

Modeling the Effects of Interventions in Infrastructures (Formulation)

Zachary Hamida Doctorant
James-A. Goulet Professeur

 Polytechnique Montréal, Canada
Département des génies civil, géologique et des mines

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*Transports,
Mobilité durable
et Électrification
des transports*

Québec 

Partenaire



Outline

Contexte & Objectifs

Intervention Information

Quantifying Interventions

Next Steps

Database of Visual Inspections

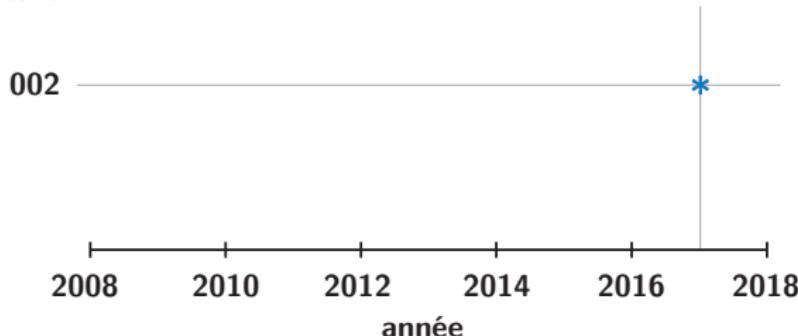
Year: 2017

Structure: #002



Database of Visual Inspections

structure



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\mathbf{b}_{002} : {type, material, DJMA, Location, ... }

Database of Visual Inspections

structure

002

2008

2010

2012

2014

2016

2018

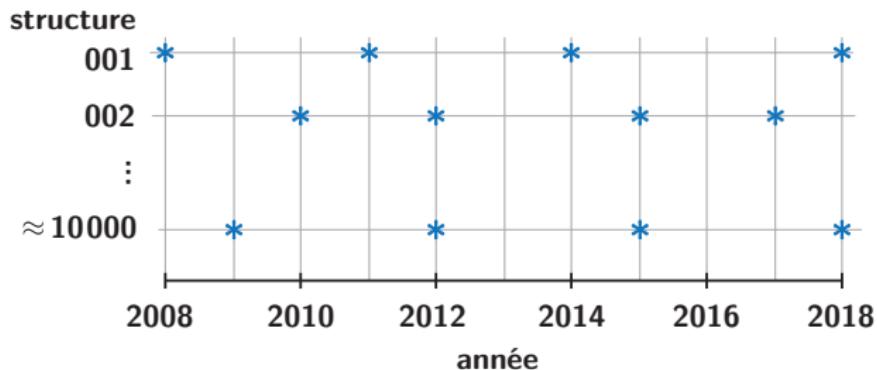
année

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Database of Visual Inspections



