



## EXERCISE SESSION 4

**Exercise 1** In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$\begin{aligned} U_{1n} &= 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n} \\ U_{2n} &= -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n} \end{aligned} \quad (1)$$

where  $tt_{in}$  and  $c_{in}$  are the travel time (in minutes) and travel cost (in CHF) for respondent  $n$ , with  $i \in \{\text{car}, \text{train}\}$ . Additionally,  $\text{income}_n$  takes value 1 if the respondent's monthly income is larger than 6000CHF and 0 otherwise, whereas  $\text{university}_n$  takes value 1 if the respondent went to the university and 0 otherwise.  $\varepsilon_{1n}, \varepsilon_{2n} \stackrel{iid}{\sim} \text{EV}(0, 1)$ .

1. Compute the probability to choose each mode for the following individuals:

Name	$tt_1$	$tt_2$	$c_1$	$c_2$	monthly income	university
Tim	22	18	2	2.1	7000	yes
Janody	120	100	10	15	3000	yes
Michel	10	50	3	5	10000	no
Meri	25	9	7	2.1	5000	no

2. Explain what the alternative specific constant in alternative 1 represents.
3. Interpret one by one all the parameters.

**Exercise 2** In a route choice case study, the utility functions are defined as follows:

$$\begin{aligned} U_1 &= ASC_1 + \beta_{length} \cdot \text{length}_1 + \varepsilon_1 \\ U_2 &= ASC_2 + \beta_{length} \cdot \text{length}_2 + \varepsilon_2 \end{aligned} \quad (2)$$

where alternatives 1 and 2 represent different routes,  $ASC_1$ ,  $ASC_2$  and  $\beta_{length}$  are parameters to be estimated and  $\text{length}_i, i \in \{1, 2\}$ , is the length of each route in kilometers.

The estimation results of a binary logit model, where  $ASC_1$  has been normalized to zero, are shown in the first column of the following table. The second column corresponds to the same specification with  $ASC_2$  normalized to zero instead:

	Logit 1	Logit 2
$ASC_1$	0	x
$ASC_2$	-2	0
$\beta_{length}$	10	x

1. Replace each  $x$  in the table with the value of the corresponding parameter.
2. What are the distributions of  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_1 - \varepsilon_2$ ?

### Exercise 3

1. Define the Box-Cox transformation.
  - (a) What modeling assumption are you testing when specifying a Box-Cox transformation of the travel cost in a model of transportation mode choice?
  - (b) Let  $\lambda$  be the parameter of the Box-Cox transformation. What particular cases do you obtain when  $\lambda = 1$  or  $\lambda = 0$ ?
2. In a model developed for the transportation mode choice in the Netherlands case study, the deterministic parts of the utilities for the car and rail alternatives are specified as:

$$\begin{aligned}
 V_{Car,n} &= ASC_{CAR} + \beta_{COST} \cdot cost_{car,n} + \beta_{TIME\_CAR} \cdot time_{car,n} \\
 V_{Rail,n} &= ASC_{RAIL} + \beta_{COST} \cdot cost_{rail,n} + \beta_{TIME\_RAIL} \cdot time_{rail,n} + \beta_{FEMALE} \cdot female_n
 \end{aligned}
 \tag{3}$$

where  $time_{car,n}$  and  $time_{rail,n}$  are the travel times for car and rail for individual  $n$ ,  $cost_{car,n}$  and  $cost_{rail,n}$  are the travel costs for the same two alternatives, and  $female_n$  takes value 1 if the individual is a female and 0 otherwise. The estimation results for this model are shown in Figure 1.

In addition to the base model, we also estimate a model with a Box-Cox transformation of the cost variables. A snapshot of the estimation results is presented in Figure 2.

Referring to the figures:

- (a) Comment and interpret the values of the estimates of both models (i.e., analyze the signs of the coefficients), and check if the estimates correspond to your expectations.
- (b) Identify what parameters are significantly different from 0 (or 1 in the case of  $\lambda$ ).

