MMAN2300: Weights and Pulleys

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Contents

1 Introduction

The purpose of this experiment was to investigate the relationship between a systems' characteristics such as the mass moment of inertia and distance of a mass from the centre of rotation of the system and its subsequent influence on the translational motion of said mass. By calculating the theoretical volume, V, density, ρ, and therefore total mass moment of inertia of the system, I_m, and deriving an equation for the time for an attached mass to fall a certain distance, $t(s)$, a relationship between I_m and its effect on the translational motion of the mass attached to the system may be realised. From the expression, $t(s)$, it was then possible to plot the distance travelled by the mass , s, against the time to travel the corresponding distance, t. Comparing the plots for both theoretical and experimental values gave insight into the efficacy of the experiment.

2 Apparatus

The apparatus that was used consisted of two flywheels (one with a hub attached) and a rope drum connected to a central shaft. These components were attached to a fixture with two low-friction mounts. The rope wound around the rope drum had a snap hook attached to the end of it, upon which the mass is attached. The required apparatuses are listed below:

- Stopwatch
- Masses D & E*
- Flywheels 7 & 12[1][2]
- Hub for flywheel 7[3]
- Rope Drum 2[4]
- Central Shaft[5]
- Rope
- Snap hook*
- Clear tube marked with distances of 0.2, 0.35, 0.5, 0.65, and 0.8m from the top

* Refer to appendix

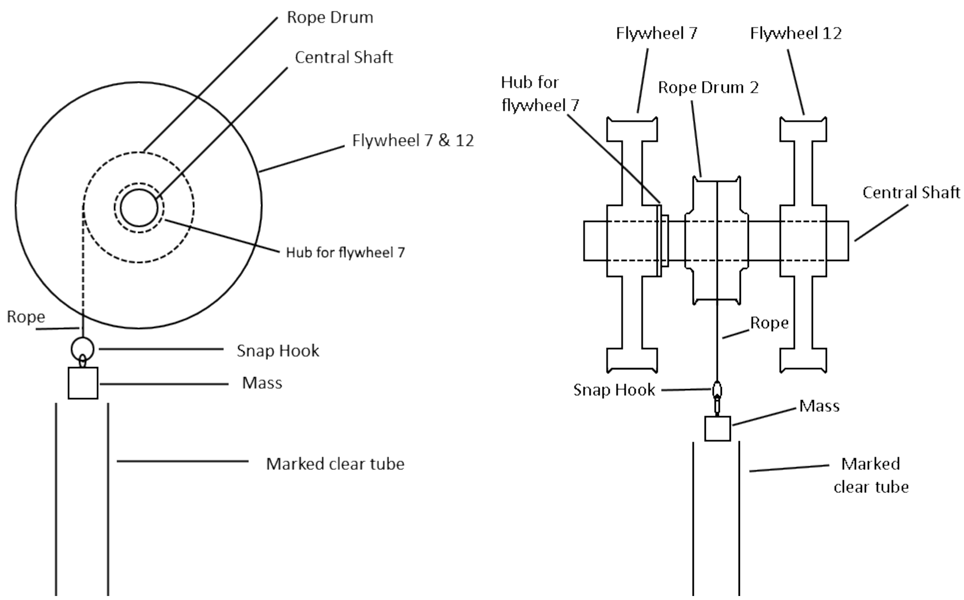


Figure 1: Schematic diagram of the experimental rig without fixture

3 Experimental Procedure

1. Attached mass D to the snap hook and moved it to the 0m marking on the tube (top of the tube).

2. Released the mass and recorded time for it to reach the 0.2m marking on the tube, repeated this two more times, resetting the mass to the top of the tube each time after waiting for the mass to come to a complete stop.

