# Deterioration Analysis of Bridges Using Network-Scale State-Space Models

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Transports, Mobilité durable et Électrification des transports Québec Partenaire

# Outline

# Database of Visual Inspections

Year: 2017
Structure: #002

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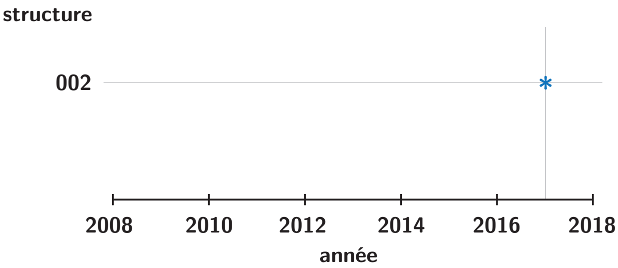


Year: 2017
Structure: #002

$\mathbf{b}_{002}$: {type, material, DJMA, Location,...}

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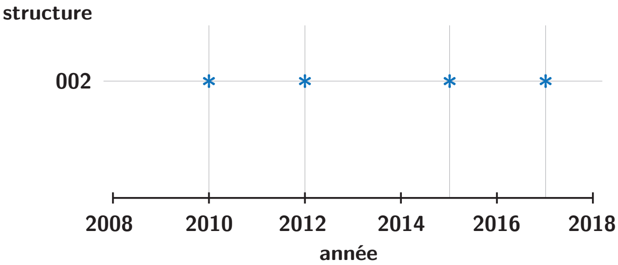


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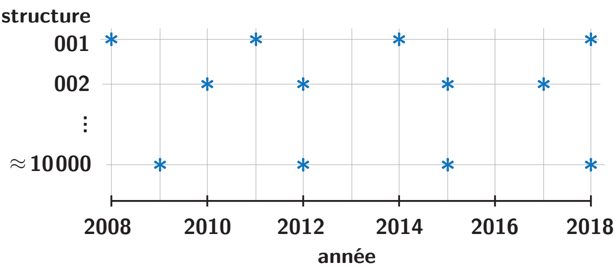


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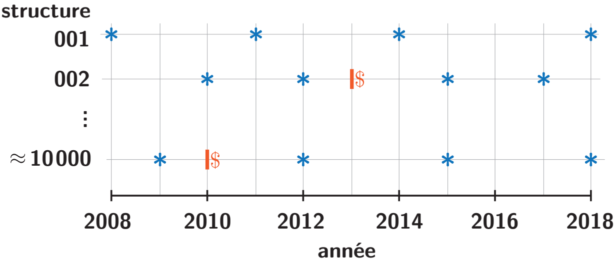


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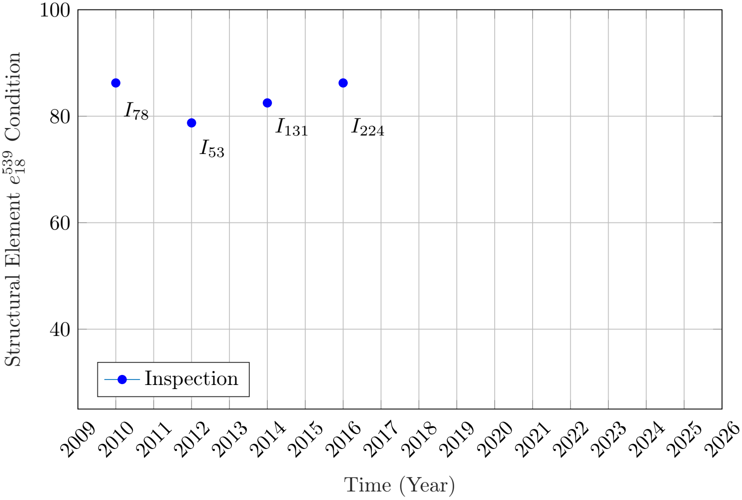
# Example: Series of Inspections on Structural Element