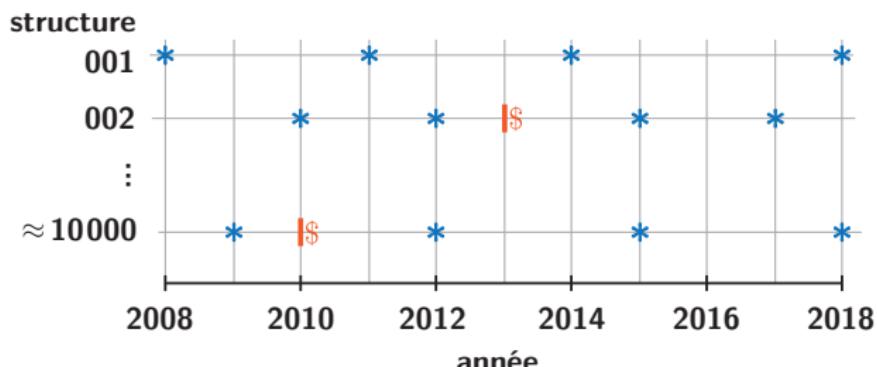
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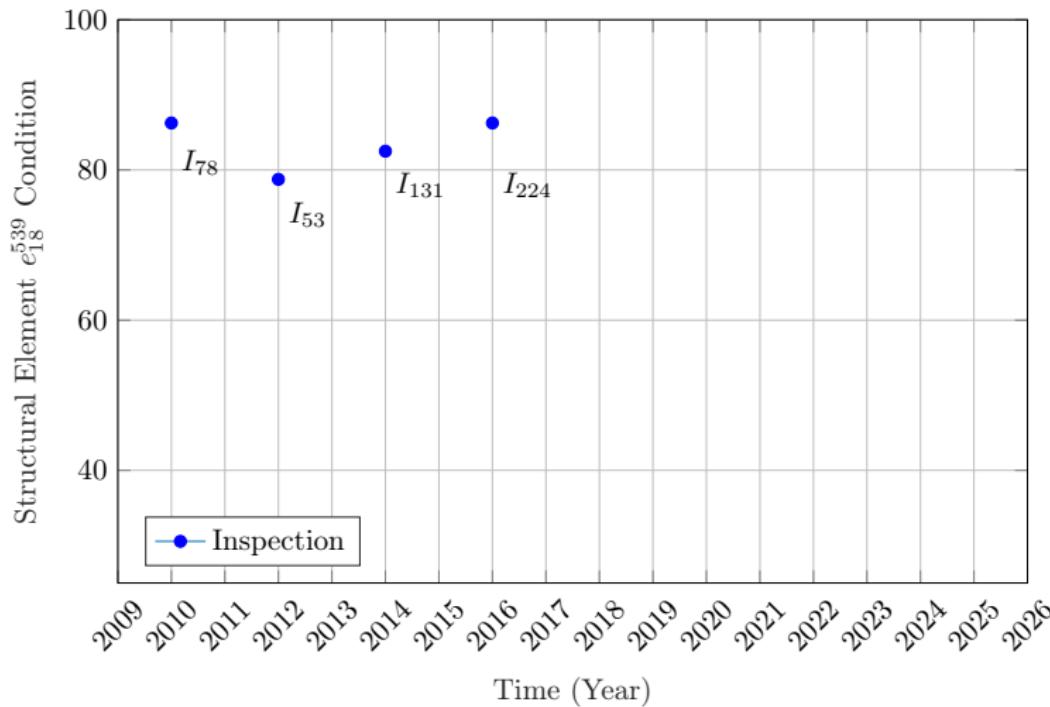


Year: 2017
Structure: #002



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Example: Series of Inspections on Structural Element



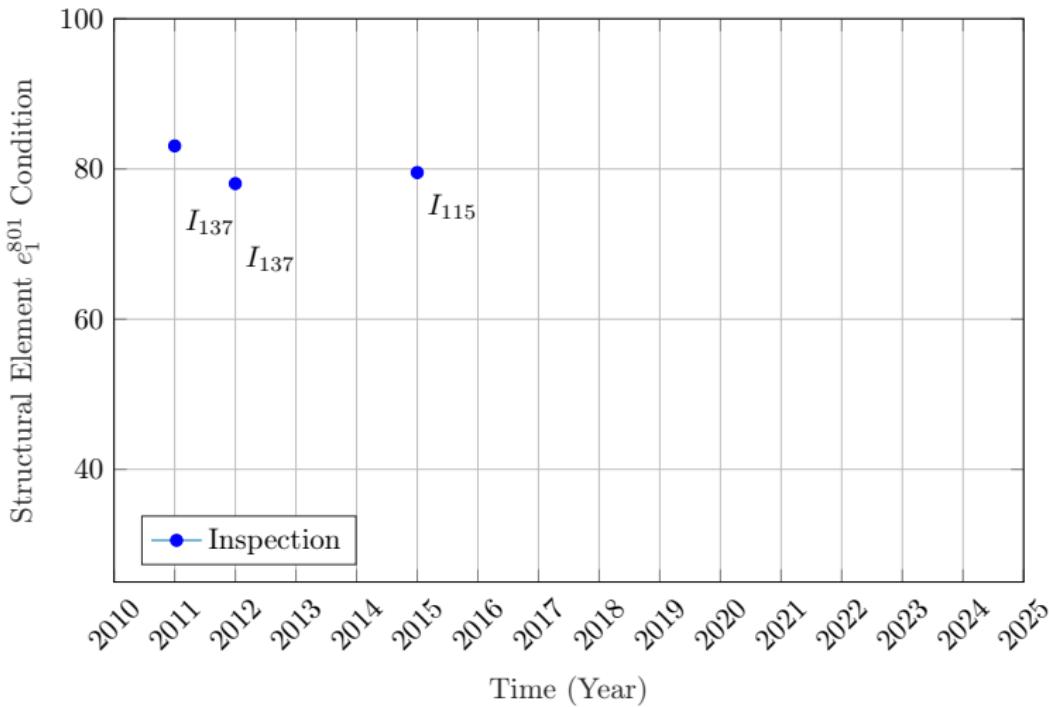
Objectives

- **Model the deterioration** behaviour based on the data from network of bridges.

