DATA 605 - Homework #15

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5/16/2020

Question #1

> Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.

$$(5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8)$$

We can utilize the equation:

$$\beta_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{X})(y_i - \overline{Y})}{\sum_{i=1}^{n}(x_i - \overline{X})^2}$$

$$\beta_1 = \frac{Cov(x, y)}{Var(x)}$$

$$\beta_0 = \overline{Y} - m\overline{X}$$

To find the regression line of the given points, we can create a dataframe.

```
df <- data.frame(x=c(5.6, 6.3, 7, 7.7, 8.4), y=c(8.8, 12.4, 14.8, 18.2, 20.8))
```

Since we have all of the information we need to fill in the formula, we can solve by doing the following calculations:

```
X <- mean(df$x)
Y <- mean(df$y)
varx <- var(df$x)
covxy <- cov(df$x, df$y)
Beta1 <- covxy/varx
Beta <- Y - (Beta1*X)

Beta
```

```
## [1] -14.8
```

```
Beta1
```

```
## [1] 4.257143
```

Therefore, our equation is $y = -14.8 + 4.26x$

We can check our work by using the `lm()` function to solve:

```r
lm_df <- lm(y ~ x, df)
summary(lm_df)
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Residuals:
##     1     2     3     4     5
## -0.24  0.38 -0.20  0.22 -0.16
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -14.8000     1.0365  -14.28 0.000744 ***
## x             4.2571     0.1466   29.04 8.97e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3246 on 3 degrees of freedom
## Multiple R-squared:  0.9965, Adjusted R-squared:  0.9953
## F-statistic: 843.1 on 1 and 3 DF,  p-value: 8.971e-05
```

SOLUTION: It is confirmed, we can see from our output that the equation is $y = -14.80 + 4.26x$

Question #2

> Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

$$f(x, y) = 24x - 6xy^2 - 8y^3$$

We can solve this in multiple parts:

$$f_x(x, y) = \frac{d}{dx}(24x - 6xy^2 - 8y^3)$$
$$f_x(x, y) = 24 - 6y^2$$
$$f_y(x, y) = \frac{d}{dx}(24x - 6xy^2 - 8y^3)$$
$$f_y(x, y) = -12xy - 24y^2$$

Therefore, we can now set $f_x(x, y)$ and $f_y(x, y)$ equal to 0 and solve:

$$0 = 24 - 6y^2$$
$$6y^2 = 24$$
$$y = 2$$

$$0 = -12xy - 24y^2$$
$$24y^2 = -12xy$$
$$-2y^2 = xy$$
$$-2y = x$$

Therefore, we know that:

When $y = 2$, $x = -4$, and when $y = -2$, $x = 4$. Since we now know x and y, we can substitute these values back into the original equation and solve:

$$f(-4,2) = 24(-4) - 6(-4)(2)^2 - 8(2)^3$$
$$f(-4,2) = -96 - 6(-4)(2)^2 - 8(2)^3$$
$$f(-4,2) = 0 - 8(2)^3$$
$$f(-4,2) = -64$$

$$f(4,-2) = 24(4) - 6(4)(-2)^2 - 8(-2)^3$$
$$f(4,-2) = 96 - 96 - 8(-8)$$
$$f(4,-2) = 64$$

With this solved, we now know that the critical points are:

$(x, y, z) = (4, -2, 64)$ and $(-4, 2, -64)$

To find the saddle points, we can utilize the Second Derivative test:

Let $D = f_{xx}(x_0, y_0) \times f_{xy}(x_0, y_0) - f_{xy}^2(x_0, y_0)$ for a P

If we find that $D < 0$, then we can say that P is a saddle point of f.

$$f_{xx}(x,y) = \frac{d}{dx}(24 - 6y^2) = 0$$
$$f_{yy}(x,y) = \frac{d}{dx}(-12xy - 24y^2) = -12x - 48y$$
$$f_{xy}(x,y) = \frac{d}{dx}(24 - 6y^2) = -12$$

And now by substituting $D(x_0, y_0)$ into our equation:

$$D(-4,2) = (0)((-12 \times -4) - (48 \times 2)) - (-12 \times 2)^2$$

$$D(-4,2) = 0 - (-12 \times 2)^2$$

$$D(-4,2) = -576$$

$$D(4,-2) = (0)((-12 \times 4) - (48 \times -2)) - (-12 \times -2)^2$$

$$D(4,-2) = 0 - (-12 \times -2)^2$$

$$D(4,-2) = -576$$

SOLUTION: To summarize, we found that our critical points are $(x, y, z) = (4, -2, 64)$ and $(-4, 2, -64)$, and since both $D(-4, 2)$ and $D(4, -2)$ are negative, we can also say that $(-4, 2)$ and $(4, 2)$ are the saddle points.

Question #3

> A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

> Step 1. Find the revenue function $R(x, y)$.

