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1. (a)

$$Z(s) = R(s) - Y(s) = \frac{1}{1+KG} R(s)$$

$$= R(s) \frac{1}{1 + K\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}} = \frac{s(\tau_1 s+1)(\tau_2 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)+K} R(s)$$

$$r(t) = tH(t) \quad \rightarrow \quad R(s) = \frac{1}{s^2}.$$

$$e_{ss} = \lim_{s\to 0} s \cdot Z(s) = \lim_{s\to 0} \frac{1}{s} \cdot \frac{s(\tau_1 s+1)(\tau_2 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)+K} = \frac{1}{K}$$

$\boxed{\therefore \text{Steady State error is } \frac{1}{K}.}$

(b). $$T(s) = \frac{Y(s)}{R(s)} = \frac{KG}{1+KG} = \frac{s(\tau_1 s+1)(\tau_2 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)+K}$$

$$= \frac{s(\tau_1 s+1)(\tau_2 s+1)}{\tau_1\tau_2 s^3 + (\tau_1+\tau_2)s^2 + s + K}$$

Use. Routh's Stability Criterion. (the close loop is stable iff there's no sign change).

s^3.	$\tau_1\tau_2$	1	0	$0 \cdots$
s^2	$(\tau_1+\tau_2)$	K	0	$0 \cdots$
s^1	$\frac{(\tau_1+\tau_2)-\tau_1\tau_2 K}{(\tau_1+\tau_2)}$	0	$\cdots$	
s^0	K	0		

Since $\tau_1, \tau_2 > 0$ we need $\frac{(\tau_1+\tau_2)-\tau_1\tau_2 K}{\tau_1+\tau_2} > 0$ and $K > 0$

$$\Leftrightarrow \frac{\tau_1\tau_2 K}{\tau_1+\tau_2} < 1 \text{ and } K>0 \Leftrightarrow K < \frac{\tau_1+\tau_2}{\tau_1\tau_2} \text{ and } K>0$$

$\boxed{\text{Therefore, the close loop is stable iff } \tau_1>0,\ \tau_2>0, \text{ and } 0 < K < \frac{\tau_1+\tau_2}{\tau_1\tau_2}}$

2. (a). According to the block diagram.

$$\left[\frac{1}{2}(R-Y)K_1\frac{s+3}{s+10}-K_2 sY\right]\frac{1}{s(s+10)}=Y$$

$$\left[\frac{K_1(s+3)}{2(s+10)}R-\frac{K_1(s+3)}{2(s+10)}Y-K_2 sY\right]=s(s+10)Y$$

$$\frac{K_1(s+3)}{2(s+10)}R=\left[\frac{K_1(s+3)}{2(s+10)}+K_2 s+s(s+10)\right]Y$$

$$G=\frac{Y}{R}=\frac{K_1 s+3K_1}{K_1(s+3)+2sK_2(s+10)+2s(s+10)^2}$$

$$E(s)=R(s)-Y(s)=R(s)\cdot\frac{2sK_2(s+10)+2s(s+10)^2}{K_1(s+3)+2sK_2(s+10)+2s(s+10)^2}$$

$$r(t)=H(t)\quad\rightarrow\quad R(s)=1/s$$

$$e_{ss}=\lim_{s\to 0}s\cdot E(s)=s\cdot\frac{1}{s}\cdot\frac{2sK_2(s+10)+2s(s+10)^2}{K_1(s+3)+2sK_2(s+10)+2s(s+10)^2}=0$$

$\boxed{\therefore\ \text{Steady state error is } 0.}$

(b) $\boxed{\text{This is a type 1 system.}}$
We calculate O.L. G: (assume input U)

$$U\cdot K_1\frac{s+3}{s+10}-YK_2 s=s(s+10)Y$$

$$UK_1\frac{s+3}{s+10}=(s(s+10)+sK_2)Y$$

$$G=\frac{Y}{U}=\frac{K_1(s+3)}{s(s+10+K_2)(s+10)}$$

Type 1 system.

