REPORT ON "QUASISYMMETRIC HARMONICS OF THE EXTERIOR ALGEBRA"

Motivated by the classic results on coinvariants of finite groups and recent studies of coinvariants of symmetric groups acting on polynomials in anticommuting (fermionic) variables (e.g., the Theta conjecture), the authors initiate the investigation of a quotient of an exterior algebra by an ideal generated by quasisymmetric functions in anticommuting variables in this paper. (The quasisymmetric functions in noncommuting variables are invariant under the action of the quasisymmetric operators studied by Hivert.) More precisely, the authors establish the following in this paper.

- The quasisymmetric polynomials in the ring R_n of polynomials in n anticommuting variables form a commutative subalgebra of R_n.
- The ring R_n is free over the ring of symmetric polynomials in R_n.
- The dimension of each homogeneous component of this quotient is given by the number of ballot sequences.
- The quotient of R_n by the ideal I_n generated by quasisymmetric polynomials has a basis indexed by ballot sequences and the dimension of each graded component is the number of standard tableaux of two-row shapes.
- The ideal I_n has a basis indexed by sequences that break the ballot condition.

This paper provides a meaningful first step toward understanding of quasisymmetric coinvariants in multiple sets of commuting and anticommuting variables and could lead to more research in this direction. Therefore **I recommend its publication in the Canadian Mathematical Bulletin**.

Below is a list of suggestions and comments on some minor issues.

- Abstract, line 2: Let R_n denote the ring of ...
- Introduction, line 1: The hyphen between Sheppard and Todd should be an em dash (use three hyphens in latex).
- Page 3, line 3: "ring symmetric polynomials" $\to$ "ring of symmetric polynomials in R_n"
- Page 4, Remark 2.2: A brief explanation for $\dot{0}$ would be helpful.
- Page 8, Lemma 2.8: There seems to be a conflict of notation with the inner product.
- Page 8, two lines above Prop. 2.9: "non-crossing partition" $\to$ "non-crossing pairing"
- Page 11, line 9: Delete "is" from "is in fact is ..."
- Page 12, Example 3.1: There should be no indentation for "and we have ..."
- Page 14, line -4: It should be "α breaking the ballot condition implies..."
- Page 15, Second proof: Change this to "Proof of Theorem 4.2" or put it right below Theorem 4.2.