STARS SHEET IV | Q 1

(i)

$$L = 4\pi R^2 \sigma T^4$$

$$\Rightarrow \log\left(\frac{L}{L_\odot}\right) = \log\left(\frac{4\pi R^2 \sigma T^4}{L_\odot}\right)$$

$$= \log\left(\frac{4\pi R^2 \sigma}{L_\odot}\right) + 4 \log T$$

$\Rightarrow$ SLOPE IS 4.

FIGURE SLOPE:

← NOTE THAT THIS GOES BACKWARDS (YAY, WE'RE ASTRONOMERS)

(ii) $\frac{L}{L_\odot} = 0.01 \Rightarrow \log\left(\frac{L}{L_\odot}\right) = -2$

Radius, reading off from the plot $\approx 0.05 R_\odot$

CHECK: (USING (4.33)

$$R = \frac{3}{2}\left(\frac{6\pi^2}{g^2}\right)^{\frac{1}{3}} \frac{\hbar^2}{G m_e (\mu m_p)^{5/3}} M^{-\frac{1}{3}} = 5 \cdot 10^{7} m$$

$: 7 \cdot 10^8 \rightarrow 0.07 R_\odot$

$M = 0.63 \cdot 2 \cdot 10^{30} kg$

$g = N = 2$

GOOD AGREEMENT WE HAVE.

Q2

(i) DEF OF DECAY CONSTANT:

$$N(t) = N_0 \exp(-\lambda t)$$

"DOMINATED BY" $\Rightarrow N \propto L$

$$L(t) = c N_0 \exp(-\lambda t) \quad \text{FOR SOME CONSTANT } c$$

$$\ln L = \ln\left(c N_0 \exp(-\lambda t)\right)$$

$$= \ln(c N_0) + \ln\left(e^{-\lambda t}\right)$$

$$= \ln(c N_0) - \lambda t$$

NOTING THAT:

$$\ln x = \log(x) / \log e$$

REWRITE:

$$\log L = \ln(c N_0) \log e - \log e \, \lambda t$$

$$\frac{d \log L}{dt} = -\log e \, \lambda$$

$$\frac{d \log L}{dt} = -0.434 \lambda$$

AS WANTED.

(ii)

$$M_{BOL} = -2.5 \log_{10} \frac{L}{L_{REFERENCE}} = -2.5 \log L + \text{Constant}$$

$$\Rightarrow \frac{d M_{BOL}}{dt} = -2.5 \frac{d \log L}{dt} = -2.5 \cdot -0.434 \lambda = 1.086 \lambda$$

Q2

(iii) $0.075 M_\odot \xrightarrow{\cdot 2\cdot 10^{30}} 1.5 \cdot 10^{29}\ \text{kg}\ {}^{56}\text{Co}$

$$\Big\downarrow \cdot \frac{1000}{56}$$

$$N_0 \approx 2.7 \cdot 10^{30}\ \text{mol Co}$$

IN FIRST x MINUTES, THIS MANY COBALT DECAYS:

$$D = N_0 - N_0 \exp(-\lambda x) = N_0(1 - \exp(-\lambda x))$$

IF EACH RELEASES ENERGY E_{REL}, TOTAL E PER UNIT TIME, IE LUMINOSITY, IMMEDIATELY AFTER FORMATION:

$$L = \lim_{x \to 0} \frac{D E_{REL}}{x} \cancel{\frac{N_0 - N_0 \exp(-\lambda x)}{x}}$$

$$= \lim_{x \to 0} \frac{N_0(1 - \exp(-\lambda x))}{x} E_{REL}$$

$$= N_0 \lambda E_{REL} = N_0 \frac{\ln 2}{\tau_{1/2}} E_{REL}$$

$$\cancel{= 2.7 \cdot 10^{30} \cdot \frac{\ln 2}{77.7 \cdot 24 \cdot 60 \cdot 60} \cdot 3.72}$$

$$= 2.7 \cdot 10^{30} \cdot \underbrace{6 \cdot 10^{23}}_{\text{mol} \to \text{HOW MANY ACTUALLY}} \cdot \frac{\ln 2}{77.7 \cdot \underbrace{24 \cdot 60 \cdot 60}_{\text{DAY} \to \text{SEC}}} \cdot \overbrace{3.72 \cdot 1.6 \cdot 10^{-6}}^{\text{MeV} \to \text{erg}}$$

$$\simeq 9.96 \cdot 10^{41} \simeq \underline{10^{42}} \frac{erg}{sec}$$

(iv)

One year after, luminosity is still proportional to how much Cobalt we've left, so:

$$L_{1YR} = L_0 \cdot \exp\left(-\lambda \cdot t\big|_{1YR}\right)$$

$$= 10^{42} \exp\left(-\ln 2 / 77.7 \cdot 365\right)$$

$$= 10^{42} \cdot 0.0385 \simeq \underline{4 \cdot 10^{40}} \frac{erg}{sec}$$

Q3

(i) HYDROSTAT EQUIL.:

$$\frac{dP}{dr} = -G\frac{\rho}{r^2}M_r$$

$$= -G\frac{\rho}{r^2}\int_0^r \rho(r')4\pi r'^2 dr' \quad / \text{REARRANGE}$$

$$\frac{1}{\rho}r^2\frac{dP}{dr} = -4\pi G\int_0^r \rho(r') r'^2 dr' \quad / \frac{d}{dr}$$

$$/ \div r^2$$

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho r^2$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

AS REQUIRED.

(ii)

$$P = K\rho^\gamma \Rightarrow \frac{dP}{dr} = K\frac{d}{dr}(\rho^\gamma) = K\gamma\rho^{\gamma-1}\frac{d\rho}{dr}$$

USING EQ. ABOVE:

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}K\gamma\rho^{\gamma-1}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

$$K\gamma\frac{1}{r^2}\frac{d}{dr}\left(r^2\rho^{\gamma-2}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

$$K\gamma\frac{1}{r^2}\left(2r\rho^{\gamma-2}\frac{d\rho}{dr} + r^2(\gamma-2)\rho^{\gamma-3}\left(\frac{d\rho}{dr}\right)^2 + r^2\rho^{\gamma-2}\frac{d^2\rho}{dr^2}\right) = -4\pi G\rho$$

$$K\gamma\left(\frac{2}{r}\rho^{\gamma-2}\frac{d\rho}{dr}+(\gamma-2)\rho^{\gamma-3}\left(\frac{d\rho}{dr}\right)^2+\rho^{\gamma-2}\frac{d^2\rho}{dr^2}\right)=-4\pi G\rho$$

$$\cancel{K\gamma\left(\frac{2}{r}\frac{d\rho}{dr}+\frac{d^2\rho}{dr^2}\right)\rho^{\gamma-2}+K\gamma(\gamma-2)\rho^{\gamma-3}\left(\frac{d\rho}{dr}\right)^2=-4\pi G\rho}$$

AND NOW IDK WHAT,
PROBABLY THERE'S EASIER WAY THAN
SOLVING THIS ABOVE.

Q4

$\rho_0 \cong 150 g/cm^3 \rightarrow 150 \, mol/cm^3 \; p^+ \& e^-$

Interparticle separation: D

$\frac{4}{3}\left(\frac{D}{2}\right)^3 \pi \cdot N \cong 1 \, cm^3$

N: PARTICLES IN $1 cm^3$

DE BROGLIE WAVELENGTH:

$E = \frac{3}{2} kT = \frac{1}{2}\frac{p^2}{m}$ — MASS OF AN ELECTRON

$\Rightarrow p = \sqrt{3kTm}$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3kTm}} = \underline{2.8 \cdot 10^{-11} m}$

$D^3 \cong \frac{6}{\pi}\frac{1}{N}$

$D \cong \sqrt[3]{\frac{6}{\pi N}} \cong \sqrt[3]{\frac{2}{N}}$

$= \sqrt[3]{\frac{2}{6 \cdot 10^{23} \cdot 150}} \cong 3 \cdot 10^{-9} cm$

$\cong \underline{3 \cdot 10^{-11} m}$

COMPARING THE TWO RESULTS WE SAY THAT YES, ELECTRONS ARE SOMEWHAT DEGENERATE.

(ii) $E = \frac{1}{2}\frac{p^2}{m} \Rightarrow p = \sqrt{2mE}$

$\lambda = \frac{h}{\sqrt{2mE}} \cong \cancel{7.5 \cdot 10^{-30} m} \; \underline{7.8 \cdot 10^{-12} m}$

Q5

(i)

$$^{12}C + {}^{16}O \longrightarrow \frac{1}{2}{}^{56}Ni$$

• MASSES OF THESE PARTICLES:

$$m(^{12}C) = m(12,6) = 12 \cdot m_u + m_{ex}\big|_{^{12}C}$$

$$= 12 \cdot 931.5 \frac{MeV}{c^2} + 0 = 1.12 \cdot 10^{10} \frac{eV}{c^2}$$

$$m(^{16}O) = m(16,8) = 16 \cdot m_u + m_{ex}\big|_{^{16}O}$$

$$= 16 \cdot 931.5 \frac{MeV}{c^2} + (-4.7) \frac{MeV}{c^2} = 1.49 \cdot 10^{10} \frac{eV}{c^2}$$

$$m(^{56}Ni) = m(56,28) = 56 \cdot m_u + m_{ex}\big|_{^{56}Ni}$$

$$= 56 \cdot 931.5 \frac{MeV}{c^2} - 53.9 \frac{MeV}{c^2} = 5.21 \cdot 10^{10} \frac{eV}{c^2}$$

• ENERGY VIEW:

$$E_{RELEASED} = E_{BEFORE} - E_{AFTER} = \left[m(^{12}C) + m(^{16}O) - \frac{1}{2}m(^{56}Ni)\right]c^2$$

$$= 5 \cdot 10^{7}\ eV \quad \text{(PER REACTION)}$$

• HOW MANY REACTIONS WE HAVE

$$1.4\ M_\odot \longrightarrow 1.4 \cdot 2 \cdot 10^{30}\ kg \xrightarrow{:(1.78 \cdot 10^{-36})} 1.57 \cdot 10^{66} \frac{eV}{c^2}$$

$$\xrightarrow{:5.21 \cdot 10^{10}} \sim 3 \cdot 10^{55}\ \text{REACTIONS}$$

• TOTAL E RELEASED:

$$E_{TOT} = \text{NUMBER OF REACTIONS} \cdot \text{ENERGY PER REACTION} = 3 \cdot 10^{55} \cdot 5 \cdot 10^{7} = 1.5 \cdot 10^{63}\ eV$$

(ii)

$$E_{RELEASED} = E_{BINDING} + E_{KINETIC}$$
$$= E_{BINDING} + \frac{1}{2} m v^2$$

$$v = \sqrt{2 \frac{E_{RELEASED} - E_{BINDING}}{m}}$$

CONVERSION TO J FROM eV

$$= \sqrt{2 \frac{1.5 \cdot 10^{63} \cdot 10^{-6} \cdot 1.6 \cdot 10^{-13} - 5 \cdot 10^{43}}{1.4 \cdot 2 \cdot 10^{30}}}$$

CONVERSION TO kg FROM SOLAR MASS

$$\simeq 1.2 \cdot 10^{7} \frac{m}{s}$$

QUICK GOOGLING IF THIS IS CONSISTENT WITH REAL RESULTS: YES, WITHIN ORDER OF MAGNITUDE.