(c) Based on result in (a)

$$Z(s) = R(s) \cdot \frac{2s(K_2)(s+10) + 2s(s+10)^2}{K_1(s+3) + 2sK_2(s+10) + 2s(s+10)^2}$$

$R(s) = 5/s^2$ (ramp velocity)

$$e_{ss} = \lim_{s \to 0} s \cdot Z(s) = \lim_{s \to 0} s \cdot \frac{5}{s^2} \cdot \frac{2sk_2(s+10) + 2s(s+10)^2}{K_1(s+3) + 2sK_2(s+10) + 2s(s+10)^2}$$

$$= 5 \cdot \frac{2k_2 \cdot 10 + 200}{3K_1}$$

Given that $k_2 = 2$, we have that $e_{ss} = \frac{1200}{3K_1}$

As shown in figure 1, the step overshoot is 17% when K_1 is set to be 1200. (Simulink model provided).

There, the system achieves 17% overshoot when $K_1 = 1200$ and $K_2 = 2$. The steady state error is

$$\boxed{e_{ss} = \frac{1200}{3K_1} = \frac{1200}{3600} = \frac{1}{3} \approx 0.33}$$

(Please see figure 1 & 2 in the next page.)

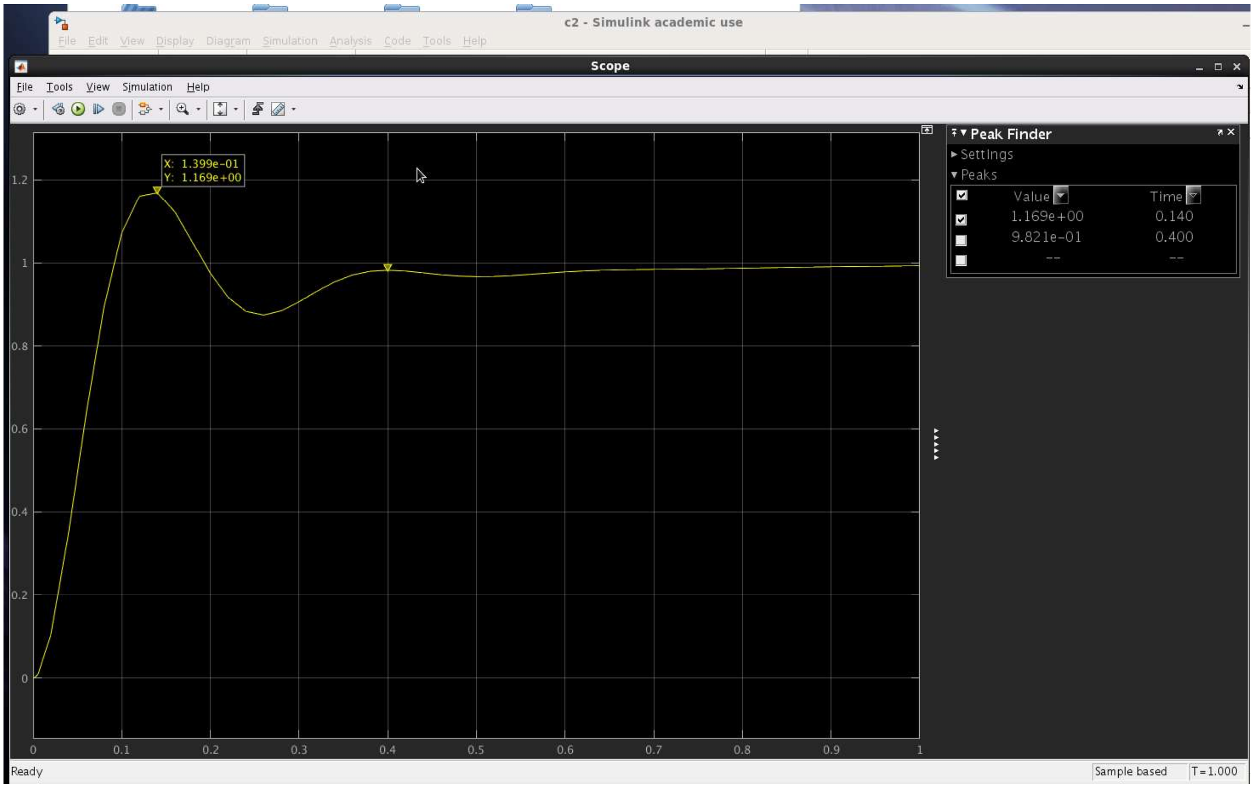


Figure 1 Q2c find K_1

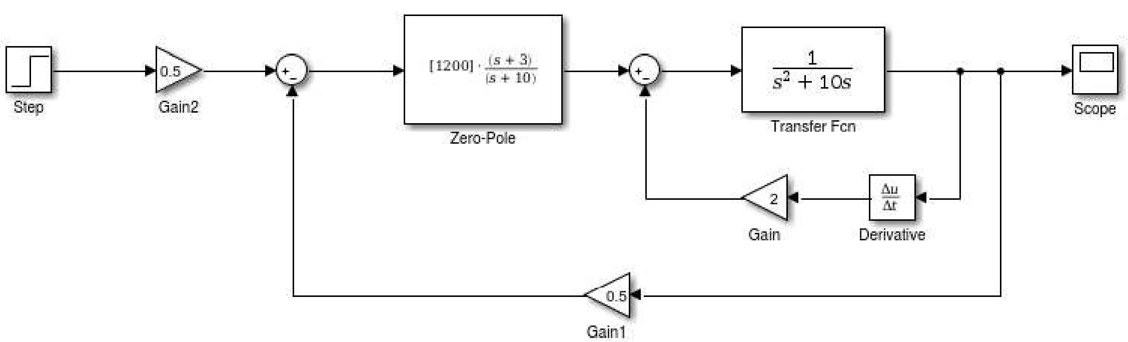


Figure 2 Q2c simulink model for finding K_1

3. (a). As $\Omega_r = 0$, according to the block diagram.

$$\left[D_t - \Omega_m \cdot K \cdot \frac{10}{0.5s+1}\right] \cdot \frac{1}{Js+b} = \Omega_m$$

Let $Y = \Omega_m$, $R = D_t$, then.