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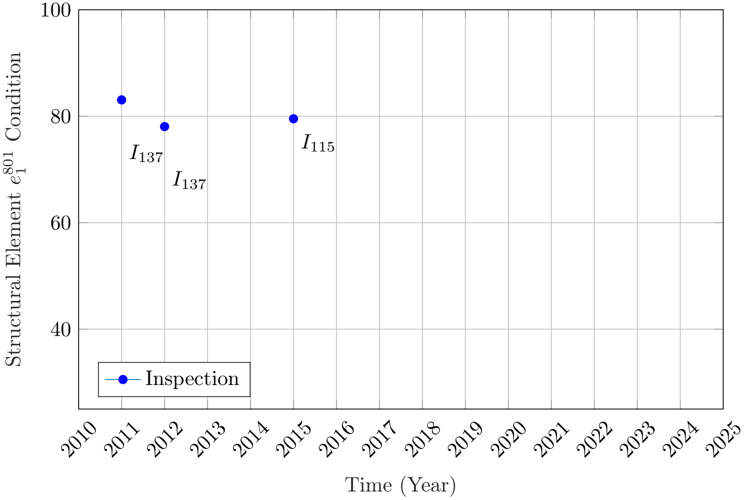
- Quantify the **effect of interventions** on structural elements.

# Modeling Deterioration: Condition Estimate



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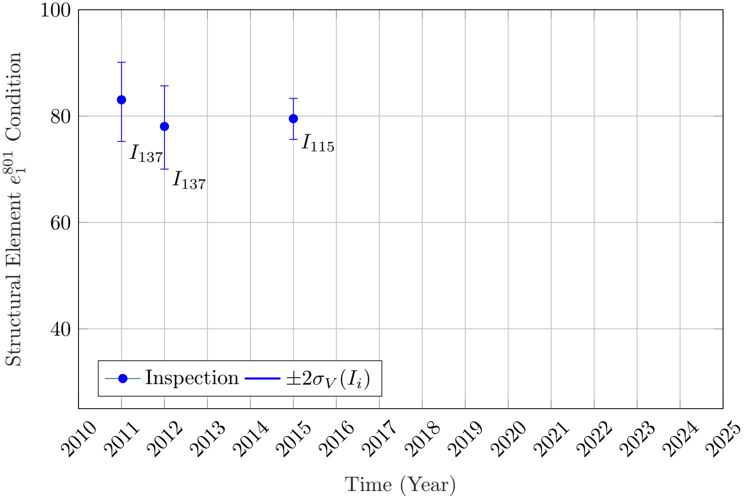

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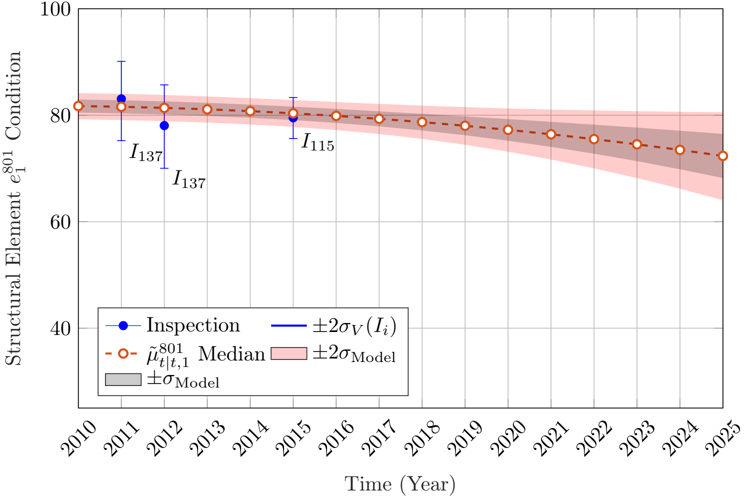

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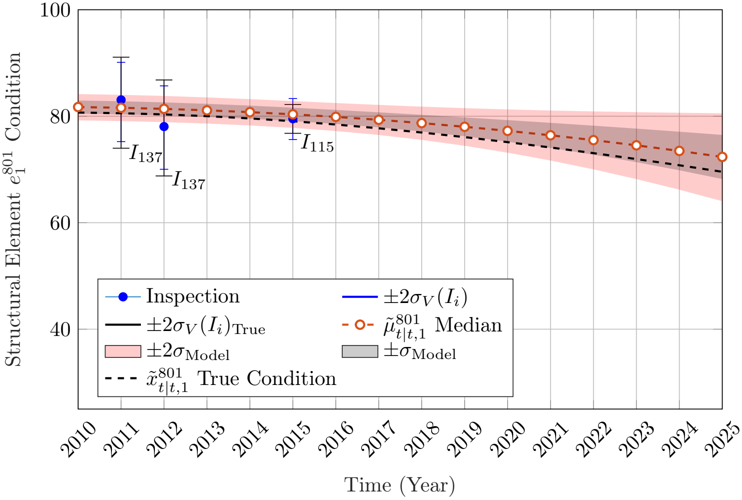

