Assignment # 5
Due: Thursday March 08, 2018, 9h00
– online submission only –

1. Derive the formula of centre of asymptotes of root loci and the asymptotical angles:

$$\begin{aligned}
\mathrm{Im}\{s\} &= (\mathrm{Re}\{s\} - \sigma_c)\tan\beta_l && (1)\\
\sigma_c &= \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} && (2)\\
\beta_l &= \frac{(2l+1)\pi}{n-m}, l = 0, 1, 2, \ldots, n-m-1 && (3)
\end{aligned}$$

[hint] For large radius R,

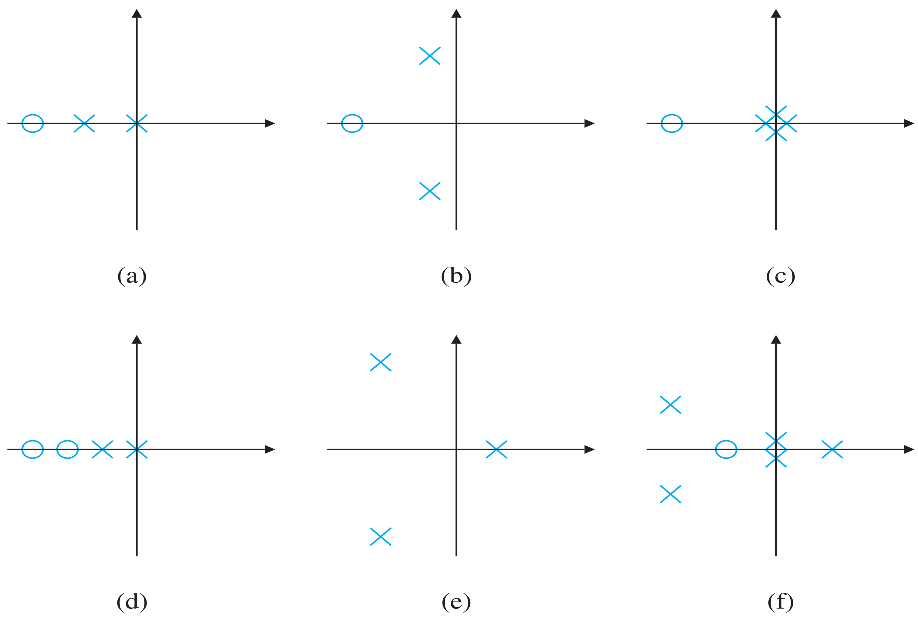
$$(\sigma_c + Re^{j\beta})^n \approx R^n e^{jn\beta} + n\sigma_c R^{n-1} e^{j(n-1)\beta}$$

2. (@Text P5.2) Roughly sketch the root loci for the pole-zero maps shown below. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros. (You may give numerical values to these poles-zeros and generate MATLAB plot to verify your sketch)

3. For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0$$

Sketch the root loci, find the asymptotes, and the value of K that the roots are located on the imaginary axis. Verify your sketch using MATLAB plot.



4. (@Text P5.15) A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

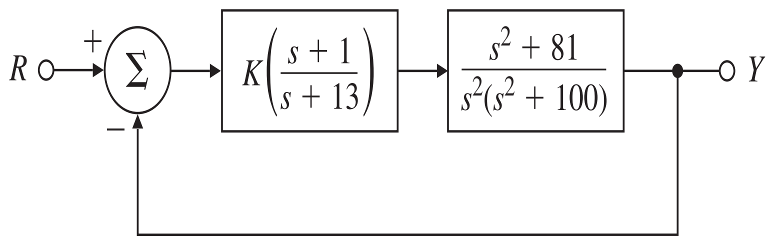
$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}$$

and the characteristic equation $1 + D(s)G(s) = 0$. Let $D(s) = k_p$ at first. Sketch the root loci for the system with parameter $K = 9.8k_p$. Verify your sketch using MATLAB plot with `axes([-4 4 -3 3])`. Suggest an alternative $D(s)$ that will result in a stable system.

5. (@Text P5.13) For the system shown below, sketch the root locus with respect to K.

   (a) Is there a value of K that will cause allot roots to have a damping ration greater than 0.5?

   (b) FInd the values of K that yield closed-loop poles with the damping ratio of $\xi = 0.707$



(c) Use MATLAB to plot the response of the resulting design to a reference step.