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CS 70, Summer 2013
Homework 2
Problem 3 [13 Points]

1. [13 Points] Fitting med students to hospitals with the stable marriage algorithm.

   (a) [5 Points] Modify the propose-and-reject alogrithm from class so that it finds a stable assignment of medical students to slots.

   $q_h = numberOfSpots$, for a hospital h
   $n = numberOfStudents$
   $s = particularStudent$
   and, $\sum q_h < n$

   Apply-and-reject Algorithm:
   On week 1, each student, s, applies to the school at the top of their preference list. Then, from those applications, each hospital, h, populates its waitlist (of length q_h) based on their respective preference lists. All students that do not make it onto the waitlist for the first week are immediately rejected, and they remove the rejecting school from their preference list. The next week, each student once again applies to the school at the top of their (possibly revised) preference list, and based on the applications it receives, each hospital updates their respective waitlists. This continues, week after week, until all the schools receive as many applications as they have spots on their waitlists. When this occurs, all students placed on the final waitlist are accepted to the school they are waitlisted at.

   (b) [7 Points] Give a proof that the above algorithm does indeed find a stable assignment. The Improvement Lemma should be applied.

   Theorem: The apply-and-reject algorithm (detailed above) always gives a stable assignment.
   Improvement Lemma: For each hospital, after its waitlist becomes full, its least favorite student on the waitlist can only improve over time.

   Direct Proof:
   It can be shown that there is no student in a rogue pairing with a hospital. Let (s,h) be some pairing of student to hospital such that s prefers some hospital h* to h. We can show that h* prefers s* to s, therefore eliminating the possibility that (s,h*) is a rogue pairing. Since h* is higher on the preference list of s than h, s must have applied to h* first and had been rejected. Utilizing the Improvement Lemma, h* has a set of students of number q_{h*}in order of prefence, and the lowest member of that list must improve over time. Therefore, h* prefers all the students on its waitlist to s, and (s,h*) is therefore not a rogue pairing.