Q6

(i) ASSUMPTIONS:

- ANGULAR MOMENTUM IS CONSERVED (NO WIND LOSS)
- ROTATIONAL KINETIC ENERGY CONSERVED

$$\frac{1}{2} I_{EARLY}\, \omega_{EARLY}^2 = \frac{1}{2} I_{LATE}\, \omega_{LATE}^2$$

$$\omega_{LATE} = \sqrt{\frac{I_{EARLY}}{I_{LATE}}}\ \omega_{EARLY}$$

~SOMEWHAT DODGY ARGUMENT

NOTING THAT

$$I = \int_{SPHERE} dm\, R^2$$

WE DO AN INTEGRAL / LOOK IT UP, ARRIVE TO:

$$I = M R^2 \cdot C$$ WHERE C IS SOME CONSTANT DEPENDING ON $\rho(r)$

SUBSTITUTE IN

$$\omega_{LATE} = \frac{R_{EARLY}}{R_{LATE}}\, \omega_{EARLY}$$

(ii)

$$\omega_{LATE} = \frac{7 \cdot 10^8}{10^7} \cdot \frac{1}{28} = \underline{\underline{2.5}}\ \frac{RAD}{DAY}$$

THIS IS LIKELY BE UPPER LIMIT, SINCE THE "NO ANGULAR MOMENTUM LOSS DUE TO WIND" WASN'T TRULY PHYSICAL ASSUMPTION.

(iii)

$$T = \frac{2\pi}{\omega} = 2\pi \frac{R_{LATE}}{R_{EARLY}} \frac{1}{\omega_{EARLY}}$$

$$= 2\pi \frac{10^4}{10^7} \frac{1}{28} \approx 2.2 \cdot 10^{-4} \text{ DAY}$$

$\cdot 24 \cdot 60 \cdot 60$

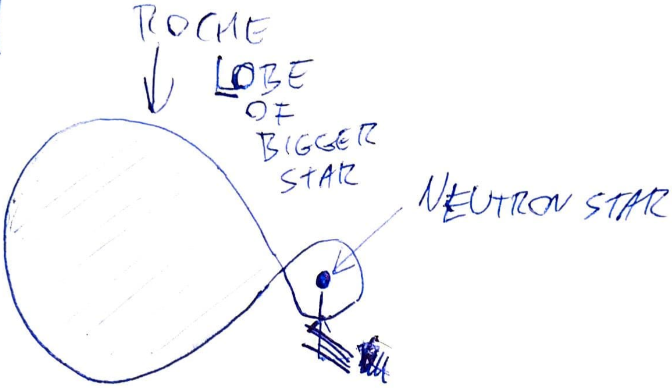
$\rightarrow \approx 20$ SEC

GOOGLING "PULSAR TYPICAL ROTATION PERIOD":

I AM AN ORDER OF MAGNITUDE HIGHER THAN WHAT WOULD BE IDEAL.

MAYBE THE HOMOLOGY ARGUMENT WAS MORE UNPHYSICAL THAN "NO WIND", MAKING MY RESULT A LOWER LIMIT AFTER ALL?

Q7



(DOES THE BIGGER STAR HAVE THIS WEIRD SHAPE DRAWN?)

(i)

Changing separation:

Roche lobes will overlap more
⇒ separation will decrease more

i.e we have a runaway process.

(ii)

~~THE GAS WOULD~~

ASSUMING I'M RIGHT ABOUT THAT THE DONATED GAS IS TURNED INTO ENERGY & RADIATED AWAY, ~~SUCH~~ LIKE IN A QUASAR:

$$L_{ACC} \sim \underbrace{\frac{1}{2}}\frac{G M_{NS} \dot{M}_{DONATION}}{R}$$

IS THIS HERE BEC. ENERGY PARTLY GOES TO HEAT, PARTLY TO RADIATION, & WE SAY ITS ~50 - 50 %?

$$= \frac{1}{2} G \frac{1.4 \cdot 2 \cdot 10^{30} \cdot \frac{10 \cdot 2 \cdot 10^{30}}{10^4 \cdot 365 \cdot 24 \cdot 60 \cdot 60}}{10^4} \approx \underline{6 \cdot 10^{35} \frac{J}{s}}$$

Q8

(i)

TOP-HEAVY MEANS:
MORE HIGH MASS STARS THAN
LOW MASS STARS

(ii)

TAKE A GLOBULAR CLUSTER WITH TOP-HEAVY MASS-FUNCTION
& MEASURE OXYGEN & IRON IN THE INTERSTELLAR MEDIUM.

