Zachary Frappier

1.1 Logic & Proofs

1.1 35. Construct truth table for each of these compound propositions

d) d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
1	1	0	1	0	1
1	0	0	0	1	1
0	1	1	0	1	1
0	0	1	1	0	1

e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

p	q	r	$\neg p$	$\neg r$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow \neg r)$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
1	1	1	0	0	1	1	0
1	1	0	0	1	1	0	1
1	0	1	0	0	0	1	1
1	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	1	0	1	1
0	0	1	1	0	1	0	1
0	0	0	1	1	1	1	0

f) $(p \oplus q) \to (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
1	1	0	0	1	1
1	0	1	1	0	0
0	1	0	1	0	0
0	0	1	0	$\cancel{0}$ 1	1

36 d) $\neg p \oplus \neg q$ d↓   e) $(p \oplus q) \vee (p \oplus \neg q)$ ~~e~~f) $(p \oplus q) \wedge (p \oplus \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \oplus \neg q$	$p \oplus q$	$p \oplus \neg q$	e↓	f
1	1	0	0	0	0	1	1	0
1	0	0	1	1	1	0	1	0
0	1	1	0	1	1	0	1	0
0	0	1	1	0	0	1	1	0

~~e) Evaluate each of these expressions~~

48

48 Evaluate each of these expressions

c) $((01010) \oplus (11011)) \oplus 01000$ d) $(11011 \vee 01010) \wedge (10001 \vee 11011)$

$$\begin{array}{r} 01010 \\ \oplus\, 11011 \\ \hline 10001 \end{array} \qquad \begin{array}{r} 10001 \\ \oplus\, 01000 \\ \hline \boxed{11001} \end{array} \qquad \begin{array}{r} 11011 \\ 01010\, \vee \\ \hline 11011 \end{array} \rightarrow \begin{array}{r} 10001 \\ 11011\, \vee \\ \hline 11011 \end{array} \qquad \begin{array}{r} 11011 \\ 11011\, \wedge \\ \hline \boxed{11011} \end{array}$$

1.3 Propositional Equivalence

20. Show $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent.

p	q	¬p	¬q	$p \leftrightarrow q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
1	1	0	0	1	1	0	1
1	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0
0	0	1	1	1	0	1	1

Two columns are the equivalent

21. Show $\neg(p \leftrightarrow q)$ & $p \leftrightarrow (\neg q)$ are logically equivalent.

p	q	¬q	$p \leftrightarrow \neg q$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$
1	1	0	0	1	0
1	0	1	1	0	1
0	1	0	1	0	1
0	0	1	0	1	0

both green col equivalent

24 Show $\neg(p \oplus q) \equiv p \leftrightarrow q$ 25 $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	¬p	$\neg p \leftrightarrow q$
1	1	0	1	1	0	0	0
1	0	1	0	0	1	0	1
0	1	1	0	0	1	1	1
0	0	0	1	1	0	1	0

35. $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$ NOT EQUIVALENT

p	q	r	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	
1	1	1	1	1	1	1	
1	1	0	1	0	0	0	
1	0	1	0	1	1	1	
1	0	0	0	1	1	1	
0	1	1	1	1	1	1	
0	1	0	1	$\boxed{0}$	0	$\boxed{1}$	← not same
0	0	1	1	1	1	1	
0	0	0	1	$\boxed{0}$	1	$\boxed{1}$	←

37. $(p \to q) \to (r \to s) \not\equiv (p \to r) \to (q \to s)$

p	q	r	s	$r \to s$	$p \to q$	$(p\to q)\to(r\to s)$	$q \to s$	$(p \to r)$	$(p\to r)\to q \to s$	
1	1	1	1	1	1	1	1	1	1	
1	1	1	0	0	1	0	1	0	0	
1	1	0	1	1	1	1	0	1	1	
1	1	0	0	1	1	1	0	0	1	
1	0	1	1	1	0	1	1	1	1	
1	0	1	0	0	0	1	1	1	1	
1	0	0	1	1	0	1	0	1	1	
1	0	0	0	1	0	1	0	1	1	
0	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	0	1	0	0	
0	1	0	1	1	1	1	1	1	1	
0	1	0	0	1	1	$\boxed{1}$	1	0	$\boxed{0}$	—
0	0	1	1	1	1	1	1	1	1	
0	0	1	0	0	1	$\boxed{0}$	1	1	$\boxed{1}$	—
0	0	0	1	1	1	1	1	1	1	
0	0	0	0	1	1	1	1	1	1	