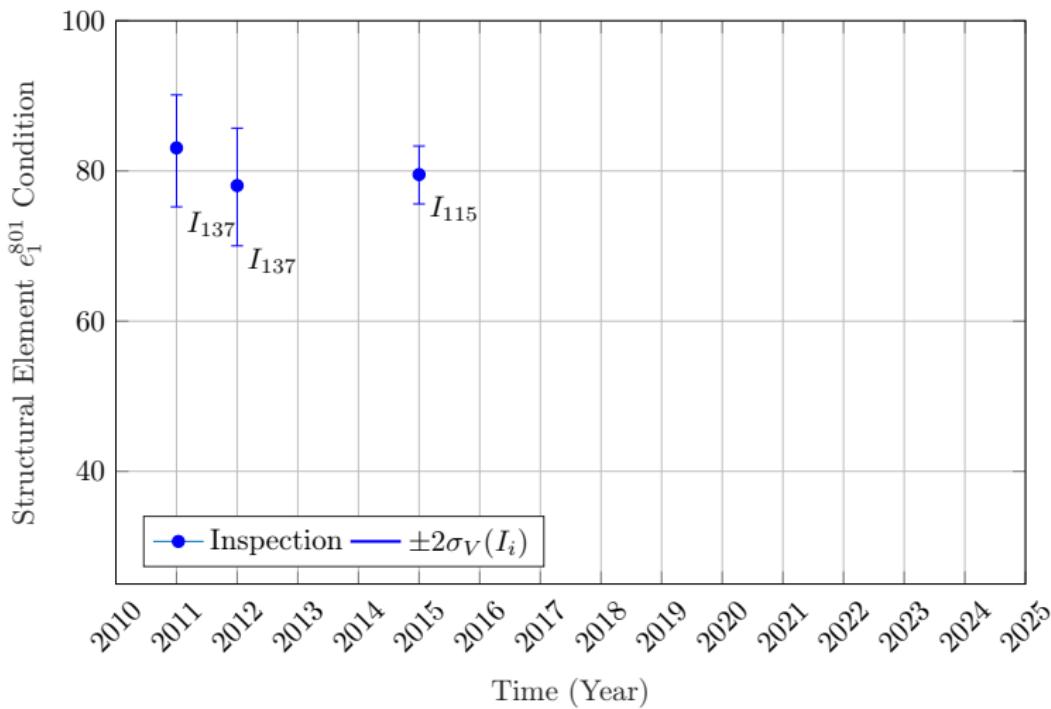
Objectives

- **Model the deterioration** behaviour based on the data from network of bridges.
- Quantify the **effect of interventions** on structural elements.