To find the revenue function, we know that the manager can sell the house and name brand for x and y dollars respectively:

$$R_x = x \times (81 - 21x + 17y)$$
$$R_x = 81x - 21x^2 + 17xy$$

$$R_y = y \times (40 + 11x - 23y)$$
$$R_y = 40y + 11xy - 23y^2$$

So we know that:

$$R(x, y) = 81x - 21x^2 + 17xy + 40y + 11xy - 23y^2$$

$$R(x, y) = 81x - 21x^2 + 28xy + 40y - 23y^2$$

SOLUTION: The revenue function is $R(x, y) = 81x - 21x^2 + 28xy + 40y - 23y^2$

> Step 2. What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

For this, we can just substitute into our revenue function and solve:

$$R(x, y) = 81x - 21x^2 + 28xy + 40y - 23y^2$$
$$R(2.3, 4.1) = 81(2.3) - 21(2.3)^2 + 28(2.3)(4.1) + 40(4.1) - 23(4.1)^2$$

$$R(2.3, 4.1) = 186.3 - 111.09 + 264.04 + 164 - 386.63$$

$$R(2.3, 4.1) = 116.62$$

SOLUTION: If the manager sells the "house" brand for 2.30 dollars and the "name" brand for 4.10 dollars, then her revenue will be 116.62 dollars.

Question #4

> A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

Since the company is committed to producing a total of 96 units of product each week, we know that:

$$x + y = 96$$

We also know that our weekly cost for both x and y is:

$$C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$$

With these in mind, we can substitute the value of y into the equation since $y = 96 - x$:

$$C(x) = \frac{1}{6}x^2 + \frac{1}{6}(96 - x)^2 + 7x + 25(96 - x) + 700$$

$$C(x) = \frac{1}{6}x^2 + \frac{1}{6}(9216 - 192x + x^2) + 7x + 25(96 - x) + 700$$

$$C(x) = \frac{1}{6}x^2 + 1536 - 32x + \frac{1}{6}x^2 + 7x + 2400 - 25x + 700$$

$$C(x) = \frac{1}{3}x^2 - 50x + 4636$$

Then, we can find $\frac{dC}{dx}$:

$$\frac{dC}{dx} = \frac{2}{3}x - 50$$

And to find x, we can solve for 0:

$$0 = \frac{2}{3}x - 50$$

$$50 = \frac{2}{3}x$$

$$x = 75$$

Using the Second Derivative test:

$$\frac{d^2C}{dx^2} = \frac{2}{3}$$

And since this is greater than 0, we know that $x = 75$ is the relative minima.

SOLUTION: Since we've found that $x = 75$ is the relative minima, we can conclude that in order to minimize the total weekly cost, the company will want to produce 75 units in Los Angeles and 21 units (96-75 units) in Denver.

Question #5

Evaluate the double integral on the given region.

$$\int\int (e^{8x+3y}) dA; R : 2 \leq x \leq 4 \ and \ 2 \leq y \leq 4$$

Write your answer in exact form without decimals.

To solve the double integral, we can do the following:

$$\int_2^4 \int_2^4 (e^{8x+3y}) \ dy \ dx$$

We can integrate the inside integral first:

$$\int_2^4 (e^{8x+3y}) \ dy$$

With u substitution:

$$u = 8x + 3y$$

$$\frac{du}{dy} = 3 \ ; \ dy = \frac{1}{3} du$$

Therefore, $\int_2^4 (e^{8x+3y}) \ dy$ can be:

$$\frac{1}{3} \int_2^4 e^u du = \frac{1}{3}(e^u|_2^4)$$

And now by substituting back in $(8x + 3y)$ in place of u and y:

$$\frac{1}{3}[e^{(8x+(3*4))} - e^{(8x-(3*2))}]$$

$$\frac{1}{3}[e^{(8x+12)} - e^{(8x+6)}]$$

Finally, we can solve the outside integral by substituting this back in:

$$\int_2^4 \frac{1}{3}[e^{(8x+12)} - e^{(8x+6)}]\ dx$$

$$\int_2^4 \frac{1}{3}[e^{(8x+12)}]\ dx - \int_2^4 \frac{1}{3}[e^{(8x+6)}]\ dx$$

$$\frac{1}{3} * \frac{1}{8}e^{(8x+12)}|_2^4 - \frac{1}{3} * \frac{1}{8}e^{(8x+6)}|_2^4$$

$$\frac{1}{24}[e^{(8(4)+12)} - e^{(8(2)+12)}] - \frac{1}{24}[e^{(8(4)+6)} - e^{(8(2)+6)}]$$

$$\frac{1}{24}[e^{(32+12)} - e^{(16+12)}] - \frac{1}{24}[e^{(32+6)} - e^{(16+6)}]$$

$$\frac{1}{24}[e^{(32+12)} - e^{(16+12)} - e^{(32+6)} + e^{(16+6)}]$$

$$\frac{1}{24}[e^{44} - e^{28} - e^{38} + e^{22}]$$

And we can now solve by calculating:

```
paste0('The answer is: ', ((1/24)*(exp(44)-exp(38)-exp(28)+exp(22))))
```

```
## [1] "The answer is: 534155947497085184"
```

```
(1/24)*(exp(44)-exp(38)-exp(28)+exp(22))
```

```
## [1] 5.341559e+17
```

SOLUTION: After evaluating the double integral, we get a value of 534,155,947,497,085,184 or 5.3411559×10^{17}