Anisotropic image segmentation by a gradient structure tensor

Prev Tutorial: Motion Deblur Filter

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Compatibility	OpenCV >= 3.0

Goal

In this tutorial you will learn:

- what the gradient structure tensor is
- how to estimate orientation and coherency of an anisotropic image by a gradient structure tensor
- how to segment an anisotropic image with a single local orientation by a gradient structure tensor

Theory

Note

The explanation is based on the books [114], [19] and [251]. Good physical explanation of a gradient structure tensor is given in [275]. Also, you can refer to a wikipedia page Structure tensor.

A anisotropic image on this page is a real world image.

What is the gradient structure tensor?

In mathematics, the gradient structure tensor (also referred to as the second-moment matrix, the second order moment tensor, the inertia tensor, etc.) is a matrix derived from the gradient of a function. It summarizes the predominant directions of the gradient in a specified neighborhood of a point, and the degree to which those directions are coherent (coherency). The gradient structure tensor is widely used in image processing and computer vision for 2D/3D image segmentation, motion detection, adaptive filtration, local image features detection, etc.

Important features of anisotropic images include orientation and coherency of a local anisotropy. In this paper we will show how to estimate orientation and coherency, and how to segment an anisotropic image with a single local orientation by a gradient structure tensor.

The gradient structure tensor of an image is a 2x2 symmetric matrix. Eigenvectors of the gradient structure tensor indicate local orientation, whereas eigenvalues give coherency (a measure of anisotropism).

The gradient structure tensor J of an image Z can be written as:

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{12} & J_{22} \end{bmatrix}$$

where $J_{11} = M[Z_x^2]$, $J_{22} = M[Z_y^2]$, $J_{12} = M[Z_x Z_y]$ - components of the tensor, $M[]$ is a symbol of mathematical expectation (we can consider this operation as averaging in a window w), Z_x and Z_y are partial derivatives of an image Z with respect to x and y.

The eigenvalues of the tensor can be found in the below formula:

$$\lambda_{1,2} = \frac{1}{2}\left[J_{11} + J_{22} \pm \sqrt{(J_{11} - J_{22})^2 + 4J_{12}^2}\right]$$

where λ_1 - largest eigenvalue, λ_2 - smallest eigenvalue.

How to estimate orientation and coherency of an anisotropic image by gradient structure tensor?

The orientation of an anisotropic image:

$$\alpha = 0.5 arctg \frac{2J_{12}}{J_{22} - J_{11}}$$

Coherency:

$$C = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$

The coherency ranges from 0 to 1. For ideal local orientation (λ_2 = 0, λ_1 > 0) it is one, for an isotropic gray value structure (λ_1 = λ_2 > 0) it is zero.

Source code

C++ Python

You can find source code in the `samples/cpp/tutorial_code/ImgProc/anisotropic_image_segmentation/anisotropic_image_segmentation.cpp` of the OpenCV source code library.

```
#include <iostream>
#include "opencv2/imgproc.hpp"
#include "opencv2/imgcodecs.hpp"

using namespace cv;
using namespace std;

void calcGST(const Mat& inputImg, Mat& imgCoherencyOut, Mat& imgOrientationOut, int w);

int main()
{
    int W = 52;             // window size is WxW
    double C_Thr = 0.43;    // threshold for coherency
    int LowThr = 35;        // threshold1 for orientation, it ranges from 0 to 180
    int HighThr = 57;       // threshold2 for orientation, it ranges from 0 to 180

    Mat imgIn = imread("input.jpg", IMREAD_GRAYSCALE);
    if (imgIn.empty()) //check whether the image is loaded or not
    {
        cout << "ERROR : Image cannot be loaded..!!" << endl;
        return -1;
    }

    Mat imgCoherency, imgOrientation;
    calcGST(imgIn, imgCoherency, imgOrientation, W);

    Mat imgCoherencyBin;
    imgCoherencyBin = imgCoherency > C_Thr;
    Mat imgOrientationBin;
    inRange(imgOrientation, Scalar(LowThr), Scalar(HighThr), imgOrientationBin);

    Mat imgBin;
    imgBin = imgCoherencyBin & imgOrientationBin;

    normalize(imgCoherency, imgCoherency, 0, 255, NORM_MINMAX);
    normalize(imgOrientation, imgOrientation, 0, 255, NORM_MINMAX);

    imwrite("result.jpg", 0.5*(imgIn + imgBin));
    imwrite("Coherency.jpg", imgCoherency);
    imwrite("Orientation.jpg", imgOrientation);
    return 0;
}
void calcGST(const Mat& inputImg, Mat& imgCoherencyOut, Mat& imgOrientationOut, int w)
{
    Mat img;
    inputImg.convertTo(img, CV_32F);

    // GST components calculation (start)
    // J =  (J11 J12; J12 J22) - GST
    Mat imgDiffX, imgDiffY, imgDiffXY;
    Sobel(img, imgDiffX, CV_32F, 1, 0, 3);
    Sobel(img, imgDiffY, CV_32F, 0, 1, 3);
    multiply(imgDiffX, imgDiffY, imgDiffXY);

    Mat imgDiffXX, imgDiffYY;
    multiply(imgDiffX, imgDiffX, imgDiffXX);
    multiply(imgDiffY, imgDiffY, imgDiffYY);

    Mat J11, J22, J12;      // J11, J22 and J12 are GST components
    boxFilter(imgDiffXX, J11, CV_32F, Size(w, w));
    boxFilter(imgDiffYY, J22, CV_32F, Size(w, w));
    boxFilter(imgDiffXY, J12, CV_32F, Size(w, w));
    // GST components calculation (stop)

    // eigenvalue calculation (start)
    // lambda1 = 0.5*(J11 + J22 + sqrt((J11-J22)^2 + 4*J12^2))
    // lambda2 = 0.5*(J11 + J22 - sqrt((J11-J22)^2 + 4*J12^2))
    Mat tmp1, tmp2, tmp3, tmp4;
    tmp1 = J11 + J22;
    tmp2 = J11 - J22;
    multiply(tmp2, tmp2, tmp2);
    multiply(J12, J12, tmp3);
    sqrt(tmp2 + 4.0 * tmp3, tmp4);
```

```
    Mat lambda1, lambda2;
    lambda1 = tmp1 + tmp4;
    lambda1 = 0.5*lambda1;      // biggest eigenvalue
    lambda2 = tmp1 - tmp4;
    lambda2 = 0.5*lambda2;      // smallest eigenvalue
    // eigenvalue calculation (stop)

    // Coherency calculation (start)
    // Coherency = (lambda1 - lambda2)/(lambda1 + lambda2)) - measure of anisotropism
    // Coherency is anisotropy degree (consistency of local orientation)
    divide(lambda1 - lambda2, lambda1 + lambda2, imgCoherencyOut);
    // Coherency calculation (stop)

    // orientation angle calculation (start)
    // tan(2*Alpha) = 2*J12/(J22 - J11)
    // Alpha = 0.5 atan2(2*J12/(J22 - J11))
    phase(J22 - J11, 2.0*J12, imgOrientationOut, true);
    imgOrientationOut = 0.5*imgOrientationOut;
    // orientation angle calculation (stop)
}
```

