Zackery Field
ID: 23031734
CS 70, Summer 2013
Homework 1
Problem 7 (12 Points)

7a [4 points]

Theorem:

$\sum\limits_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ holds for $n \geq 1$, $n \in \mathbb{Z}$

Proof by simple induction:

Base Case:

$n = 1$

$\sum\limits_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1+1}$

$\frac{1}{2} = \frac{1}{2}$

Inductive Hypothesis:

Assume P(n). That is $\sum\limits_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$.

Inductive Step:

Assume P(n+1). That is $\sum\limits_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{n+1}{(n+1)+1}$.

$$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{n+1}{(n+1)+1} \tag{1}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} + \frac{1}{(n+1)((n+1)+1)} = \frac{n+1}{n+2} \tag{2}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n+1}{n+2} - \frac{1}{(n+1)(n+2)} \tag{3}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{(n+1)^2 - 1}{(n+1)(n+2)} \tag{4}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{((n+1)-1)((n+1)+1)}{(n+1)(n+2)} \tag{5}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{(n)(n+2)}{(n+1)(n+2)} \tag{6}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \tag{7}$$

$$\tag{8}$$

7b [4 points]

Theorem:

$\forall n \in \mathbb{N}, 5|(8^n - 3^n)$

Proof by simple induction:

Lemma:

For any numbers a, x, y. If x and y are divisible by z, then ax + y is divisible by z.

Base Case:

$n = 1$

$5|(8^n - 3^n)$
$5|(8^1 - 3^1)$
$5|(8 - 3)$
$5|5$

Inductive Hypothesis:

Assume P(n): That is, $5|(8^n - 3^n)$

Inductive Step:

Assume P(n+1): That is, $5|(8^{(n+1)} - 3^{(n+1)})$

$$
\begin{align}
8^{(n+1)} - 3^{(n+1)} \tag{9}\\
(8^n * 8) - (3^n * 3) \tag{10}\\
(8^n * 3) + (8^n * 5) - (3^n * 3) \tag{11}\\
3(8^n - 3^n) + (8^n * 5) \tag{12}\\
5|8^n - 3^n \tag{13}\\
5|8^n * 5 \tag{14}
\end{align}
$$

And, by the lemma, $3(8^n - 3^n) + (8^n * 5)$ is also divisible by 5.

7c [4 points]

Theorem: