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Residential Telephone Services

1 Estimation of a Nested Logit Model

NB: All model files are available as python scripts (`.py`) or Jupyter notebooks (`.ipynb`)

Files to use with the telephone dataset

Model file:	*MNL_Tel_generic and GEV_Tel_NL_unrestricted*
Data file:	*telephone.dat*

We start by giving some examples of possible nesting structures for the Nested Logit (NL) model in Figure 1. The `GEV_Tel_NL_unrestricted` file describes the first (top left) nesting structure shown in Figure 1. The utility functions are specified as follows

$$\begin{aligned}
V_{BM} &= \mathrm{ASC}_{BM} + \beta_{cost}\ln(cost_{BM}) \\
V_{SM} &= \beta_{cost}\ln(cost_{SM}) \\
V_{LF} &= \mathrm{ASC}_{LF} + \beta_{cost}\ln(cost_{LF}) \\
V_{EF} &= \mathrm{ASC}_{EF} + \beta_{cost}\ln(cost_{EF}) \\
V_{MF} &= \mathrm{ASC}_{MF} + \beta_{cost}\ln(cost_{MF}).
\end{aligned}$$

The estimation results of the NL model are shown in Table 1. To be consistent with random utility theory, we assume the inequality $\frac{\mu}{\mu_m} < 1$, with μ being normalized to 1, holds, implying $\mu_m > 1$. To verify the NL model, we compare it to its restriction, the MNL model (*MNL_Tel_generic*), by testing null hypothesis $H_0 : \mu_{meas} = \mu_{flat} = 1$. As the MNL model is a restricted version of the NL model, we perform a likelihood ratio test:

$$-2(\mathcal{L}_R - \mathcal{L}_U) = -2(-477.557 + 473.219) = 8.676$$

The test statistic is asymptotically χ^2 distributed with 2 degrees of freedom since there are 2 restrictions. Since $8.676 > 5.991$ (the critical value of the χ^2 distribution with 2 degrees of

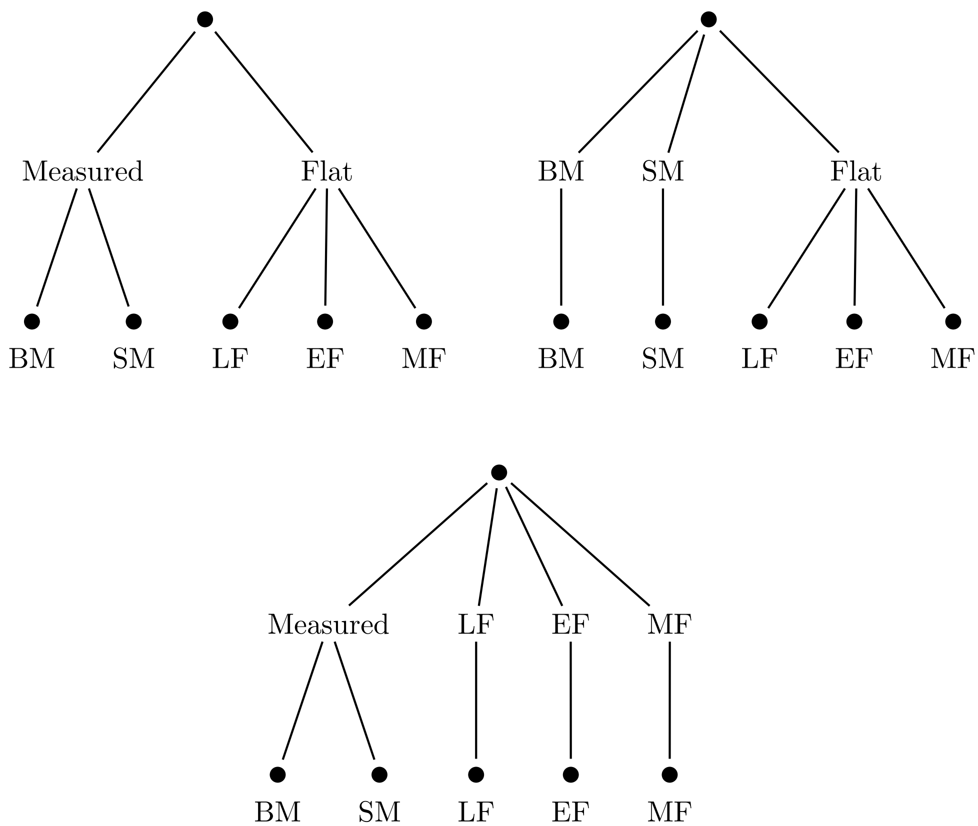

Figure 1: The possible nesting structures

NL with generic attributes					
Parameter number	Parameter name	Parameter estimate	Robust standard error	Robust *t stat. 0*	Robust *t stat. 1*
1	ASC_{BM}	-0.378	0.117	-3.22	
2	ASC_{LF}	0.893	0.158	5.64	
3	ASC_{EF}	0.847	0.391	2.17	
4	ASC_{MF}	1.41	0.238	5.90	
5	β_{cost}	-1.49	0.243	-6.13	
6	μ_{meas}	2.06	0.573	3.60	1.85
7	μ_{flat}	2.29	0.763	3.00	1.69

Summary statistics
Number of observations = 434
$\mathcal{L}(0) = -560.250$
$\mathcal{L}(\hat{\beta}) = -473.219$
$\bar{\rho}^2 = 0.143$

Table 1: NL with generic attributes (first nesting structure)

freedom at a 95 % level of confidence), we can reject the null hypothesis H_0 and accept the NL model.

