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CS 70, Summer 2013
Homework 2
Problem 1 [9 Points]

1. [9 Points]

(a) [3 Points]In a stable marriage instance, if a man M and a woman W each put each other at the top of their respective preference lists, then M must be paired with W in every stable pairing.

True. For the sake of contradiction, let there be a stable pairing of two men and two women defined as: M paired with W', and W paired with M'. This pairing forms a rogue couple, namely, (M,W).

(b) [3 Points] In a stable marriage instance with at least two men and two women, if man M and woman W put each other at the bottom of their respective prefernce lists, then M cannot be paired with W in any stable pairing.

Take a stable marriage instance to be defined by the following tables, where A,B,C stand for 3 men, and 1,2,3, stand for 3 women.

Men prefernce:

A	1	2	3
B	2	3	1
C	1	3	2

Women Preference:

1	C	B	A
2	B	A	C
3	B	C	A

In this case there is a stable pairing: {(B,2),(C,1),(A,3)}, where A and 3 both put each other at the bottom of their respective preference lists, and they still ended up in a stable pairing together.

(c) [3 Points] For every $n \geq 2$, there exists a stable marriage instance with n men and n women that has an unstable pairing in which every umathced man-woman pair is a rogue couple.

Let the following tables describe a preference list for a marriage instance,

Men prefernce:

A	1	2	3
B	2	3	1
C	3	1	2

Women Preference:

1	A	C	B
2	B	A	C
3	B	C	A

A pairing where each unmatched-pair is a rogue couple can be described by: {(A,3),(B,1),(C,2)}. Observing that this case can be constructed for $n = 3$, it can be shown that pairing structure will hold for the general case, $n \geq 2$. For any instance of the form:

Men's preference list: $\begin{matrix} M_1 & \cdots & W_1 \\ M_2 & \cdots & W_2 \\ \vdots & \ddots & \vdots \\ M_n & \cdots & W_n \end{matrix}$ Women's preference list: $\begin{matrix} W_1 & \cdots & M_1 \\ W_2 & \cdots & M_2 \\ \vdots & \ddots & \vdots \\ W_n & \cdots & M_n \end{matrix}$

There exists a pairing $\{(M_1, W_1), (M_2, W_2), \cdots, (M_n, W_n)\}$, where each unpaired couple is rogue.