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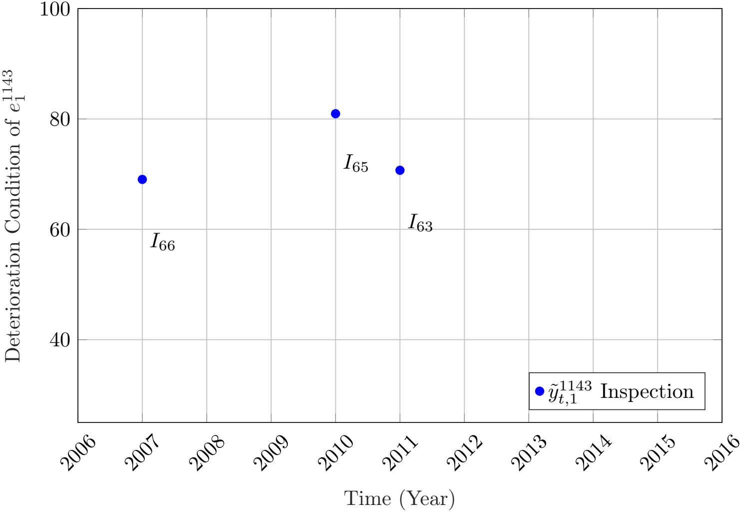

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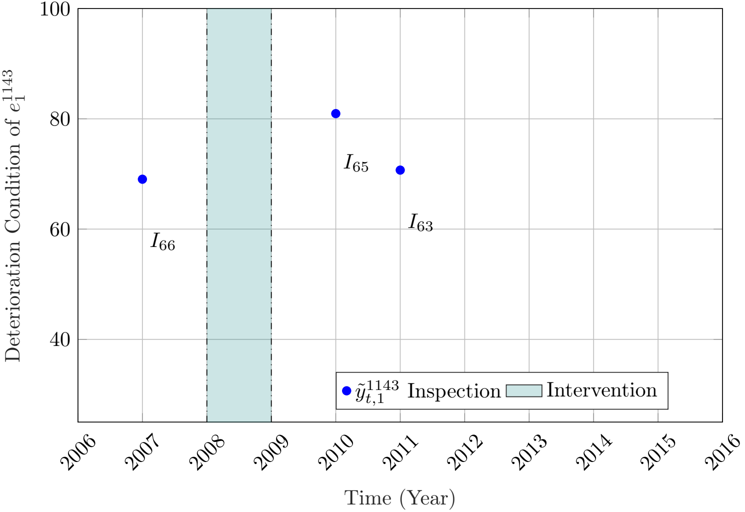

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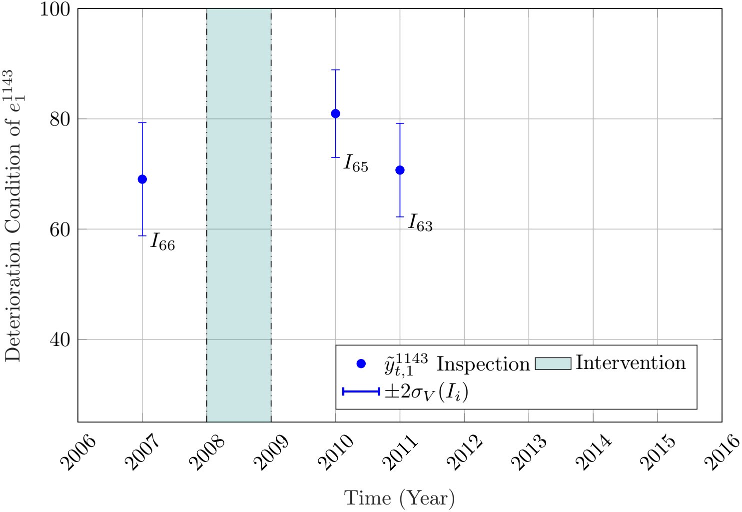