$$R - Y\left(K\frac{10}{0.5s+1}\right) = (Js+b)Y$$

$$R = Y\left(Js + b + K\frac{10}{0.5s+1}\right)$$

$$\frac{Y}{R} = \frac{0.5s+1}{(Js+b)(0.5s+1) + 10K}$$ $J = 0.1\ kg\,m^2$, $b = 1\ Nms$

$$E(s) = R(s) \cdot \frac{0.5s+1}{(Js+b)(0.5s+1)+10K}$$ $\Rightarrow R(s) = \frac{1}{s}$ (step-dist.)

$$e_{ss} = \lim_{s\to 0} \frac{1}{s} \cdot s\,E(s) = \frac{1}{10K+1}$$

If we want $e_{ss} \leq 0.01$, the $\frac{1}{10K+1} \leq 0.01$

$$\Leftrightarrow\ 1 \leq 0.1K + 0.01$$
$$\Leftrightarrow\ K \geq 0.99 \times 10 = 9.9.$$

$$\boxed{\therefore \text{Need } K \geq 9.9}$$

(b). Let $K = 9.9$, $Y = \Omega_m$, $R = \Omega_r$

$$(R - Y)\,K\frac{10}{0.5s+1}\,\frac{1}{Js+b} = Y$$

$$T = \frac{Y}{R} = \frac{10K}{(Js+b)(0.5s+1)+10K}$$

$$= \frac{99}{(0.1s+1)(0.5s+1)+99} = \frac{1980}{s^2 + 12s + 2000}$$

compare with standard expression De: $s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n = \sqrt{2000} = 44.7 \qquad \zeta = \frac{12}{2\sqrt{2000}} = 0.13$

roots: $s_1 = -\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2} = -6 + 2i\sqrt{491}$ see figure 3 & 4.
$s_2 = -\zeta\omega_n - i\omega_n\sqrt{1-\zeta^2} = -6 - 2i\sqrt{491}$

