Thesis Proposal: Network-Scale Structural Health Monitoring Using Machine Learning

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Presentation Outline

Context

Objectives

Existing Methods

Proposed Framework

Analyses

Next Steps & Plan

Database from Network of Bridges

Year: 2017
Structure: #002

Database from Network of Bridges

Year: 2017
Structure: #002

$$\mathbf{x}_{002} = \begin{Bmatrix} \text{type} \\ \text{material} \\ \text{\# vehicle} \\ \text{Location} \\ \vdots \end{Bmatrix}$$

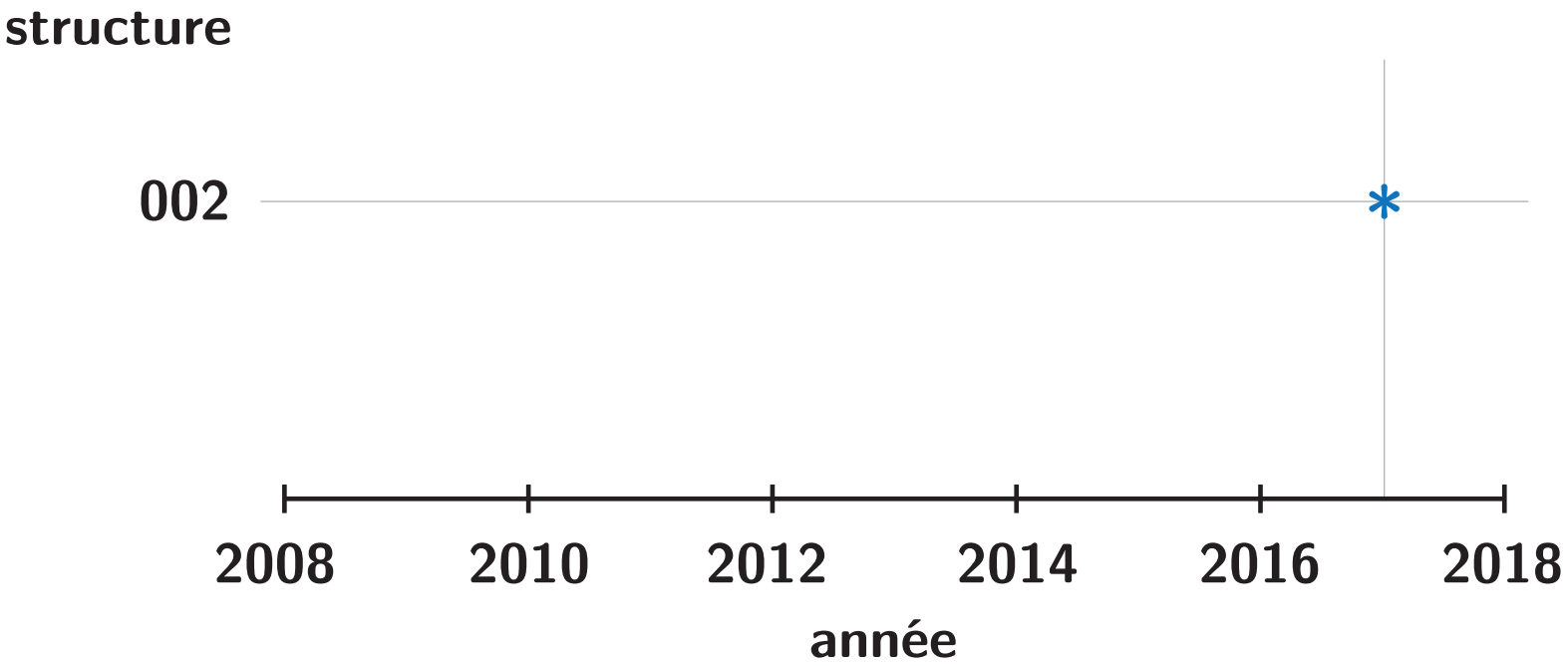
Database from Network of Bridges

Year: 2017
Structure: #002

$$\mathbf{x}_{002} = \begin{Bmatrix} \text{type} \\ \text{material} \\ \#\ \text{vehicle} \\ \text{Location} \\ \vdots \end{Bmatrix}$$

Element	$\tilde{y}_a$	$\tilde{y}_b$	$\tilde{y}_c$	$\tilde{y}_d$
1	80%	10%	10%	0%
2	.	.	.	.
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
≈ 20	.	.	.	.

Database of Visual Inspections



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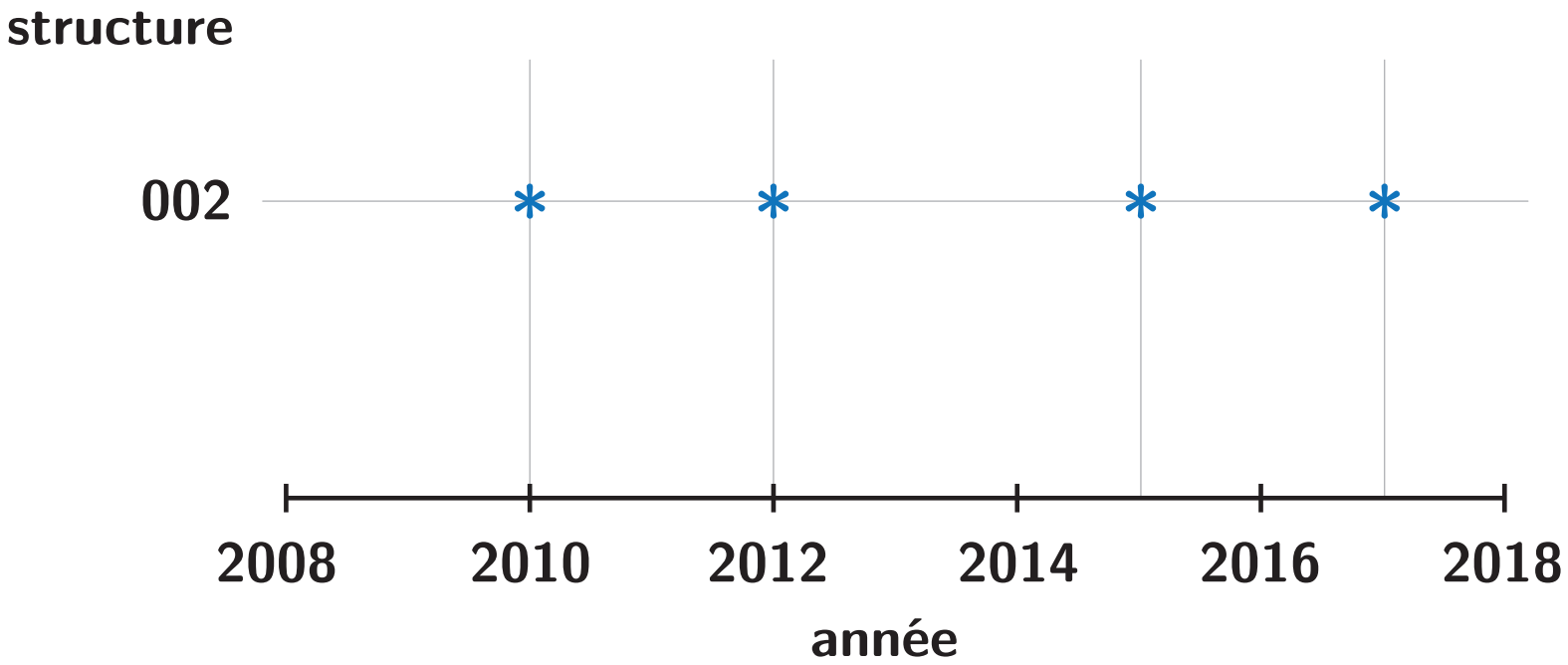

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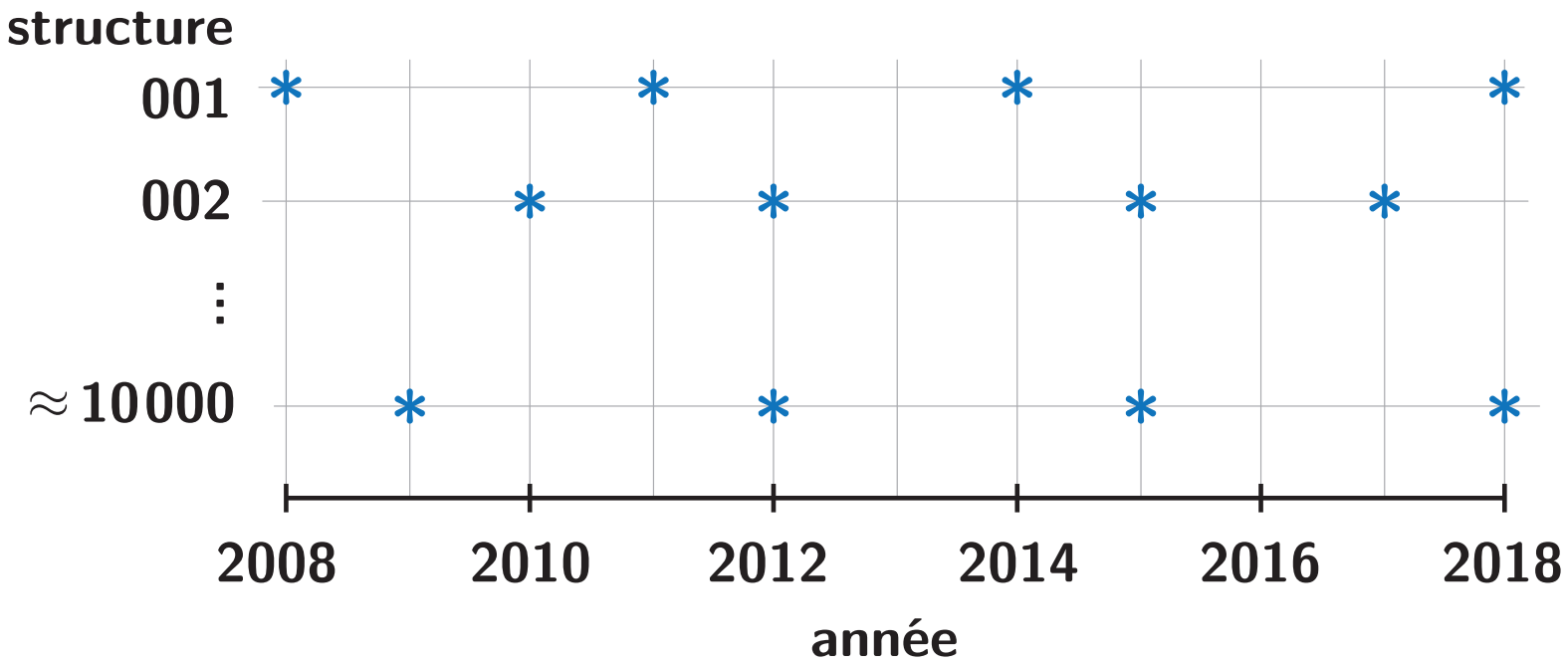