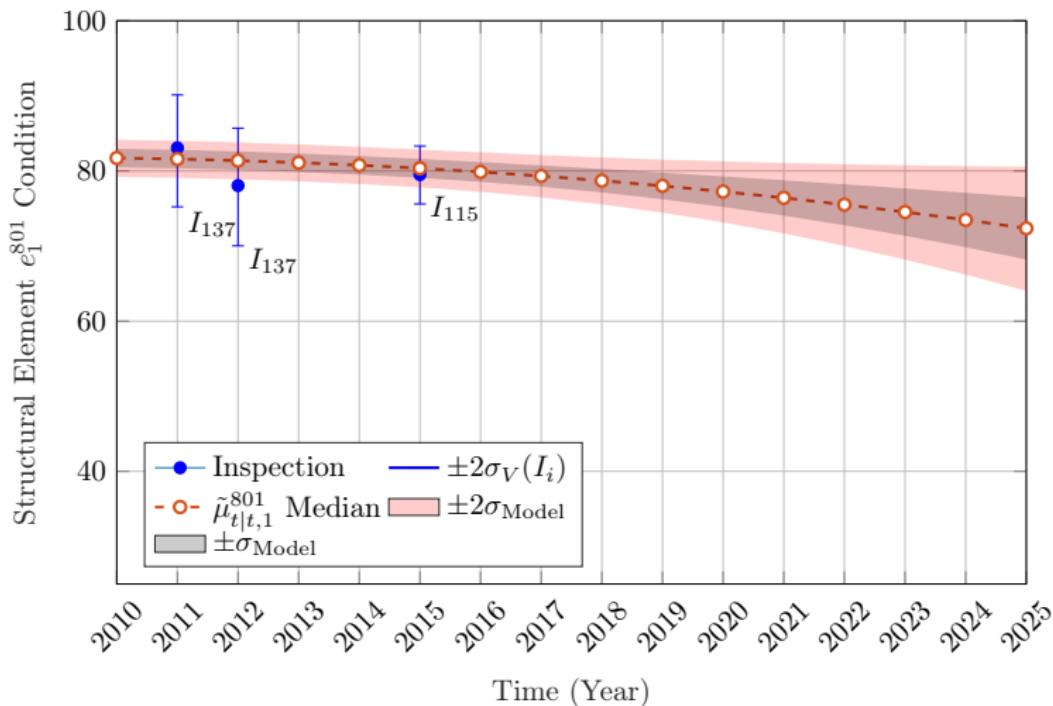
Deterioration Model Performance: Condition Estimate



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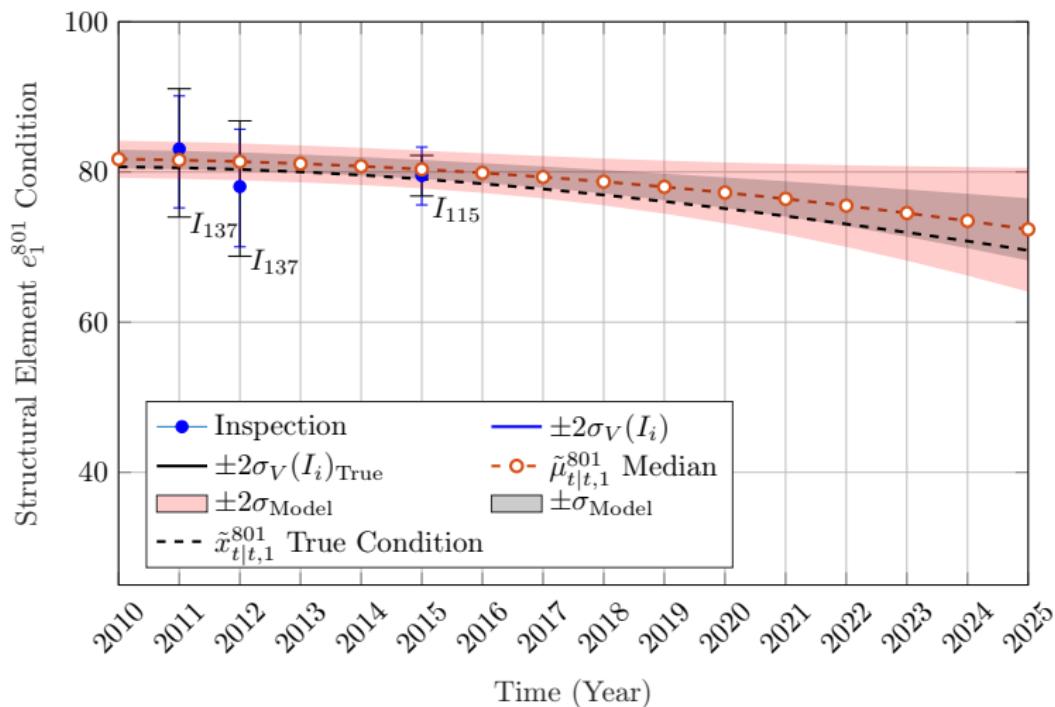


