Zackery Field
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CS 70, Summer 2013
Homework 1
Problem 3 (9 Points)

3a. [3 points]

Theorem: $\neg\forall x\exists y(P(x) \Rightarrow \neg Q(x,y)) \equiv \exists x\forall y(P(x) \wedge Q(x,y))$

Proof:

$$\neg\forall x\exists y(P(x) \Rightarrow \neg Q(x,y)) \equiv \exists x\forall y(P(x) \wedge Q(x,y)) \tag{1}$$
$$\exists x\neg\exists y(P(x) \Rightarrow \neg Q(x,y)) \equiv \exists x\forall y(P(x) \wedge Q(x,y)) \tag{2}$$
$$\exists x\forall y\neg(P(x) \Rightarrow \neg Q(x,y)) \equiv \exists x\forall y(P(x) \wedge Q(x,y)) \tag{3}$$
$$\neg(P(x) \Rightarrow \neg Q(x,y)) \equiv (P(x) \wedge Q(x,y)) \tag{4}$$

Proof by Truth Table:

P	Q	$\neg(P(x) \Rightarrow \neg Q(x,y))$	$(P(x) \wedge Q(x,y))$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

After removing the equivalent quantifiers, the truth table demonstrates that the equivalence holds.

3b. [3 points]

Theorem: $\forall x\exists y(P(x) \Rightarrow Q(x,y)) \equiv \forall(P(x) \Rightarrow (\exists Q(x,y))$

Proof:

$$\forall x\exists y(P(x) \Rightarrow Q(x,y)) \equiv \forall x(P(x) \Rightarrow (\exists Q(x,y)) \tag{5}$$
$$\forall x\exists y(\neg P(x) \vee Q(x,y)) \equiv \forall x(\neg P(x) \vee (\exists Q(x,y)) \tag{6}$$
$$\forall x((\exists y\neg P(x)) \vee (\exists y(x,y))) \equiv \forall x(\neg P(x) \vee (\exists Q(x,y)) \tag{7}$$
$$\forall x(\neg P(x) \vee (\exists y(x,y))) \equiv \forall x(\neg P(x) \vee (\exists Q(x,y)) \tag{8}$$

Distributing the quantifiers across disjunction statements, and removing the $\exists y$ from $P(x)$ demonstrates that the equivalence holds.

3c. [3 points]

Theorem: $\forall x \exists y (Q(x,y) \Rightarrow P(x)) \equiv \forall x ((\exists y Q(x,y)) \Rightarrow P(x))$

Proof:

$$\forall x \exists y (Q(x,y) \Rightarrow P(x)) \equiv \forall x ((\exists y Q(x,y)) \Rightarrow P(x)) \tag{9}$$
$$\forall x \exists y (\neg Q(x,y) \vee P(x)) \equiv \forall x (\neg(\exists y Q(x,y)) \vee P(x)) \tag{10}$$
$$\forall x (\exists y \neg Q(x,y) \vee \exists y P(x)) \equiv \forall x (\neg(\exists y Q(x,y)) \vee P(x)) \tag{11}$$
$$\forall x (\exists y \neg Q(x,y) \vee P(x)) \not\equiv \forall x (\forall y \neg Q(x,y) \vee P(x)) \tag{12}$$

The equivalence does not hold due to the difference in quantifier associated with Q(x,y).