PQM IV Q1 (I)

$$H(t) = H_0 + \Delta_s(t)$$

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \qquad \Delta_s(t) = \begin{pmatrix} 0 & V_0 e^{i\omega t} \\ V_0 e^{-i\omega t} & 0 \end{pmatrix}$$

$$P(|1\rangle \to |2\rangle) = a^*_{(t)} a(t)$$

$$a(t) = \frac{1}{i\hbar} \int_0^t e^{i\omega_{12} t'} \langle 1|\Delta(t')|2\rangle dt'$$

$$\omega_{12} = \frac{E_1 - E_2}{\hbar}$$

$$\langle 1|\Delta(t')|2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^+ V_0 \begin{pmatrix} 0 & e^{i\omega t'} \\ e^{-i\omega t'} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= (1 \quad 0) V_0 \begin{pmatrix} e^{i\omega t'} \\ 0 \end{pmatrix} = V_0 e^{i\omega t'}$$

WE HAVE:

$$a(t) = \frac{1}{i\hbar} \int_0^t e^{i \frac{E_1 - E_2}{\hbar} t'} V_0 e^{i\omega t'} dt' =$$

$$= \frac{V_0}{i\hbar} \int_0^t e^{i(E_1 - E_2 + \hbar\omega) t'/\hbar} dt' =$$

$$= \frac{V_0}{\cancel{i\hbar}} \frac{\cancel{\hbar}}{\cancel{i}(E_1 - E_2 + \hbar\omega)} \left(e^{i(E_1 - E_2 + \hbar\omega) t/\hbar} - 1 \right)$$

$$= - \frac{V_0}{E_1 - E_2 + \hbar\omega} \left[e^{i(E_1 - E_2 + \hbar\omega) t/\hbar} - 1 \right]$$

QM IV Q1 (II)

LET $X = E_1 - E_2 + \hbar\omega$

THEN:

$$a^* a = +\frac{V_0^2}{X^2}\left(e^{-iXt/\hbar} - 1\right)\left(e^{iXt/\hbar} - 1\right)$$

$$= \frac{V_0^2}{X^2}\left(1 - e^{iXt/\hbar} - e^{-iXt/\hbar} + 1\right)$$

$$= \frac{V_0^2}{X^2}\left[2 - \left(e^{iXt/\hbar} + e^{-iXt/\hbar}\right)\right]$$

$$= \frac{V_0^2}{X^2} 2\left[1 - \cos(Xt/\hbar)\right]$$

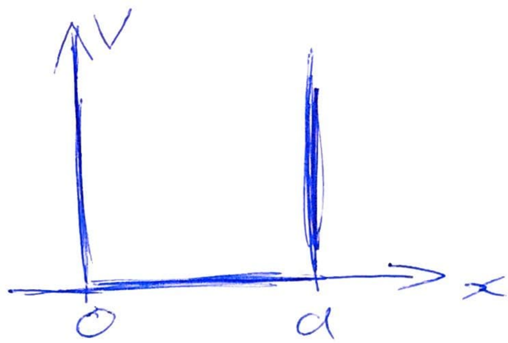
$$= \frac{V_0^2}{X^2} 2 \cdot 2 \sin^2\left(\frac{Xt}{2\hbar}\right)$$

$$= \frac{4V_0^2}{(E_1 - E_2 + \hbar\omega)^2} \sin^2\left(\frac{(E_1 - E_2 + \hbar\omega)t}{2\hbar}\right)$$

PQM IV Q2 (I)

START FROM 1D CASE:

TDSE:



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\underbrace{V\psi}_{\overset{\shortparallel}{0}\text{ IN BOX}}=E\psi \qquad \psi(0)=\psi(L)=0$$

$$\Rightarrow \psi(x)=A\sin\left(\frac{n\pi x}{a}\right) \qquad n=1,2,3\ldots$$

$$E_n=-\frac{\hbar^2}{2m}\left(-\frac{n^2\pi^2}{a^2}\right)=\frac{\pi^2\hbar^2}{2ma^2}n^2$$

$$\int_0^a \psi^*\psi\,dx=1 \Rightarrow A=\sqrt{\frac{2}{a}}$$

3D CASE:
(IE PARTICLE IN A BOX)

$$E_{klm}=\frac{\pi^2\hbar^2}{2ma^2}\left(k^2+l^2+m^2\right)$$

$$E_{INITIAL}=\frac{3\pi^2\hbar^2}{2ma^2}\Rightarrow k=l=m=1$$

$$E_{LATE}=\frac{6\pi^2\hbar^2}{2ma^2}\Rightarrow k,l,m=2,1,1$$

$$P(E_{INITIAL}\rightarrow E_{LATE})=a^*(t)a(t)$$

$$\cancel{a(t)=\frac{1}{i\hbar}\int_0^t e^{i\frac{E_{LATE}-E_{INITIAL}}{\hbar}t'}\langle k|\Delta(t')|}$$

$$a(t)=\frac{1}{i\hbar}\int_0^t e^{i\frac{E_{LATE}-E_{INITIAL}}{\hbar}t'}\langle LATE|\Delta(t')|INITIAL\rangle\,dt'$$

PQM IV | Q2 | (II) LETS APPLY FIELD IN X DIRECTION.

$$\underline{H} = \underline{H}_0 + \underline{\Delta}$$

$$\underline{\Delta} = (eEX, 0, 0)$$

$$\langle 2,1,1 | \underline{\Delta} | 1,1,1 \rangle =$$

$$= \langle 2 | eEX | 1 \rangle + \cancel{\langle 1 | 0 | 1 \rangle}^{\to 0} + \cancel{\langle 1 | 0 | 1 \rangle}^{\to 0}$$

$$= \int_0^a \frac{2}{a} \sin\left(\frac{2\pi x}{a}\right) x \sin\left(\frac{\pi x}{a}\right) dx =$$ ALGEBRA STEPS/COMPUTER

$$= -\frac{16a}{9\pi^2}$$

$$a(t) = \frac{1}{i\hbar} \int_0^t e^{i\frac{E_{LATE} - E_{INIT}}{\hbar} t'} \left(-\frac{16a}{9\pi^2}\right) dt'$$

$$= \frac{1}{i\cancel{\hbar}} \left(-\frac{16a}{9\pi^2}\right) \frac{\cancel{\hbar}}{i(E_{LATE} - E_{INIT})} \left(e^{i\frac{E_{LATE} - E_{INIT}}{\hbar} t} - 1\right)$$

~~$a^*(t) a(t) = \frac{16a}{9\pi}$~~

$$a^* a = \left(\frac{16a}{9\pi^2}\right)^2 \frac{1}{(E_{LATE} - E_{INIT})^2} \left(e^{i\frac{E_{LATE} - E_{INIT}}{\hbar} t} - 1\right)\left(e^{-i\frac{E_{LATE} - E_{INIT}}{\hbar} t} - 1\right)$$

$$= \left(\frac{16a}{9\pi^2}\right)^2 \frac{1}{E_{LATE} - E_{INIT}} 4 \sin^2\left(\frac{E_{LATE} - E_{INIT}}{\hbar} t\right)$$

PQM IV Q3 | (I)

$$P(|\text{GROUND}\rangle \to |\text{LATE}\rangle) = a^* a$$

$$a(t) = \int_0^t e^{i \frac{E_{LATE} - E_{GROUND}}{\hbar} t'} \langle \text{LATE} | \Delta(t') | \text{GROUND} \rangle dt'$$

WE HAVE:

$$\Delta(t') \propto X = A + A^+$$

SO:

$$\langle \text{LATE} | \Delta(t') | \text{GROUND} \rangle \propto \langle \text{LATE} | A + A^+ | \text{GROUND} \rangle$$

$$= \langle \text{LATE} | A | \text{GROUND} \rangle + \underbrace{\langle \text{LATE} | A^+ | \text{GROUND} \rangle}_{\to 0}$$

$$= \langle n | A | 0 \rangle = \delta_{1n}$$

(A LOWERS n BY 1, IF n IS NOT 1, IT IS NOT LOWERED TO 0, IE RESULT WILL BE 0)

$\Rightarrow$ TRANSITION ONLY ALLOWED TO FIRST EXCITED STATE.