=> !! See last page for plots of roots and time response of output.

c) According to the formula for 2nd order system

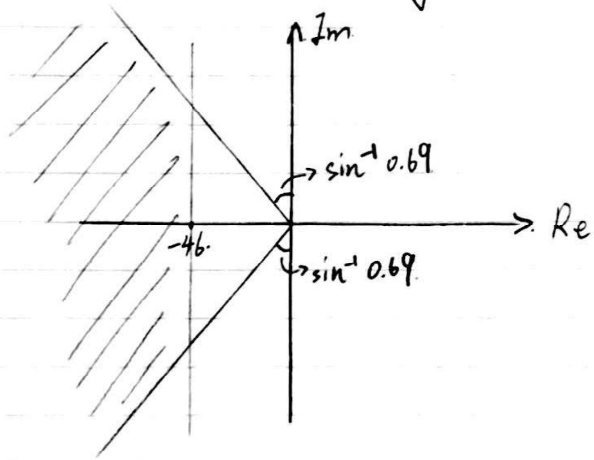
Settling time $T_s = 4.6/\zeta\omega_n$.

for $T_s \le 0.1$, we require $\zeta\omega_n \ge 46$.

Overshoot $M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} \times 100\%$.

for $M_p \le 5\%$, we require that $\zeta > 0.69$

=> Plot:



d) for a PID controller, we have that $K = k_p + s k_d$.

let $Y = \Omega_m$, $R = \Omega_r$.

$$(R-Y)(k_p + s k_d)\left(\frac{10}{0.5s+1} \cdot \frac{1}{Js+b}\right) = Y$$

$$T = \frac{Y}{R} = \frac{10(k_p + s k_d)}{(Js+b)(0.5s+1) + 10(k_p + s k_d)} \qquad \begin{cases} J = 0.1 \\ b = 1 \end{cases}$$

$$= \frac{200(k_p + s k_d)}{s^2 + (12 + 200 k_d)s + 20 + 200 k_p}$$

Use a PD controller, any values for ω_n and ζ can be achieved. We can use Simulink to find values of k_p and k_d that satisfy the requirement.

The values can be $k_d = 0.25$; $k_p = 9.9$.

See figure 5 and 6 for corresponding time domain simulation and simulink model. (See last page)

(e). Similar to (a), let D_t = R, Ω_m = Y, we find transfer function $\frac{Y}{R}$ (i.e. $\frac{\Omega_m}{D_t}$).
disturbance

$$\left[D_t - \Omega_m \cdot K \cdot \frac{10}{0.5s+1}\right] \cdot \frac{1}{Js+b} = \Omega_m$$

$$\frac{Y}{R} = \frac{0.5s+1}{(0.1s+1)(0.5s+1)+10K} \qquad K = k_d s + k_p$$

$$= \frac{20(0.5s+1)}{s^2+(12+200k_d)s+20+200k_p}$$

As $\Omega_r = 0$,

$$E(s) = -Y = \frac{+20(0.5s+1)}{s^2+(12+200k_d)s+20+200k_p} \cdot R.$$

Assume step disturbance $R = 1/s$.

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{+20(0.5s+1)}{s^2+(12+200k_d)s+20+200k_p}$$

$$= \frac{+20}{20+200k_p} = \frac{1}{1+10k_p}$$

⇒ ∴ Steady state error is $\frac{1}{1+10k_p}$.

⇒ The steady state error can be eliminated entirely by adding a intergral term $\frac{k_i}{s}$ to K.