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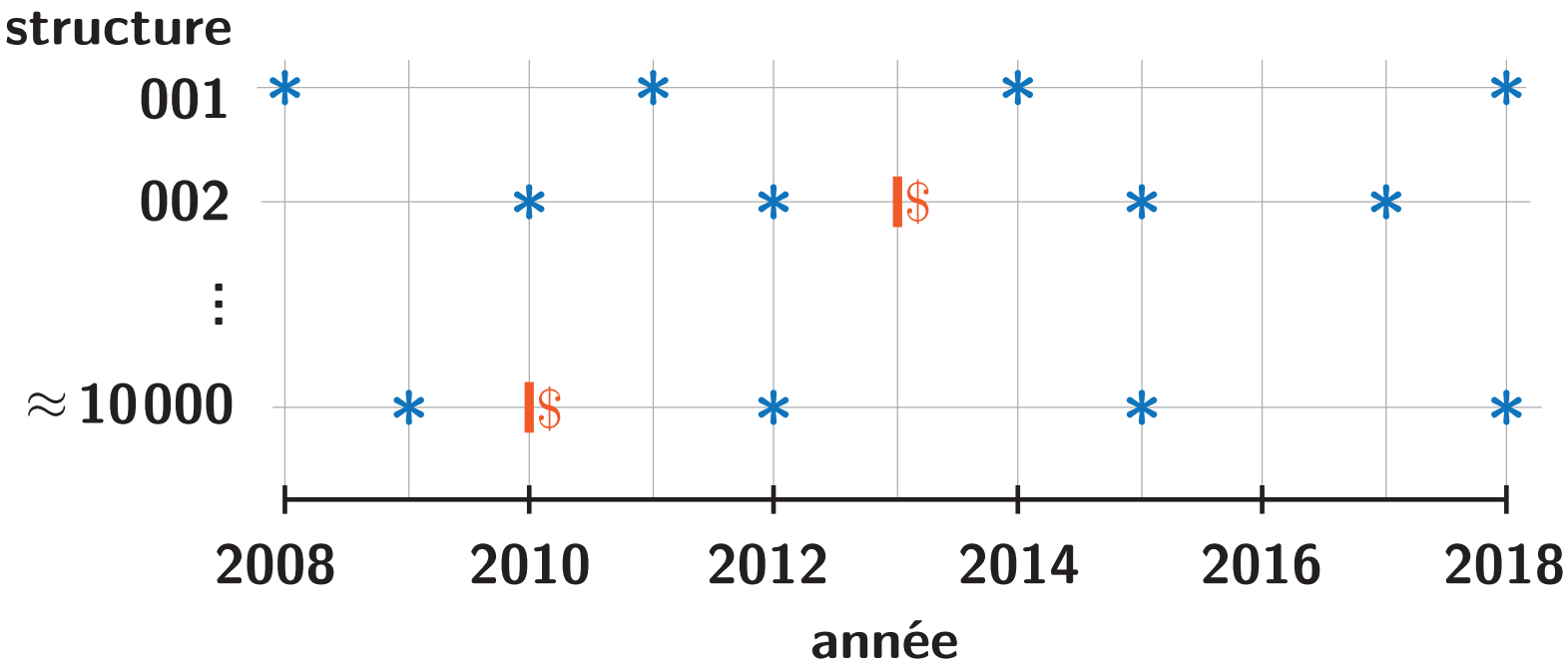


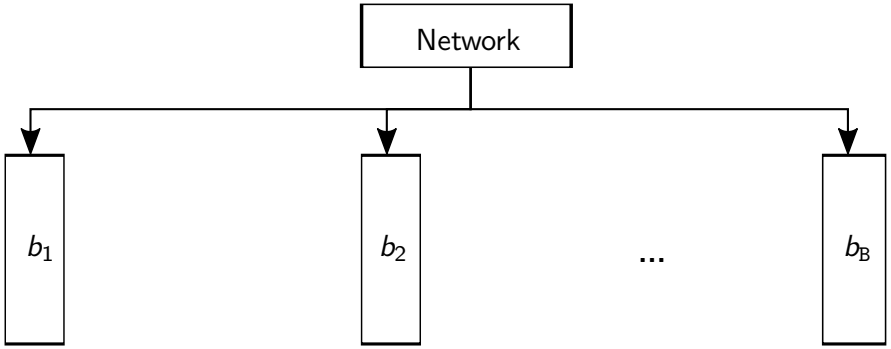
Hierarchy of Information

Network

Network Level

Country	Inspcetion Code
Province	Other...

Hierarchy of Information



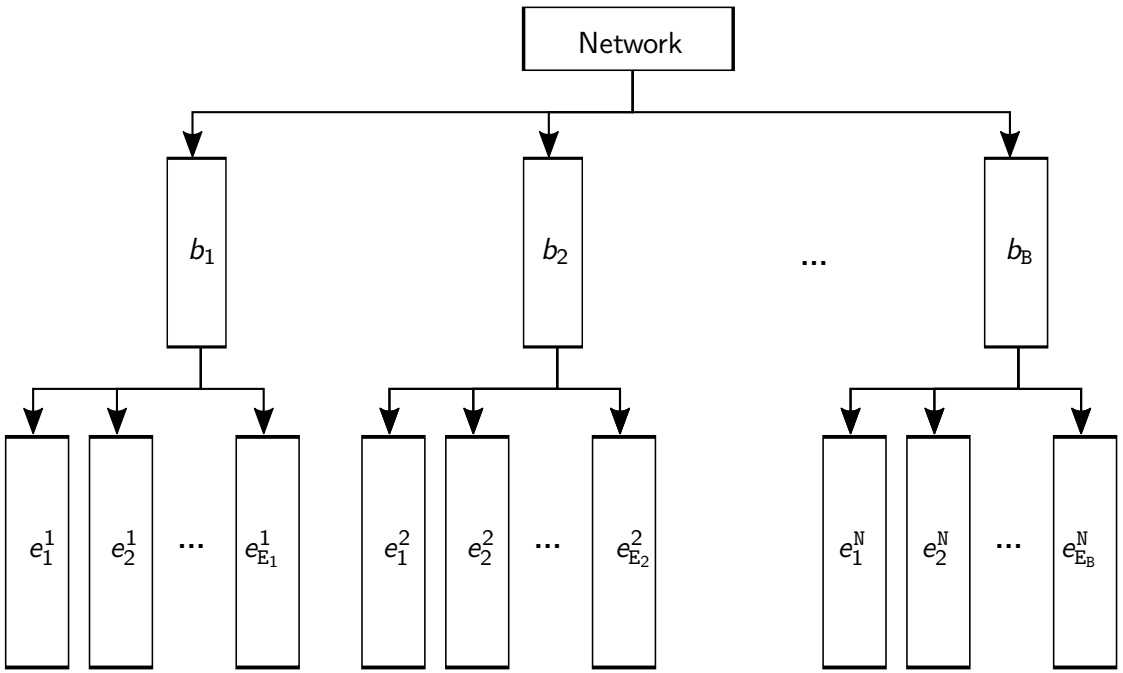
Network Level

Country	Inspcet. Code
Province	Other...

Bridge Level

Geolocation	Const. Year
Traffic Loads	Other...

Hierarchy of Information



Network Level

Country	Inspcet. Code
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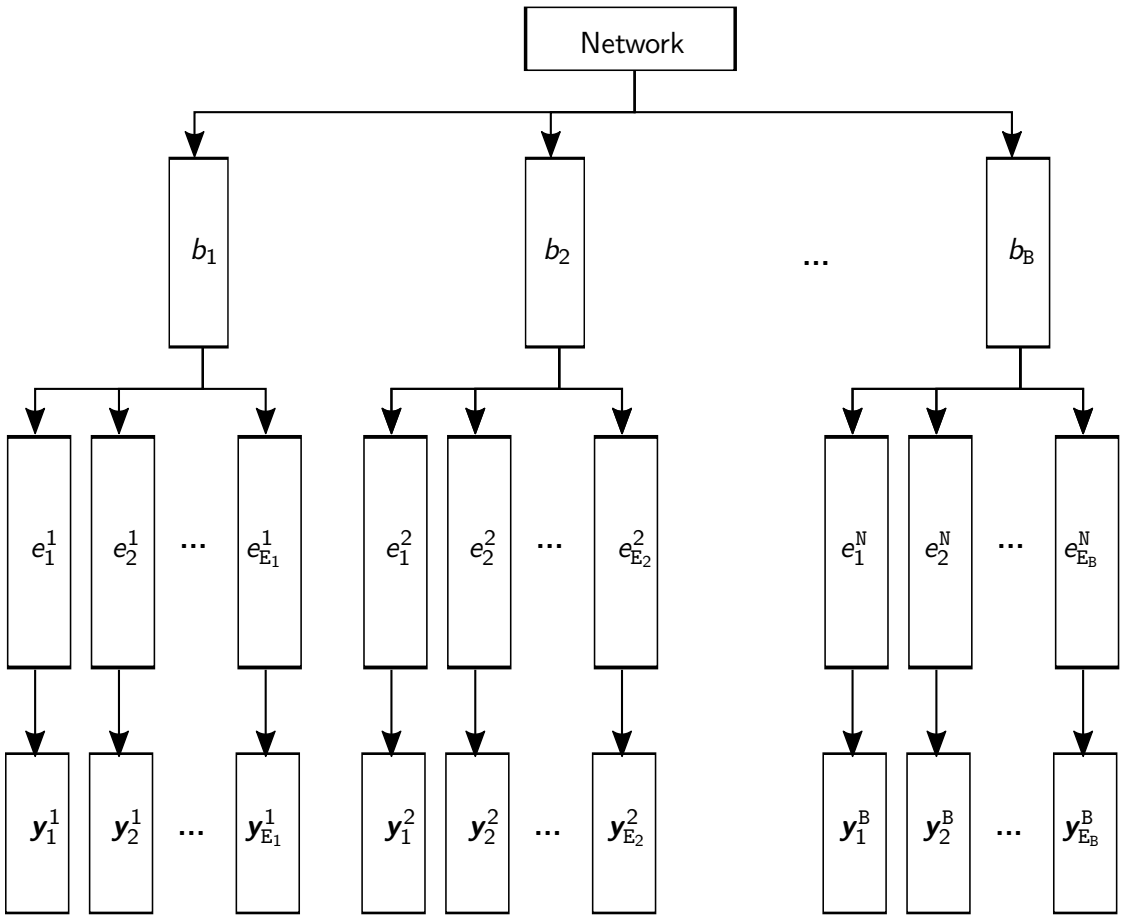
Element Level

Material	Location
Type	Other...

