



## EXERCISE SESSION 4 (Solution)

**Exercise 1** In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$\begin{aligned} U_{1n} &= 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n} \\ U_{2n} &= -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n} \end{aligned} \quad (1)$$

where  $tt_{in}$  and  $c_{in}$  are the travel time (in minutes) and travel cost (in CHF) for respondent  $n$ , with  $i \in \{\text{car, train}\}$ . Additionally,  $\text{income}_n$  takes value 1 if the respondent's monthly income is larger than 6000CHF and 0 otherwise, whereas  $\text{university}_n$  takes value 1 if the respondent went to the university and 0 otherwise.  $\varepsilon_{1n}, \varepsilon_{2n} \stackrel{iid}{\sim} \text{EV}(0, 1)$ .

1. Compute the probability to choose each mode for the following individuals:

Name	$tt_1$	$tt_2$	$c_1$	$c_2$	monthly income	university
Tim	22	18	2	2.1	7000	yes
Janody	120	100	10	15	3000	yes
Michel	10	50	3	5	10000	no
Meri	25	9	7	2.1	5000	no

**Solution:** The choice probability of individual  $n$  choosing mode 1 is given as:

$$P_n(1) = \frac{\exp(V_{1n})}{\exp(V_{1n}) + \exp(V_{2n})} = \frac{\exp(U_{1n} - \varepsilon_{1n})}{\exp(U_{1n} - \varepsilon_{1n}) + \exp(U_{2n} - \varepsilon_{2n})}$$

and the probability of  $n$  choosing 2 can be obtained in the same way or by calculating  $P_n(2) = 1 - P_n(1)$ . The solutions, therefore, are the followings:

$$\begin{aligned} P_{\text{Tim}}(1) &= 0.66, P_{\text{Tim}}(2) = 0.34 \\ P_{\text{Janody}}(1) &= 0.24, P_{\text{Janody}}(2) = 0.76 \\ P_{\text{Michel}}(1) &= 0.90, P_{\text{Michel}}(2) = 0.10 \\ P_{\text{Meri}}(1) &= 0.52, P_{\text{Meri}}(2) = 0.48 \end{aligned}$$

2. Explain what the alternative specific constant in alternative 1 represents.

**Solution:** The mean of the difference  $\varepsilon_{1n} - \varepsilon_{2n}$ . It can be interpreted such that the alternative specific constant for an alternative captures the average impact on utility of all factors that are not included in the model.

3. Interpret one by one all the parameters.

**Solution:**

- Coefficient of car travel time ( $= -0.03$ ). It is negative as expected. The increase of car travel time leads to the decrease of car utility.
- Coefficient of car travel cost ( $= -0.06$ ). It is negative as expected. The increase of car travel cost leads to the decrease of car utility.
- Coefficient of income dummy variable ( $= 0.5$ ). It is positive as (un)expected. Travelers with high income are more likely to choose car.
- Coefficient of train travel time ( $= -0.02$ ). It is negative as expected. The increase of train travel time leads to the decrease of train utility.
- Coefficient of train travel cost ( $= -0.0375$ ). It is negative as expected. The increase of train travel cost leads to the decrease of train utility.
- Coefficient of university dummy variable ( $= 0.5$ ). It is positive as (un)expected. Travelers who went to a university are more likely to choose train.
- The value of time for car is  $(-0.03 / -0.06) \cdot 60 = 30$  [CHF/h], and that for train is  $(-0.02 / -0.0375) \cdot 60 = 32$  [CHF/h]. Travelers would be willing to pay more money to decrease their travel time when traveling by train.

**Exercise 2** In a route choice case study, the utility functions are defined as follows:

$$\begin{aligned} U_1 &= ASC_1 + \beta_{length} \cdot length_1 + \varepsilon_1 \\ U_2 &= ASC_2 + \beta_{length} \cdot length_2 + \varepsilon_2 \end{aligned} \quad (2)$$

where alternatives 1 and 2 represent different routes,  $ASC_1$ ,  $ASC_2$  and  $\beta_{length}$  are parameters to be estimated and  $length_i, i \in \{1, 2\}$ , is the length of each route in kilometers.