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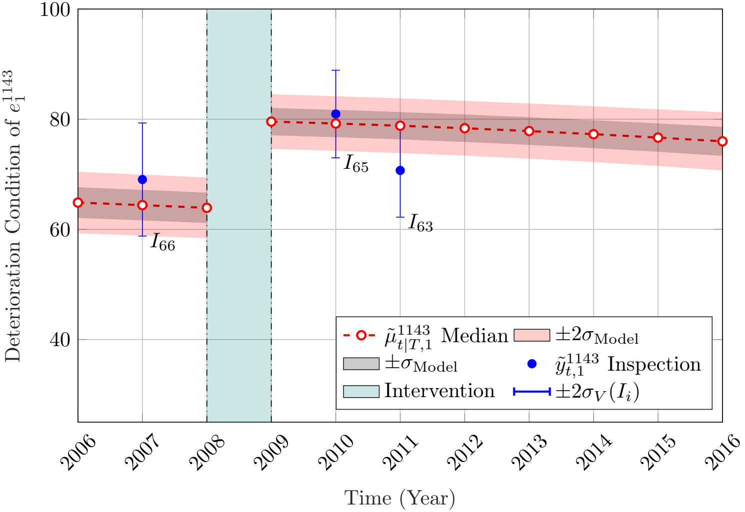

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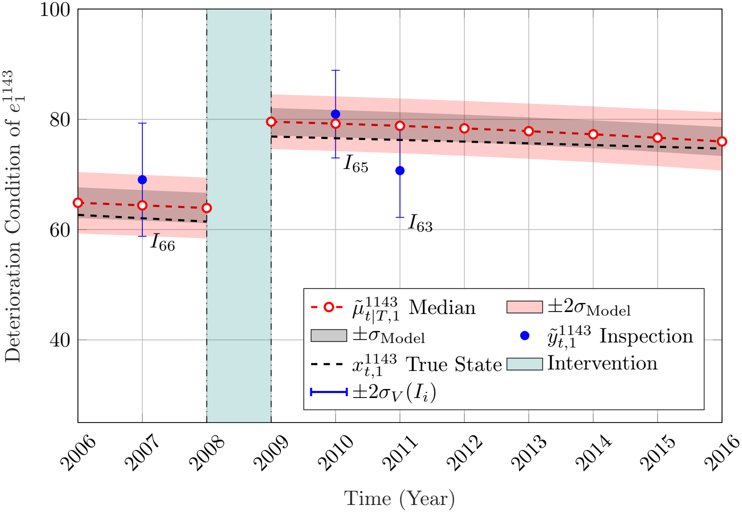

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$b_j$

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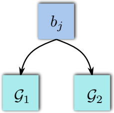

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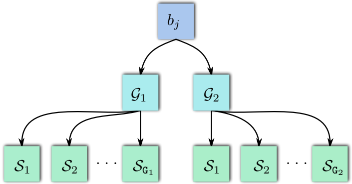

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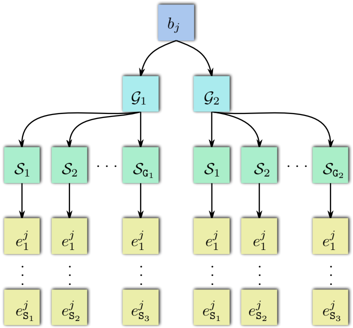

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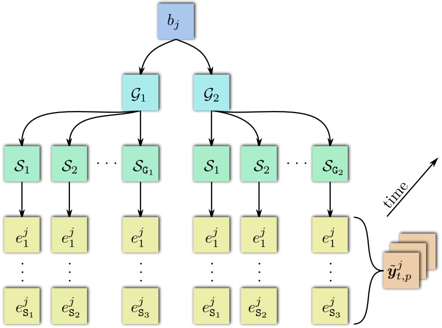

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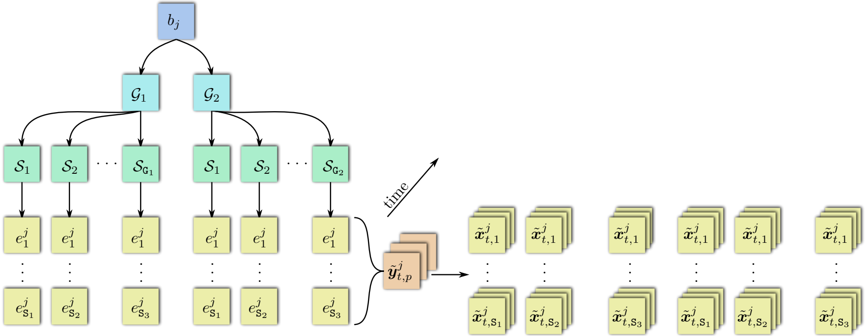