Hierarchy of Information



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Inspection Data

Condition	Inspector
Inspection Time	

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- Model the effect of interventions
- Verify & validate the performance of the model with real and synthetic data

Section Outline

Existing Methods

3.1 Discrete Markov Model

3.2 Regression-based Methods

Discrete Markov Model (DMM)

Describes time-evolution of a system in which, given the present, the future is independent of the past.

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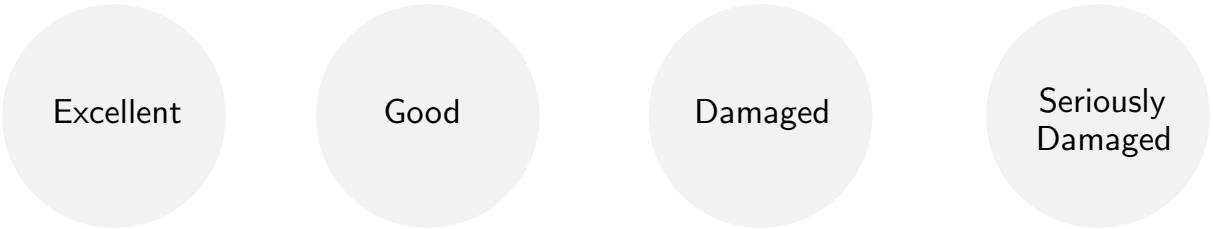
1. The system states $\mathcal{X} = \{x_1, x_2, \ldots, x_{\mathtt{X}}\}$
2. The transition matrix $\boldsymbol{Z} \in [0,1]^{\mathtt{X}\times\mathtt{X}}$

Example of Discrete Markov Model (DMM)



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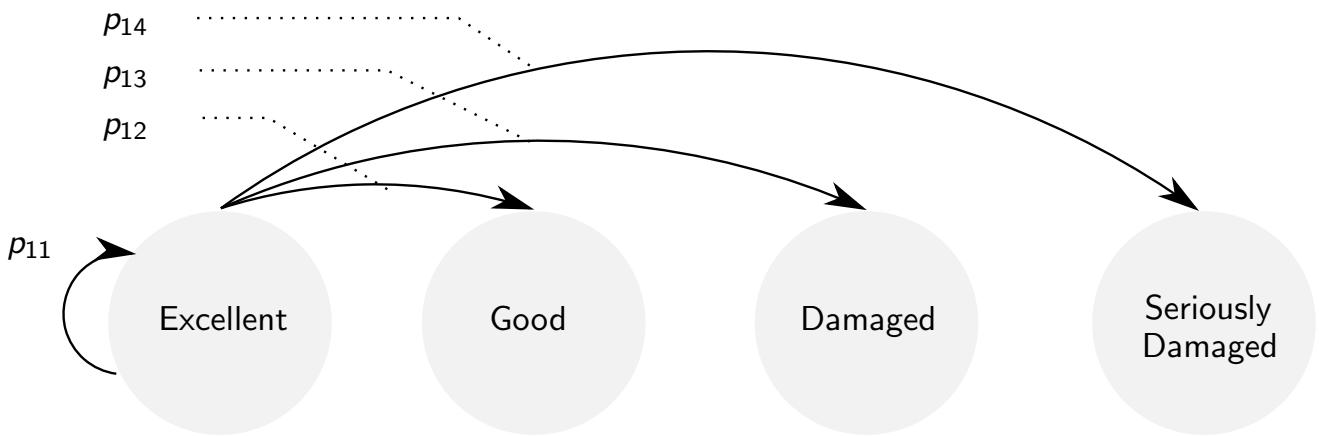

$$p\left(x_j^{t+1} | x_i^t\right) = \boldsymbol{Z} = \begin{array}{cc} \begin{array}{cccc} x_1^{t+1} & x_2^{t+1} & x_3^{t+1} & x_4^{t+1} \end{array} & \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ & & & \\ & & & \end{bmatrix} & x_1^t \end{array}$$

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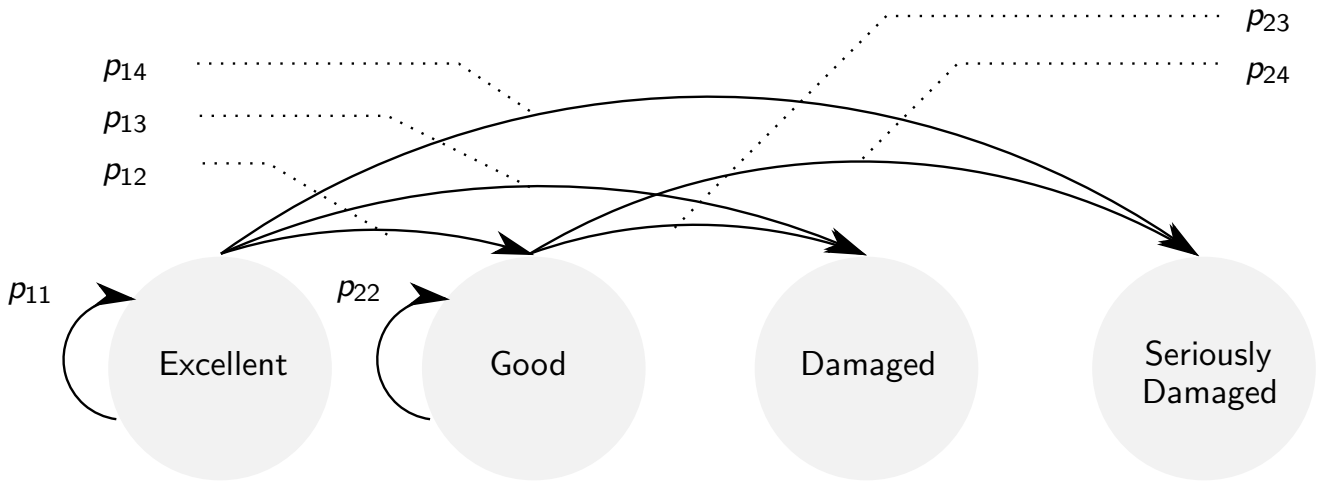

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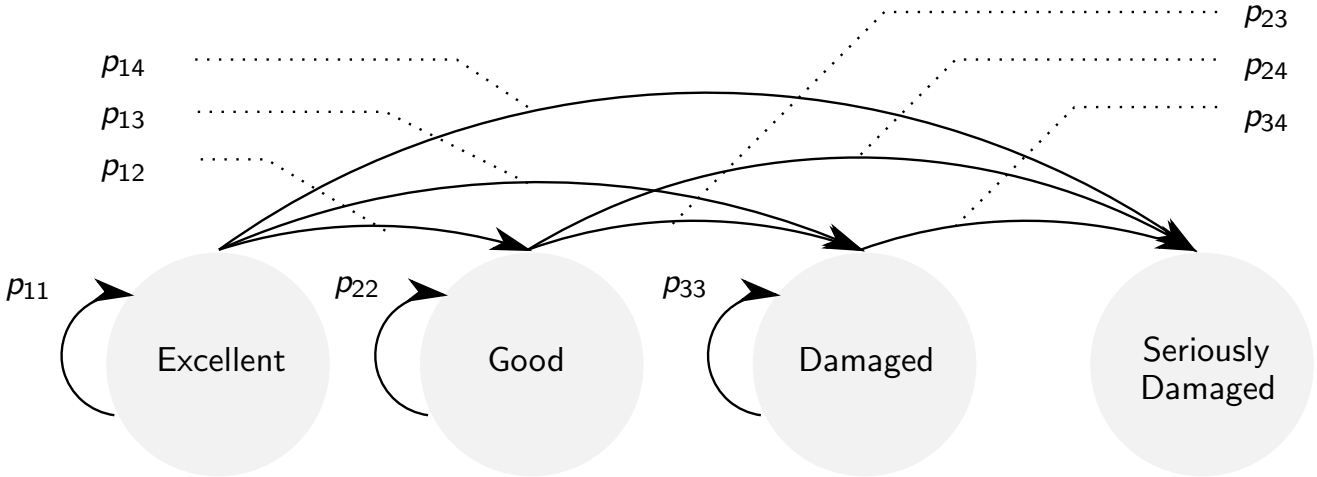

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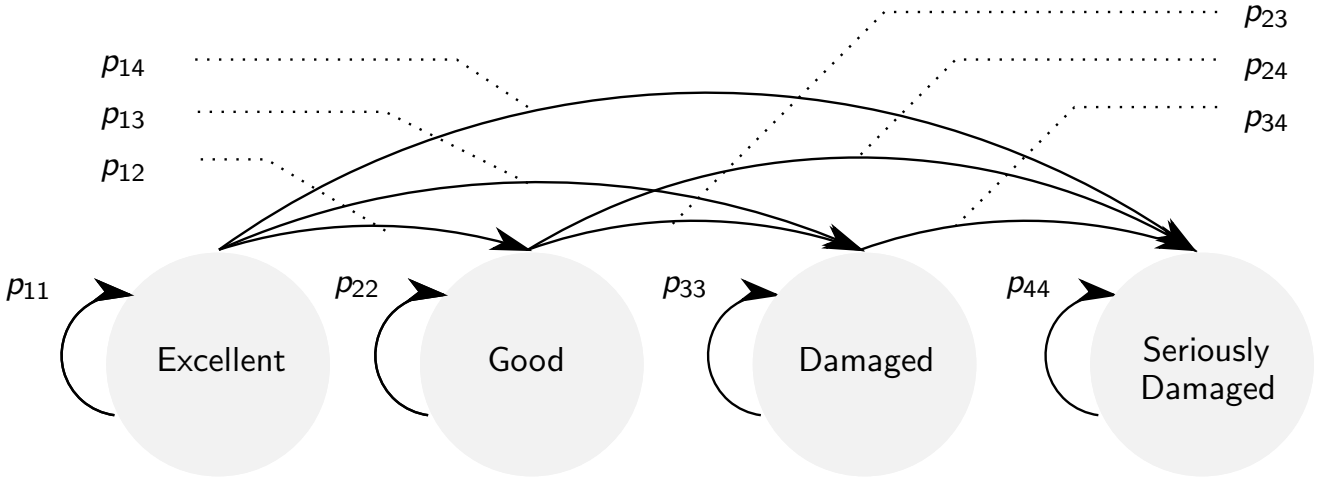

$$p\left(x_j^{t+1}|x_i^t\right) = \boldsymbol{Z} = \begin{array}{c} \begin{array}{cccc} x_1^{t+1} & x_2^{t+1} & x_3^{t+1} & x_4^{t+1} \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & p_{44} \end{bmatrix} \end{array} \begin{array}{c} \\ x_1^t \\ x_2^t \\ x_3^t \\ x_4^t \end{array}$$

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2. Not factoring structural attributes → too many model parameters
3. Limited to estimating the deterioration condition, no deterioration speed or acceleration.

Regression Methods

- Described mathematically by: $\boldsymbol{y} = g(\boldsymbol{x}) + \boldsymbol{\epsilon}$

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- Relies on the inspections $\boldsymbol{y}$ & structural attributes $\boldsymbol{x}$

Key Limitations: Difficult to capture the temporal dependance from short *time-series* (2 to 5 observations)

Section Outline

Proposed Framework

4.1 Basis of the Proposed Deterioration Model
4.2 Deterioration Condition : $\{\tilde{y}_a, \tilde{y}_b, \tilde{y}_c, \tilde{y}_d\} \rightarrow \tilde{y}$
4.3 Observations Uncertainty
4.4 Deterioration Modeling Framework
4.5 Model Parameter Estimation

Deterioration Behaviour Described by Kinematics

$$\overbrace{x_t = x_{t-1} + \dot{x}_{t-1}\Delta t + \frac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w \quad \text{(Condition)}}^{\text{Kinematic Equations}}$$

Deterioration Behaviour Described by Kinematics

$$\overbrace{\begin{aligned} x_t &= x_{t-1} + \dot{x}_{t-1}\Delta t + \tfrac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w \quad \text{(Condition)} \\ \dot{x}_t &= \dot{x}_{t-1} + \ddot{x}_{t-1}\Delta t + \dot{w} \quad \text{(Speed)} \end{aligned}}^{\text{Kinematic Equations}}$$

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$$\overbrace{\begin{aligned} x_t &= x_{t-1} + \dot{x}_{t-1}\Delta t + \tfrac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w && \text{(Condition)} \\ \dot{x}_t &= \dot{x}_{t-1} + \ddot{x}_{t-1}\Delta t + \dot{w} && \text{(Speed)} \\ \ddot{x}_t &= \ddot{x}_{t-1} + \ddot{w} && \text{(Acceleration)} \end{aligned}}^{\text{Kinematic Equations}}$$

Deterioration Behaviour Described by Kinematics

$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}}_{\boldsymbol{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \\ \ddot{x}_{t-1} \end{bmatrix}}_{\boldsymbol{x}_{t-1}} + \underbrace{\begin{bmatrix} w_t \\ \dot{w}_t \\ \ddot{w}_t \end{bmatrix}}_{\boldsymbol{w}_t}$$

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Method: State-Space Model

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$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}}_{\boldsymbol{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \\ \ddot{x}_{t-1} \end{bmatrix}}_{\boldsymbol{x}_{t-1}} + \underbrace{\begin{bmatrix} w_t \\ \dot{w}_t \\ \ddot{w}_t \end{bmatrix}}_{\boldsymbol{w}_t}$$

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$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}}_{\boldsymbol{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \\ \ddot{x}_{t-1} \end{bmatrix}}_{\boldsymbol{x}_{t-1}} + \underbrace{\begin{bmatrix} w_t \\ \dot{w}_t \\ \ddot{w}_t \end{bmatrix}}_{\boldsymbol{w}_t}$$

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$$\overbrace{\boldsymbol{y}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t}^{\textbf{observation model}}$$

Deterioration Behaviour Described by Kinematics

$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}}_{\boldsymbol{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \\ \ddot{x}_{t-1} \end{bmatrix}}_{\boldsymbol{x}_{t-1}} + \underbrace{\begin{bmatrix} w_t \\ \dot{w}_t \\ \ddot{w}_t \end{bmatrix}}_{\boldsymbol{w}_t}$$

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Deterioration Condition : $\{\tilde{y}_a, \tilde{y}_b, \tilde{y}_c, \tilde{y}_d\} \rightarrow \tilde{y}$

Year: 2018
Structure: #002

Element	$\tilde{y}_a$	$\tilde{y}_b$	$\tilde{y}_c$	$\tilde{y}_d$	$\tilde{y}$
1	80%	10%	10%	0%	?
2	.	.	.	.	.
⋮	⋮	⋮	⋮	⋮	
≈ 20	.	.	.	.	.

Deterioration Condition : $\{\tilde{y}_a, \tilde{y}_b, \tilde{y}_c, \tilde{y}_d\} \rightarrow \tilde{y}$

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Element	$\tilde{y}_a$	$\tilde{y}_b$	$\tilde{y}_c$	$\tilde{y}_d$	$\tilde{y}$
1	80%	10%	10%	0%	?
2	.	.	.	.	.
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
≈ 20	.	.	.	.	.

$$\tilde{y} = \overbrace{100}^{\omega_1} \times \overbrace{80\%}^{\tilde{y}_a}$$

Deterioration Condition : $\{\tilde{y}_a, \tilde{y}_b, \tilde{y}_c, \tilde{y}_d\} \rightarrow \tilde{y}$

Year: 2018
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Element	$\tilde{y}_a$	$\tilde{y}_b$	$\tilde{y}_c$	$\tilde{y}_d$	$\tilde{y}$
1	80%	10%	10%	0%	?
2	.	.	.	.	.
⋮	⋮	⋮	⋮	⋮	
≈ 20	.	.	.	.	.

$$\tilde{y} = \overbrace{100}^{\omega_1} \times \overbrace{80\%}^{\tilde{y}_a} + \overbrace{75}^{\omega_2} \times \overbrace{10\%}^{\tilde{y}_b}$$

Deterioration Condition : $\{\tilde{y}_a, \tilde{y}_b, \tilde{y}_c, \tilde{y}_d\} \rightarrow \tilde{y}$

Year: 2018
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Element	$\tilde{y}_a$	$\tilde{y}_b$	$\tilde{y}_c$	$\tilde{y}_d$	$\tilde{y}$
1	80%	10%	10%	0%	?
2	.	.	.	.	.
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
≈ 20	.	.	.	.	.

$$\tilde{y} = \overbrace{100}^{\omega_1} \times \overbrace{80\%}^{\tilde{y}_a} + \overbrace{75}^{\omega_2} \times \overbrace{10\%}^{\tilde{y}_b} + \overbrace{50}^{\omega_3} \times \overbrace{10\%}^{\tilde{y}_c}$$

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1	80%	10%	10%	0%	?
2	.	.	.	.	.
⋮	⋮	⋮	⋮	⋮	
≈ 20	.	.	.	.	.

$$\tilde{y} = \overbrace{100}^{\omega_1} \times \overbrace{80\%}^{\tilde{y}_a} + \overbrace{75}^{\omega_2} \times \overbrace{10\%}^{\tilde{y}_b} + \overbrace{50}^{\omega_3} \times \overbrace{10\%}^{\tilde{y}_c} + \overbrace{25}^{\omega_4} \times \overbrace{0\%}^{\tilde{y}_d}$$

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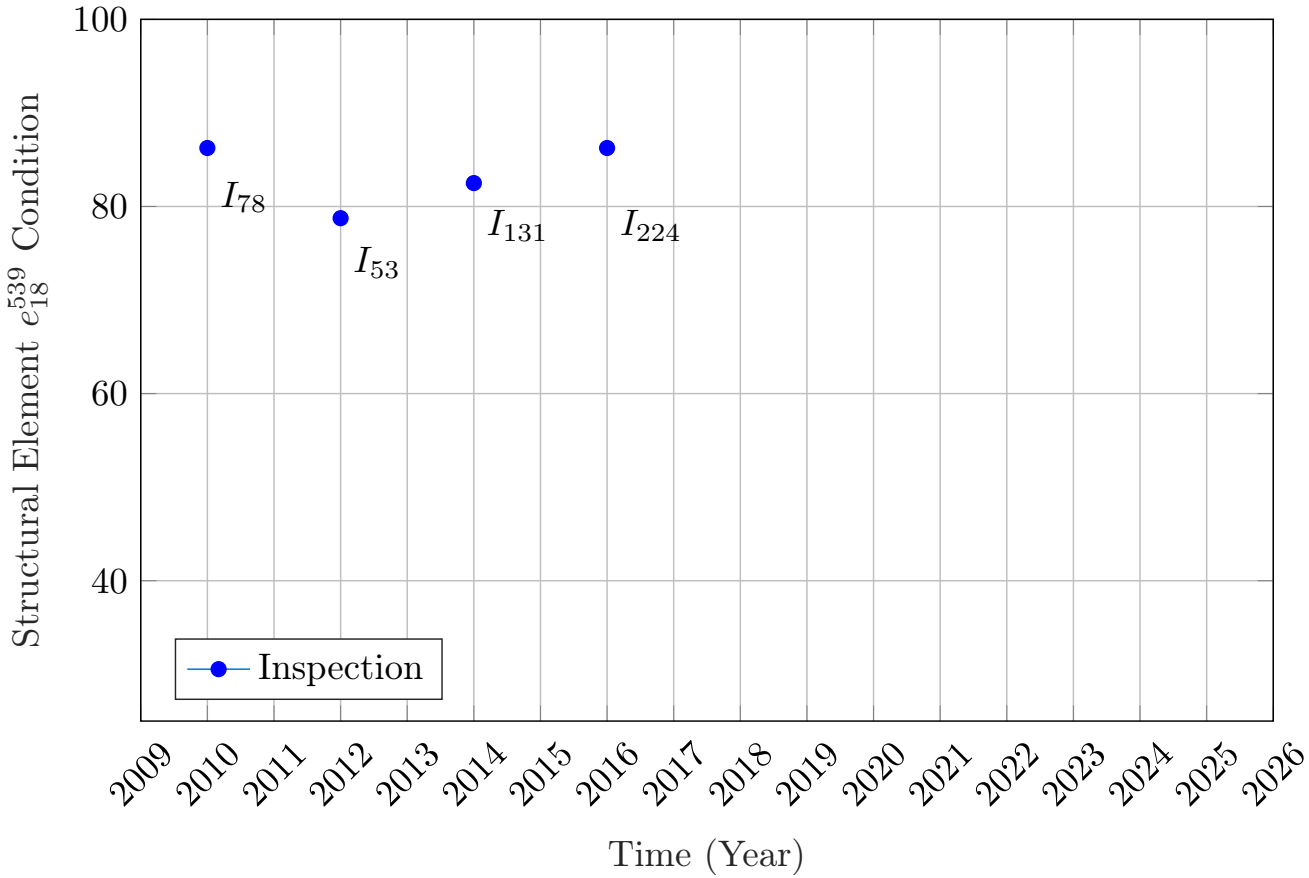
Element	$\tilde{y}_a$	$\tilde{y}_b$	$\tilde{y}_c$	$\tilde{y}_d$	$\tilde{y}$
1	80%	10%	10%	0%	**92.5**
2	.	.	.	.	.
⋮	⋮	⋮	⋮	⋮	
≈ 20	.	.	.	.	.

$$\begin{aligned}\tilde{y} &= \overbrace{100}^{\omega_1} \times \overbrace{80\%}^{\tilde{y}_a} + \overbrace{75}^{\omega_2} \times \overbrace{10\%}^{\tilde{y}_b} + \overbrace{50}^{\omega_3} \times \overbrace{10\%}^{\tilde{y}_c} + \overbrace{25}^{\omega_4} \times \overbrace{0\%}^{\tilde{y}_d} \\ &= \mathbf{92.5} \in [25, 100]\end{aligned}$$

Uncertainty of Observations (Inspectors)



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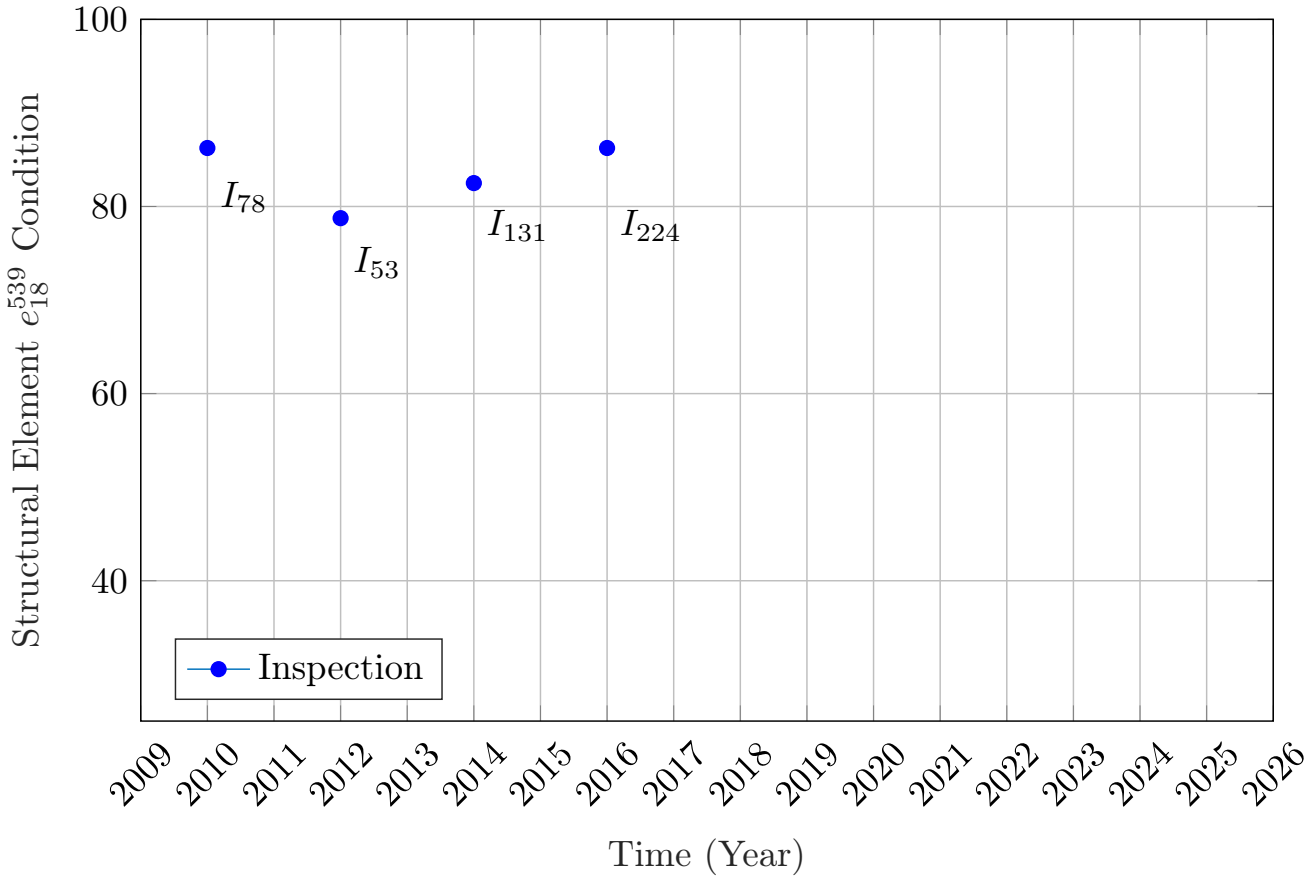

The Uncertainty of Observations $\tilde{y}$ Depends on the Inspector Responsible for the Evaluation

Uncertainty of Observations (Inspectors)

Proposed Solution:

Uncertainty of Observations (Inspectors)

Proposed Solution: Modify the observation model

$$\overbrace{\boldsymbol{y}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t}^{\text{observation model}}, \underbrace{\boldsymbol{v}_t : \boldsymbol{V} \sim \mathcal{N}(\boldsymbol{v}; \boldsymbol{0}, \boldsymbol{R}_t)}_{\text{observation error}}$$

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$$\overbrace{\boldsymbol{y}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t}^{\text{observation model}}, \underbrace{\boldsymbol{v}_t : \boldsymbol{V}(l_i) \sim \mathcal{N}(\boldsymbol{v}; \boldsymbol{0}, \boldsymbol{R}_t(l_i))}_{\text{observation error}}$$

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$$\underbrace{l_i \in [l_1, l_2, \ldots, l_{\mathrm{I}}] = \mathcal{I}}_{\text{inspectors}}$$

Uncertainty of Observations (Condition)

Source: Google images

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$\tilde{y} = 25$ $\tilde{y} = 100$

Uncertainty of Observations (Condition)

Source: Google images

$\tilde{y} = 25$ $\tilde{y} = 100$

The Uncertainty of Observations $\tilde{y}$ Depends on the Deterioration State

Proposed Solution:

Proposed Solution: Space Transformation

Mapping a point from an original space to a point in a transformed space that has different properties $g : \tilde{\mathbb{X}} \rightarrow \mathbb{X}$

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Space Transformation Function Characteristics:

1. Uncertainty dependent on the state $\rightarrow$ Non-linear Transformation

Proposed Solution: Space Transformation

Mapping a point from an original space to a point in a transformed space that has different properties $g : \tilde{\mathbb{X}} \rightarrow \mathbb{X}$

Space Transformation Function Characteristics:

1. Uncertainty dependent on the state $\rightarrow$ Non-linear Transformation
2. Bound the deterioration condition estimate $\tilde{x} \in [25, 100] \rightarrow$ Step Function

Proposed Transformation Function

$$g(x)=\begin{cases} \frac{1}{\Gamma(\alpha)}\int_0^{x^{\frac{1}{\alpha}}} t^{\alpha-1}e^{-t}dt, & \frac{u+l}{2} < x \leq u, \\ x, & x = \frac{u+l}{2}, \\ -\frac{1}{\Gamma(\alpha)}\int_0^{x^{\frac{1}{\alpha}}} t^{\alpha-1}e^{-t}dt, & l \leq x < \frac{u+l}{2}, \end{cases}$$