71. Explain steps of compound proposition for sudoku puzzle.

1) Let $p(i, j, n)$ where i = row j = col n = number

2) every row contains every number $\bigwedge_{i=1}^{9}\bigwedge_{n=1}^{9}\bigvee_{j=1}^{9} p(i,j,n)$

71

3) every col contains every number $\bigwedge_{j=1}^{9}\bigwedge_{n=1}^{9}\bigvee_{i=1}^{9} p(i,j,n)$

72) 4) every 3x3 block contains every number $\bigwedge_{r=0}^{2}\bigwedge_{s=0}^{2}\bigwedge_{n=1}^{9}\bigvee_{i=1}^{3}\bigvee_{j=1}^{3} p(3r+i, 3s+j, n)$

1.4 Predicates & Quantifiers

12. let $Q(x)$ be statement $x+1 > 2x$, if domain all integers, find truth

a) $Q(0) = 0+1 > 2(0) = 1 > 0$ True

b) $Q(-1) = -1+1 > 2(-1) = 0 > -2$ True

c) $Q(1) = 1+1 > 2(1) = 2 > 2$ False

d) $\exists x Q(x)$ = there exist a number x to make true = True

e) $\forall x Q(x)$ = for all values x, it is True = False

f) $\exists x \neg Q(x)$ = there exist an x it is false = True

g) $\forall x \neg Q(x)$ = for all x values it is false = False

35. Express negation of each these statements $\forall \to \exists$ $< \to \geq$ $\wedge \to \vee$

e) $\forall x ((x < -1) \vee (x > 2))$

$\boxed{\exists x ((x \geq -1) \wedge (x \leq 2))}$

f) $\exists x ((x < 4) \vee (x > 7))$

$\boxed{\forall x ((x \geq 4) \wedge (x \leq 7))}$

1.5 Nested Quantifiers

33. Rewrite statements so negations only appear within predicates

d) $\neg(\exists x \exists y \neg P(x,y) \wedge \forall x \forall y\, Q(x,y))$ $\exists \to \forall$ $\wedge \to \vee$ $\neg\neg = \cancel{\neg}$

$\boxed{(\forall x \forall y\, P(x,y) \vee \exists x \exists y \neg Q(x,y))}$

e) $\neg\forall x\, (\exists y \forall z\, P(x,y,z) \wedge \exists z \forall y\, P(x,y,z))$

$\boxed{(\exists x \forall y \exists z \neg P(x,y,z) \vee \forall z \exists y \neg P(x,y,z))}$

45. Determine truth value $\forall x \exists y (xy = 1)$ for all x, exist a y where $xy = 1$

a) nonzero positive real numbers any int x, y = fraction $\frac{1}{x}$ = True

b) nonzero integers No b/c y needs to be fraction = False

c) nonzero real numbers same as a, but if x = neg, y = neg = True

1.8 Proof Methods & Strategy

17. Show that each statement shows there is unique x s.t $P(x)$ = true $\exists! x P(x)$

a) $\exists x \forall y\, (P(y) \leftrightarrow x = y)$

$P(y) \to x = y$: if $P(y)$ is true, then $x = y$ uniqueness

$x = y \to P(y)$: if x is equal to y, then $P(y)$ is true existence

b) $\exists x\, P(x) \wedge \forall x \forall y\, (P(x) \wedge P(y) \to x = y)$

$\exists x P(x)$: there exist x for $P(x)$ to be true existence

$\wedge \forall x \forall y$: and all x and y $(P(x) \wedge P(y) \to x = y)$ $P(x)$ & $P(y)$ are true then $x = y$

c) $\exists x\, (P(x) \wedge \forall y\, (P(y) \to x = y)$

there exists an x, $P(x)$ is true and all y $P(y)$ is true then $x = y$

27. $1, 2, \ldots 2n$ n = odd $|j - k|$ prove n must be odd.

$= n(2n+1)$

any even # will result even, all odd # result odd

$\therefore$ b/c n = odd result will always be odd.