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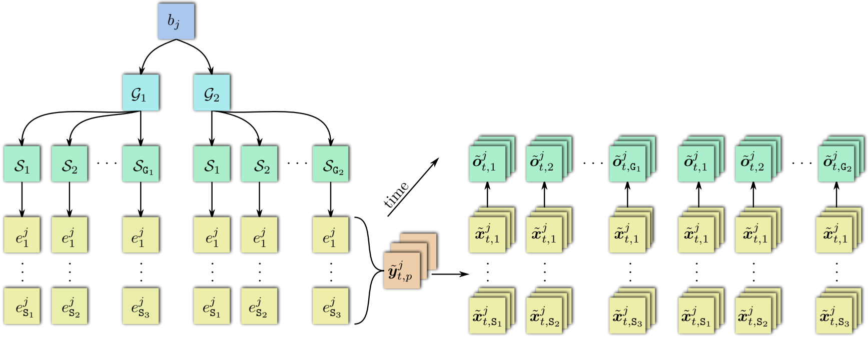

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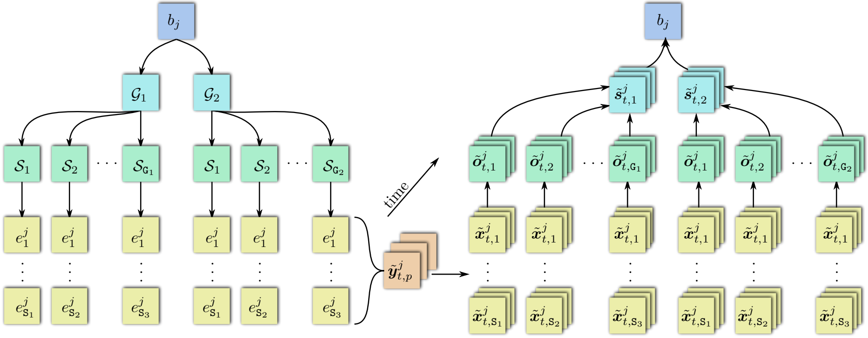

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$$\mathcal{L}(h) = \sum_{t=1}^{\mathtt{T}_p} \ln f(y^j_{t,p} | y^j_{1:t-1,p}, h, \boldsymbol{\theta}),$$

where the effect of interventions $\boldsymbol{\delta}_t$ associated with each type $h$ is known.

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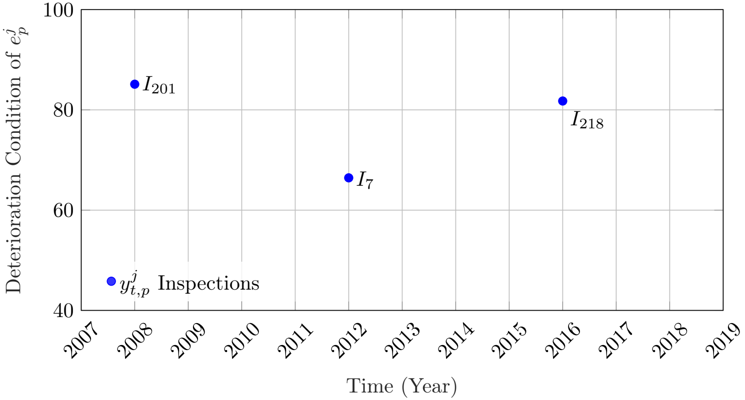

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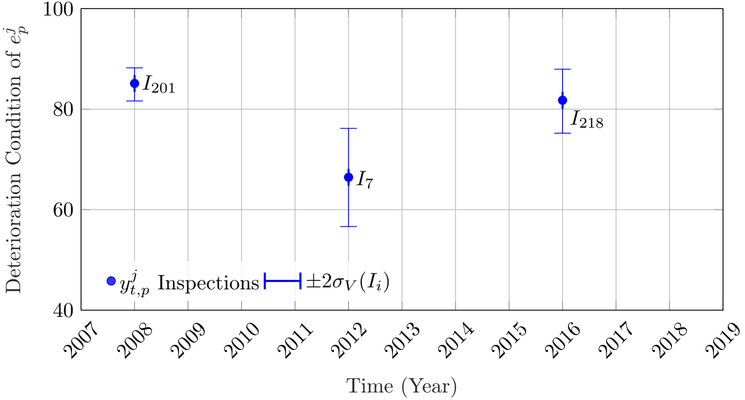

