(r). TWO CONDITIONS:

- LAWS OF PHYSICS ARE THE SAME IN ALL INERTIAL FRAMES (ESPECIALLY: LENGTH & TIME MEASUREMENTS)

- SPEED OF LIGHT IS CONSTANT IN ALL INERTIAL FRAMES.

When we actually do the lorentz transform, we have to take care to align our axes correctly.

• EQUATION OF MOTION

Acceleration 4-vector: $a^\mu = \dfrac{Du^\mu}{D\tau}$ where u^μ is the 4-velocity.

4-VELOCITY & 4-ACCELERATION ARE PERPENDICULAR.

$$\text{\textcircled{$\blacksquare$}} \quad g_{\mu\nu}\, a^\mu u^\nu = 0$$

$$\frac{Du^\mu}{D\tau} = \frac{du^\mu}{d\tau} \qquad \text{IN CARTESIAN COORDINATES.}$$

NOTING THAT:
$$u^\mu = \gamma_u (c, \vec{u})$$

AND THAT:
$$dt = \gamma_u\, d\tau$$

REWRITE:

$$\frac{du^{\nu}}{d\tau} = \frac{d}{\left(\frac{dt}{\gamma_u}\right)}\left(c\,\gamma_u,\ \vec{u}\,\gamma_u\right)$$

$$= \gamma_u \frac{d}{dt}\left(c\,\gamma_u,\ \vec{u}\,\gamma_u\right)$$

EXPAND DERIVATIVE:

$$\frac{d}{dt}\gamma_u = \frac{d}{dt}\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} = -\frac{1}{2}\left(1-\frac{\vec{u}\cdot\vec{u}}{c^2}\right)^{-\frac{3}{2}}\frac{-2\vec{u}}{c^2}\frac{d\vec{u}}{dt}$$

$$= \frac{\gamma_u^{+3}}{c^2}\,\vec{u}\cdot\vec{a}$$

ARRIVE TO:
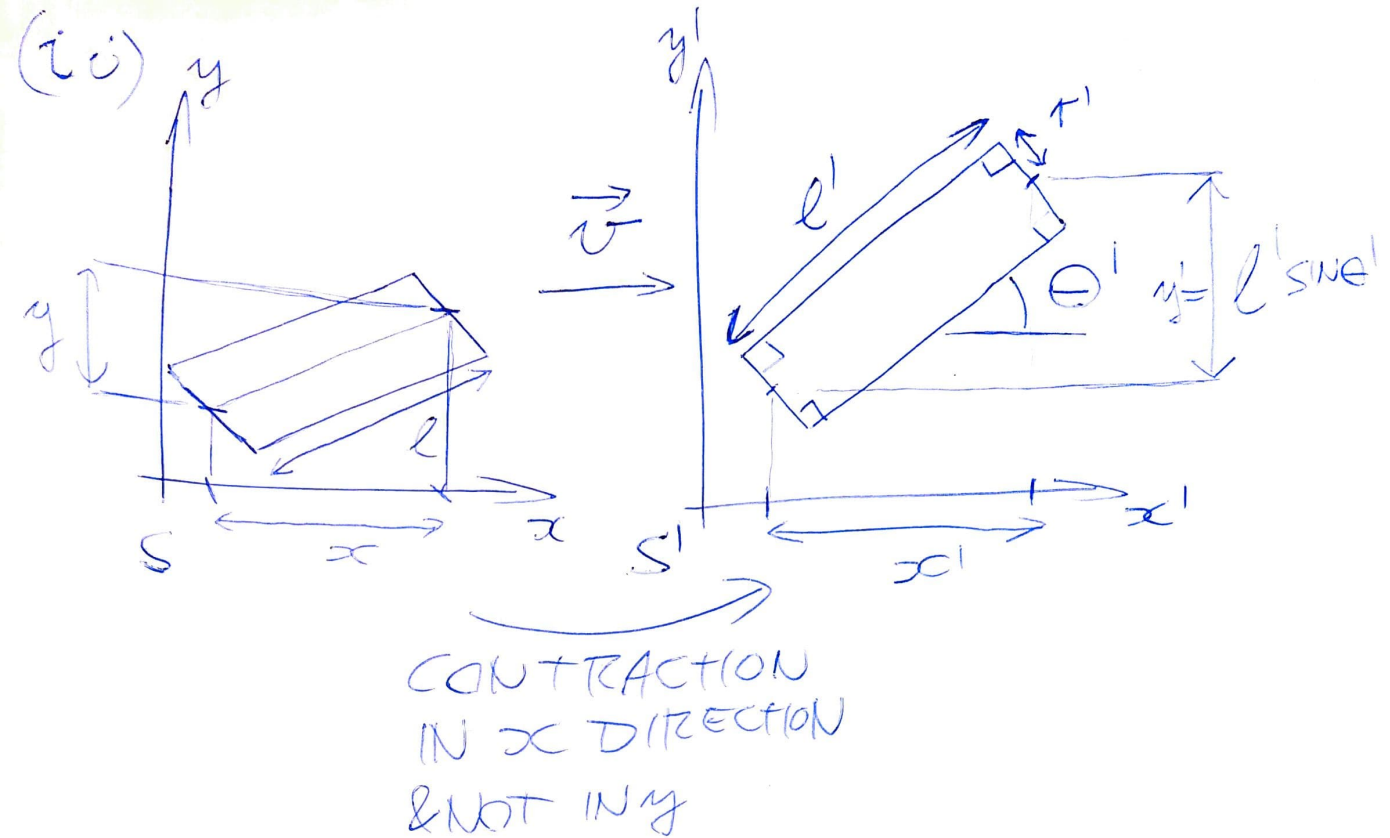
$$\frac{du^{\nu}}{d\tau} = \cancel{\gamma_u \frac{d}{dt}}$$

$$= \gamma_u\left(\frac{\gamma_u^3}{c^2}\,\vec{u}\cdot\vec{a}\cdot c,\ \gamma_u\vec{a} + \vec{u}\,\frac{\gamma_u^3}{c^2}\,\vec{u}\cdot\vec{a}\right)$$

$$= \gamma_u^2\left(\frac{\gamma_u^2}{c}\,\vec{u}\cdot\vec{a},\ \vec{a} + \frac{\gamma_u^2}{c^2}(\vec{u}\cdot\vec{a})\,\vec{u}\right)$$

IF MOVING ALONG z AXIS, $\vec{u} = dz/dt$.

IS THIS AN EQUATION OF MOTION THOUGH?

(i ⅴ)

CONTRACTION
IN x DIRECTION
& NOT IN y

$$l = l(l', \theta', \beta)$$

$$x' \gamma = x$$

CHECK:

x' IS THE LENGTH CONTRACTED,
SO $x' < x$, $\gamma > 1$, SO
SMALLER x GAMMA = BIGGER.

COORDINATE DISTANCES ALONG y
DIRECTION IS INVARIANT.

$$l^2 = (x' \gamma)^2 + (l' \sin \theta')^2$$

$$\Rightarrow l = \left(l'^2 \cos^2 \theta' \left[\frac{1}{1 - \beta^2} \right] + l'^2 \sin^2 \theta' \right)^{\frac{1}{2}}$$
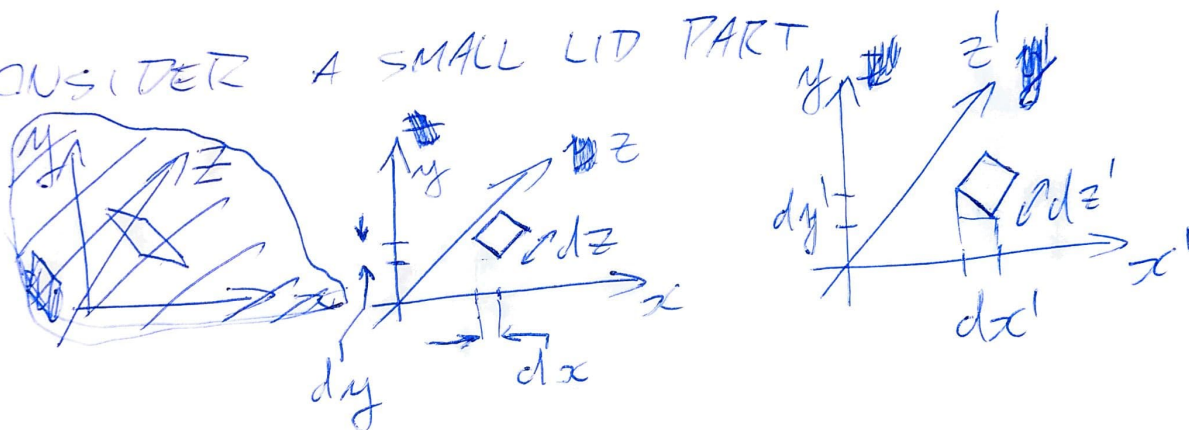
"DRAW THE CYLINDER" BIT:

SEE PREVIOUS PAGE.

Explanation: I elongated it in S along x direction so when it is length contracted in S', it forms a proper cylinder.

