Exercises for Statistical Analysis of Network Data - Sheet 2

1. Assume we draw observations from a configuration model with

$$\pi_i = \frac{\theta_n}{i^\gamma}\ , \quad i = 1, ..., n.$$

Calculate the degrees from the realized adjacency matrix $\mathbf{A}$ via

$$d_i = \sum_{j \neq i} A_{ij}.$$

(a) Calculate the expectation of d_i. You may use the result that for large n (Gradshteyn + Ryzkin 0.233, and an integral squeezing argument):

$$\sum_{j=1}^{n} \frac{1}{j^p} \approx \begin{cases} \zeta(p) & \text{if } p > 1 \\ \log(n)(1 + o(1)) & \text{if } p = 0 \\ \frac{n^{1-p}}{1-p} & \text{if } 0 < p < 1 \end{cases}$$

(b) Calculate the variance of d_i.

(c) Calculate the dispersion of d_i. Are there instances where the dispersion approaches unity?

(d) Calculate $\mathrm{Cov}(d_i, d_j)$.

2. Show that the degree–corrected stochastic blockmodel is exchangeable.

3. Calculate the expected degree of node i from the random dot product graph.

4. Reformulate the stochastic block model as a random dot product graph. How do we select d the latent dimension of the RDGP relative to k the number of blocks.

5. What is the size of the automorphism group of an edge?

6. What is the size of the automorphism group of a triangle?