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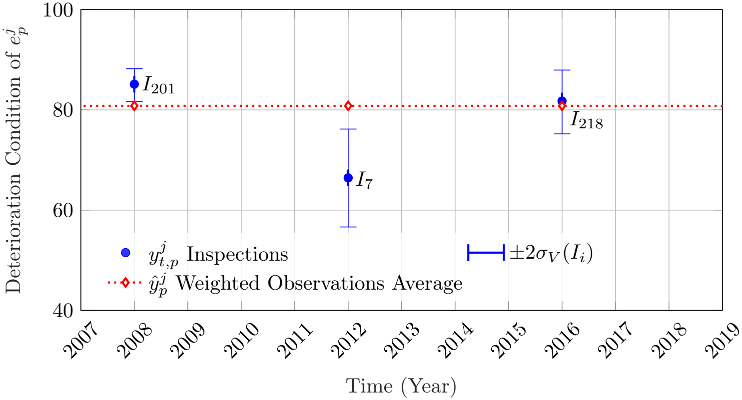

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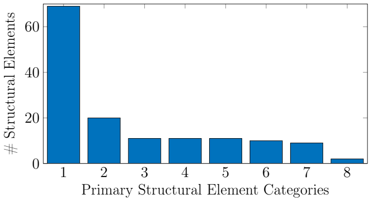

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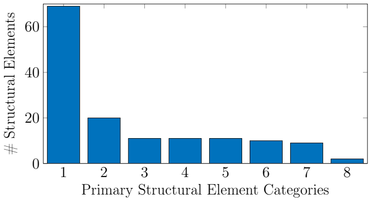


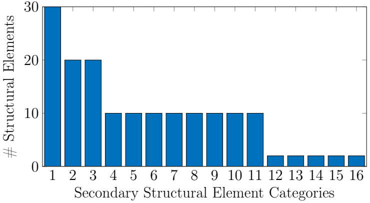


| Order | Primary $\mathcal{G}_1$ | Secondary $\mathcal{G}_2$ |
|---|---|---|
| 1 | $\mathcal{S}_{1,1}$: Beams | $\mathcal{S}_{2,1}$: Bumpers |
| 2 | $\mathcal{S}_{1,2}$: External Sides | $\mathcal{S}_{2,2}$: Abutment Backwall |
| 3 | $\mathcal{S}_{1,3}$: Bearing pad | $\mathcal{S}_{2,3}$: Median Barrier |

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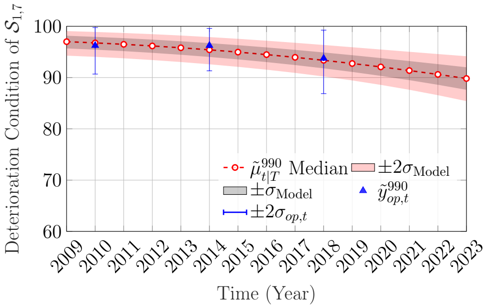

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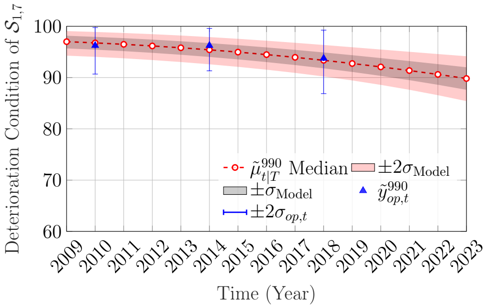


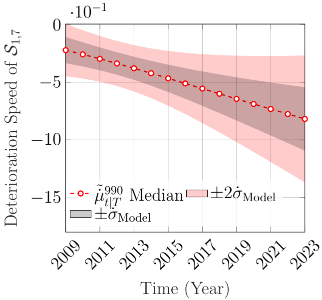

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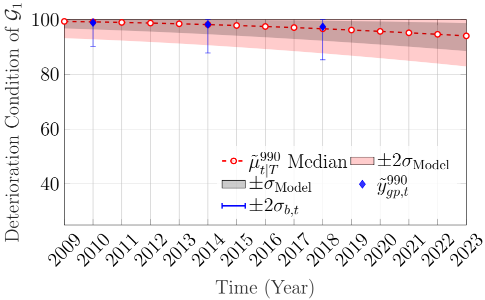

