Exercises for Statistical analysis of network data – Sheet 1

1. Please write down both the adjacency matrix and the incidence matrix of the plotted graphs in 1(a)–(f) as well as their edge list. Are they known graph types? Please identify the graphs if you can.

2. Please plot the graphs, and write down the adjacency matrices, given by the edge lists of

   (a) $E_1 = \{(1,2),(1,3),(2,3)\}$.

   (b) $E_2 = \{(1,5),(2,5),(3,5),(4,5)\}$.

   (c) $E_3 = \{(1,2),(2,3),(3,4),(4,5)\}$.

   (d) $E_4 = \{(1,2),(2,3),(3,4),(4,5),(5,6),(6,1)\}$

3. Assume that A_{ij} is generated from an Erdős-Rényi network with edge probability ρ.

   (a) Determine the distribution of a degree d_i for a fixed value i.

   (b) What is the expectation and variance of d_i?

   (c) What is the dispersion of d_i, namely $\mathrm{Var}\{d_i\}/\mathrm{E}\{d_i\}$? For a Poisson random variable the dispersion is unity. How would ρ need to scale for the dispersion to behave like a Poisson random variable?

   (d) Repeat these calculations for the inhomogeneous random graph with edge probabilities $\{p_{ij}\}$

4. The number of triangles in an Erdős-Rényi network with edge probability ρ and adjacency matrix $A = (a_{ij})$ is calculated as

$$T = \mathrm{trace}(A^3)/6 = \frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ij}a_{jk}a_{ki}.$$

   Determine the expectation of T. Why is it appropriate to divide by 6?

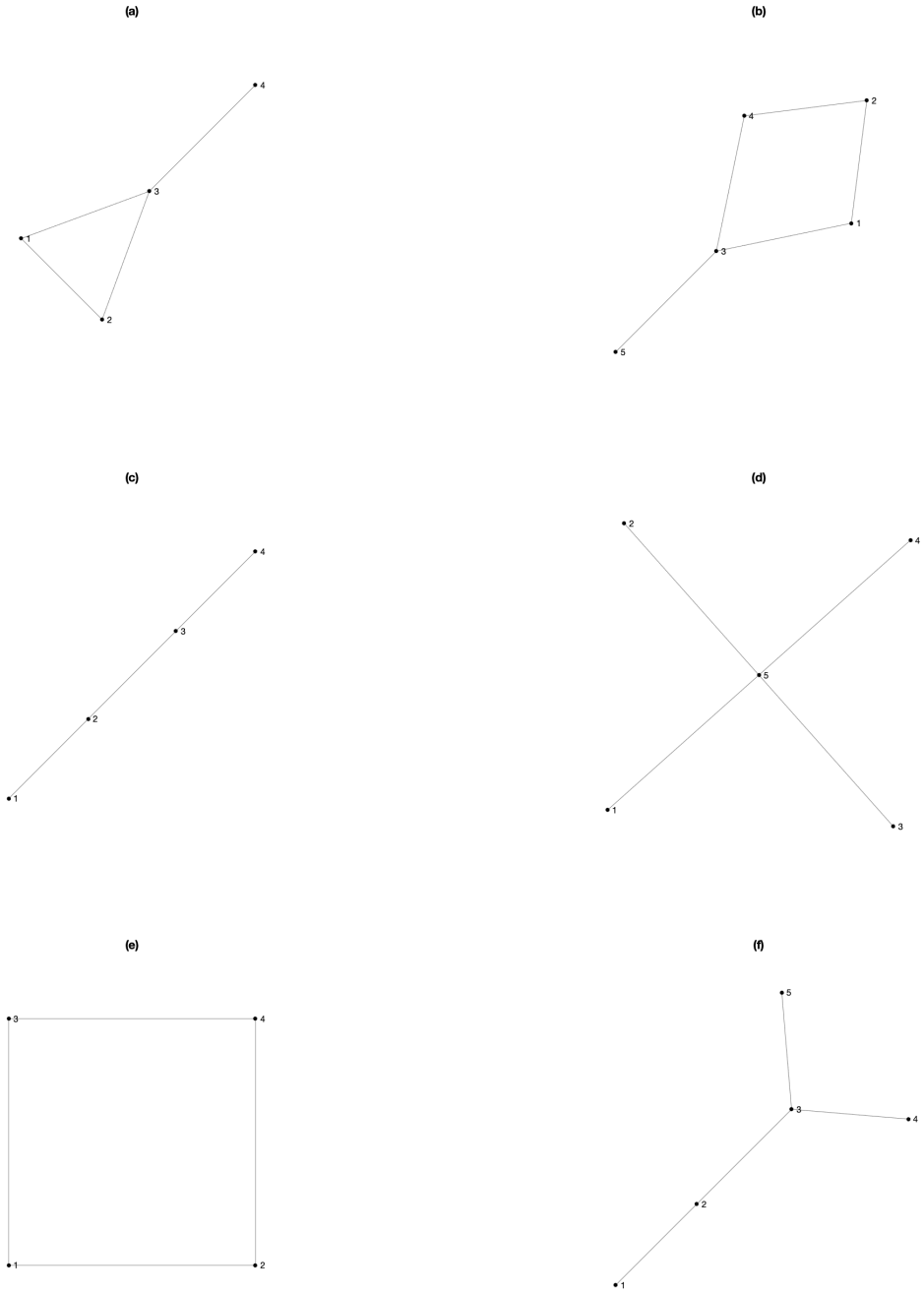

Figure 1: A number of simple graphs.