|_{EXPLOSION} = \frac{GM_1' M_2}{a} \qquad PE\Big|_{LATER} \approx 0$$

$$\frac{1}{2}M_1' v_1^2 + \frac{1}{2}M_2 v_2^2 + \frac{GM_1' M_2}{a} = \frac{1}{2}M_1' v_1^{FINAL^2} + \frac{1}{2}M_2 v_2^{FINAL^2}$$

CONSERVATION OF LINEAR MOMENTUM

$$M_1' v_1^{FINAL} = M_2 v_2^{FINAL} \Rightarrow v_2^{FINAL} = \frac{M_1'}{M_2} v_1^{FINAL}$$

$$v_1^{FINAL} = \frac{M_2}{M_1'} v_2^{FINAL}$$

SUB IN FOR v_1 & v_2; M_1', M_2

$$\frac{1}{2}(1.4M_\odot)\frac{G}{a}M_\odot + \frac{1}{2}(5M_\odot)\frac{G}{a}16M_\odot + \frac{G\,1.4\cdot 5M_\odot^2}{a} =$$

$$\cancel{= \frac{1}{2}\,1.4M_\odot \left(\frac{1.4M_\odot}{5M_\odot}\right) \frac{1.4M_\odot}{5M_\odot}}$$

$$= \frac{1}{2}(1.4M_\odot)\left(\frac{5M_\odot}{1.4M_\odot} v_2^{FINAL}\right)^2 + \frac{1}{2}(5M_\odot)\, v_2^{FINAL^2}$$

$$47.7\frac{G}{a}M_\odot^{\cancel{2}} = 11.43\cancel{M_\odot}\, v_2^{FINAL^2}$$

$$4.173\frac{GM_\odot}{v_2^{FINAL^2}} = a$$

CHECK UNITS: $\frac{N\cancel{m^2}^{1}\,\frac{1}{kg^{\cancel{2}}}\,kg}{\left(\frac{m}{s}\right)^2} = \frac{kg\,\frac{m}{s^2}\,\frac{1}{kg}}{\left(\frac{1}{s}\right)^2} = m$ GOOD.

$$a = 4.173 \cdot \frac{G \cdot 2 \cdot 10^{30}}{(4.75 \cdot 10^{9})^2} = 2.47 \cdot 10^{11}$$

$$\simeq \underline{\underline{2.5 \cdot 10^{11}\,m}}$$

$$\simeq 1.6\,AU$$

$$v_1^{FINAL} = \underbrace{\frac{M_2}{M_1}}_{L} v_2^{FINAL}$$

$$L = \frac{5}{1.4} \simeq 3.6$$

$$v_{NS} = 3.6 \times v_{SECONDARY}$$

I WOULD LOOK 3.6 ARCMIN IN THE OTHER DIRECTION.