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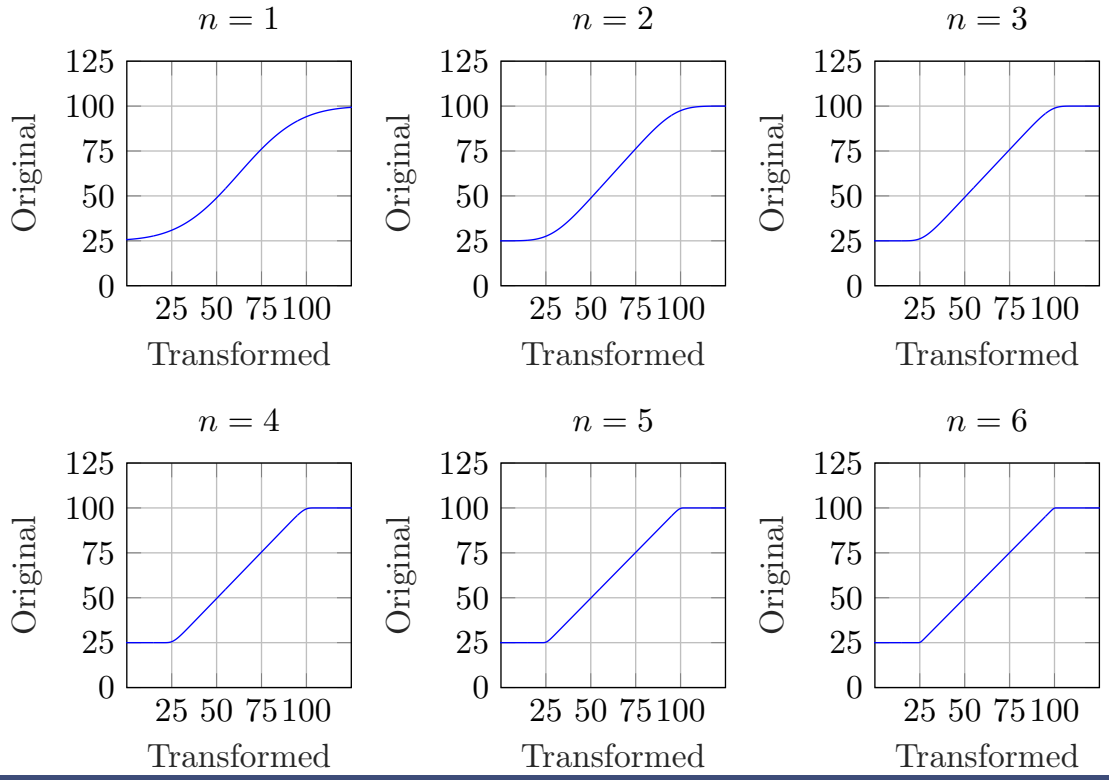
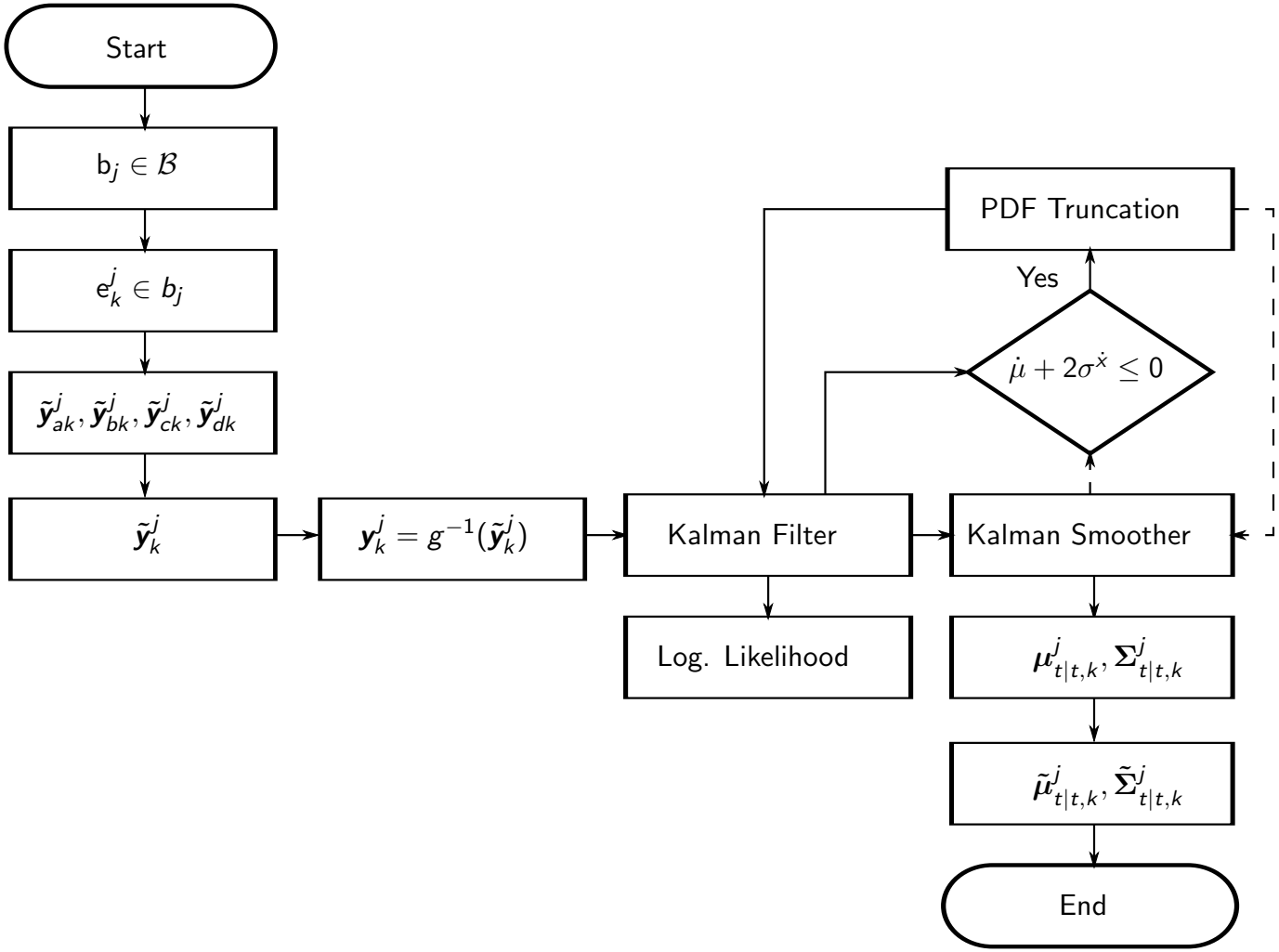


Deterioration Model Illustration



Model Parameter Estimation

$$\mathcal{P} = \left\{ \underbrace{\sigma_v(l_1), \sigma_v(l_2), \cdots, \sigma_v(l_{\mathrm{I}})}_{\text{Inspector std.}}, \overbrace{\sigma_w}^{\text{Process error std.}}, \underbrace{n}_{\text{Transform. Param.}}, \overbrace{\dot{\mu}_0, \ddot{\mu}_0, \sigma_0^{x}, \sigma_0^{\dot{x}}, \sigma_0^{\ddot{x}}}^{\text{Initial state.}} \right\}$$

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$$\mathcal{L}(\mathcal{P}) = \sum_{j=1}^{\mathtt{B}} \sum_{k=1}^{\mathtt{E}_j} \sum_{t=1}^{\mathtt{T}_k} \left(\ln p(y_{t,k}^j | y_{1:t-1,k}^j, \mathcal{P}) \times \left| \frac{\partial}{\partial x} g(x = y_{t,k}^j, \mathcal{P}) \right|^{-1} \right)$$

Section Outline

Analyses

5.1 Verification: Synthetic Dataset
5.2 Examples of Synthetic Inspection Data
5.3 Deterioration Model Performance
5.4 Aspects to improve
5.5 Examples of Real Inspection Data

Characteristics of Synthetic Dataset

1. Same number of structural elements

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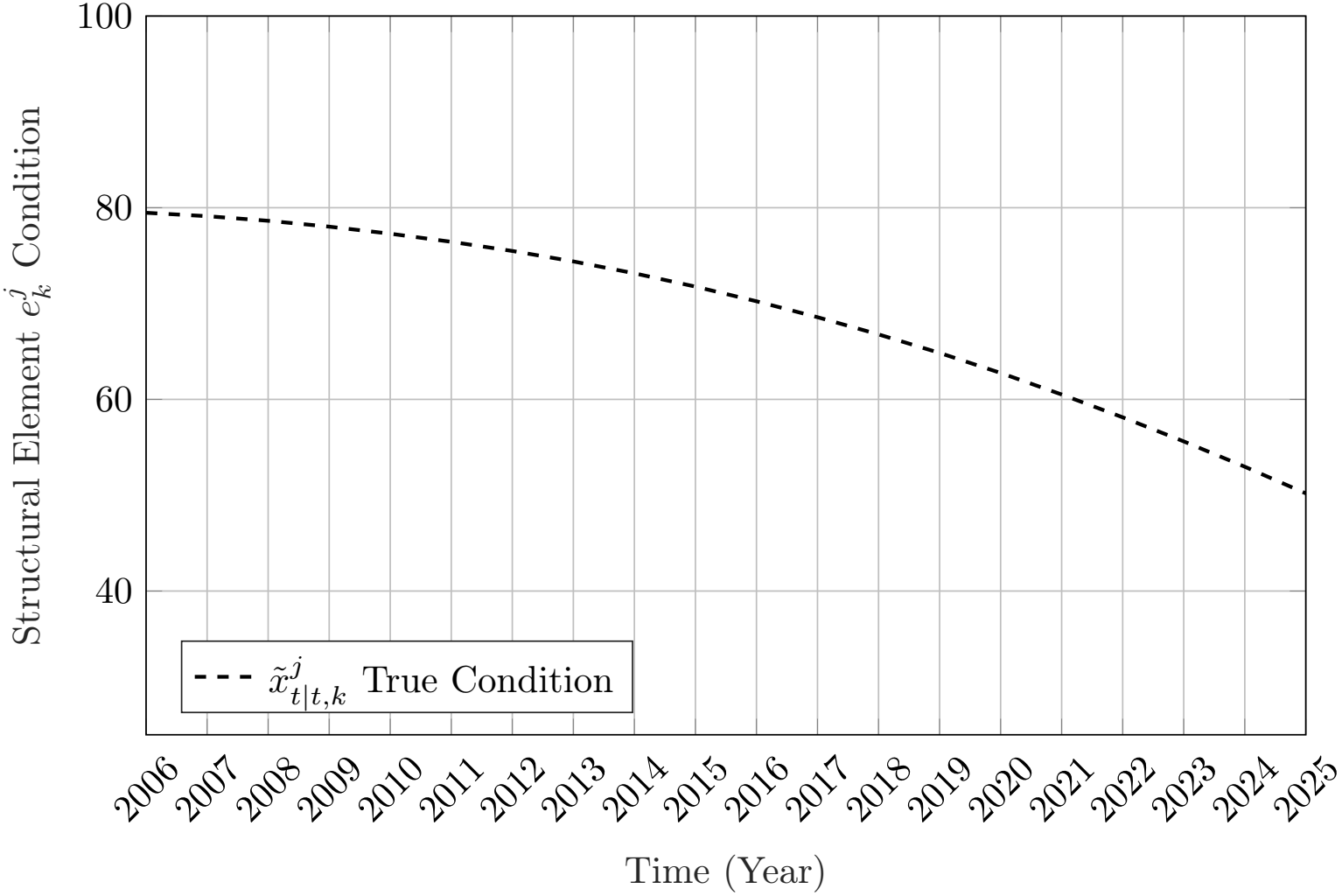
Objective: Verify the Predictive Capacity of the Model

