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Кр 2.1

Алгем, 2 семестр

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Задача 1.2.
Найти репер Френе для кривой $\vec{r}(t) = \left\{\frac{1+t}{1-t},\ \frac{1}{1-t^2}, \frac{1}{1+t}\right\}$.

Решение:

$$\begin{array}{cccc} & x & y & z \\ \vec{r}'(t) & \frac{2}{(t-1)^2} & \frac{2t}{(t^2-1)^2} & -\frac{1}{(t+1)^2} \\ \vec{r}''(t) & -\frac{4}{(t-1)^3} & \frac{2(t^2-1)^3-8t^2(t^2-1)^2}{(t^2-1)^3(t^2-1)^2} & \frac{2}{(t+1)^3} \end{array}$$

Найдем репер Френе:

$$\begin{cases} \vec{\tau}(t) = \dfrac{\vec{r}(t)'}{|\vec{r}(t)'|} \\ \vec{\nu}(t) = \dfrac{(\vec{r}' \times \vec{r}''(t)) \times \vec{r}'(t)}{|(\vec{r}' \times \vec{r}''(t)) \times \vec{r}'(t)|} \\ \vec{\beta}(t) = \dfrac{\vec{r}' \times \vec{r}''(t)}{|\vec{r}' \times \vec{r}''(t)|} \end{cases}$$

$$\vec{\tau}(t) = \frac{\{\frac{2}{(t-1)^2}, \frac{2t}{(t^2-1)^2}, -\frac{1}{(t+1)^2}\}}{\sqrt{(\frac{2}{(t-1)^2})^2+(\frac{2t}{(t^2-1)^2})^2+(\frac{1}{(t+1)^2})^2}} = \{\frac{2(t+1)^2}{\sqrt{4t^2+4(t+1)^4+(t-1)^4}}, \frac{2t}{\sqrt{4t^2+4(t+1)^4+(t-1)^4}}, -\frac{(t-1)^2}{\sqrt{4t^2+4(t+1)^4+(t-1)^4}}\}$$

$$\vec{\beta}(t) = \frac{\{\begin{vmatrix} \frac{2t}{(t^2-1)^2} & -\frac{1}{(t+1)^2} \\ \frac{2(t^2-1)^3-8t^2(t^2-1)^2}{(t^2-1)^3(t^2-1)^2} & \frac{2}{(t+1)^3} \end{vmatrix}, \begin{vmatrix} \frac{2}{(t-1)^2} & -\frac{1}{(t+1)^2} \\ -\frac{4}{(t-1)^3} & \frac{2}{(t+1)^3} \end{vmatrix}, \begin{vmatrix} \frac{2}{(t-1)^2} & \frac{2t}{(t^2-1)^2} \\ -\frac{4}{(t-1)^3} & \frac{2(t^2-1)^3-8t^2(t^2-1)^2}{(t^2-1)^3(t^2-1)^2} \end{vmatrix}\}}{\sqrt{\begin{vmatrix} \frac{2t}{(t^2-1)^2} & -\frac{1}{(t+1)^2} \\ \frac{2(t^2-1)^3-8t^2(t^2-1)^2}{(t^2-1)^3(t^2-1)^2} & \frac{2}{(t+1)^3} \end{vmatrix}^2 + \begin{vmatrix} \frac{2}{(t-1)^2} & -\frac{1}{(t+1)^2} \\ -\frac{4}{(t-1)^3} & \frac{2}{(t+1)^3} \end{vmatrix}^2 + \begin{vmatrix} \frac{2}{(t-1)^2} & \frac{2t}{(t^2-1)^2} \\ -\frac{4}{(t-1)^3} & \frac{2(t^2-1)^3-8t^2(t^2-1)^2}{(t^2-1)^3(t^2-1)^2} \end{vmatrix}^2}} =$$

$$= \frac{\{-\frac{2}{(t^2-1)^3}, -\frac{8}{(t^2-1)^3}, -\frac{4}{(t^2-1)^3}\}}{\sqrt{(\frac{2}{(t^2-1)^3})^2+(\frac{8}{(t^2-1)^3})^2+(\frac{4}{(t^2-1)^3})^2}} = \left\{-\frac{\sqrt{21}}{21}, -\frac{4\sqrt{21}}{21}, -\frac{2\sqrt{21}}{21}\right\}$$

$$\vec{\nu}(t) = \frac{\{\frac{8(t^2-t+1)}{(t^2-1)^5}, \frac{2(5t^2+6t+5)}{(t^2-1)^5}, \frac{4(t^2+7t+4)}{(t^2-1)^5}\}}{\sqrt{(\frac{8(t^2-t+1)}{(t^2-1)^5})^2+(\frac{2(5t^2+6t+5)}{(t^2-1)^5})^2+(\frac{4(t^2+7t+4)}{(t^2-1)^5})^2}} =$$

$$= \{\frac{4(t^2-t+1)}{\sqrt{45t^4+84t^3+362t^2+252t+105}}, \frac{5t^2+6t+5}{\sqrt{45t^4+84t^3+362t^2+252t+105}}, \frac{2(t^2+t+4}{\sqrt{45t^4+84t^3+362t^2+252t+105}}\}$$

ОтВеТ:
$$\begin{cases} \vec{\tau}(t) = \{\frac{2(t+1)^2}{\sqrt{4t^2+4(t+1)^4+(t-1)^4}}, \frac{2t}{\sqrt{4t^2+4(t+1)^4+(t-1)^4}}, -\frac{(t-1)^2}{\sqrt{4t^2+4(t+1)^4+(t-1)^4}}\} \\ \vec{\beta}(t) = \{-\frac{\sqrt{21}}{21}, -\frac{4\sqrt{21}}{21}, -\frac{2\sqrt{21}}{21}\} \\ \vec{\nu}(t) = \{\frac{4(t^2-t+1)}{\sqrt{45t^4+84t^3+362t^2+252t+105}}, \frac{5t^2+6t+5}{\sqrt{45t^4+84t^3+362t^2+252t+105}}, \frac{2(t^2+t+4}{\sqrt{45t^4+84t^3+362t^2+252t+105}}\} \end{cases}$$