Deterioration Model Performance: Condition Estimate

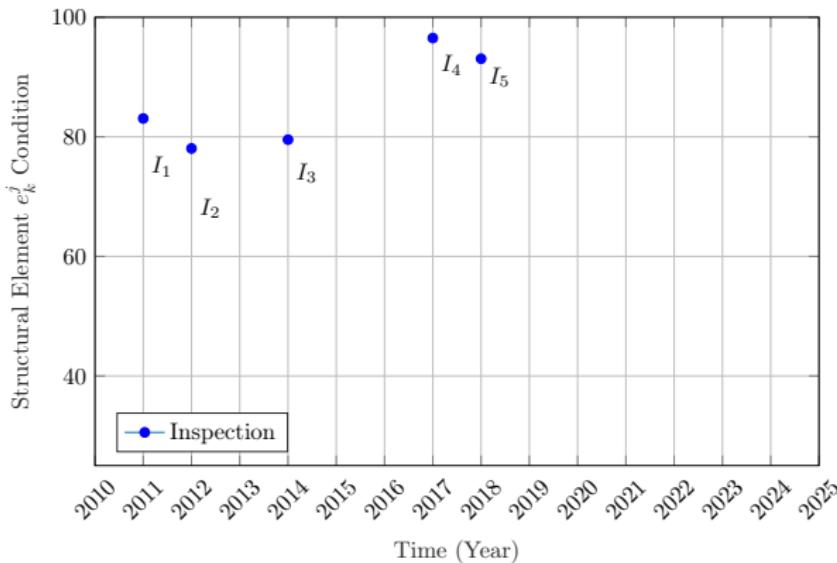


Project Progress

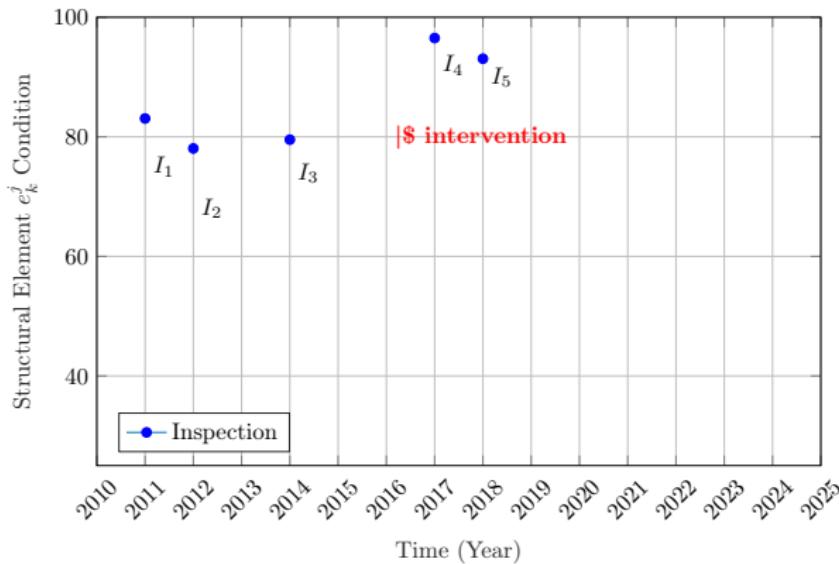
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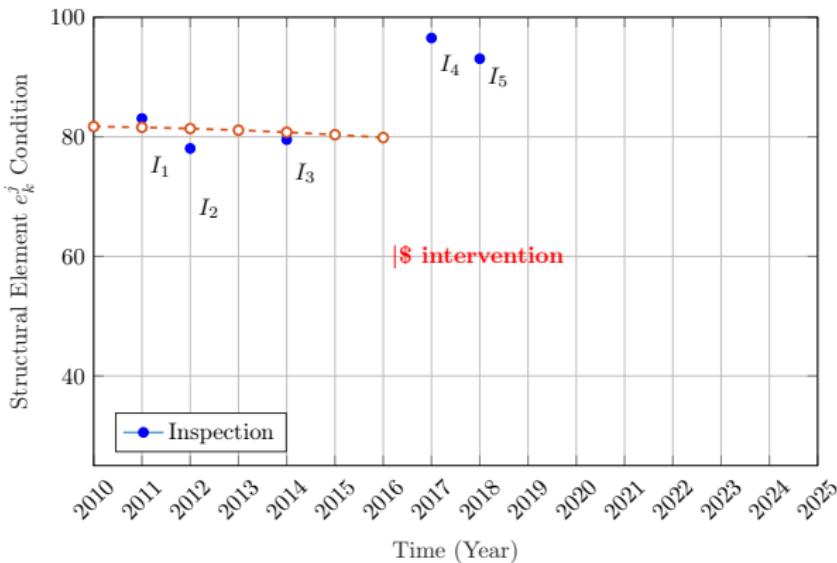
Model the Effect of Interventions