Explanation

C++ Python

An anisotropic image segmentation algorithm consists of a gradient structure tensor calculation, an orientation calculation, a coherency calculation and an orientation and coherency thresholding:

```
    Mat imgCoherency, imgOrientation;
    calcGST(imgIn, imgCoherency, imgOrientation, W);

    Mat imgCoherencyBin;
    imgCoherencyBin = imgCoherency > C_Thr;
    Mat imgOrientationBin;
    inRange(imgOrientation, Scalar(LowThr), Scalar(HighThr), imgOrientationBin);

    Mat imgBin;
    imgBin = imgCoherencyBin & imgOrientationBin;
```

A function calcGST() calculates orientation and coherency by using a gradient structure tensor. An input parameter w defines a window size:

```
void calcGST(const Mat& inputImg, Mat& imgCoherencyOut, Mat& imgOrientationOut, int w)
{
    Mat img;
    inputImg.convertTo(img, CV_32F);

    // GST components calculation (start)
    // J =  (J11 J12; J12 J22) - GST
    Mat imgDiffX, imgDiffY, imgDiffXY;
    Sobel(img, imgDiffX, CV_32F, 1, 0, 3);
    Sobel(img, imgDiffY, CV_32F, 0, 1, 3);
    multiply(imgDiffX, imgDiffY, imgDiffXY);

    Mat imgDiffXX, imgDiffYY;
    multiply(imgDiffX, imgDiffX, imgDiffXX);
    multiply(imgDiffY, imgDiffY, imgDiffYY);

    Mat J11, J22, J12;      // J11, J22 and J12 are GST components
    boxFilter(imgDiffXX, J11, CV_32F, Size(w, w));
    boxFilter(imgDiffYY, J22, CV_32F, Size(w, w));
    boxFilter(imgDiffXY, J12, CV_32F, Size(w, w));
    // GST components calculation (stop)

    // eigenvalue calculation (start)
    // lambda1 = 0.5*(J11 + J22 + sqrt((J11-J22)^2 + 4*J12^2))
    // lambda2 = 0.5*(J11 + J22 - sqrt((J11-J22)^2 + 4*J12^2))
    Mat tmp1, tmp2, tmp3, tmp4;
    tmp1 = J11 + J22;
    tmp2 = J11 - J22;
    multiply(tmp2, tmp2, tmp2);
    multiply(J12, J12, tmp3);
    sqrt(tmp2 + 4.0 * tmp3, tmp4);

    Mat lambda1, lambda2;
    lambda1 = tmp1 + tmp4;
    lambda1 = 0.5*lambda1;      // biggest eigenvalue
    lambda2 = tmp1 - tmp4;
    lambda2 = 0.5*lambda2;      // smallest eigenvalue
    // eigenvalue calculation (stop)
```

```
    // Coherency calculation (start)
    // Coherency = (lambda1 - lambda2)/(lambda1 + lambda2)) - measure of anisotropism
    // Coherency is anisotropy degree (consistency of local orientation)
    divide(lambda1 - lambda2, lambda1 + lambda2, imgCoherencyOut);
    // Coherency calculation (stop)

    // orientation angle calculation (start)
    // tan(2*Alpha) = 2*J12/(J22 - J11)
    // Alpha = 0.5 atan2(2*J12/(J22 - J11))
    phase(J22 - J11, 2.0*J12, imgOrientationOut, true);
    imgOrientationOut = 0.5*imgOrientationOut;
    // orientation angle calculation (stop)
}
```