3. Repeated (2) for markings of 0.35m, 0.5m, 0.65m and 0.8m.

4. Replaced mass D with mass E and repeated steps (2) and (3).

4 Results

In order to determine the validity of the experimental results, a theoretical time was obtained to for comparison. To do this, the theoretical mass moment of inertia of the system was calculated first as below, separating the system into its constituents:

Flywheel 12

Simplifying the shape (removing the small nibs on the outer ring of the flywheel) and separating the flywheel into 3 concentric cylinders, as shown below:

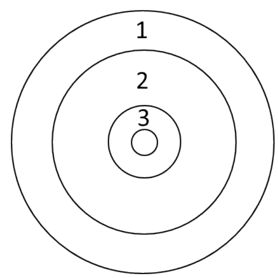


Figure 2: Schematic of simplified flywheel with labelled individual concentric cylinders

The second moment of area was found for each concentric cylinder using:

$$I_z = \frac{\pi}{2}(R^4 - r^4) \tag{1}$$

where R is the outer radius, and r is the inner radius. Then the mass moment of inertia was found using the relationship:

$$I_m = I_z \rho t \tag{2}$$

where ρ is the density of the flywheel, and t is the thickness of the cylinder in question. In order the calculate the density of the flywheel, the volume had to be calculated first, which is the sum of the volumes of each concentric cylinder. That is, $V = \sum_{n=1}^{3} V_n$, so $\rho = \dfrac{m}{\sum_{n=1}^{3} V_n}$. The volume was found to be:

$$V = \sum_{n=1}^{3} V_n = V_1 + V_2 + V_3 \tag{3}$$

$$V_1 = \frac{\pi}{4}(D_1^2 - D_2^2)t_1 = \frac{\pi}{4}(0.35^2 - 0.285^2) * 0.055 = 1.78295 \text{ x } 10^{-3} \text{ m}^3$$
$$V_2 = \frac{\pi}{4}(D_2^2 - D_3^2)t_2 = \frac{\pi}{4}(0.285^2 - 0.07^2) * 0.02 = 1.19891 \text{ x } 10^{-3} \text{ m}^3$$
$$V_3 = \frac{\pi}{4}(D_3^2 - D_O^2)t_3 = \frac{\pi}{4}(0.07^2 - 0.0254^2) * 0.055 = 183.7959 \text{ x } 10^{-6} \text{ m}^3$$

$$\therefore V = 1.78295 \text{ x } 10^{-3} + 1.19891 \text{ x } 10^{-3} + 183.7959 \text{ x } 10^{-6}$$
$$\rightarrow V = 3.165658 \text{ x } 10^{-3} \text{ m}^3$$

$$\text{Since } m = 8.65\text{kg}^*\text{, then } \rho = \frac{m}{V} = \frac{8.65}{3.165658 \text{ x } 10^{-3}} = 2732.449135 \text{ kg/m}^3$$

Now, using Eq. (1) the second moment of area for each cylinder was found:

$$(I_z)_1 = \frac{\pi}{2}(r_1^4 - r_2^4) = \frac{\pi}{2}(0.175^4 - 0.1425^4) = 825.527 \text{ x } 10^{-6} \text{ m}^4$$
$$(I_z)_2 = \frac{\pi}{2}(r_2^4 - r_3^4) = \frac{\pi}{2}(0.1425^4 - 0.035^4) = 645.3509 \text{ x } 10^{-6} \text{ m}^4$$
$$(I_z)_3 = \frac{\pi}{2}(r_3^4 - r_O^4) = \frac{\pi}{2}(0.035^4 - 0.0127^4) = 2.31631 \text{ x } 10^{-6} \text{ m}^4$$

From Eq. (2), the mass moment of inertia I_m was found for each cylinder:

$$(I_m)_1 = (I_z)_1\rho t_1 = (825.527 \text{ x } 10^{-6})(2732.449135)(0.055) = 0.124064 \text{ kgm}^2$$
$$(I_m)_2 = (I_z)_2\rho t_2 = (645.3509 \text{ x } 10^{-6})(2732.449135)(0.02) = 0.035268 \text{ kgm}^2$$
$$(I_m)_3 = (I_z)_3\rho t_3 = (2.31631 \text{ x } 10^{-6})(2732.449135)(0.055) = 0.000348 \text{ kgm}^2$$

So, the mass moment of inertia of flywheel 12 is the sum of the mass moment of inertia of each of the concentric cylinders that make up the flywheel. That is:

$$I_m = \sum_{n=1}^{3}(I_m)_n \tag{4}$$

$$I_m = 0.124064 + 0.035268 + 0.000348$$
$$\boxed{I_m = 0.15968 \text{ kgm}^2}$$

Flywheel 7 Hub

For the flywheel 7 hub, an assumption had to be made as to the material that was used, the assumption was that the hub was made of the same steel as the central shaft, and so it was assumed that $\rho \approxeq 7800$ kg/m^3. The hub was separated into concentric cylinder as with flywheel 12, but this time it was separated into just two. The volume of the hub was found to be:

$$V = 47.7572 \text{ x } 10^{-6} \text{ m}^3$$

The mass given was both the hub mass and flywheel mass combined, to separate the two, the following relationship was used:

$$m = \rho V \tag{5}$$

$$\therefore m = (7800)(47.7572 \text{ x } 10^{-6}) = 0.3725 \text{ kg}$$

Using Eq. (2), (3) and (4) [with n=1→2 instead of 3], and ignoring the 4 screw holes in the second moment of area equation (the effect of the 4 holes on the overall moment of inertia is negligible), the mass moment of inertia of the hub was found to be:

$$\boxed{I_m = 188.3039 \text{ x } 10^{-6} \text{ kgm}^2}$$

Flywheel 7

Using the same method as with flywheel 12, the volume of flywheel 7 was found to be:

$$V = 3.46266 \text{ x } 10^{-3} \text{ m}^3$$

From the mass of the hub found above, and the total mass of the hub and flywheel combined, the mass of the flywheel can be found as:

$$m = 2.51 - 0.3725 = 2.1375\text{kg}$$
$$\therefore \rho = \frac{m}{V} = \frac{2.1375}{3.46266 \text{ x } 10^{-3}} = 617.300 \text{ kg/m}^3$$

Using Eq. (2), (3) and (4), and as with the flywheel hub, ignoring the 4 screw holes, the mass moment of inertia of the flywheel was calculated as:

$$\boxed{I_m = 0.042553 \text{ kgm}^2}$$

Rope Drum 2

As with the flywheel hub, the rope drum was separated into two concentric cylinders, and the volume was found to be:

$$V = 738.7177 \text{ x } 10^{-6} \text{ m}^3$$
$$\therefore \rho = \frac{m}{V} = \frac{2.1}{738.7177 \text{ x } 10^{-6}} = 2842.76385 \text{ kgm}^2$$

Once again, Eq. (2), (3) and (4) were used to obtain the mass moment of inertia of the rope drum:

$$\boxed{I_m = 0.0058271 \text{ kgm}^2}$$

Central Shaft

The last component to determine the mass moment of inertia for is the central shaft of the system, being a solid cylinder the second moment of area is simply:

$$I_z = \frac{\pi}{2} r^4 \tag{6}$$

The volume and density of the shaft were calculated as:

$$V = \tfrac{\pi}{4} d^2 l = \tfrac{\pi}{4}(0.0254)^2(0.41) = 207.75 \text{ x } 10^{-6} \text{ m}^3$$
$$\therefore \rho = \frac{m}{V} = \frac{1.62}{207.75 \text{ x } 10^{-6}} = 7797.831 \text{ kg/m}^3$$

From these values, the mass moment of inertia can be determined:

$$I_z = \tfrac{\pi}{2}(0.0127^4) = 40.8634 \text{ x } 10^{-9} \text{ m}^4$$
$$I_m = I_z \rho t = (40.8634 \text{ x } 10^{-9})(7797.831)(0.41)$$
$$\boxed{I_m = 0.0001306 \text{ kgm}^2}$$

And so, from the above I_m values, the mass moment of inertia of the entire system was determined by the sum of the mass moment of inertia of the individual constituents. The value was calculated to be:

$$I_{m,total} = 0.15968 + 188.3039 \text{ x } 10^{-6} + 0.042553 + 0.0058271 + 0.0001306$$
$$\boxed{I_{m,total} = 0.208379 \text{ kgm}^2}$$

