HOMEWORK #13

① USE INTEGRATION BY SUBSTITUTION TO SOLVE THE INTEGRAL BELOW.

$$\int 4e^{-7x}\,dx \qquad \begin{aligned} u &= -7x \\ du &= -7\,\tfrac{d}{dx} \end{aligned}$$

$$\frac{du}{dx} = -7 \longrightarrow \frac{1}{dx} \cdot \frac{-7}{du} \longrightarrow dx = \frac{1}{-7}\,du$$

THEREFORE,

$$\int 4e^{u} \cdot \frac{1}{7}\,du$$

$$\int -\frac{4}{7} e^{u}\,du$$

$$-\frac{4}{7}\int e^{u}\,du \longrightarrow -\frac{4}{7}(e^{u}+c)$$

$$\boxed{= -\frac{4}{7}e^{-7x} + c}$$

ANSWER

② BIOLOGISTS ARE TREATING A POND CONTAMINATED WITH BACTERIA. THE LEVEL OF CONTAMINATION IS CHANGING AT A RATE OF

$\frac{dN}{dt} = \frac{3150}{t^4} - 220$ BACTERIA PER CUBIC CENTIMETER PER DAY,

WHERE t IS THE NUMBER OF DAYS SINCE TREATMENT BEGAN.

FIND A FUNCTION $N(t)$ TO ESTIMATE THE LEVEL OF CONTAMINATION IF THE LEVEL AFTER 1 DAY WAS 6530 BACTERIA PER CUBIC CENTIMETER.

SINCE $\frac{dN}{dt} = \frac{3150}{t^4} - 220$ AND $t(1) = 6530$

NEXT PAGE

$$N' = \frac{3150}{t^4} - 220$$

$$\int N'\,dt = \int \left(\frac{3150}{t^4} - 220\right) dt$$

$$N(t) = 3150 \int \frac{1}{t^4} + -220 \int dt$$

$$N(t) = 3150 \cdot \int t^{-4} + -220 \int dt$$

$$N(t) = 3150 \cdot \frac{1}{3} t^{-3} - 220t + C$$

$$N(t) = \frac{1050}{t^3} - 220t + C$$

WHEN $t = 1$, WE CAN FIND C:

$$6530 = \frac{1050}{1^3} - 220(1) + C$$

$$6530 = 1050 - 220 + C$$

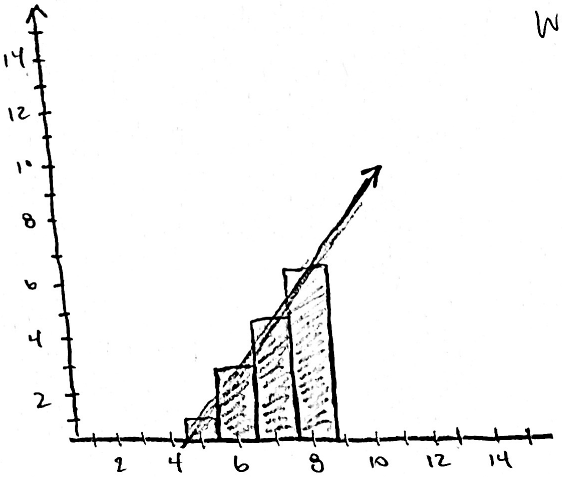
$$5700 = C$$

ANSWER

THEREFORE, OUR FUNCTION IS:

$$\boxed{N(t) = \frac{1050}{t^3} - 220t + 5700}$$

(3) FIND THE TOTAL AREA OF THE RED RECTANGLES IN THE FIGURE BELOW, WHERE THE EQUATION OF THE LINE IS $f(x) = 2x - 9$:



WE CAN EVALUATE THE INTEGRAL:

$$\int_{4.5}^{8.5} 2x - 9$$

NEXT PAGE

$$\int_{4.5}^{8.5} 2x-9 \longrightarrow 2\int_{4.5}^{8.5} x + \int_{4.5}^{8.5} -9dx \longrightarrow 2\frac{x^2}{2} - 9x \Big|_{4.5}^{8.5}$$

$$= \left[2\frac{(8.5)^2}{2} - 9(8.5)\right] - \left[2\frac{(4.5)^2}{2} - 9(4.5)\right]$$

$$= (72.25 - 76.5) - (20.25 - 40.5)$$

$$= -4.25 + 20.25 \boxed{= 16}$$

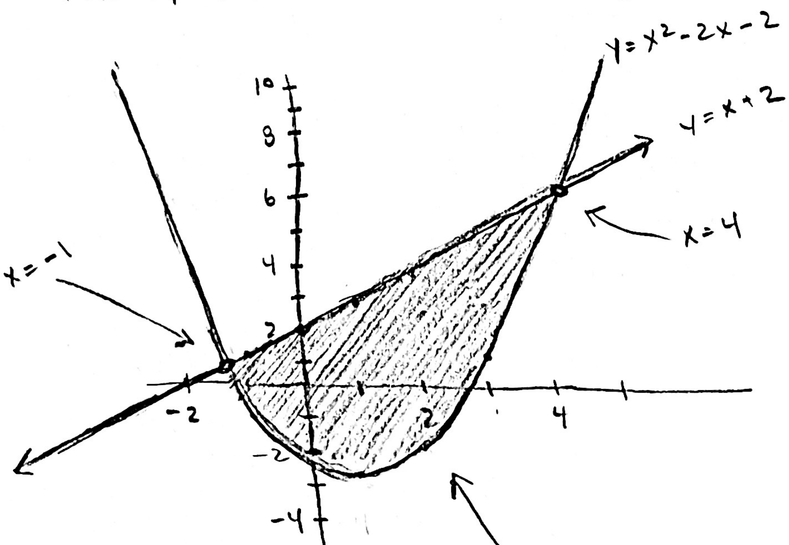
ANSWER

(4) FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS OF THE GIVEN EQUATIONS:

$$y = x^2 - 2x - 2, \quad y = x + 2$$

x	y
-2	6
-1	-1
0	-2
1	-3
2	-2
3	1

FIRST, WE WILL DRAW THESE LINES:



x	y
-2	0
-1	1
0	2
1	3
2	4

WE WILL FIND THE AREA OF THE SHADED REGION

* BOUNDED AT x = -1 AND x = 4

NEXT PAGE

$$\int_{-1}^{4} (x+2)\,dx - \int_{-1}^{4} (x^2-2x-2)\,dx$$

$$\int_{-1}^{4} (x+2-x^2+2x+2)\,dx$$

$$\int_{-1}^{4} (-x^2+3x+4)\,dx \longrightarrow -\frac{x^3}{3}+\frac{3x^2}{2}+4x \Big|_{-1}^{4}$$

NOW EVALUATE IN BOUNDS:

$$-\frac{4^3}{3}+\frac{3(4)^2}{2}+4(4) - \left(-\frac{1^3}{3}+\frac{3(1)^2}{2}+4(1)\right)$$

$$= -\frac{64}{3}+\frac{48}{2}+16-\left(-\frac{1}{3}+\frac{3}{2}+4\right)$$

$$= -\frac{65}{3}+\frac{45}{2}+20 = -21.66+22.5+20$$

$$\approx 20.83$$

ANSWER

$\boxed{\text{THE AREA IS ABOUT } 20.83 \text{ UNITS}^2}$

NEXT PAGE

(5) A BEAUTY SUPPLY STORE EXPECTS TO SELL 110 FLAT IRONS DURING THE NEXT YEAR. IT COSTS \$3.75 TO STORE ONE FLAT IRON FOR ONE YEAR. THERE IS A FIXED COST OF \$8.25 FOR EACH ORDER. FIND THE LOT SIZE AND THE NUMBER OF ORDERS PER YEAR THAT WILL MINIMIZE INVENTORY COSTS.