## Formulas

*Car utility:*  $ASC\_CAR * one + BETA\_COST * car\_cost\_euro + BETA\_TIME\_CAR * car\_time$

*Rail utility:*  $ASC\_RAIL * one + BETA\_COST * rail\_cost\_euro + BETA\_TIME\_RAIL * rail\_time + BETA\_FEMALE * gender$

## Estimation report

Number of estimated parameters: 5  
Sample size: 228  
Excluded observations: 1511  
Init log likelihood: -158.038  
Final log likelihood: -115.880  
Likelihood ratio test for the init. model: 84.314  
Rho-square for the init. model: 0.267  
Rho-square-bar for the init. model: 0.235  
Akaike Information Criterion: 241.761  
Bayesian Information Criterion: 258.908  
Final gradient norm: +1.921e-05  
Diagnostic: CFSQP: Normal termination. Obj: 6.05545e-06 Const: 6.05545e-06  
Iterations: 12  
Data processing time: 00:00  
Run time: 00:00  
Nbr of threads: 12

## Estimated parameters

Click on the headers of the columns to sort the table [\[Credits\]](#)

Name	Value	Std err	t-test	p-value		Robust Std err	Robust t-test	p-value	
ASC_CAR	2.85	1.09	2.62	0.01		1.02	2.80	0.01	
BETA_COST	-0.130	0.0251	-5.17	0.00		0.0265	-4.89	0.00	
BETA_FEMALE	0.675	0.330	2.05	0.04		0.329	2.05	0.04	
BETA_TIME_CAR	-2.34	0.489	-4.78	0.00		0.495	-4.73	0.00	
BETA_TIME_RAIL	-0.529	0.418	-1.27	0.20	*	0.414	-1.28	0.20	*

Figure 1: Estimation results of the base model

## Formulas

*Car utility:*  $ASC\_CAR * one + ( BETA\_COST * ( ( car\_cost\_euro ^ LAMBDA ) - ( 1 ) ) ) / LAMBDA + BETA\_TIME\_CAR * car\_time$   
*Rail utility:*  $ASC\_RAIL * one + BETA\_COST * ( ( ( rail\_cost\_euro ^ LAMBDA ) - ( 1 ) ) / LAMBDA ) + BETA\_TIME\_RAIL * rail\_time + BETA\_FEMALE * gender$

## Estimation report

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Number of estimated parameters: 6
      Sample size: 228
      Excluded observations: 1511
      Init log likelihood: -158.038
      Final log likelihood: -113.265
Likelihood ratio test for the init. model: 89.546
      Rho-square for the init. model: 0.283
      Rho-square-bar for the init. model: 0.245
      Akaike Information Criterion: 238.530
      Bayesian Information Criterion: 259.106
      Final gradient norm: +5.939e-05
      Diagnostic: CFSQP: Normal termination. Obj: 6.05545e-06 Const: 6.05545e-06
      Iterations: 24
      Data processing time: 00:00
      Run time: 00:00
      Nbr of threads: 12

```

## Estimated parameters

Click on the headers of the columns to sort the table [\[Credits\]](#)

Name	Value	Std err	t-test	p-value	Robust Std err	Robust t-test	p-value	
ASC_CAR	2.64	1.09	2.41	0.02	1.03	2.56	0.01	
BETA_COST	-0.544	0.266	-2.05	0.04	0.249	-2.19	0.03	
BETA_FEMALE	0.735	0.338	2.18	0.03	0.334	2.20	0.03	
BETA_TIME_CAR	-2.42	0.500	-4.84	0.00	0.509	-4.76	0.00	
BETA_TIME_RAIL	-0.616	0.427	-1.44	0.15	* 0.423	-1.46	0.15	*
LAMBDA	0.400	0.224	1.78	0.07	* 0.211	1.90	0.06	*

Figure 2: Estimation results of model with a Box-Cox transformation

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