Once the mass moment of inertia of the system was determined, an expression for the time for a mass to fall as a function of distance, $t(s)$, was derived. The free body diagram of the system is shown below:

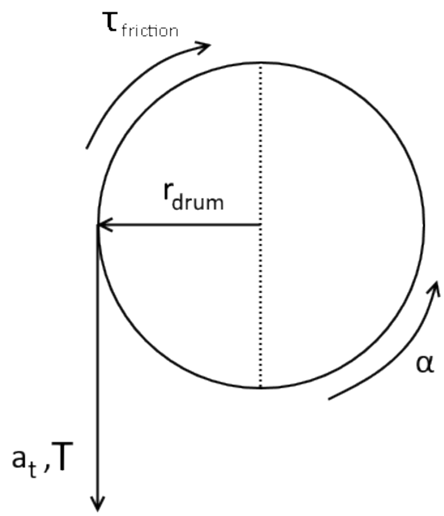


Figure 3: Free-body diagram of rope drum

From Newton's Second Law, the following expressions were found:

$$\circlearrowleft + \sum \tau = I\alpha \tag{7}$$

$$\rightarrow T r_{drum} - \tau_{friction} = I_{m,total}\alpha$$

Where $\tau_{friction} = 0.04$ Nm; $T = mg$; $m = m_{D||E} + m_{snaphook}$

$$\therefore \alpha = \frac{mgr_{drum} - 0.04}{I_{m,total}}$$

Translational motion is described by the equation:

$$s(t) = s_0 + v_0 t + \tfrac{1}{2}at^2$$

Where $a = a_t = \alpha r_{drum}$, and the mass was released at $s = 0$m from rest, so $s_0 = v_0 = 0$, taking downwards as positive, then the expression becomes:

$$s(t) = \frac{1}{2}t^2\alpha r_{drum} = \frac{t^2 r_{drum}}{2I_{m,total}}(mgr_{drum} - 0.04) \tag{8}$$

The above equation was rearranged to obtain an expression of time as a function of distance:

$$t(s) = \sqrt{\frac{2I_{m,total}s}{r_{drum}(mgr_{drum} - 0.04)}} \tag{9}$$

Applying Eq. (9) with $r_{drum} = 0.075$m, $I_{m,total} = 0.208379$ kgm^2, $g = 9.81$ m/s^2, the following theoretical values were obtained and tabulated:

Table 1: Time for mass D to fall a certain distance

Test	Distance (m)	t_{exp} 1 (s)	t_{exp} 2 (s)	Avg. t_{exp} (s)	t_{theo} (s)	$\lvert\Delta t/t\rvert$ (%)
1 & 2	0.20	1.41	1.35	1.38	1.446	4.56
3 & 4	0.35	2.05	1.91	1.98	1.913	3.5
5 & 6	0.50	2.29	2.28	2.285	2.286	0.044
7 & 8	0.65	2.65	2.71	2.68	2.606	2.84
9 & 10	0.80	3.17	3.20	3.185	2.892	10.13

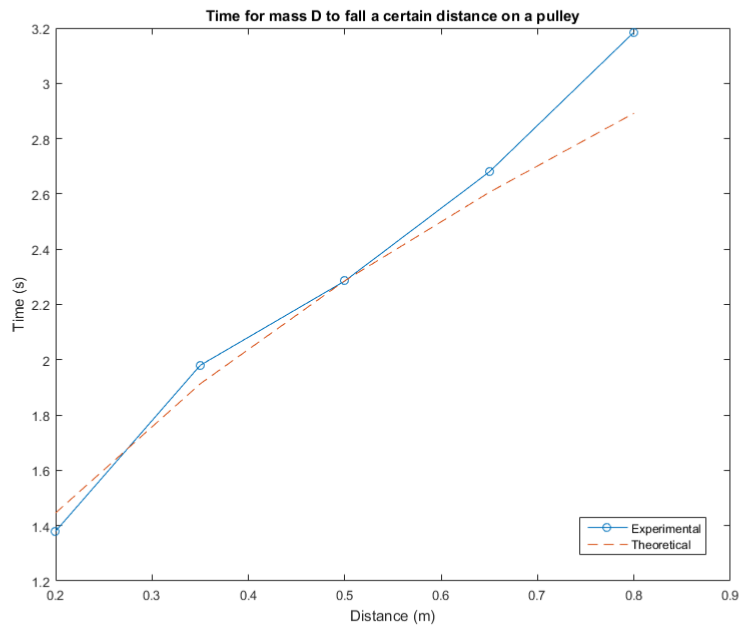
Note: t_{exp} is experimental time and t_{theo} is theoretical time, $|\Delta t/t|$ is the percentage error between avg. t_{exp} and t_{theo};
also, for mass D, $m = 0.75 + 0.0027 = 0.777$ kg