"DERIVE AN EXPRESSION FOR THE AREA"

CONSIDER A SMALL LID PART

$$dy = dy'$$
$$dz = dz'$$
$$dx = dx' \bigcirc \cdot \gamma$$

$$dA = \sqrt{dx^2 + dy^2}\, dz$$

$$dA' = \sqrt{dx'^2 + dy'^2}\, dz'$$

$$A = A' \frac{dA}{dA'} = \gamma'^2 \pi \frac{\sqrt{dx^2 + dy^2}}{\sqrt{dx'^2 + dy'^2}}$$

$$= \gamma'^2 \pi \frac{\sqrt{dx'^2 \gamma^2 + dy'^2}}{\sqrt{dx'^2 + dy'^2}}$$
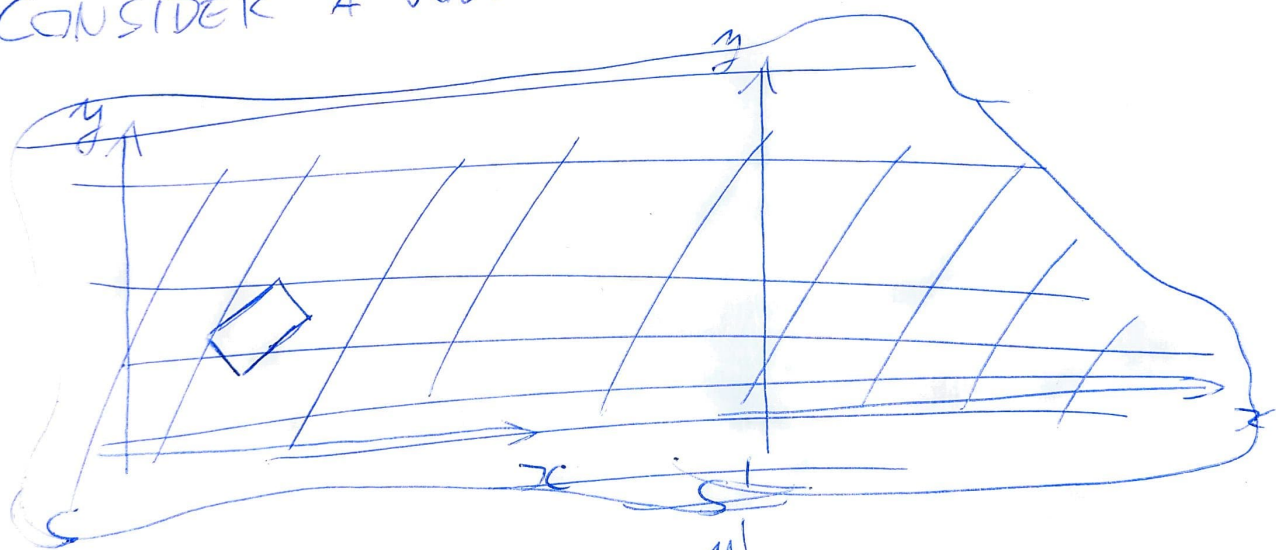
$$= A'^2 \pi \frac{\sqrt{dx'^2 \gamma^2 + \tan^2\theta' dx'^2}}{\sqrt{dx'^2 + \tan^2\theta' dx'^2}}$$

$$= A'^2 \pi \frac{\gamma^2 + \tan^2\theta'}{\sqrt{1 + \tan^2\theta'}}$$

$$= A'^2 \pi \left(\gamma^2 + \tan^2\theta'\right) \cos\theta'$$

$$= A'^2 \pi \left(\frac{1}{1-\beta^2} + \tan^2\theta'\right) \cos\theta'$$
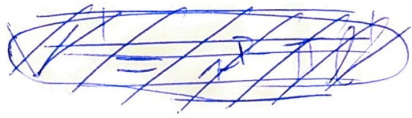
CONSIDER A VOLUME ELEMENT:

VOLUME ELEMENT FROM THE SIDE.

OUR VOLUME ELEMENT LOOKED FROM THE SIDE
GETS CONTRACTED BY A FACTOR OF γ
IN ONE DIRECTION.

OTHER DIRECTIONS ARE UNCHANGED.

$$V' = r'^2 \pi \ell'$$

$$V = \gamma \, r'^2 \pi \ell'$$

$$V = \gamma V' = \frac{1}{\sqrt{1-\beta^2}} V'$$

THOUGHTS:

— HAVEN'T USED EXPRESSION IN HINT.

— MY COORDINATE SYSTEM ENDED UP BEING
LEFT-HANDED, SORRY.