3. (a). As $\Omega_r = 0$, according to the block diagram.

$$\left[D_t - \Omega_m \cdot K \cdot \frac{10}{0.5s+1}\right] \cdot \frac{1}{Js+b} = \Omega_m$$

Let $Y = \Omega_m$, $R = D_t$, then.

$$R - Y\left(K\frac{10}{0.5s+1}\right) = (Js+b)Y$$

$$R = Y\left(Js + b + K\frac{10}{0.5s+1}\right)$$

$$\frac{Y}{R} = \frac{0.5s+1}{(Js+b)(0.5s+1) + 10K} \qquad J = 0.1 \ kg\,m^2, \quad b = 1 \ Nms$$

$$E(s) = R(s) \cdot \frac{0.5s+1}{(Js+b)(0.5s+1)+10K} \quad \Rightarrow R(s) = \frac{1}{s} \text{ (step-dist.)}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s} \cdot s E(s) = \frac{1}{10K+1}$$

If we want $e_{ss} \le 0.01$, the $\frac{1}{10K+1} \le 0.01$

$$\Leftrightarrow \quad 1 \le 0.1K + 0.01$$
$$\Leftrightarrow \quad K \ge 0.99 \times 10 = 9.9.$$

$\boxed{\therefore \text{Need } K \ge 9.9}$

(b). Let $K = 9.9$, $Y = \Omega_m$, $R = \Omega_r$

$$(R - Y)\, K \frac{10}{0.5s+1} \; \frac{1}{Js+b} = Y$$

$$T = \frac{Y}{R} = \frac{10K}{(Js+b)(0.5s+1)+10K}$$

$$= \frac{99}{(0.1s+1)(0.5s+1)+99} = \frac{1980}{s^2 + 12s + 2000}$$

compare with standard expression De: $s^2 + 2\zeta\omega_n s + \omega_n^2$

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$\omega_n = \sqrt{2000} = 44.7$ $\qquad \zeta = \frac{12}{2\sqrt{2000}} = 0.13$

roots: $s_1 = -\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2} = -6 + 2i\sqrt{491}$ see figure 3 & 4.

$s_2 = -\zeta\omega_n - i\omega_n\sqrt{1-\zeta^2} = -6 - 2i\sqrt{491}$

⇒ !! See last page for plots of roots and time response of output.

c) According to the formula for 2nd order system

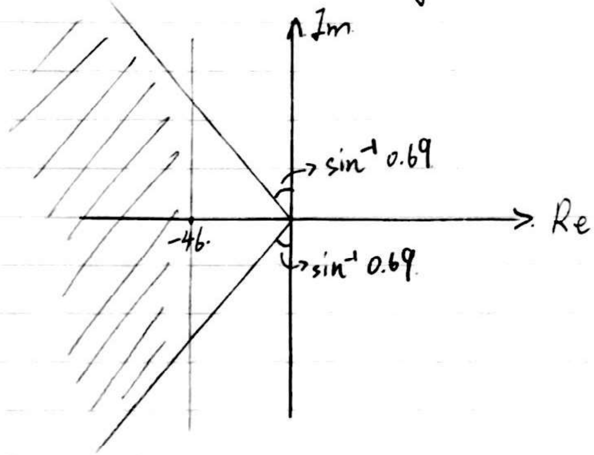
Settling time $T_s = 4.6/\zeta\omega_n$.

for $T_s \le 0.1$, we require $\zeta\omega_n \ge 46$.

Overshoot $M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \times 100\%$.

for $M_p \le 5\%$, we require that $\zeta > 0.69$

⇒ Plot:



d) For a PID controller, we have that $K = k_p + s k_d$.

let $Y = \Omega_m$, $R = \Omega_r$.

$$(R - Y)(k_p + s k_d)\left(\frac{10}{0.5s+1} \cdot \frac{1}{Js+b}\right) = Y$$

$$T = \frac{Y}{R} = \frac{10(k_p + s k_d)}{(Js+b)(0.5s+1) + 10(k_p + s k_d)} \qquad \begin{cases} J = 0.1 \\ b = 1 \end{cases}$$

$$= \frac{200(k_p + s k_d)}{s^2 + (12 + 200 k_d)s + 20 + 200 k_p}$$

Use a PD controller, any values for ω_n and ζ can be achieved. We can use Simulink to find values of k_p and k_d that satisfy the requirement.

The values can be $k_d = 0.25$; $k_p = 9.9$.

See figure 5 and 6 for corresponding time domain simulation and simulink model. (See last page) (e). Similar to (a), let $D_t = R$ (disturbance), $\Omega_m = Y$, we find transfer function $\frac{Y}{R}$ (i.e. $\frac{\Omega_m}{D_t}$).

$$\left[D_t - \Omega_m \cdot K \cdot \frac{10}{0.5s+1}\right] \cdot \frac{1}{Js+b} = \Omega_m$$

$$\frac{Y}{R} = \frac{0.5s+1}{(0.1s+1)(0.5s+1)+10K} \qquad K = k_d s + k_p$$

$$= \frac{20(0.5s+1)}{s^2+(12+200k_d)s+20+200k_p}$$

As $\Omega_r = 0$,

$$E(s) = Y = \frac{+20(0.5s+1)}{s^2+(12+200k_d)s+20+200k_p} \cdot R.$$

Assume step disturbance $R = 1/s$.

$$e_{ss} = \lim_{s\to0} sE(s) = \lim_{s\to0} \frac{+20(0.5s+1)}{s^2+(12+200k_d)s+20+200k_p}$$

$$= \frac{+20}{20+200k_p} = \frac{1}{1+10k_p}$$

$\Rightarrow$ $\therefore$ Steady state error is $\frac{1}{1+10k_p}$.

$\Rightarrow$ The steady state error can be eliminated entirely by adding a intergral term $\frac{k_i}{s}$ to K.