AER372 Assignment 4 Yuchen Wu [redacted]

1. (a)

$$Z(s) = R(s) - Y(s) = \frac{1}{1+KG} R(s)$$

$$= R(s) \frac{1}{1+K\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}} = \frac{s(\tau_1 s+1)(\tau_2 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)+K} R(s)$$

$$r(t) = tH(t) \to R(s) = \frac{1}{s^2}.$$

$$e_{ss} = \lim_{s\to 0} s \cdot Z(s) = \lim_{s \to 0} \frac{1}{s} \cdot \frac{s(\tau_1 s+1)(\tau_2 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)+K} = \frac{1}{K}$$

$\boxed{\therefore \text{ Steady State error is } \frac{1}{K}.}$

(b). $$T(s) = \frac{Y(s)}{R(s)} = \frac{KG}{1+KG} = \frac{s(\tau_1 s+1)(\tau_2 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)+K}$$

$$= \frac{s(\tau_1 s+1)(\tau_2 s+1)}{\tau_1\tau_2 s^3 + (\tau_1+\tau_2)s^2 + s + K}$$

Use. Routh's Stability Criterion. (the close loop is stable iff there's no sign change).

s^3.	$\tau_1\tau_2$	1	0	0 ...
s^2	$(\tau_1+\tau_2)$	K	0	0 ...
s^1	$\frac{(\tau_1+\tau_2)-\tau_1\tau_2 K}{(\tau_1+\tau_2)}$	0 ...		
s^0	K	0		

Since $\tau_1, \tau_2 > 0$ we need $\frac{(\tau_1+\tau_2)-\tau_1\tau_2 K}{\tau_1+\tau_2} > 0$ and $K > 0$

$$\Leftrightarrow \frac{\tau_1\tau_2 K}{\tau_1+\tau_2} < 1 \text{ and } K>0 \Leftrightarrow K < \frac{\tau_1+\tau_2}{\tau_1\tau_2} \text{ and } K > 0$$

Therefore, the close loop is stable iff $\tau_1 > 0$, $\tau_2 > 0$, and $0 < K < \frac{\tau_1+\tau_2}{\tau_1\tau_2}$

2. (a). According to the block diagram.

$$\left[\frac{1}{2}(R-Y)K_1\frac{s+3}{s+10}-K_2 sY\right]\frac{1}{s(s+10)}=Y$$

$$\left[\frac{K_1(s+3)}{2(s+10)}R-\frac{K_1(s+3)}{2(s+10)}Y-K_2 sY\right]=s(s+10)Y$$

$$\frac{K_1(s+3)}{2(s+10)}R=\left[\frac{K_1(s+3)}{2(s+10)}+K_2 s+s(s+10)\right]Y$$

$$G=\frac{Y}{R}=\frac{K_1 s+3K_1}{K_1(s+3)+2sK_2(s+10)+2s(s+10)^2}$$

$$E(s)=R(s)-Y(s)=R(s)\cdot\frac{2sK_2(s+10)+2s(s+10)^2}{K_1(s+3)+2sK_2(s+10)+2s(s+10)^2}$$

$$r(t)=H(t)\quad\rightarrow\quad R(s)=\frac{1}{s}$$

$$e_{ss}=\lim_{s\to0}s\cdot E(s)=s\cdot\frac{1}{s}\cdot\frac{2sK_2(s+10)+2s(s+10)^2}{K_1(s+3)+2sK_2(s+10)+2s(s+10)^2}=0$$

$\boxed{\therefore\text{ Steady state error is } 0.}$

(b) $\boxed{\text{This is a type 1 system.}}$

We calculate O.L G: (assume input U)

$$U\cdot K_1\frac{s+3}{s+10}-YK_2 s=s(s+10)Y$$

$$U\,K_1\frac{s+3}{s+10}=(s(s+10)+sK_2)Y$$

$$G=\frac{Y}{U}=\frac{K_1(s+3)}{s(s+10+K_2)(s+10)}$$

Type 1 system.

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(c) Based on result in (a)

$$Z(s) = R(s) \cdot \frac{2s(K_2)(s+10) + 2s(s+10)^2}{K_1(s+3) + 2sK_2(s+10) + 2s(s+10)^2}$$

$R(s) : 5/s^2$ (ramp velocity)

$$e_{ss} = \lim_{s \to 0} s \cdot Z(s) = \lim_{s \to 0} s \cdot \frac{5}{s^2} \cdot \frac{2sk_2(s+10) + 2s(s+10)^2}{K_1(s+3) + 2sK_2(s+10) + 2s(s+10)^2}$$

$$= 5 \cdot \frac{2k_2 \cdot 10 + 200}{3K_1}$$

Given that $K_2 = 2$, we have that $e_{ss} = \frac{1200}{3K_1}$

As shown in figure 1, the step overshoot is 17% when K_1 is set to be 1200. (Simulink model provided).

There, the system achieves 17% overshoot when $K_1 = 1200$ and $K_2 = 2$. The steady state error is

$$\boxed{e_{ss} = \frac{1200}{3K_1} = \frac{1200}{3600} = \frac{1}{3} \approx 0.33}$$

(Please see figure 1 & 2 in the next page.)