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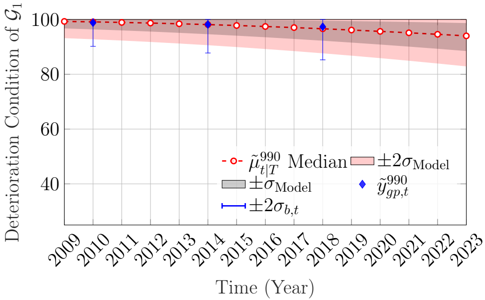


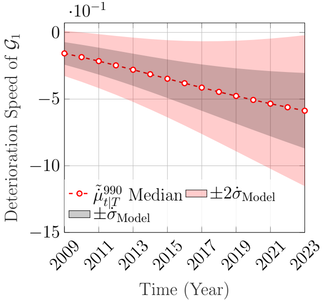

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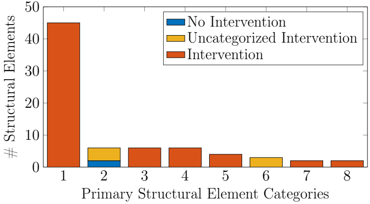

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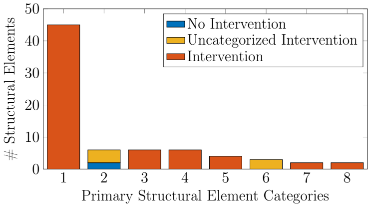


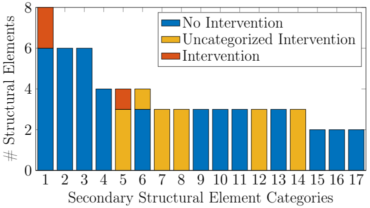


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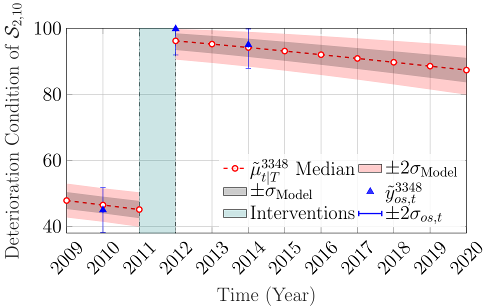

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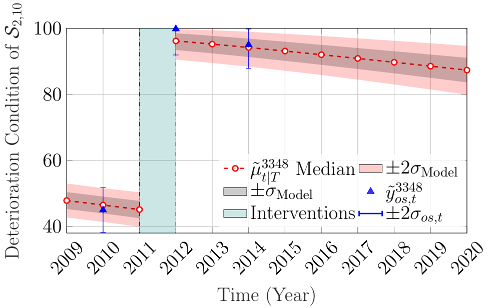


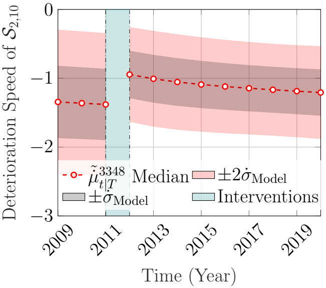

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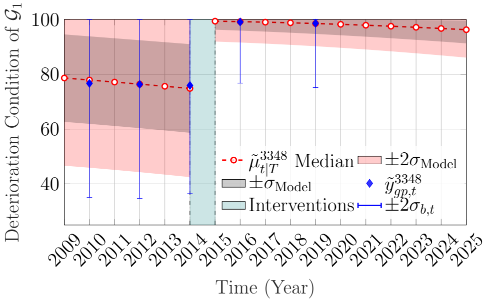

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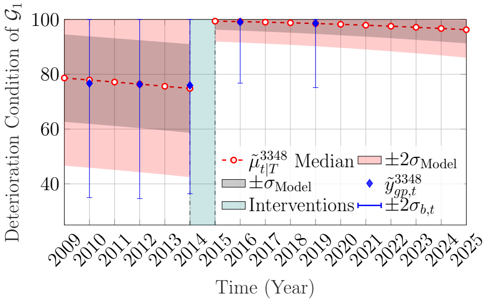


