$\boxed{4.6(\mathrm{I})}$ • CIRCULAR ORBITS: $v_r = 0$ EVERYWHERE

$$S = \int f \, d\,\text{VELOCITY SPACE}$$

• USE SPC IN VELOCITY SPACE:

$$v_r = v\cos\alpha \qquad v: 0 \to \infty$$
$$v_\theta = v\sin\alpha\cos\beta \qquad \alpha: 0 \to \pi$$
$$v_\phi = v\sin\alpha\sin\beta \qquad \beta: 0 \to 2\pi$$

$$d^3\underline{v} = v^2 \sin\alpha \, dv \, d\alpha \, d\beta$$

$$S = \int_{v=0}^{\infty}\int_{\alpha=0}^{\pi}\int_{\beta=0}^{2\pi} f(\Psi - \tfrac{1}{2}v^2)\, v^2 \sin\alpha \, dv\, d\alpha\, d\beta$$

$$= \int_{v=0}^{\infty} f(\Psi - \tfrac{1}{2}v^2)\, v^2 dv \int_{\alpha=0}^{\pi} \sin\alpha\, d\alpha \int_{\beta=0}^{2\pi} d\beta = 4\pi \int_0^\infty f v^2 dv = \begin{cases} S_0 & r < R \\ 0 & r \geq R \end{cases}$$

WE WANT

WE HAVE:

$$\int_0^\infty f v^2 dv = \begin{cases} \dfrac{S_0}{4\pi} & r < R \quad \text{CASE I} \\ 0 & r \geq R \quad \text{CASE II} \end{cases}$$

~~CASE I:~~

~~let~~ $\cancel{f(r<R) = Cv^n}$

~~then:~~

$$\cancel{\int_0^\infty C v^n v^2 dv = C\left[\frac{1}{n+3} v^{n+3}\right]_0^\infty}$$

SDSG
4.6(II)

CASE I:

let f be such:
$$\lim_{v\to\infty}\left(fv^2\right)=0 \quad \& \quad \lim_{v\to 0}\left(fv^2\right)=0$$

then:
$$\int_0^\infty fv^2\,dv \text{ is CONVERGENT.}$$

Scale f so that: $\int_0^\infty fv^2\,dv=\frac{S_0}{4\pi}$

Such f certainly exists.

CASE II:
$$\int_0^\infty fv^2\,dv=0 \quad \forall r \geq R$$
let $f=0$ & then this is satisfied.

"Do you think a similar distribution function might be constructed for any density distribution $S(r)$?"

Certainly not, for example: some $S(r)$ distributions create potentials ~~other~~ which make circular orbits not possible, because they are so steep.

DSG
4.6 (III)

How to test DF of globular clusters?

- if they are close & we can resolve them: use luminosity - matter relations
- if we cannot resolve them: obtain upper bound for speeds of stars on circular orbit, compare results with redshift measurements (?)