

Задача 5.

Построить касательную плоскость эллипсоида $x^2 + 2y^2 + 4z^2 = 22$, параллельную плоскости $4x + 2y + 4z = 1$.

Решение: Пусть α - касательная пл-ть, и пусть $\beta = 4x + 2y + 4z - 1 = 0$. Эллипсоид: $x^2 + 2y^2 + 4z^2 - 22 = 0$.

$$\left\{ \begin{array}{l} \alpha : F'_x(x_0, y_0, z_0)(x - x_0) + F'_y(x_0, y_0, z_0)(y - y_0) + F'_z(x_0, y_0, z_0)(z - z_0) = 0 \\ F'_x(x_0, y_0, z_0) = 2x_0 \\ F'_y(x_0, y_0, z_0) = 4y_0 \\ F'_z(x_0, y_0, z_0) = 8z_0 \end{array} \right\} \Rightarrow \vec{n}_\alpha = \{2x_0, 4y_0, 8z_0\} - \text{нормаль к } \alpha$$

Нормаль к $\beta : \vec{n}_\beta = \{4, 2, 4\}$

$$\left. \begin{array}{l} \beta \parallel \alpha \\ \vec{n}_\beta \perp \beta \\ \vec{n}_\alpha \perp \alpha \end{array} \right\} \Rightarrow \vec{n}_\beta \perp \alpha \& \vec{n}_\beta \perp \beta \Rightarrow \vec{n}_\alpha \cdot \lambda = \vec{n}_\beta \Rightarrow \{\lambda 2x_0, \lambda 4y_0, \lambda 8z_0\} = \{4, 2, 4\}$$

$$\left\{ \begin{array}{l} \lambda 2x_0 = 4 \\ \lambda 4y_0 = 2 \\ \lambda 8z_0 = 4 \end{array} \Rightarrow \left\{ \begin{array}{l} \lambda x_0 = 2 \\ \lambda 2y_0 = 1 \\ \lambda 2z_0 = 1 \end{array} \right\} \Rightarrow \lambda 2y_0 = \lambda 2z_0 \Rightarrow y_0 = z_0 \right\} \Rightarrow 4x_0 = y_0 = z_0$$
$$\left\{ \begin{array}{l} \lambda x_0 = 2 \\ \lambda 4y_0 = 2 \end{array} \right\} \Rightarrow \lambda x_0 = \lambda 4y_0 \Rightarrow x_0 = 4y_0$$

Подставляем y_0 в уравнение эллипсоида:

$$(4y_0)^2 + 2(y_0)^2 + 4(y_0)^2 - 22 = 0 \Rightarrow 16y_0^2 + 2y_0^2 + 4y_0^2 - 22 = 0 \Rightarrow 22y_0^2 = 22 \Rightarrow y_0^2 = 1$$

$$(1) \left\{ \begin{array}{l} y_0 = 1 \\ z_0 = 1 \\ x_0 = 4 \end{array} \Rightarrow \left\{ \begin{array}{l} F'_x(x_0, y_0, z_0) = 8 \\ F'_y(x_0, y_0, z_0) = 4 \\ F'_z(x_0, y_0, z_0) = 8 \end{array} \right\} \Rightarrow 4(x-8) + 2(y-4) + 4(z-8) - 1 = 0 \Rightarrow 4x + 2y + 4z - 73 = 0$$

$$(2) \left\{ \begin{array}{l} y_0 = -1 \\ z_0 = -1 \\ x_0 = -4 \end{array} \Rightarrow \left\{ \begin{array}{l} F'_x(x_0, y_0, z_0) = -8 \\ F'_y(x_0, y_0, z_0) = -4 \\ F'_z(x_0, y_0, z_0) = -8 \end{array} \right\} \Rightarrow 4(x-8) + 2(y-4) + 4(z-8) - 1 = 0 \Rightarrow 4x + 2y + 4z + 71 = 0$$

Ответ: $4x + 8y + z + 71 = 0$ и $4x + 2y + 4z - 73 = 0$