29. Prove a conjecture about decimal digits of the final digit of a fourth power.

~~use forward reasoning to show x is nonzero real #,~~ $\cancel{x^2+1/x^2} \geq 2$

let $n/10$, k = quotient l = remainder

$n = 10k + l$ compute n^4 in 10 cases.

$0^4 = 0$ $1^4 = 1$ $2^4 = 16$ $3^4 = 81$ $4^4 = 256$ $5^4 = 625$ $6^4 = 1296$

$7^4 = 2401$ $8^4 = 4096$ $9^4 = 6561$ last digits = $\{0, 1, 5, 6\}$

~~35~~

35. Prove $n = abc$ where a, b, c pos int then $a \leq \sqrt[3]{n}, b \leq \sqrt[3]{n}, c \leq \sqrt[3]{n}$

if it is not true that $a \leq \sqrt[3]{n}$, $b \leq \sqrt[3]{n}$, $c \leq \sqrt[3]{n}$

then the opposite $a > \sqrt[3]{n}$, $b > \sqrt[3]{n}$, $c > \sqrt[3]{n}$

$abc^3 < (\sqrt[3]{n})^3$ ← negation $abc > \sqrt[3]{n}$

$abc^3 < n \neq n = abc$

39. Let $S = x_1y_1 + x_2y_2 + \dots x_ny_n$ wher x, y different sequences of pos real #s.
each containing n elements.

a) Show S takes its Maximum value over all orderings of the two sequences when both sequences are sorted.

let sorted order be $a_1 \leq a_2 \leq \dots a_n$ & $b_1 \leq b_2 \leq \dots b_n$

$S = \sum_{i=1}^{n} a_ib_i \underset{?\geq}{} \sum a_jb_j$

b) Show S takes minimal value: $S = \sum_{i=1} a_ib_i \leq \sum a_jb_j$

2.1 Sets

9. For each set determine if 2 is element of set

a) $\{x \in R \mid x$ is int greater than $1\}$ since $2 > 1$, yes true.

b) $\{x \in R \mid x$ is the square of an int$\}$ $\sqrt{x} = 2$, No,

c) $\{2, \{2\}\}$ Set contains 2 and another set. an Yes

d) $\{\{2\}, \{\{2\}\}\}$ Set contains 2 other sets No

e) $\{\{2\}, \{2, \{2\}\}\}$ Set contains 2 more sets No

f) $\{\{\{2\}\}\}$, Set contains single set with more inside No

25. How many elements belonging to each set where a and b are distinct elements.

a) $P(\{a, b, \{a, b\}\})$ Power set: includes $\emptyset$ set, cardinality 2^a $a = \#$ elements

elements: 3, $a, b, \{a, b\}$ $2^3 = \boxed{8 \text{ elements}}$

b) $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$ elements: 4: $\emptyset, a, \{a\}, \{\{a\}\}$: $2^4 = \boxed{16 \text{ elements}}$

c) $P(P(\emptyset))$ Elements: 1: $P(\emptyset)$: $2^1 = \boxed{2 \text{ elements}}$

48. Find truth sets of each predicates where domain is set of int

a) $P(x)$: $x^3 \geq 1$ $\sqrt[3]{x} \geq 1$ $\sqrt[3]{x}$ $\boxed{[1, \infty)}$

b) $Q(x)$: $x^2 = 2$ $\sqrt{x^2} = \sqrt{2}$ $x = \sqrt{2}$ $\boxed{\{\sqrt{2}, -\sqrt{2}\}}$

c) $R(x)$: $x < x^2$ → $0 < x$,

$0 < x^2 - x$ $1 < x = (1, \infty)$

$0 < x(x-1)$ $\boxed{(1, \infty) \cup (-\infty, 0)}$

49. ordered pairs. two ordered pairs are equal if and only if if their first and second elements are equal.

We can construct ordered pairs using basic notation.

Show (a,b) to be $\{\{a\},\{a,b\}\}$ then $(a,b)=(c,d)$ if only if $a=c$ and $b=d$.

Case 1: $a \neq b$

1.1 if $a \neq b$, then $\{\{a\},\{a,b\}\}$ has two elements, set $\{a\}$ & set $\{a,b\}$

1.2 Set $\{\{c\},\{c,d\}\}$ must follow same. So $c \neq d$

1.3 Since $\{a\}$, $\{c\}$ are singletons implies $\{a\}=\{c\}$

1.4 Since $a \neq b$ in $\{a,b\}$, $c \neq d$ in $\{c,d\}$, $\{a,b\}=\{c,d\}$

Case 2: $a=b$ Proof by contradiction.