DO THE SAME FOR A NOT TOP-HEAVY GLOBULAR
CLUSTER.

(OXYGEN IN ISM MAINLY PUT THERE BY HEAVY STARS,
UNLIKE IRON, WHICH IS PRODUCED BY LOW-MASS
STARS)

COMPARE SPECTRA OF THE ABOVE TWO MEASUREMENTS WITH THAT OF THE DISTANT GALAXY'S
ISM: WHICH ONE IS IT MORE SIMILAR TO?

(I DON'T KNOW HOW TO MEASURE ISM SPECTRA
OF DISTANT GALAXY THOUGH, WITHOUT GETTING
LIGHT FROM THE STARS THEMSELVES THOUGH)

$\boxed{\text{Q9}}$

$$N(M) = M^{-\alpha}\Big|_{\alpha=-2.35}$$

(i)

AFTER 10 GYR, STARS HEAVIER THAN $1\,M_\odot$ HAVE DIED.

$$M_{\substack{\text{INITIAL}\\\text{STARS}\\\text{PER UNIT}\\\text{VOLUME}}} = \int_{0.1}^{60} N(M)\cdot M\,dM$$

$$= \int_{0.1}^{60} M^{-\alpha}\cdot M\,dM = \int_{0.1}^{60} M^{-\alpha+1}\,dM = \left[\frac{1}{-\alpha+2}M^{-\alpha+2}\right]_{0.1}^{60}\Bigg|_{\alpha=2.35}$$

$$= 5.71\,M_\odot$$

$$M_{\substack{\text{10 GYR}\\\text{LATER}\\\text{PER UNIT}\\\text{VOLUME}}} = \int_{0.1}^{1} N(M)M\,dM + \int_{1}^{8} N(M)M\frac{1}{5}\,dM + \int_{8}^{60} N(M)\,1.4\,dM$$

$$= \int_{0.1}^{1} M^{-\alpha+1}\,dM + \int_{1}^{8} M^{-\alpha+1}\,dM\frac{1}{5} + \int_{8}^{60} M^{-\alpha}\cdot dM\cdot 1.4$$

$$= \left[\frac{1}{-\alpha+2}M^{-\alpha+2}\right]_{0.1}^{1} + \left[\frac{1}{-\alpha+2}M^{-\alpha+2}\right]_{1}^{8}\frac{1}{5} + \left[\frac{1}{-\alpha+1}M^{-\alpha+1}\right]_{8}^{60}\cdot 1.4$$

$$\cancel{= \frac{1}{-\alpha+2}\left(\left[M^{-\alpha+2}\right]_{0.1}^{1} + \left[M^{-\alpha+2}\right]_{1}^{8}\frac{1}{5}\right) + \frac{1}{-\alpha+1}\left[M^{-\alpha+1}\right]}$$

$= 3.81\ M_\odot$

$$\text{RETURNED TO ISM FRACTION} = \frac{M_{INITIAL} - M_{LEFT}}{M_{INITIAL}} = \frac{5.71 - 3.81}{5.71} \simeq \underline{\underline{0.33}}$$

(ii) This is a bit higher than I expected but believable.

Q10

(i)

THIS MANY IRON ATOMS WE MAKE $= \frac{M_C}{M_{FE}} \cong \frac{1.4 \cdot 2 \cdot 10^{30}\ kg}{56 \cdot 10^{-3} : (6 \cdot 10^{23})\ kg} = 3 \cdot 10^{55}$

$E_{\text{TOTAL ABSORBED}} = \frac{3}{4} \cdot 3 \cdot 10^{55} \cdot 124.4 \cong \underline{2.8 \cdot 10^{57}\ MeV}$

(ii)

NUMBER OF PROTONS $=$ NUMBER OF IRONS $\cdot 56 = 1.68 \cdot 10^{57}$

$\cdot 10\ MeV \rightarrow \underline{1.68 \cdot 10^{58}\ MeV}$

(iii)

OHMM, I FORGOT, HOW DO I READ OFF LUMINOSITY OF A GIVEN MASS STAR FROM HRD?