LETS DO THIS EXACTLY NOW.

$$\langle \text{LATE} | \Delta | \text{GROUND} \rangle =$$

$$= \langle \text{LATE} | \frac{q\mathcal{E}}{\sqrt{\pi}\tau} \exp\left(-\frac{t^2}{\tau^2}\right) \sqrt{\frac{\hbar}{2m\omega}} \hat{a} | \text{GROUND} \rangle$$

$$= \frac{q\mathcal{E}}{\sqrt{\pi}\tau} \sqrt{\frac{\hbar}{2m\omega}} \exp\left(-\frac{t^2}{\tau^2}\right) \delta_{1n}$$

PQM IV Q3 (ii)

$$a(t) = \int_0^t e^{i\frac{E_{LATE} - E_{GROUND}}{\hbar}t'} \frac{q\mathcal{E}}{\sqrt{\pi}\tau}\sqrt{\frac{\hbar}{2m\omega}} \exp\left(-\frac{t'^2}{\tau^2}\right) dt'$$

$$E_{LATE} - E_{GROUND} = \hbar\omega\left(1+\frac{1}{2}\right) - \hbar\omega\frac{1}{2} = \hbar\omega$$

$$\Rightarrow a(t) = \frac{q\mathcal{E}}{\sqrt{\pi}\tau}\sqrt{\frac{\hbar}{2m\omega}} \int_0^{t} e^{i\omega t'} e^{-\frac{t'^2}{\tau^2}} dt'$$

"FROM VERY EARLY TO VERY LATE TIMES" $\Rightarrow$

$\Rightarrow$ LIMITS SHOULD BE: $-\infty, \infty$

$$a(t) = \frac{q\mathcal{E}}{\sqrt{\pi}\tau}\sqrt{\frac{\hbar}{2m\omega}} \int_{-\infty}^{\infty} e^{i\omega t'} e^{-\frac{t'^2}{\tau^2}} dt' =$$

AFTER ~~LOTS~~ SOME OF ALGEBRA/ COMPUTER

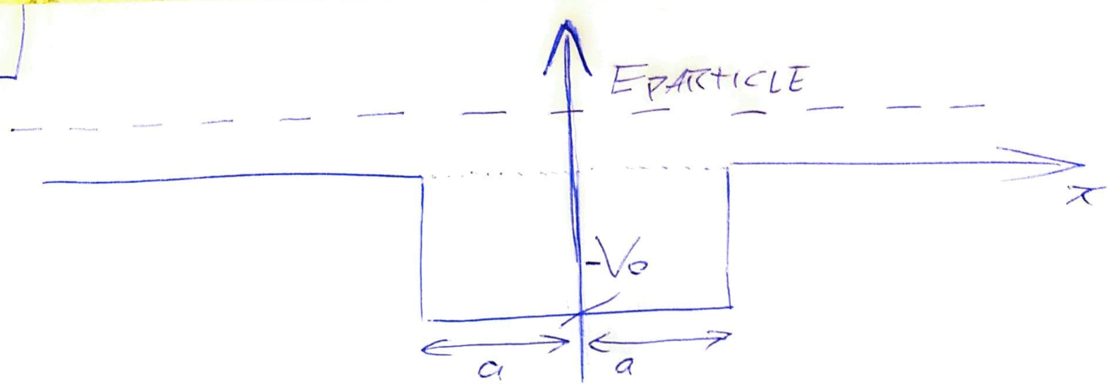
$$= \frac{q\mathcal{E}}{\cancel{\sqrt{\pi}\tau}}\sqrt{\frac{\hbar}{2m\omega}}\,\cancel{\sqrt{\pi}}\,\cancel{\tau} \exp\left(-\frac{1}{4}\tau^2\omega^2\right)$$

$$= q\mathcal{E}\sqrt{\frac{\hbar}{2m\omega}} \exp\left(-\frac{1}{4}\tau^2\omega^2\right)$$

$$a^*a = q^2\mathcal{E}^2\frac{\hbar}{2m\omega}\exp\left(-\frac{1}{2}\tau^2\omega^2\right)$$

I AM UNSURE IF I WAS RIGHT TO SEND THE LOWER LIMIT TO $-\infty$ IN THE INTEGRAL ABOVE.

