Algorithms.

Definition An algorithm is a finite sequence of precise instructions for performing a computation or for solving a problem.

Searching Algorithms.

Linear Search Algorithm

procedure linear search (x: int, $a_1, \cdots, a_n$: (distinct) int)

```
i := 1
while ( i ≤ n and x ≠ a_i )
    i := i + 1
if i ≤ n then location := i
else location := 0
return location ( i  if  x = a_i
                  0  if  x is not found )
```

Definition (Big-O notation)

Let f and g be functions from $\mathbb{R}$ to $\mathbb{R}$.

We write $f(x) = O(g(x))$ if there are constants C, k such that $|f(x)| \leq C|g(x)|$ whenever $x > k$.

Discussions. ① In the worst case scenario, how many comparisons

② If the input array may have some identical numbers, which index does the algorithm find?

Binary Search Algorithm (Python - bisect_left / bisect_right)

```
def bisect_left(a, x)   # a = [a_0, a_1, ..., a_{n-1}]
    lo = 0
    hi = len(a) - 1   # n-1
    while lo < hi:
        mid = (lo + hi) // 2
        if a[mid] < x:
            lo = mid + 1
        else:
            hi = mid
    return lo.
def bisect_right(a, x)  # a = [a_0, a_1, ..., a_{n-1}]
    lo = 0
    hi = len(a) - 1   # n-1
    while lo < hi:
        mid = (lo + hi) // 2
        if x < a[mid]:
            hi = mid
        else:
            lo = mid + 1
    return lo
```

Discussions. ① How many comparisons?

② If the input array may have some identical numbers, which index does the algorithm find?

Sorting Algorithms

Bubble sort: Array of numbers into increasing order by successively comparing adjacent elements, interchanging them if they are in the wrong order

Procedure bubblesort ($a_1, \ldots, a_n$: real numbers, $n \geq 2$)

for $i = 1$ to $n-1$

  for $j = 1$ to $n-i$

    if $a_j > a_{j+1}$ then interchange a_j and a_{j+1}

Insertion Sort: Array of numbers into increasing order by iteratively inserting each element of an unsorted list into its correct position in a sorted portion of the list

procedure insertionsort ($a_0, \ldots, a_{n-1}$: real number, $n \geq 2$)

for $i = 0$ to $n-1$

  key = a_i

  $j = i-1$

  while $j \geq 0$ and key $< a_j$

    interchange a_j, a_{j+1}

    $j := j-1$

Discussions ① How many comparisons?

② How many swaps?