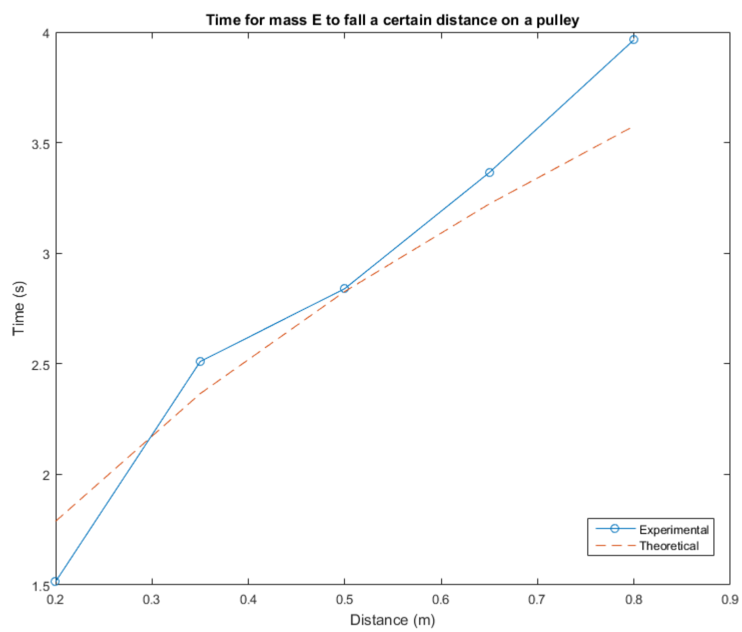
Table 2: Time for mass E to fall a certain distance

Test	Distance (m)	t_{exp} 1 (s)	t_{exp} 2 (s)	Avg. t_{exp} (s)	t_{theo} (s)	$\lvert\Delta t/t\rvert$ (%)
1 & 2	0.20	1.52	1.51	1.515	1.788	15.27
3 & 4	0.35	2.59	2.43	2.51	2.365	6.13
5 & 6	0.50	2.85	2.83	2.84	2.827	0.46
7 & 8	0.65	3.33	3.40	3.365	3.223	4.41
9 & 10	0.80	3.96	3.97	3.965	3.575	10.91

Note: Mass E is $m = 0.5 + 0.0027 = 0.527$ kg

Plotting the above data, graphs were generated for both mass D and E respectively, with the distance travelled by the mass on the x-axis, and the time taken to travel this distance on the y-axis.





As can be seen from the above graphs, there is an obvious discrepancy between some of the experimental and theoretical values, the reasons for this will be discussed in the next section.

