CHAPTER R. C10

IN VECTOR SPACE $\mathbb{C}^3$, COMPUTE THE VECTOR REPRESENTATION $\rho_B(v)$ FOR THE BASES B AND VECTOR v BELOW.

$$B = \left\{ \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right\} \qquad v = \begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}$$

SINCE,

$$a_1 \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} + b_1 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + c_1 \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}$$

THIS BECOMES A SYSTEM OF LINEAR EQUATIONS, AND WE CAN CREATE AN AUGMENTED MATRIX:

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 11 \\ -2 & 3 & 5 & 5 \\ 2 & 1 & 2 & 8 \end{array}\right]$$

CONVERT TO RREF

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 11 \\ -2 & 3 & 5 & 5 \\ 2 & 1 & 2 & 8 \end{array}\right] \xrightarrow{R_1+R_2} \begin{bmatrix} 2 & 1 & 3 & 11 \\ 0 & 4 & 8 & 16 \\ 2 & 1 & 2 & 8 \end{bmatrix} \xrightarrow{R_1-R_3} \begin{bmatrix} 2 & 1 & 3 & 11 \\ 0 & 4 & 8 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow{1/8 R_2} \begin{bmatrix} 2 & 1 & 3 & 11 \\ 0 & 1/2 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_3-R_2} \begin{bmatrix} 2 & 1 & 3 & 11 \\ 0 & -1/2 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 & 3 & 11 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow{R_1-3R_3} \begin{bmatrix} 2 & 1 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_1-R_2} \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{1/2 R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

THEREFORE,

$$\boxed{\rho_B(v) = \rho_B\left(\begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}\right) = \rho_B\left(2\begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} + (-2)\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}}$$