File to develop using the same dataset as before
Model file: GEV_Tel_NL_restricted

Another possible specification is to set the μ_m's of the two nests equal to each other. This can be done by constraining the two nest coefficients to be equal. The estimation results for this specification are shown in Table 2.

Files to develop using the same dataset as before
Model file: GEV_Tel_NL_unrestricted_2 and GEV_Tel_NL_unrestricted_3

The estimation results for the other two nesting structures shown in Figure 1 (top right and bottom) are provided in Tables 3 and 4.

NL with linear constraints					
Parameter number	Parameter name	Parameter estimate	standard error	*t stat.* 0	*t stat.* 1
1	ASC_{BM}	-0.368	0.113	-3.26	
2	ASC_{LF}	0.882	0.154	5.74	
3	ASC_{EF}	0.833	0.401	2.08	
4	ASC_{MF}	1.39	0.232	5.96	
5	β_{cost}	-1.50	0.243	-6.18	
6	μ_{meas}	2.16	0.563	3.84	2.06
7	μ_{flat}	2.16	0.563	3.84	2.06

Summary statistics
Number of observations = 434
$\mathcal{L}(0) = -560.250$
$\mathcal{L}(\hat{\beta}) = -473.288$
$\bar{\rho}^2 = 0.145$

Table 2: NL with linear constraint on the nest parameters

NL with generic attributes					
Parameter number	Parameter name	Parameter estimate	Robust standard error	Robust *t stat. 0*	Robust *t stat. 1*
1	ASC_{BM}	-0.680	0.150	-4.54	
2	ASC_{LF}	1.20	0.141	8.56	
3	ASC_{EF}	1.14	0.450	2.54	
4	ASC_{MF}	1.80	0.221	8.14	
5	β_{cost}	-1.75	0.249	-7.00	
6	μ_{flat}	1.89	0.662	2.86	1.34

Summary statistics
Number of observations = 434
$\mathcal{L}(0) = -560.250$
$\mathcal{L}(\hat{\beta}) = -475.792$
$\bar{\rho}^2 = 0.140$

Table 3: NL with generic attributes (second nesting structure)

NL with generic attributes					
Parameter number	Parameter name	Parameter estimate	Robust standard error	Robust *t stat. 0*	Robust *t stat. 1*
1	ASC_{BM}	-0.497	0.140	-3.54	
2	ASC_{LF}	0.977	0.187	5.22	
3	ASC_{EF}	0.777	0.703	1.11	
4	ASC_{MF}	1.44	0.300	4.79	
5	β_{cost}	-1.88	0.217	-8.69	
6	μ_{meas}	1.61	0.406	3.97	1.50
Summary statistics					
Number of observations = 434					
$\mathcal{L}(0) = -560.250$					
$\mathcal{L}(\hat{\beta}) = -476.145$					
$\bar{\rho}^2 = 0.139$					

Table 4: NL with generic attributes (third nesting structure)

2 Estimation of a Cross-Nested Logit Model with Fixed Alphas

File to use with the same dataset as before
Model file: GEV_Tel_CNL_fix.

In this section we specify a different Cross-Nested Logit (CNL) model using fixed degrees of membership. The specifications presented hereafter are mainly for demonstration purposes. However, an assumption that might make sense is that the local flat alternative (LF) is likely to be correlated with both measured and flat options. Like the measured plans, the LF plan is a reasonable option for users with relatively basic needs. So, the LF option may belong to both nests. Based on this hypothesis, the proposed cross-nested structure is shown in Figure 2.

We present the CNL model with the same deterministic utility functions as before. The nest parameters are constrained to 2.16, the value that we obtain from the restricted NL estimation, in order to allow for a clearer focus on the cross-nested estimation structure.

Note that we define α_{CNL} so that the LF alternative belongs equally to both the flat and the measured nests. This assumption will be relaxed in the next section. Thus, CNL with fixed α's is a restricted model of CNL with variable α's. The estimation results are shown in Table 5.