The below code applies a thresholds LowThr and HighThr to image orientation and a threshold C_Thr to image coherency calculated by the previous function. LowThr and HighThr define orientation range:

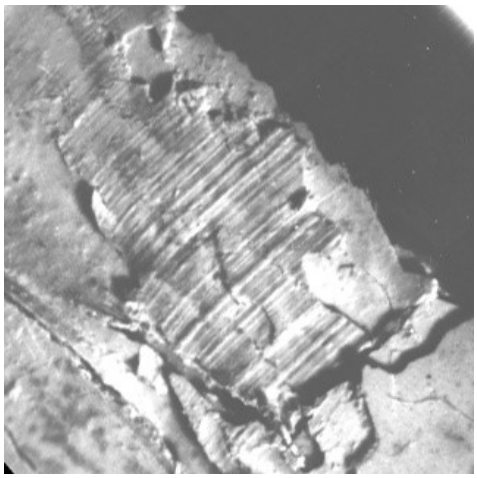
```
    Mat imgCoherencyBin;
    imgCoherencyBin = imgCoherency > C_Thr;
    Mat imgOrientationBin;
    inRange(imgOrientation, Scalar(LowThr), Scalar(HighThr), imgOrientationBin);
```

And finally we combine thresholding results:

```
    Mat imgBin;
    imgBin = imgCoherencyBin & imgOrientationBin;
```

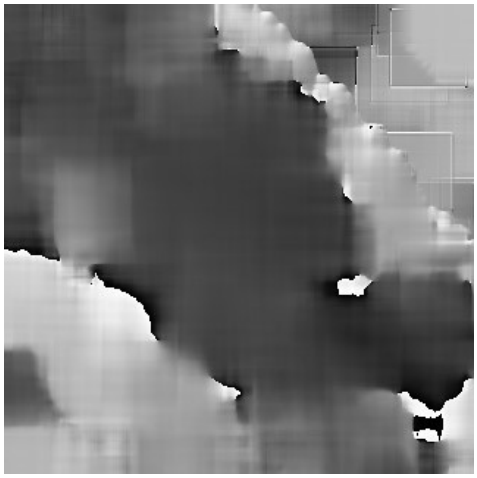
Result

Below you can see the real anisotropic image with single direction:

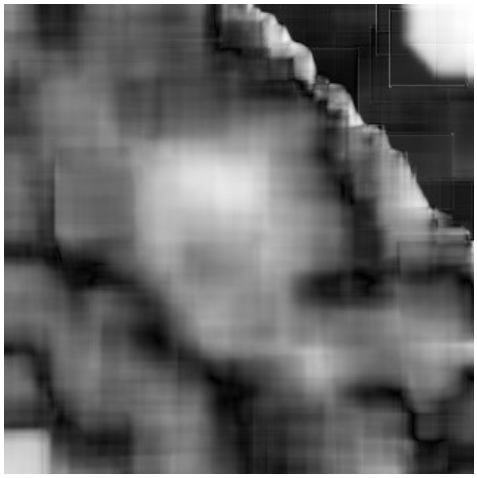


Anisotropic image with the single direction

Below you can see the orientation and coherency of the anisotropic image:

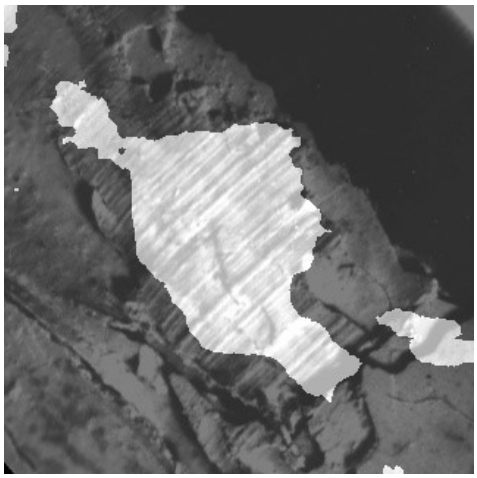


Orientation



Coherency

Below you can see the segmentation result:



Segmentation result

The result has been computed with w = 52, C_Thr = 0.43, LowThr = 35, HighThr = 57. We can see that the algorithm selected only the areas with one single direction.

References

- Structure tensor - structure tensor description on the wikipedia