2.1 if $a=b$, then $\{\{a\},\{a,b\}\} = \{a\}$

2.2 similarly $c=d$ $\{\{c\},\{c,d\}\} = \{c\}$

2.3 Since $\{\{a\}\}=\{\{c\}\}$, $a=c$ = Contradiction.

2.2 Set operations

32. does $A=B$ if A, B, C are:

a) $A \cup C = B \cup C$? $\rightarrow \{\{a\},\{c\}\} = \{\{B\},\{c\}\}$ example $a=1$, $b=2$ $c=\{1,2\}$

$1 \cup \{1,2\} = \{1,2\} \neq 2 \cup \{1,2\} = \{1,2\}$ $\boxed{\text{NO}}$

b) $A \cap C = B \cap C$? $\rightarrow \{\{a\},\{c\}\} = \{\{b\},\{c\}\}$ $a=\{1,2\}$ $b=\{1,3\}$ $c=\{1\}$

$\{1,2\} \cap \{1\} = \{1\} \neq \{1,3\} \cap \{1\} = \{1\}$ $\boxed{\text{NO}}$

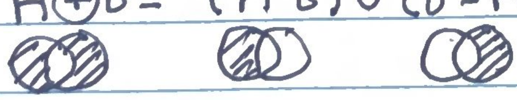
c) $(A \cup C \wedge B \cup C) \wedge (B \cap C \wedge A \cap C)$ $A \cap C \wedge B \cap C$: $\in C$ $\exists$ in A & B

$A \cup C \wedge B \cup C$: $\in A$ & $\in B$ $\exists$ in C

41. Show $A \oplus B = (A \cup B) - (A \cap B)$ $A \oplus B$: elements of A or B but not belonging to both (sames)

$A \cup B$: Elements both A, B $-$ $A \cap B$ elements belonging to both

42 Show $A \oplus B = (A-B) \cup (B-A)$ $(A-B)$ Elements of A minus Elements shared with A

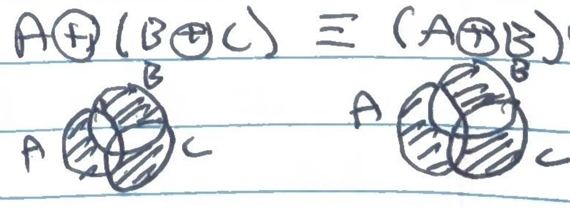


$(B-A)$ Elements of B minus elements shared with A.

union $(A-B)$ & $(B-A)$ = $A \oplus B$ = Elements of A & B but not shared between them

46. does Associative rule apply to $A \oplus (B \oplus C) \equiv (A \oplus B) \oplus C$

$A \oplus (B \oplus C)$: elements of A, not belonging to (Elements of B not belonging to C)



=

$(A+B)+C$: elements of A not belonging to B, not belonging to elements C.

$\boxed{\text{yes, Assosiative rules apply to nor } \oplus}$

47. Suppose A, B, C are sets s.t. $A \oplus C = B \oplus C$, must $A = B$?
yes b/c symmetric difference  
preserves uniqueness of both sets

53. Let $A_i = \{1, 2, \ldots i\}$ for $i = 1, 2, 3 \ldots$ Find
a) $\bigcup_{i=1}^{n} A_i$ $\quad A_1 \cup A_2 \ldots \cup A_i$: all element of future sets included $= [1, \infty)$
b) $\bigcap_{i=1}^{n} A_i$ $\quad A_1 \cap A_2 \ldots$: excludes future set's elements $= [1]$

56. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ for every positive int i when
a) $A_i = \{i, i+1, i+2, \ldots\}$. $\quad A_1 = \{1, 2, \ldots\}$ $\quad A_2 = \{2, 3, \ldots\}$
$\bigcup_{i=1}^{\infty}$ union, would be all elements so $\{i, 1, 2 \ldots \infty\}$
$\bigcap_{i=1}^{\infty}$ intersect, would only be shared elements, only empty $= \boxed{\{\emptyset\}}$
b) $A_i = \{0, i\}$
$\bigcup_{i=1}^{\infty}$ union, includes all elements $= \{0, 1 \ldots \infty\}$
$\bigcap_{i=1}^{\infty}$ intersect, only common $= \{\emptyset\}$
c) $A_i = (0, i)$ set of real #'s x w/ $0 < x < i$
$\bigcup_{i=1}^{\infty}$ $(0,1) \cup (0,2)$ union of real #s $= (0, \infty)$
$\bigcap_{i=1}^{\infty}$ intersect of all the sets, all include $= (0, 1)$
d) $A_i = (i, \infty)$ set real #'s x w/ $x > i$
$\bigcup_{i=1}^{\infty} = (1, \infty) \cup (2, \infty) \ldots$ $= (1, \infty)$
$\bigcap_{i=1}^{\infty} = (1, \infty) \cap (2, \infty) \ldots$ $=$ only $\emptyset$

2.3 Functions

46. Let f be function to A, B S + T subsets of B.
a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
$f^{-1}(S \cup T) = f(x) \in S \cup f(x) \in (T) = f^{-1}(S) \cup f^{-1}(T)$
b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$
$f^{-1}(S \cap T) = f(x)(S \cap T) = f(x) \in S \cap f(x) \in T = f^{-1}(S) \cup f^{-1}(T)$

2.4 Sequences & Summations

29. What are the values of these sums?
a) $\sum_{k=1}^{5} (k+1)$: $2 + 3 + 4 + 5 + 6$ $= \boxed{20}$
b) $\sum_{j=0}^{4} (-2)^j$: $1 + -2 + 4 + -8 + 16$ $= \boxed{11}$
c) $\sum_{i=1}^{10} (3)$: $3 + 3 + 3 \ldots$: $(3 * 10)$ $= \boxed{30}$
d) $\sum_{j=0}^{8} (2^{j+1} - 2^j)$: $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256$
$= \boxed{511}$

30. what are values of these sums when $S = \{1, 3, 5, 7\}$

a) $\sum_{j \in S} j =$ $1+3+5+7 = \boxed{16}$

b) $\sum_{j \in S} j^2 :$ $1^2 + 9 + 25 + 49 = \boxed{84}$

c) $\sum_{j \in S} 1/j :$ $1 + 1/3 + 1/5 + 1/7 = \boxed{16/105}$

d) $\sum_{j \in S} 1 =$ $1+1+1+1 = \boxed{4}$

46. Express $n!$ using product notation

$0! = 1$

$n! = \prod_{k=1}^{n} k$ $\quad \Pi$: Capital pi : Product of all elements $f(x) = k$ rang. 1-n

47. Find $\sum_{j=0}^{4} j!$, $0! + 1! + 2! + 3! + 4! = 1+1+2+6+24 = \boxed{34}$

48. Find $\prod_{j=0}^{4} j!$, $0! * 1! * 2! * 3! * 4! = 1*1*2*6*24 = \boxed{288}$

2-5 Cardinality of Sets

20. Show $|A| = |B|$ & $|B| = |C|$, then $|A| = |C|$

(1) $\in A = \in B$ (2) $\therefore$ Can substitute $|A|$ w/ $|B|$, (3) So $\in A = \in C$, (4) hence $|A| = |C|$

21. Show that if A, B, and C are sets s.t $|A| \leq |B|$ & $|B| \leq |C|$, then $|A| \leq |C|$

$|A| \leq |B|$ is one to one $f : |A| \leq |B|$

$|B| \leq |C|$ also one to one $g : |B| \leq |C|$

So $f \circ g$, since one to one $|A| \leq |C|$

23. Show that if A is an infinite set, then it contains a countably infinite subset

(1) Axiom of choice: for non empty sets, possible to select one element

(2) Set $A = \{1, 2, \ldots \infty\}$ each subset a has one element

(3) $a_n = A \setminus \{a_i\}$ each subset also infinite

(4) Subset $S = \{a_1, a_2, \ldots a_n\}$

37. Show that the set of all programs in a computer language is countable

let program language be set A, s.t Program 1 = A_1 Program 2 = A_2...

each string of symbols = subset range n

So if A = countable, then $n \subseteq A$ is also countable

38. Show $\{0,1,2,3,4,5,6,7,8,9\}$ is uncountable ← Cantor's Diagonalization

Let $S = \mathbb{Z}^+ = \{1, 2 .. \infty\}$ to set $D = \{0, 1, 2 ..\}$

Since S is countable and has one-to-one correspondence to D

D = infinite sequence of digits 0-9

$D_1 = \{S_{11}, S_{12}, S_{13}, \ldots\}$ $D_2 = \{S_{21}, S_{22}, S_{23} \ldots\}$ ... $D_n = \{S_{n1}, n_2\}$ ...

