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Кр 2.1
Алгем, 2 семестр

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Задача 1.1.
Найти репер Френе для кривой $x = t, y = t^2, z = e^t$.

Решение:
$$\begin{array}{cccc} & x & y & z \\ \vec{r}'(t) & 1 & 2t & e^t \\ \vec{r}''(t) & 0 & 2 & e^t \end{array}$$

Найдем репер Френе:
$$\begin{cases} \vec{\tau}(t) = \dfrac{\vec{r}(t)'}{|\vec{r}(t)'|} \\ \vec{\nu}(t) = \dfrac{(\vec{r}' \times \vec{r}''(t)) \times \vec{r}'(t)}{|(\vec{r}' \times \vec{r}''(t)) \times \vec{r}'(t)|} \\ \vec{\beta}(t) = \dfrac{\vec{r}' \times \vec{r}''(t)}{|\vec{r}' \times \vec{r}''(t)|} \end{cases}$$

$\vec{\tau}(t) = \frac{\{1,2t,e^t\}}{\sqrt{1+4t^2+e^{2t}}} = \{\frac{1}{\sqrt{1+4t^2+e^{2t}}}, \frac{2t}{\sqrt{1+4t^2+e^{2t}}}, \frac{e^t}{\sqrt{1+4t^2+e^{2t}}}\}$

$$\vec{\beta}(t) = \frac{\{\begin{vmatrix} 2t & e^t \\ 2 & e^t \end{vmatrix}, \begin{vmatrix} 1 & e^t \\ 0 & e^t \end{vmatrix}, \begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix}\}}{\sqrt{\begin{vmatrix} 2t & e^t \\ 2 & e^t \end{vmatrix}^2 + \begin{vmatrix} 1 & e^t \\ 0 & e^t \end{vmatrix}^2 + \begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix}^2}} =$$

$= \frac{\{2e^t(t-1),e^t,2\}}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}} = \{\frac{2e^t(t-1)}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}}, \frac{e^t}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}}, \frac{2}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}}\}$

$$\vec{\nu}(t) = \frac{\{2e^t(t-1),e^t,2\}\times\{1,2t,e^t\}}{|\{2e^t(t-1),e^t,2\}\times\{1,2t,e^t\}|} = \frac{\begin{vmatrix} e^t & 2 \\ 2t & e^t \end{vmatrix}, \begin{vmatrix} 2e^t(t-1) & 2 \\ 1 & e^t \end{vmatrix}, \begin{vmatrix} 2e^t(t-1) & e^t \\ 1 & 2t \end{vmatrix}}{\sqrt{\begin{vmatrix} e^t & 2 \\ 2t & e^t \end{vmatrix}^2 + \begin{vmatrix} 2e^t(t-1) & 2 \\ 1 & e^t \end{vmatrix}^2 + \begin{vmatrix} 2e^t(t-1) & e^t \\ 1 & 2t \end{vmatrix}^2}} =$$

$= \frac{\{e^{2t}-4t,2e^{2t}(t-1)-2,4e^t(t^2-t)\}}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}} =$

$= \{\frac{e^{2t}-4t}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}}, \frac{2e^{2t}(t-1)-2}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}}, \frac{4e^t(t^2-t)}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}}\}$

Ответ:
$$\begin{cases} \vec{\tau}(t) = \{\dfrac{1}{\sqrt{1+4t^2+e^{2t}}}, \dfrac{2t}{\sqrt{1+4t^2+e^{2t}}}, \dfrac{e^t}{\sqrt{1+4t^2+e^{2t}}}\} \\ \vec{\nu}(t) = \{\dfrac{e^{2t}-4t}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}}, \\ \dfrac{2e^{2t}(t-1)-2}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}}, \\ \dfrac{4e^t(t^2-t)}{\sqrt{(e^{2t}-4t)^2+(2e^{2t}(t-1)-2)^2+(4e^t(t^2-t))^2}}\} \\ \vec{\beta}(t) = \{\dfrac{2e^t(t-1)}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}}, \dfrac{e^t}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}}, \dfrac{2}{\sqrt{4e^{2t}(t-1)^2+e^{2t}+4}}\} \end{cases}$$