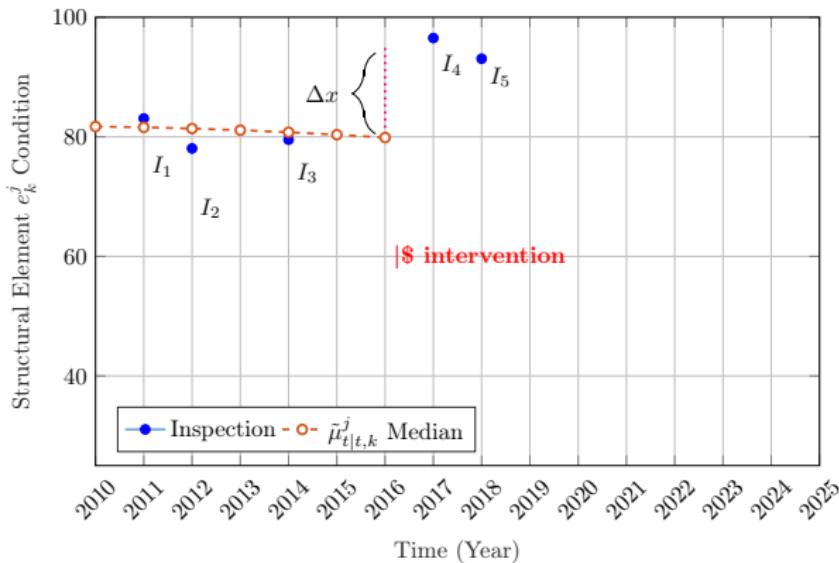
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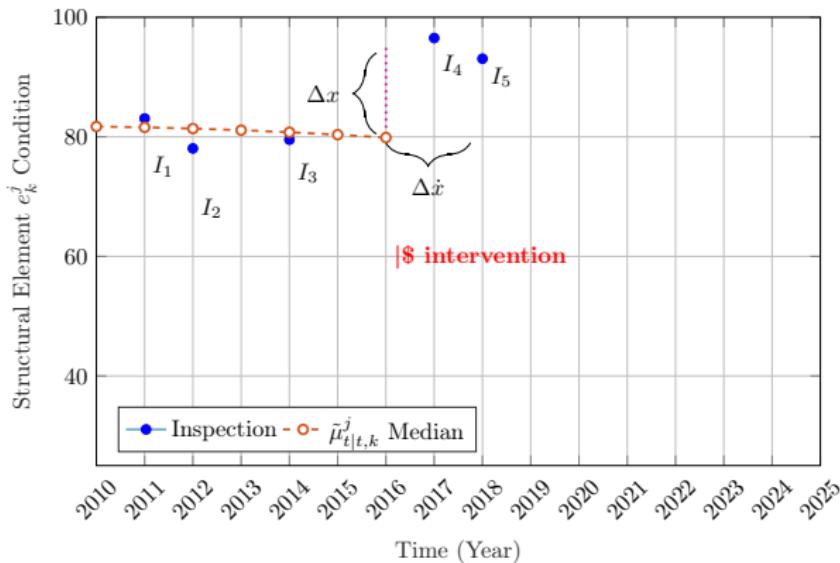
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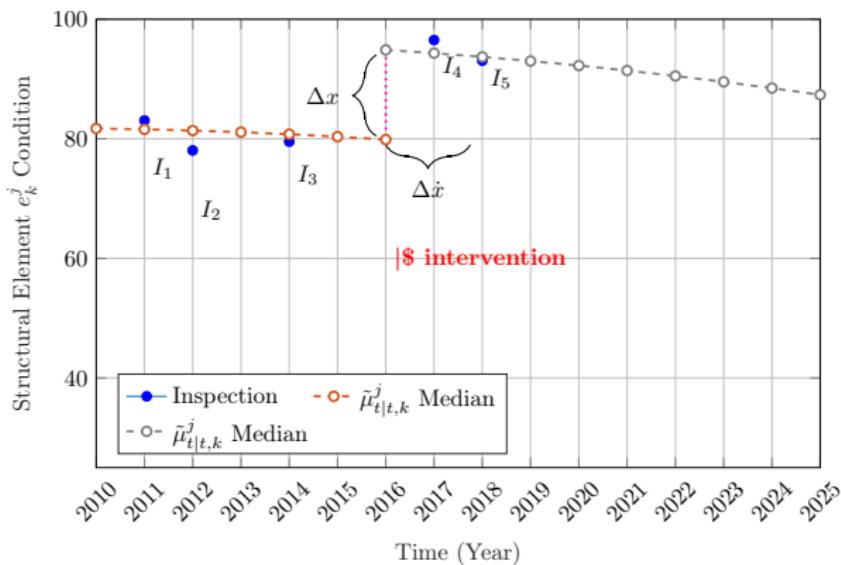
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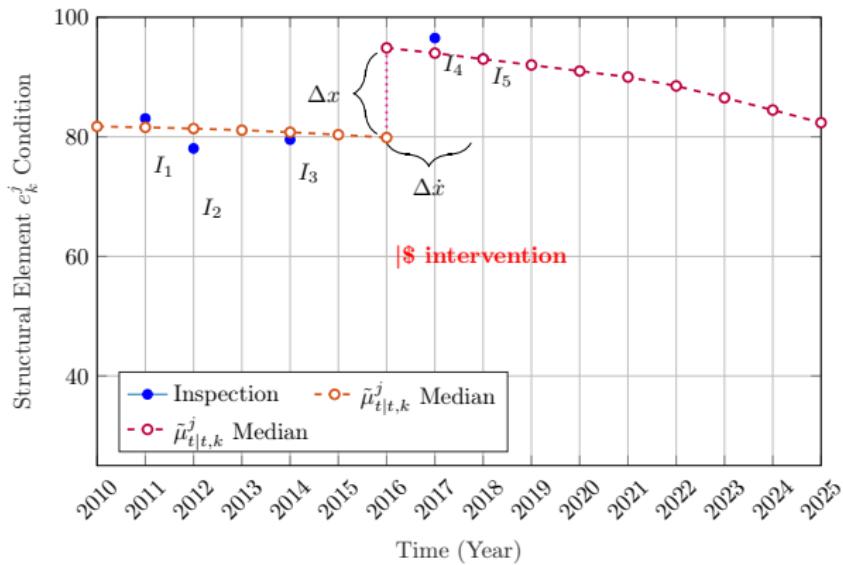