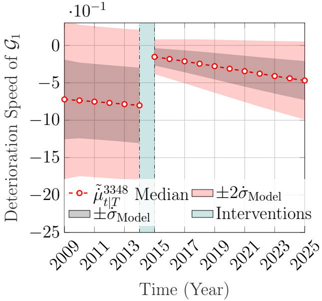

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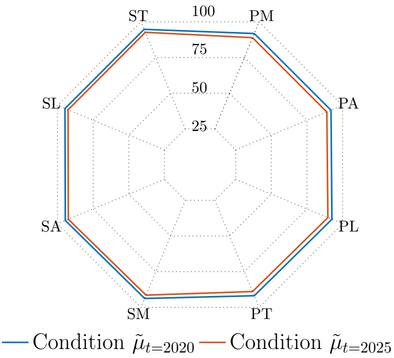

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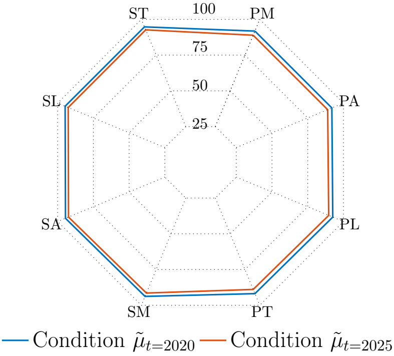
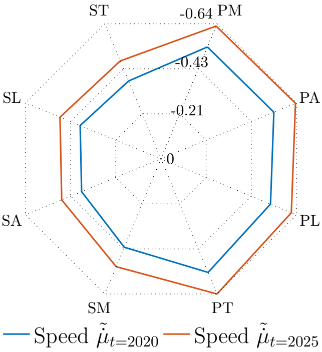
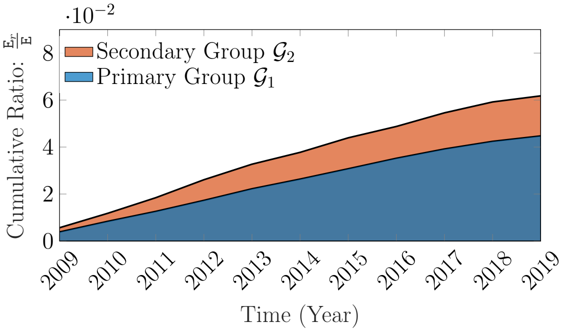


# Intervention per Year:



# Cost vs.# Interventions:

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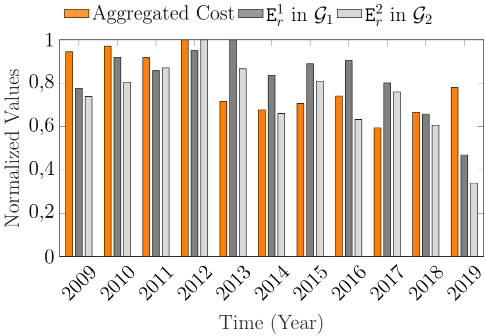

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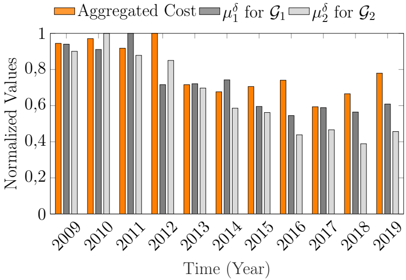

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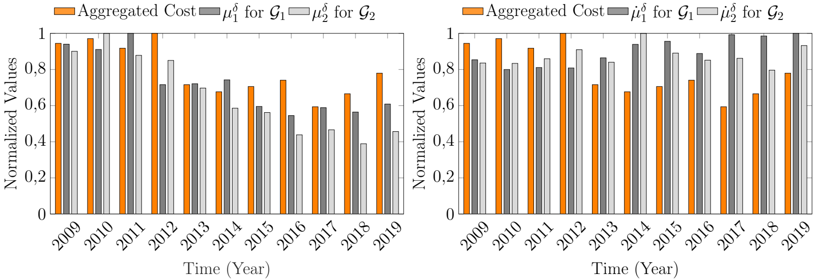

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- Analysis are based on the visually inspected elements only.