The estimation results of a binary logit model, where  $ASC_1$  has been normalized to zero, are shown in the first column of the following table. The second column corresponds to the same specification with  $ASC_2$  normalized to zero instead:

	Logit 1	Logit 2
$ASC_1$	0	x
$ASC_2$	-2	0
$\beta_{length}$	10	y

1. Replace  $x$  and  $y$  in the table with the value of the corresponding parameter.

**Solution:**  $x = 2, y = 10$ .

2. What are the distributions of  $\varepsilon_1, \varepsilon_2$  and  $\varepsilon_1 - \varepsilon_2$ ?

**Solution:**  $\varepsilon_1, \varepsilon_2$  are the extreme value distributions:  $\varepsilon_1 \sim \text{EV}(0, \mu), \varepsilon_2 \sim \text{EV}(0, \mu)$ , which are independent with the same scale parameter  $\mu$  (i.i.d.);  $\varepsilon_1 - \varepsilon_2$  follows a logistic distribution:  $\varepsilon_1 - \varepsilon_2 \sim \text{Logistic}(0, \mu)$ .

### Exercise 3

1. Define the Box-Cox transformation.

- (a) What modeling assumption are you testing when specifying a Box-Cox transformation of the travel cost in a model of transportation mode choice?

**Solution:** Non-linearity of the influence of the travel cost on the utility function.

- (b) Let  $\lambda$  be the parameter of the Box-Cox transformation. What particular cases do you obtain when  $\lambda = 1$  or  $\lambda = 0$ ?

**Solution:** The Box-Cox transformation transforms a positive variable to a function of parameter  $\lambda$ :  $x(\lambda) = \frac{x^\lambda - 1}{\lambda}$ . The cases when  $\lambda = 1$  or  $\lambda = 0$  are as follows:

$$x(1) = \frac{x^1 - 1}{1} = x - 1$$

$$x(0) = \lim_{\lambda \rightarrow 0} \frac{x^\lambda - 1}{\lambda} = \ln x$$

2. In a model developed for the transportation mode choice in the Netherlands case study, the deterministic parts of the utilities for the car and rail alternatives are specified as:

$$\begin{aligned} V_{Car,n} &= ASC_{CAR} + \beta_{COST} \cdot cost_{car,n} + \beta_{TIME\_CAR} \cdot time_{car,n} \\ V_{Rail,n} &= ASC_{RAIL} + \beta_{COST} \cdot cost_{rail,n} + \beta_{TIME\_RAIL} \cdot time_{rail,n} + \beta_{FEMALE} \cdot female_n \end{aligned} \quad (3)$$

where  $time_{car,n}$  and  $time_{rail,n}$  are the travel times for car and rail for individual  $n$ ,  $cost_{car,n}$  and  $cost_{rail,n}$  are the travel costs for the same two alternatives, and  $female_n$  takes value 1 if the individual is a female and 0 otherwise. The estimation results for this model are shown in Figure 1.

In addition to the base model, we also estimate a model with a Box-Cox transformation of the cost variables. A snapshot of the estimation results is presented in Figure 2.

Referring to the figures:

- (a) Comment and interpret the values of the estimates of both models (i.e., analyze the signs of the coefficients), and check if the estimates correspond to your expectations.

**Solution:** Provide interpretations in the similar way to Exercise 1.3. With respect to the parameter of Box-Cox transformation,  $\lambda$  is estimated as 0.400. In that case, the utility function is not linear-in-parameters anymore but concave in travel cost.

- (b) Identify what parameters are significantly different from 0 (or 1 in the case of  $\lambda$ ).

**Solution:** We do it with the  $t$  test. The value of the  $t$  statistic is:

$$\frac{\hat{\beta} - \beta_0}{\sigma}$$

where  $\hat{\beta}$  is the estimated value of the parameter,  $\sigma$  is its associated standard error, and  $\beta_0$  is the reference value. The threshold values of the significant level of 95 % are  $\pm 1.96$ . Note that, in the case where  $\beta_0 = 0$  the  $t$  statistics are provided in the table of estimated parameters of Biogeme, but you need to calculate it by yourself if  $\beta_0 \neq 0$ . In this exercise, the  $t$  statistic for  $\lambda$  of the latter model is calculated as:

$$\frac{0.400 - 1}{0.224} = -2.68 < -1.96.$$

Given that, the parameters that are significant are:

- $ASC_{CAR}, \beta_{COST}, \beta_{FEMALE}, \beta_{TIME\_CAR}$  for the base model,
- $ASC_{CAR}, \beta_{COST}, \beta_{FEMALE}, \beta_{TIME\_CAR}, \lambda$  for the model with the Box-Cox transformation.