THE RELATIONSHIP CAN BE DEFINED:

$nx = 110$

C = COST
n = ORDERS / YR
x = IRONS / ORDER

AND $C = 8.25n + 3.75\frac{x}{2}$

$$C = 8.25n + 3.75\frac{\frac{110}{n}}{2} = 8.25n + \frac{412.5}{n}\cdot\frac{1}{2}$$

$$C = 8.25n + \frac{412.5}{2n} = 8.25n + \frac{206.25}{n}$$

WE MUST THEN TAKE THE DERIVATIVE:

$$\frac{d}{dn}\left[8.25n + \frac{206.25}{n}\right]$$

$$8.25\cdot\frac{d}{dn}[n] - 206.25\cdot\frac{d}{dn}\left[\frac{1}{n}\right]$$

$$8.25\cdot 1 - 206.25\cdot\frac{1}{n^2}$$

$$C' = 8.25 - \frac{206.25}{n^2}$$

$$-8.25 = -\frac{206.25}{n^2} \longrightarrow \frac{206.25}{8.25} = n^2$$

$$25 = n^2$$

$$\boxed{5 = n}$$

NEXT PAGE

SINCE $n=5$, WE CAN THEN SOLVE FOR x:

$$(5)x = 110$$

$$\boxed{x = 22}$$

ANSWER

THEREFORE, TO MINIMIZE INVENTORY COSTS, YOU'LL WANT TO PLACE 5 ORDERS PER YEAR AND A LOT SIZE OF 22 IRONS.

(6) USE INTEGRATION BY PARTS TO SOLVE THE INTEGRAL BELOW:

$$\int \ln(9x) \cdot x^6 \, dx$$

WE CAN USE THE FOLLOWING TO SOLVE:

$$\int u \, dv = uv - \int v \, du \quad \text{WHERE,}$$

$$u = \ln(9x) \qquad v = \frac{1}{7}x^7$$

$$\frac{du}{dx} = \frac{d}{dx}\left(\ln(9x)\right) = \frac{1}{x}$$

$$\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{7}x^7\right) = x^6$$

$$= \ln(9x)\left(\frac{1}{7}x^7\right) - \int \left(\frac{1}{7}x^7\right)\left(\frac{1}{x}\right) dx$$

$$= \frac{\ln(9x)\,x^7}{7} - \int \left(\frac{1}{7}x^7\right)\left(\frac{1}{x}\right) dx$$

$$= \frac{\ln(9x)\,x^7}{7} - \frac{1}{7}\int \frac{x^7}{x}\, dx$$

$$= \frac{\ln(9x)\,x^7}{7} - \frac{1}{7}\int x^6 \, dx$$

NEXT PAGE

$$= \frac{\ln(9x)x^7}{7} - \frac{1}{7}\left(\frac{x^7}{7}\right) + c$$

$$= \frac{\ln(9x)x^7}{7} - \frac{x^7}{49} + c$$

ANSWER

$$= \frac{x^7 \ln(9x)}{7} - \frac{x^7}{49} + c$$

$$\boxed{= \frac{x^7}{7}\left(\ln(9x) - \frac{1}{7}\right) + c}$$

(7) DETERMINE WHETHER $f(x)$ IS A PROBABILITY DENSITY FUNCTION ON THE INTERVAL $[1, e^6]$. IF NOT, DETERMINE THE VALUE OF THE DEFINITE INTEGRAL:

$$f(x) = \frac{1}{6x}$$

WE CAN EVALUATE BY BOUNDS:

$$\int_1^{e^6} \frac{1}{6x}\,dx \longrightarrow \frac{1}{6}\int_1^{e^6} \frac{1}{x}\,dx$$

$$= \frac{1}{6}\ln(x)\Big|_1^{e^6}$$

$$= \frac{1}{6}\left(\ln(e^6) - \ln(1)\right)$$

$$= \frac{\ln(e^6)}{6} - \frac{\ln(1)}{6}$$

$$= \frac{6}{6} - \frac{0}{6}$$

ANSWER

$$= 1 - 0 \ \boxed{= 1}$$

ANSWER

$$\int_1^{e^6} f(x)\,dx = \int_1^{e^6} \left(\frac{1}{6x}\right)dx = 1.$$

THEREFORE, $f(x)$ IS A PROBABILITY DENSITY FUNCTION ON $[1, e^6]$.