STARS
SHEET III

Q1

(I)

(i) NOTING THAT $\frac{d\ln P}{dP} = \frac{1}{P}$ & $\frac{d\ln\tau}{d\tau} = \frac{1}{\tau}$

REARRANGE:

$$\frac{d\ln P}{d\ln T} = \frac{(dP)}{(d\tau)} \cdot \frac{\tau}{P}$$

THIS IS CLAIMED TO BE EQUAL TO:

$$\frac{dP}{d\tau} \cdot \frac{\tau}{P} = \frac{g\,\tau}{\kappa P}$$

$$\Rightarrow \frac{dP}{d\tau} = \frac{g}{\kappa}$$

HYDROSTAT. EQUILIB (EQ. 8.23) $\Rightarrow dP = -\rho g\, dr$

COMBINE:

$$-\rho \cancel{g} \frac{dr}{d\tau} = \cancel{g}/\kappa$$

$$-\rho\kappa\, dr = d\tau$$

OPTICAL DEPTH DEF: $\tau = \int_0^r \kappa \rho\, dr$ (EQ. 5.10)

$$\Rightarrow \frac{d\tau}{dr} = \kappa\rho \Rightarrow d\tau = \rho\kappa\, dr$$

SO APART FROM A MINUS SIGN, WE'VE ARRIVED TO SOMETHING WHICH IS TRUE $\Rightarrow$ ORIGINAL CLAIM IS TRUE.

STARS SHEET (III) | Q1 | (ii)

NOTING THAT

$$\ln(x) = \frac{\log(x)}{\log e}$$

WE HAVE:

$$\frac{d\ln P}{d\ln t} = \frac{d\left(\frac{\log P}{\log e}\right)}{d\left(\frac{\log t}{\log e}\right)} = \frac{d\log P}{d\log t} \Rightarrow$$ SECOND EQUALITY IS ALSO TRUE.

(iii)

STARS SHEET III | Q3 | (I)

i/ FULLY RADIATIVE $\Rightarrow$ ALL ENERGY PRODUCED INSIDE UNDERGOES RADIATIVE TRANSPORT

LUMINOSITY AT r, IE POWER RADIATION AT A CERTAIN DEPTH IS THEREFORE EQUAL TO RATE OF ENERGY PRODUCTION WITHIN SPHERE OF RADIUS r.

IE
$$L(r) = \varepsilon m(r)$$

WHERE $m(r)$ IS ENCLOSED MASS WITHIN RADIUS r.

EQ 8.23

CONSIDER:

$$\frac{dT}{dP} = \underbrace{\frac{dT}{dr}}_{\text{EQ. 8.1}} \frac{dr}{dP} = -\frac{3}{4}\frac{1}{ac}\frac{\kappa \cancel{\rho}}{T^3}\frac{L(r)}{4\pi r^2} \cdot \frac{-1}{\cancel{\rho} g}$$

EQ. 8.1

$$= \frac{3}{16}\frac{1}{\pi}\frac{1}{ac}\frac{\kappa \varepsilon m}{r^2 g}\frac{1}{T^3}$$

USE: $G\frac{m}{r^2} = g$

$$= \frac{3}{16}\frac{1}{\pi}\frac{1}{ac}\frac{\kappa \varepsilon \cancel{m}}{\cancel{r^2}}\frac{\cancel{r^2}}{G\cancel{m}}\frac{1}{T^3}$$

$$= \frac{3\kappa\varepsilon}{16\pi a c T^3}$$ AS REQUIRED.

SHEET III | Q3 | (II)

ii/ START FROM:

$$\frac{dT}{dP} = \frac{3\kappa\varepsilon}{16\pi acGT^3}$$

/ - MULTIPLY UP WITH DENOMINATORS
/ - INTEGRATE

$$16\pi acG\frac{T^4}{4} = 3\kappa\varepsilon P$$

IE $\frac{T^4}{4} = \frac{3\kappa\varepsilon P}{16\pi acG}$

TAYLOR EXPAND IN P AROUND SURFACE:

$$\frac{T^4}{4} \cong \left.\frac{T^4}{4}\right|_{SURFACE} + \left.\frac{d\left(\frac{T^4}{4}\right)}{dP}\right|_{SURFACE}(P-P_0)$$

$$\cong \frac{T_0^4}{4} + \frac{3\kappa\varepsilon}{16\pi acG}(P-P_0)$$

INTERIOR OF STAR: $P \gg P_0$, $T \gg T_0$

$$\Rightarrow \frac{T^4}{4} \cong \frac{3\kappa\varepsilon}{16\pi acG}P \Rightarrow P \cong \underbrace{\frac{4}{3}\frac{\pi acG}{\kappa\varepsilon}}T^4$$

$$P \cong cT^4$$

I REALLY DID JUST FIRST-ORDER TAYLOR EXPAND A FUNCTION WHICH IS TRUE ON THE SURFACE ONLY, WITHOUT TAKING INTO ACCOUNT ANYTHING RELATED TO THE SPHERICAL NATURE OF THE PROBLEM, AND THEN I EXPECT THIS TO HOLD IN THE INTERIOR? NICE.