Generating Synthetic Inspection Data



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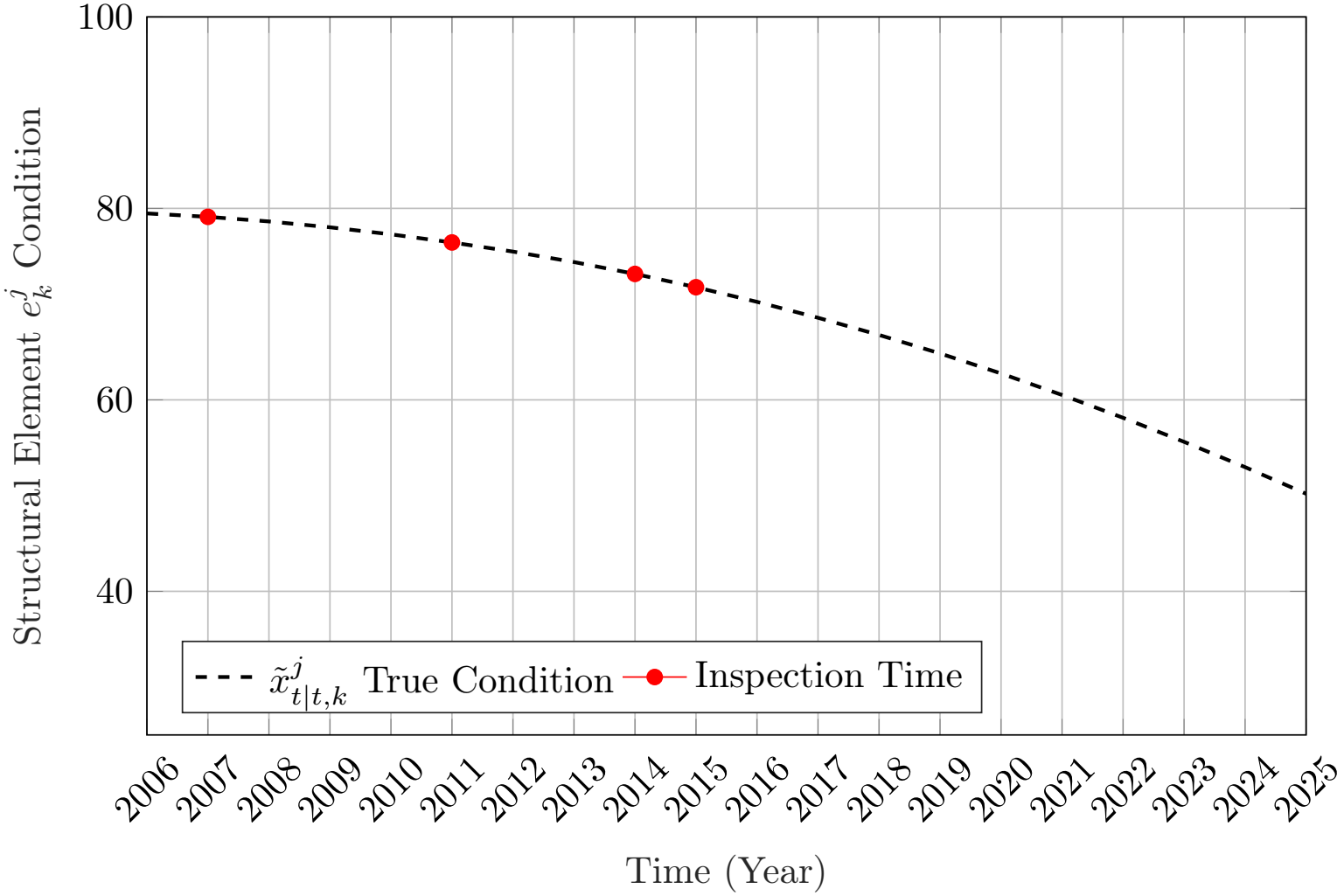

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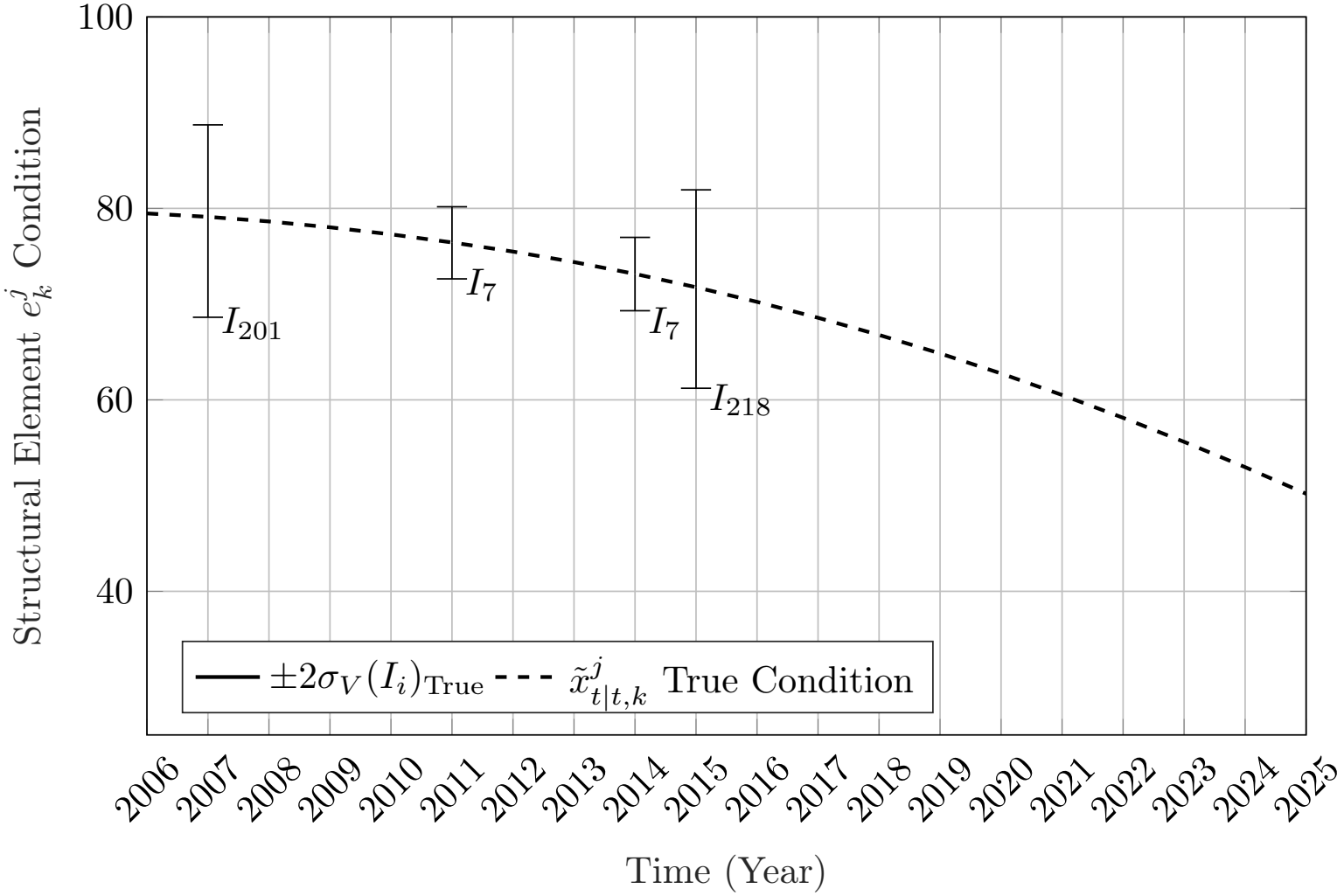

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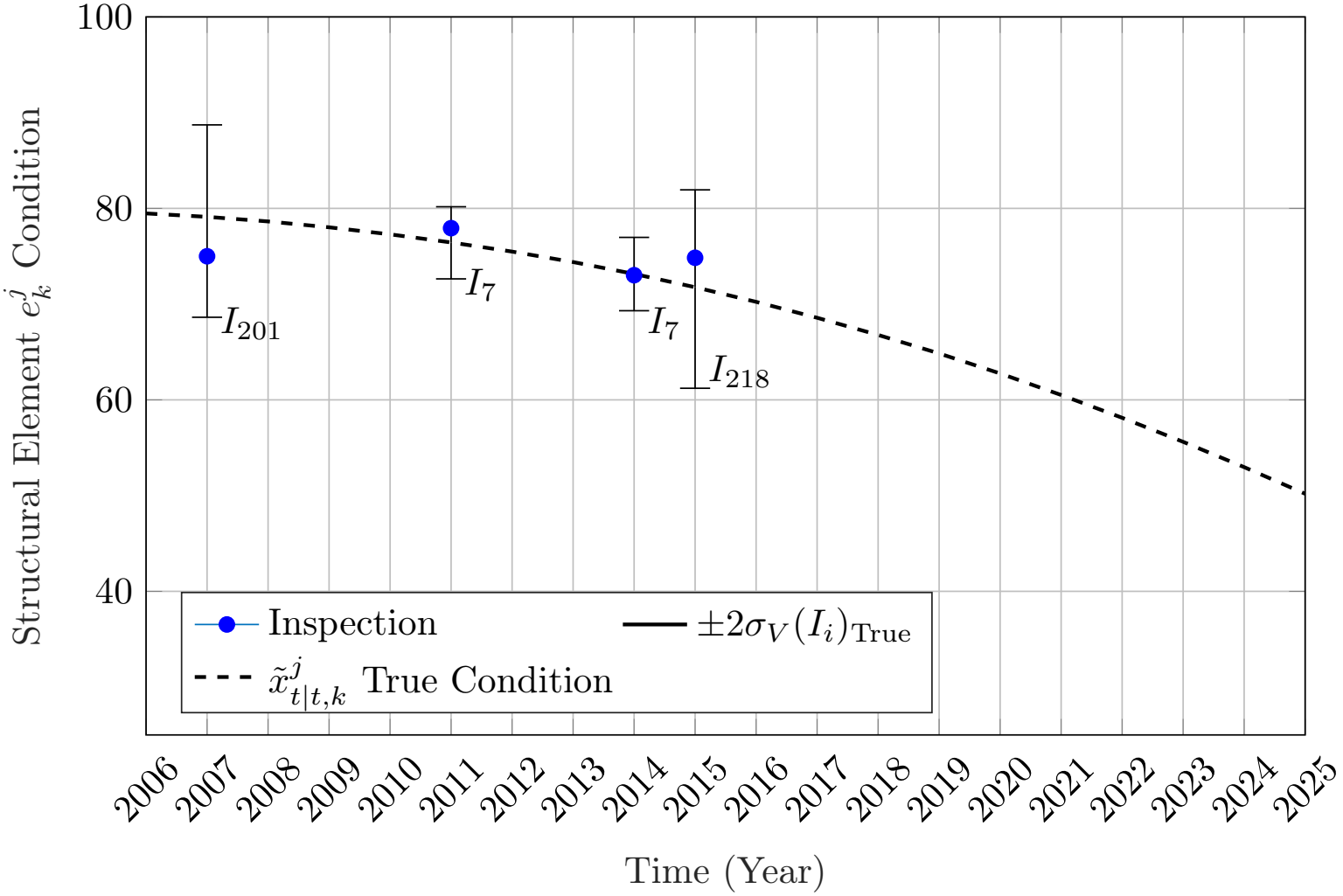