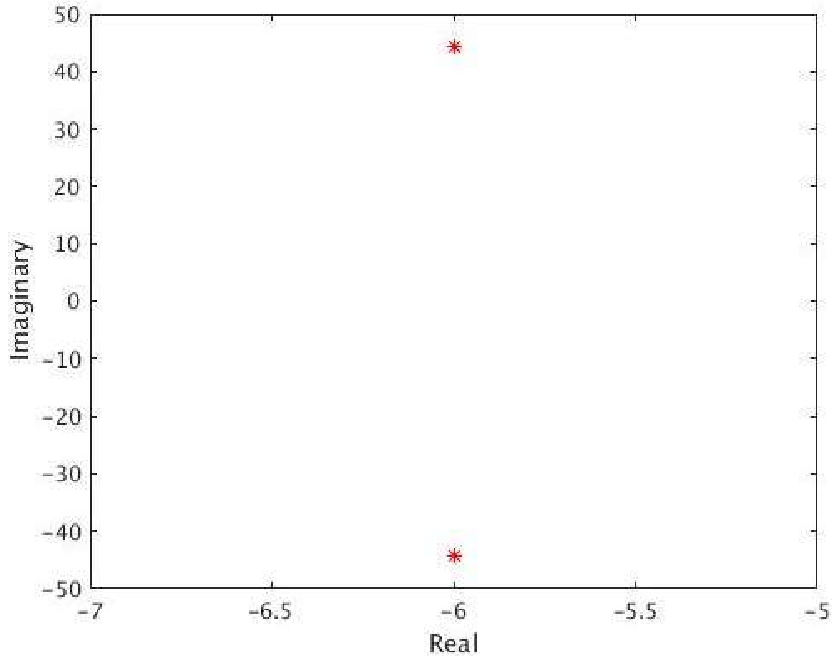


Figure 3 Q3b roots of closed-loop system

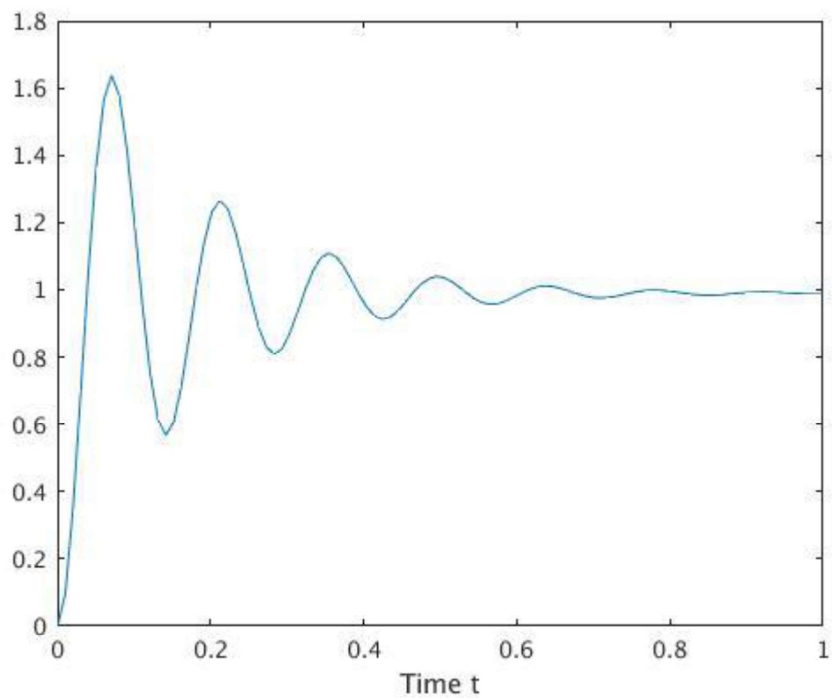


Figure 4 Q3b time response of the output for a step reference input with K=9.9