PQM IV Q4



MOVE TO PARTICLE'S FRAME.

SWITCH ON PERTURBATION AT $t=0$,

SWITCH OFF AT $\frac{2a}{v}$

HERE I FIND TROUBLESOME TO CONCEPTUALLY CONNECT ABSORBTION-EMISSION (TYPICAL FERMI'S RULE APPLICATION AREA) TO REFLECTION - TRANSMISSION.

PQM IV Q5 / I /

$$H = \omega\hbar\left(A^+A + \frac{1}{2}\right) + \hbar\left(f^*(t)A + f(t)A^+\right)$$

$$= H_0 + \underbrace{\hbar\left(f^*(t)A + f(t)A^+\right)}_{\delta H(t)}$$

LET'S DEFINE: $\mathcal{U}_0(t) = \exp\left(-\frac{iH_0 t}{\hbar}\right)$

LET'S HAVE A STATE $|\psi(t)\rangle$ WHICH EVOLVES ACCORDING TO $H(t)$.

IF WE DEFINE $|\tilde{\psi}(t)\rangle$ THE FOLLOWING WAY, IT'LL EVOLVE ACCORDING TO δH ONLY:

$$|\tilde{\psi}(t)\rangle = \mathcal{U}^+(t)|\psi(t)\rangle$$

SCHRÖDINGER EQ. FOR $|\tilde{\psi}(t)\rangle$:

$$i\hbar\frac{d}{dt}|\tilde{\psi}(t)\rangle = i\hbar\frac{d}{dt}\left[\exp\left(+\frac{iH_0 t}{\hbar}\right)|\psi(t)\rangle\right]$$

$$= i\hbar\frac{iH_0}{\hbar}\underbrace{\exp\left(\frac{iH_0 t}{\hbar}\right)|\psi(t)\rangle}_{|\tilde{\psi}(t)\rangle} + i\hbar\exp\left(\frac{iH_0 t}{\hbar}\right)\frac{d}{dt}|\psi(t)\rangle$$

SCHRÖDINGER FOR $|\psi(t)\rangle$: $i\hbar\frac{d}{dt}|\psi(t)\rangle = (H_0 + \delta H)|\psi(t)\rangle$

USING THIS, REWRITE LINE ABOVE.

$$= -H_0|\tilde{\psi}(t)\rangle + \exp\left(\frac{iH_0 t}{\hbar}\right)\underbrace{(H_0 + \delta H)}_{\text{II}}\exp\left(-\frac{iH_0 t}{\hbar}\right)|\tilde{\psi}(t)\rangle$$

THESE COMMUTE & CANCEL W/ THIS

$$= \exp\left(\frac{iH_0 t}{\hbar}\right)\delta H(t)\exp\left(-\frac{iH_0 t}{\hbar}\right)|\tilde{\psi}(t)\rangle$$

PQM IV Q 5(II)

$$= \exp(i\omega t)\,\delta H(t)\,\exp(-i\omega t)\,|\tilde{\psi}(t)\rangle$$

$$= \underbrace{\exp(i\omega t)\,\delta H(t)}\,|\psi(t)\rangle$$

THIS IS THE HAMILTONIAN GOVERNING THE TIME-DEPENDENT INTERACTION.

$$\Rightarrow V_I(t) = \exp(i\omega t)\,\delta H(t)$$

$$= \exp(i\omega t)\,\hbar\,(f^* A + f A^+)$$

$$= \hbar\,(e^{i\omega t} f(t) A + e^{i\omega t} f A^+)$$

$$= \hbar\,(\tilde{f}^* A + \tilde{f} A^+)$$

AS REQUIRED

(LARGELY INSPIRED BY MIT OCW NOTES)