## Formulas

*Car utility:*  $ASC\_CAR * one + BETA\_COST * car\_cost\_euro + BETA\_TIME\_CAR * car\_time$

*Rail utility:*  $ASC\_RAIL * one + BETA\_COST * rail\_cost\_euro + BETA\_TIME\_RAIL * rail\_time + BETA\_FEMALE * gender$

## Estimation report

Number of estimated parameters: 5  
Sample size: 228  
Excluded observations: 1511  
Init log likelihood: -158.038  
Final log likelihood: -115.880  
Likelihood ratio test for the init. model: 84.314  
Rho-square for the init. model: 0.267  
Rho-square-bar for the init. model: 0.235  
Akaike Information Criterion: 241.761  
Bayesian Information Criterion: 258.908  
Final gradient norm: +1.921e-05  
Diagnostic: CFSQP: Normal termination. Obj: 6.05545e-06 Const: 6.05545e-06  
Iterations: 12  
Data processing time: 00:00  
Run time: 00:00  
Nbr of threads: 12

## Estimated parameters

Click on the headers of the columns to sort the table [\[Credits\]](#)

Name	Value	Std err	t-test	p-value		Robust Std err	Robust t-test	p-value	
ASC_CAR	2.85	1.09	2.62	0.01		1.02	2.80	0.01	
BETA_COST	-0.130	0.0251	-5.17	0.00		0.0265	-4.89	0.00	
BETA_FEMALE	0.675	0.330	2.05	0.04		0.329	2.05	0.04	
BETA_TIME_CAR	-2.34	0.489	-4.78	0.00		0.495	-4.73	0.00	
BETA_TIME_RAIL	-0.529	0.418	-1.27	0.20	*	0.414	-1.28	0.20	*

Figure 1: Estimation results of the base model

## Formulas

*Car utility:*  $ASC\_CAR * one + ( BETA\_COST * ( ( car\_cost\_euro ^ LAMBDA ) - ( 1 ) ) ) / LAMBDA + BETA\_TIME\_CAR * car\_time$   
*Rail utility:*  $ASC\_RAIL * one + BETA\_COST * ( ( ( rail\_cost\_euro ^ LAMBDA ) - ( 1 ) ) / LAMBDA ) + BETA\_TIME\_RAIL * rail\_time + BETA\_FEMALE * gender$

## Estimation report

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Number of estimated parameters: 6
      Sample size: 228
      Excluded observations: 1511
      Init log likelihood: -158.038
      Final log likelihood: -113.265
Likelihood ratio test for the init. model: 89.546
      Rho-square for the init. model: 0.283
      Rho-square-bar for the init. model: 0.245
      Akaike Information Criterion: 238.530
      Bayesian Information Criterion: 259.106
      Final gradient norm: +5.939e-05
      Diagnostic: CFSQP: Normal termination. Obj: 6.05545e-06 Const: 6.05545e-06
      Iterations: 24
      Data processing time: 00:00
      Run time: 00:00
      Nbr of threads: 12

```

## Estimated parameters

Click on the headers of the columns to sort the table [\[Credits\]](#)

Name	Value	Std err	t-test	p-value	Robust Std err	Robust t-test	p-value	
ASC_CAR	2.64	1.09	2.41	0.02	1.03	2.56	0.01	
BETA_COST	-0.544	0.266	-2.05	0.04	0.249	-2.19	0.03	
BETA_FEMALE	0.735	0.338	2.18	0.03	0.334	2.20	0.03	
BETA_TIME_CAR	-2.42	0.500	-4.84	0.00	0.509	-4.76	0.00	
BETA_TIME_RAIL	-0.616	0.427	-1.44	0.15	* 0.423	-1.46	0.15	*
LAMBDA	0.400	0.224	1.78	0.07	* 0.211	1.90	0.06	*

Figure 2: Estimation results of model with a Box-Cox transformation

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