STARS SHEET III | Q3 | (III)

~~FAILED ATTEMPT~~

(iii)

$$P = \underbrace{\frac{SKT}{\mu m_H}}_{\text{GAS PRESSURE}} + \underbrace{\frac{1}{3} a T^4}_{\text{RADIATION PRESSURE}}$$

RESULT FROM PART (ii):

$$P \cong \frac{4}{3} \frac{\pi a c G}{K \varepsilon} T^4$$

SUBSTITUTE IN:

$$\frac{SKT}{\mu m_H} + \frac{1}{3} a T^4 = \frac{4}{3} \frac{\pi a c G}{K \varepsilon} T^4$$

THIS IS CLAIMED TO BE EQUAL TO, PROVIDED $\varepsilon < \frac{4\pi c G}{K}$:

$$= K S^{\frac{4}{3}}$$

REARRANGE

$$\frac{SKT}{\mu m_H} = \frac{1}{3} a T^4 \left(\frac{4\pi c G}{K \varepsilon} - 1 \right)$$

REARRANGE

$$S = \frac{1}{3} a \frac{\mu m_H}{K} \left(\frac{4\pi c G}{K \varepsilon} - 1 \right) T^3$$

$\downarrow (\;)^{\frac{4}{3}}$

$$S^{\frac{4}{3}} = \left[\frac{1}{3} a \frac{\mu m_H}{K} \left(\frac{4\pi c G}{K \varepsilon} - 1 \right) \right]^{\frac{4}{3}} T^4$$

AND NOW WHAT.

SHEET III | Q3

$S^{\frac{4}{3}} =$ SOME CONSTANT WHICH IS ONLY POSITIVE, IE PHYSICALLY ACCEPTABLE IF $\frac{4\pi CG}{K\varepsilon} - 1 > 0$ $\cdot T^4$

NOTING THAT: $P \propto T^4$, WE CAN WRITE:

$$S^{\frac{4}{3}} \propto P, \text{ IF } \frac{4\pi CG}{K\varepsilon} - 1 > 0 \text{ IE } \frac{4\pi CG}{K} > 0$$

$$\Rightarrow P = K S^{\frac{4}{3}} \text{ IF } \frac{4\pi CG}{K} > 0.$$

STARS SHEET III | Q4 (i)

ENERGY TRANSFER IN CONVECTING STARS:

$$\frac{dT}{dr} = -\frac{\gamma - 1}{\gamma} \frac{\mu m_H}{R} \frac{GM(r)}{r^2}$$

ON THE SURFACE, I THINK WE SHOLD HAVE:

$$0 = -\frac{\gamma - 1}{\gamma} \frac{\mu m_H}{R} \frac{GM}{R^2}$$

BECAUSE THERE IS NO HEAT TRANSFER ANYMORE AT THE SURFACE.

THIS IS NOT TRUE HOWEVER, UNLESS $\gamma = 1$, BUT THAT'S NOT TRUE.

SO WHAT'S GOING ON HERE?

STARS SHEET III | Q4 | (I)

(ii)

$$BC \Rightarrow \sqrt{\frac{KP}{G}} = \cancel{M^{\frac{1}{2}}} M^{\frac{1}{2}} R^{-1}$$

$$K \equiv PT^{-\frac{5}{2}} = \frac{GM}{R^2}\frac{1}{K}\left(\frac{P\mu m_H}{\rho k}\right)^{-\frac{5}{2}}$$

$$= \frac{GM}{R^2}\frac{1}{K}\frac{(\rho k)^{\frac{2}{5}}}{(P\mu m_H)^{\frac{2}{5}}}$$

OK THIS DIDN'T WORK.

STARS SHEET III | Q5 | (I)

(i)

MASS OF CLOUD $= \frac{4}{3}\pi R^3 \rho$

PROTON MASS

$= \frac{4}{3}\pi (3 \cdot 10^{16})^3 \cdot \frac{100 \cdot 10^6 \cdot m_p}{1}$

$\cong 1.9 \cdot 10^{31}$ kg

O STAR IS PROBABLY MAIN SEQUENCE.
BASED ON HR DIAGRAM & USING THAT SUN IS TYPE G, WE CONCLUDE THAT IT IS LESS LUMINOUS THAN SUN $\Rightarrow$ MASS OF O STAR < SUN MASS

SUN MASS $= 1.9 \cdot 10^{30}$ kg

MASS OF O STAR < MASS OF SUN < MASS OF CLOUD

$\downarrow$ BY A MAGNITUDE

$\Rightarrow$ MASS OF O STAR $\ll$ MASS OF CLOUD.

$\Rightarrow$ IGNORE THE MASS OF THE STAR, CALCULATE JEANS MASS, DECIDE UPON THAT.

STARS SHEET III | Q5 | (II)

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi \rho_0}\right)^{\frac{1}{2}}$$

$\mu = 1$, $m_H = m_{PROTON}$, $\rho_0 = 100 \cdot 10^6 \cdot m_{PROTON}$

$$= 1.8 \cdot 10^{37}$$

CLOUD MASS $\ll$ JEANS MASS $\Rightarrow$ REGION IS STABLE.

(ii)

$z = 3 \Rightarrow$ EARLY (ISH) UNIVERSE

$\Rightarrow$ NOT MUCH METAL

~~$\Rightarrow$ HII ABSORPTION~~

$\Rightarrow$ HYDROGEN SPECTRAL FEATURES ARE EXPECTED TO BE DOMINANT.