5 Discussion

As is obvious from the plots shown in the results, a significant inconsistency was present between the theoretical and experimental results. This can be attributed to a variety of factors that include human reaction speed, rounding and inaccuracy of theoretical calculations due to the simplification of the flywheels and flywheel hub (ignoring screw holes and small nibs on the outer rings) as well as ignoring the collar and locking collar on the central shaft. The most influential component was human reaction speed; this was immediately realised as a significant source of error during the experiment when the results received varied quite significantly between identical tests. It made logical sense to assume this, as can be seen by the $0.044 \leq |\frac{\Delta t}{t}| \leq 15.27$ spread, with the least accurate results being the first and last tests for each mass, this is due to the immediate reaction required for a 0.2m reading, and that the mass was at its highest velocity for 0.8m, meaning it was more difficult to get accurate timing. In an attempt to mitigate the effect this had on the final results, multiple tests were taken with the data then averaged to give a more representative and accurate answer. However, in order for human reaction speed to be a non-factor, a very large sample size of results would have to be taken, which is something that was not done here and therefore it remains a large factor in the discrepancy of the results. Another contributing source of error was within the theoretical calculations for the mass moment of inertia of the system. During the calculation for the second moment of area, I_z, the screw holes and small outer rings present on the flywheels, flywheel hub, and rope drum were not considered in order to simplify the calculations. Individually, the impact this would have on the final result of each mass moment of inertia is minuscule, however, since the total mass moment of inertia of the system is the sum of the mass moment of inertia of each of the components that make up the system (see Eq. 4), the error in the calculation compounds, and becomes potentially impactful. Due to this, the theoretical values in Table 1. and 2. are not entirely representative of the true theoretical values, and as such, neither is the percentage error, $|\frac{\Delta t}{t}|$.

6 Conclusion

Conclusively, as is obvious from the errors listed in detail above, the results obtained spread over too large of an error margin (that is, $0.044 \leq |\frac{\Delta t}{t}| \leq 15.27$) to be convincingly accurate. While most were within a reasonable percentage error ($< 10\%$), and the average error being a completely justifiable 5.8254%, several outliers and rapid fluctations in the percentage error meant the results were too unpredictable, and so it would be impossible to account for this within calculations were they to be used. Overall, the system set up provided an accurate foundation upon which to perform the experiment, and so the improvements to be made to the experiment lie solely within the recording of data, and theoretical calculation of the mass moment of inertia of the system. In order to improve the accuracy of the recording of the test data, a much larger sample size of results should be taken, averaging them to remove the influence of potential outlier measurements. More specifically, as the number of recordings, $N \to \infty$, $t_{exp} \to t_{theo}$, the outcome being a much more accurate experiment. Moreover, the precision of the theoretical calculations can be improved by not simplifying the shape of the components, and including any extreme imperfections in the overall structure.

7 Appendix

7.1 Appendix A

A table of the dimensions and characteristics of each of the components that comprise the system used for the experiment.

Table 3: Component dimensions and characteristics

Component	Number or Letter	Measured Mass (kg)	Theoretical Volume (m^3)	Density (kg/m^3)	Theoretical Mass Moment of Inertia (kgm^2)
Rope Drum	2	2.1	738.72 x 10^{-6}	2842.76	0.0058271
Snap Hook	-	0.027	-	-	-
Suspended mass 1	D	0.75	-	-	-
Suspended mass 2	E	0.5	-	-	-
Flywheel on one end	7	2.1375	3.46266 x 10^{-3}	617.3	0.042553
Flywheel Hub	-	0.3725	47.7572 x 10^{-6}	7800	188.3039 x 10^{-6}
Flywheel on other end	12	8.65	3.165658 x 10^{-3}	2732.449	0.15968
Shaft	-	1.62	207.75 x 10^{-6}	7797.831	0.0001306
Total rotating components	-	-	-	-	0.208379

References

[1] School of Mechanical and Manufacturing Engineering UNSW 2016, Flywheel No. 7, CAD document distributed in Engineering Mechanics 2 MMAN2300 at UNSW Australia on 4 October 2016

[2] School of Mechanical and Manufacturing Engineering UNSW 2016, Flywheel No. 12, CAD document distributed in Engineering Mechanics 2 MMAN2300 at UNSW Australia on 4 October 2016

[3] School of Mechanical and Manufacturing Engineering UNSW 2016, Hub For Flywheel No. 7 & 8, CAD document distributed in Engineering Mechanics 2 MMAN2300 at UNSW Australia on 4 October 2016

[4] School of Mechanical and Manufacturing Engineering UNSW 2016, Rope Drum No. 2, CAD document distributed in Engineering Mechanics 2 MMAN2300 at UNSW Australia on 4 October 2016

[5] School of Mechanical and Manufacturing Engineering UNSW 2016, Central Shaft, CAD document distributed in Engineering Mechanics 2 MMAN2300 at UNSW Australia on 4 October 2016