D_1 = infinite set functions & uncountable

39. use exercise 37 & 38 to show that there are functions that are not computable

37: Shows theres a countable set from a countable set

38: Shows uncountable set can come from countable set.

These two examples show both countable & uncountable can be obtained

2.6 Matrices

17. Let A & B be two $n \times n$ matricies. Show that

a) $(A+B)^t = A^t + B^t$

let $A=[a_{ij}]$ & $B=[b_{ij}]$ then $A+B=[a_{ij}+b_{ij}]$

therefor $(A+B)^t = [a_{ji}+b_{ji}] = [a_{ij}]+[b_{ij}] = A^t + B^t$

b) $(AB)^t = B^t A^t$

let $A=[a_{ij}]$ & $B=[b_{ij}]$, then $AB = \sum_q [a_{iq} b_{jq}]$

therefor $B^t A^t = \sum_q b_{qi} a_{jq} = \sum a_{jq} b_{qi} = (AB)^t$

24. a) Show system of linear equations

$a_{11}x_1 + a_{12}x_2 \cdots a_{1n}x_n = b$

$\vdots$

$a_{n1}x_1 + a_{n2}x_2 \cdots a_{nn}x_n = b_n$.

in variables $x_1, x_2 \ldots x_n$ can be expressed as $Ax=B$,

where $A=[a_{ij}]$, x is $n \times 1$ matrix w/ x_i ith entry

$B =$ $n \times 1$ matrix w/ b_i entry in its ith row

$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \\ a_{n1} & & a_{nn} \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ a = coefficient matrix, x = variable M

B = constant matrix

$(ax) = \begin{bmatrix} a_{11}x_1 + \cdots a_{1n}x_n \\ a_{n1}x_1 + \cdots a_{nn}x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ $\rightarrow$ So $Ax = B$

b) Show $A=[a_{ij}]$ is invertible so $X = A^{-1}B$, Let $Ax = B$

multiply by A^{-1} $A^{-1}Ax = A^{-1}B$

$A^{-1}A = I = 1$ $IX = A^{-1}B$ $\longrightarrow$ $X = A^{-1}B$

33. Let A, B, C be $m \times n$ zero-one matricies

a) $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

Let $A=[a_{ij}]$ $B=[b_{ij}]$ & $C=[c_{ij}]$

$A \vee (B \wedge C) = [a_{ij}] \vee [b_{ij} \wedge c_{ij}] = [a_{ij} \vee b_{ij}] \wedge [a_{ij} \vee c_{ij}] = (A \vee B) \wedge (A \vee C)$

b) $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

Let $A=[a_{ij}]$ $B=[b_{ij}]$ & $C=[c_{ij}]$

$A \wedge (B \vee C) = [a_{ij}] \wedge [b_{ij} \vee c_{ij}] = [a_{ij} \wedge b_{ij}] \vee [a_{ij} \wedge c_{ij}]$

35 Assume: A is an $m \times p$ zero-one matrix

B is an $p \times k$ zero-one matrix

C is an $k \times n$ zero-one matrix

Show $A \odot (B \odot C) = (A \odot B) \odot C$

let $A = [a_{ij}]$ $B = [b_{ij}]$ & $C = [c_{ij}]$

Method 1 $A \odot (B \odot C) = [\vee_q a_{iq} \wedge (\vee_r (b_{qr} \wedge c_{rj}))] = [\vee_q \vee_r (a_{iq} \wedge b_{qr} \wedge c_{rj})]$

$= [\vee_r \vee_q (a_{iq} \wedge b_{qr} \wedge c_{rj})] = [\vee_r (\vee_q (a_{iq} \wedge b_{qj})) \wedge c_{rj}]$

$= (A \odot B) \odot C$

Method 2

$A \odot (B \odot C) = [a_{ij}] \odot [b_{ij} \odot c_{ij}] = [a_{ij} \odot b_{ij}] + [c_{ij}] = (A \odot B) \odot C$