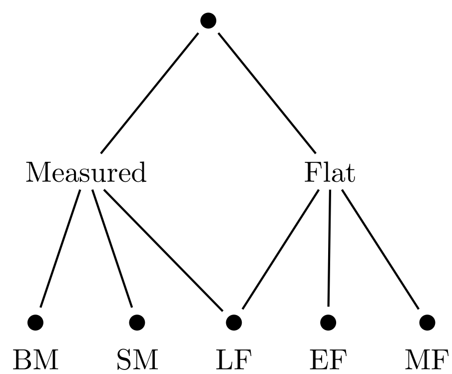

Figure 2: The cross-nested structure

CNL with α_{CNL} fixed				
Parameter number	Parameter name	Parameter estimate	Robust standard error	Robust *t stat. 0*
1	ASC_{BM}	-0.356	0.0730	-4.87
2	ASC_{LF}	0.867	0.0870	9.96
3	ASC_{EF}	0.465	0.395	1.18
4	ASC_{MF}	0.791	0.161	4.90
5	β_{cost}	-1.24	0.132	-9.41
Summary statistics				
Number of observations = 434				
$\mathcal{L}(0) = -560.250$				
$\mathcal{L}(\hat{\beta}) = -480.146$				
$\bar{\rho}^2 = 0.208$				

Table 5: CNL with α_{CNL} fixed

Cross-Nested Logit Model with Variable Alphas

File to develop using the same dataset as before
Model file: GEV_Tel_CNL_var

In the previous section, we assumed that the LF alternative belongs equally to the measured nest and the flat nest by fixing α_{LF_meas} and α_{LF_flat} to be equal to 0.5. This assumption can be relaxed in order to estimate the share of LF in each nest during the estimation of the model parameters. From the results presented in Table 6, we see that the alternative LF has a very small share in the measured nest.

Note that, in both CNL specifications presented in the previous and the present section, we have imposed the condition

$$\sum_m \alpha_{jm} = 1$$

Such a condition is not necessary for the validity of the model. It is imposed for identification purposes.

To select between the restricted NL and CNL model with variable α's, we can test the null hypothesis $H_0 : \alpha_{LF\_flat} = 1$. As there is a single restriction, we can use either a t-test or a likelihood ratio test, that are asymptotically equivalent. The t-statistic with respect to 1 is -0.807, which indicates that α_{LF_flat} is not significantly different from 1, and hence we accept the null hypothesis (restricted NL model) and reject the CNL model with variable α's.
We can also do a likelihood ratio test as follows. The test statistic for the null hypothesis is given by

$$-2(\mathcal{L}_R - \mathcal{L}_U) = -2(-473.288 + 473.250) = 0.076$$

where the restricted model is the NL model and the unrestricted model is the CNL model. The test statistic is asymptotically χ^2 distributed with 1 degree of freedom since there is 1 restriction. Since $0.076 < 3.841$ (the critical value of the χ^2 distribution with 1 degree of freedom at a 95 % level of confidence), we accept the null hypothesis (restricted NL model) and reject the CNL model with variable α's. We can thus conclude that the LF alternative is correlated only with the flat nest but not with the measured nest.

To select between the CNL model with fixed α's and the CNL model with variable α's, we can test the null hypothesis $H_0 : \alpha_{LF\_flat} = 0.5$. As there is a single restriction, we can use either a t-test or a likelihood ratio test, that are asymptotically equivalent. The t-statistic with respect to 0.5 is 5.04, which indicates that α_{LF_flat} is significantly different from 0.5, and hence we reject the null hypothesis (CNL model with fixed α's) and accept the CNL model with variable α's. We can also do a likelihood ratio test as follows. The test statistic for the null hypothesis is

CNL with α_{CNL} variable					
Parameter number	Parameter name	Parameter estimate	Robust standard error	Robust *t stat. 0*	Robust *t stat. 1*
1	ASC_{BM}	-0.368	0.0759	-4.85	
2	ASC_{LF}	0.943	0.128	7.39	
3	ASC_{EF}	0.827	0.395	2.09	
4	ASC_{MF}	1.37	0.192	7.17	
5	β_{cost}	-1.49	0.155	-9.65	
6	α_{LF_flat}	0.931	0.0855	10.89	-0.807
Summary statistics					
Number of observations = 434					
$\mathcal{L}(0) = -560.250$					
$\mathcal{L}(\hat{\beta}) = -473.250$					
$\bar{\rho}^2 = 0.218$					

Table 6: CNL with α_{CNL} variable

given by

$$-2(\mathcal{L}_R - \mathcal{L}_U) = -2(-480.146 + 473.250) = 13.792$$

where the restricted model is the CNL model with fixed α's and the unrestricted model is the CNL model with variable α's. The test statistic is asymptotically χ^2 distributed with 1 degree of freedom since there is 1 restriction. Since 13.792 > 3.841 (the critical value of the χ^2 distribution with 1 degree of freedom at a 95 % level of confidence), we reject the null hypothesis (CNL model with fixed α's) and accept the CNL model with variable α's.

As the restricted NL model is preferred to the CNL model with variable α's, and the CNL model with variable α's is preferred to the CNL model with fixed α's, we select the restricted NL model over the CNL models.

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