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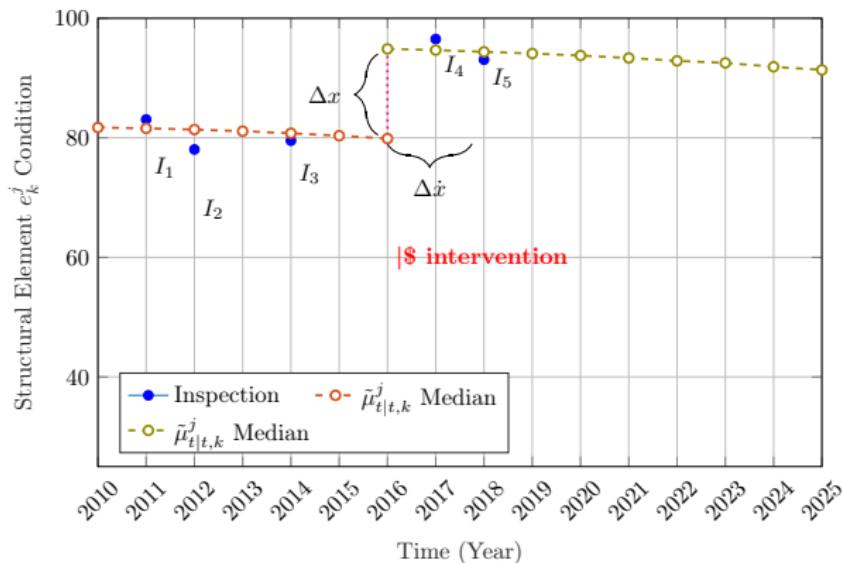
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Model the Effect of Interventions



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The main benefit:

Research Questions:

Following an intervention strategy h_1 :

Health:



Deterioration Time Delay:



The main benefit:

- Find an intervention strategy such that:
 - 1 . Cost effective (Minimize Cost).
 - 2 . Delays the replacement.