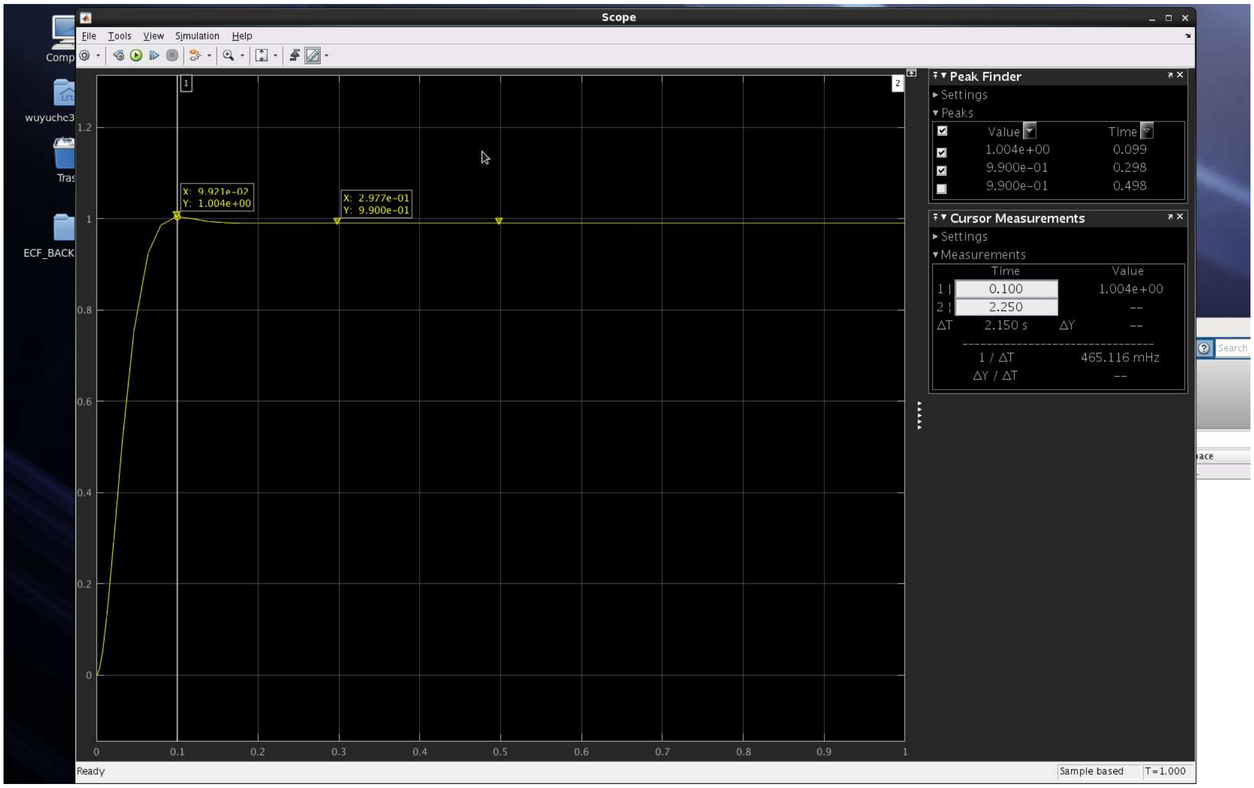


Figure 5 Q3d find values of kd and kp

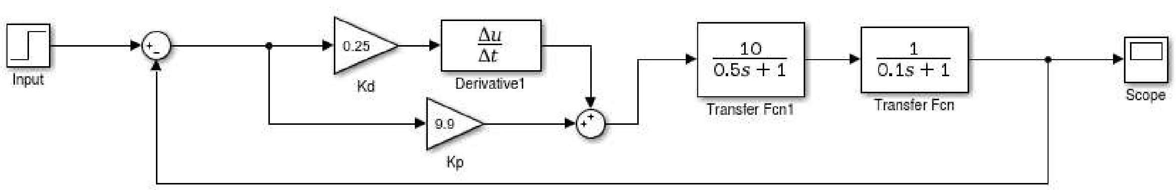


Figure 6 Q3d Simulink model for finding kd and kp