📍 Plan de la section

Intervention Information

- 2.1 Existing Intervention Database
 - 2.2 Challenges & Opportunities
-

Existing Intervention Database

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- Uncategorized intervention data (**Not mentioned in the database**).

Contexte & Objectifs
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Challenges & Oportunties

Intervention Information
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Quantifying Interventions
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Next Steps
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Challenges:

Challenges & Opportunities

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- It is not possible to categorize the type of intervention for Uncategorized interventions .

Contexte & Objectifs
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Challenges & Oportunites

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Next Steps
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Opportunities:

We know:

- The deterioration state of the structural element before intervention.
- The observations post-intervention.
- A single network of structures → same (similar) interventions triggers (thresholds).
- Interventions can be categorized according to their effect.

Contexte & Objectifs
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Challenges & Oportunites

Intervention Information
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Quantifying Interventions
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Next Steps
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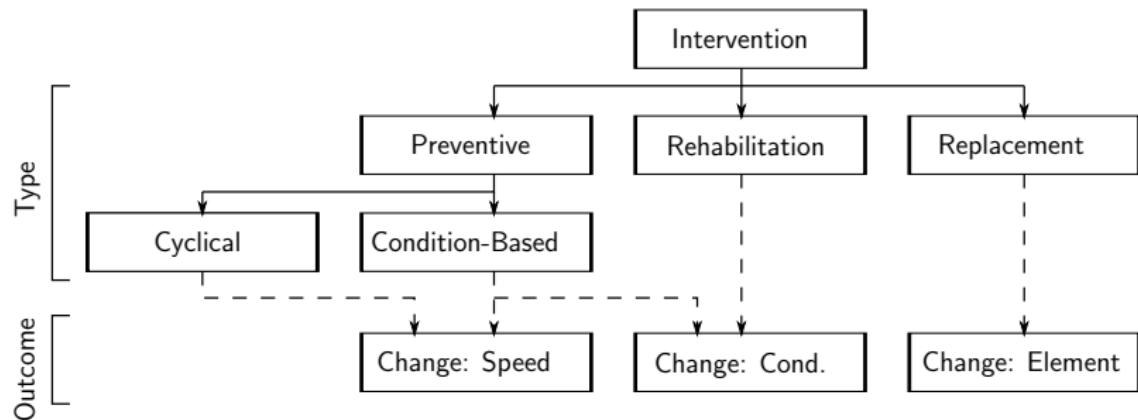
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2. The structural element condition: $C \geq 20\%$ or $D > 0\%$.

Intervention Hierarchy:



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Formulation

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The transition model:

$$\mathbf{x}_{t|t-1} = \mathbf{A}\mathbf{x}_{t-1|t-1} + \mathbf{w}_t : \mathbf{W} \sim \mathcal{N}(\mathbf{w}_t; \mathbf{0}, \mathbf{Q}),$$

Quantifying Interventions:

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With:

$$\mathbf{A}_{t<\tau} = \begin{bmatrix} \mathbf{A}^{ki} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}, \quad \mathbf{A}_{t \geq \tau} = \begin{bmatrix} \mathbf{A}^{ki} & \mathbf{I}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix},$$

$$\mathbf{A}^{ki} = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}.$$

Quantifying Interventions:

The covariance matrix \mathbf{Q}_t :

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$$\mathbf{Q}^{\text{ki}} = \begin{bmatrix} \frac{\Delta t^5}{20} & \frac{\Delta t^4}{8} & \frac{\Delta t^3}{6} \\ \frac{\Delta t^4}{8} & \frac{\Delta t^3}{3} & \frac{\Delta t^2}{2} \\ \frac{\Delta t^3}{6} & \frac{\Delta t^2}{2} & \Delta t \end{bmatrix}.$$

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Problem: very few observations

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