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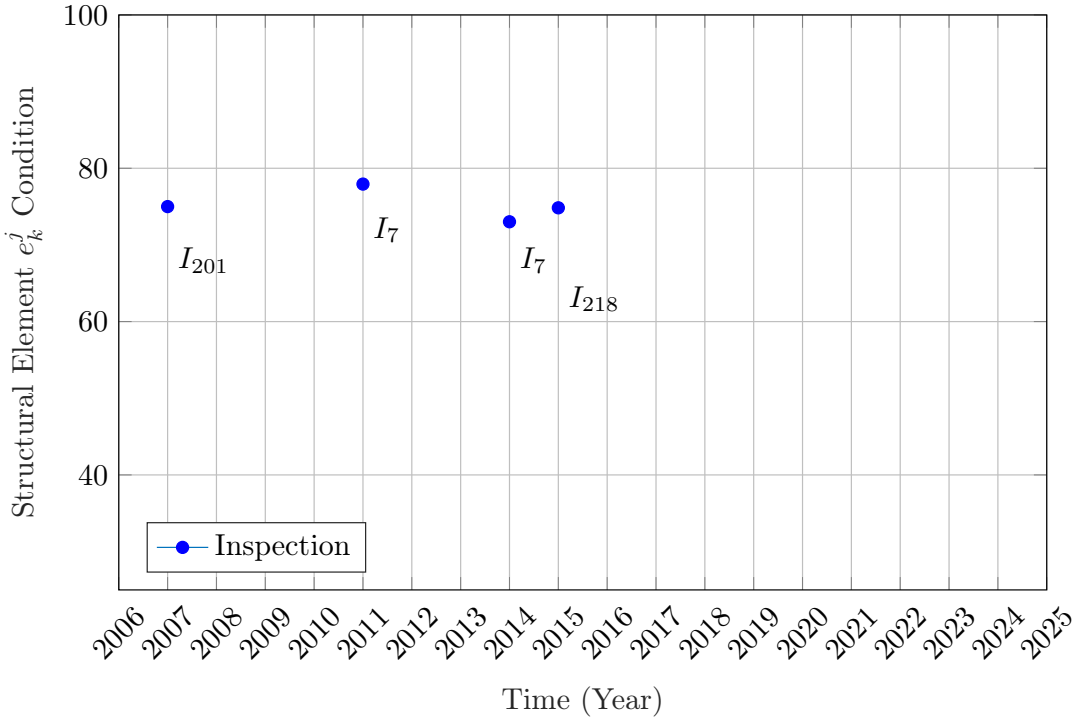
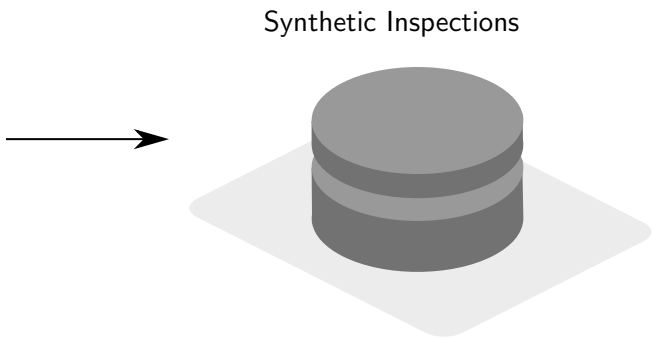
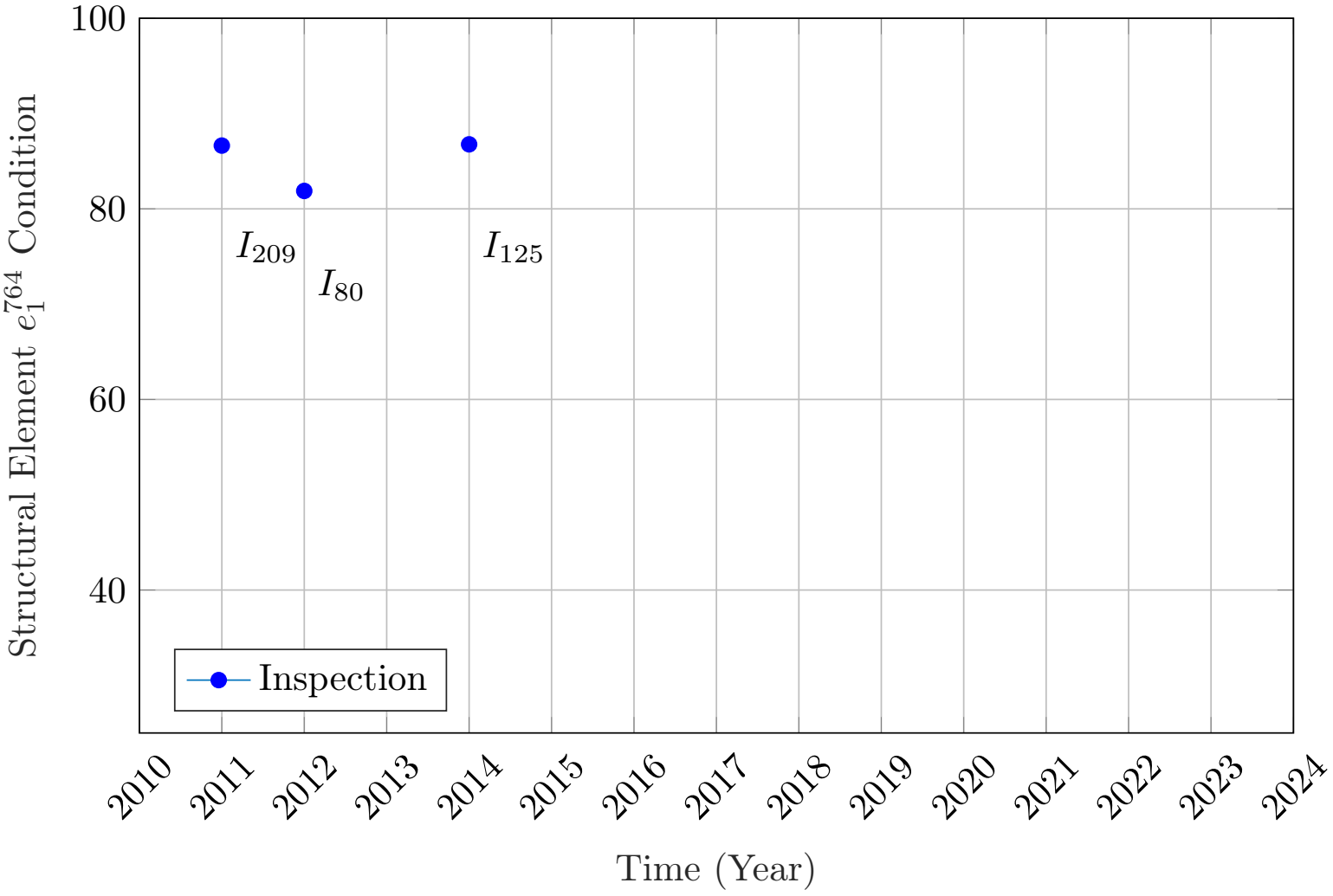




Example of Synthetic Data: Poor Performance



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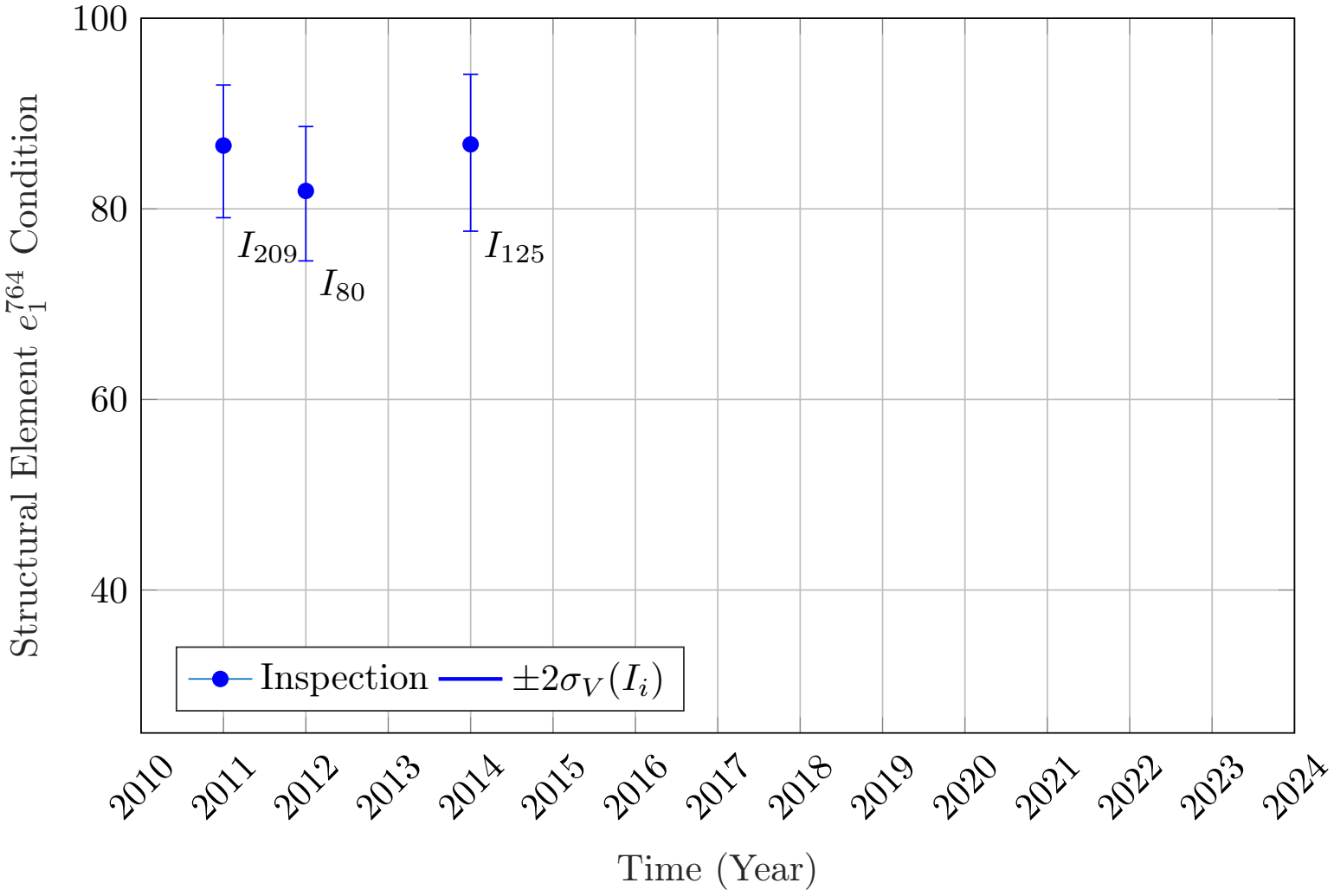

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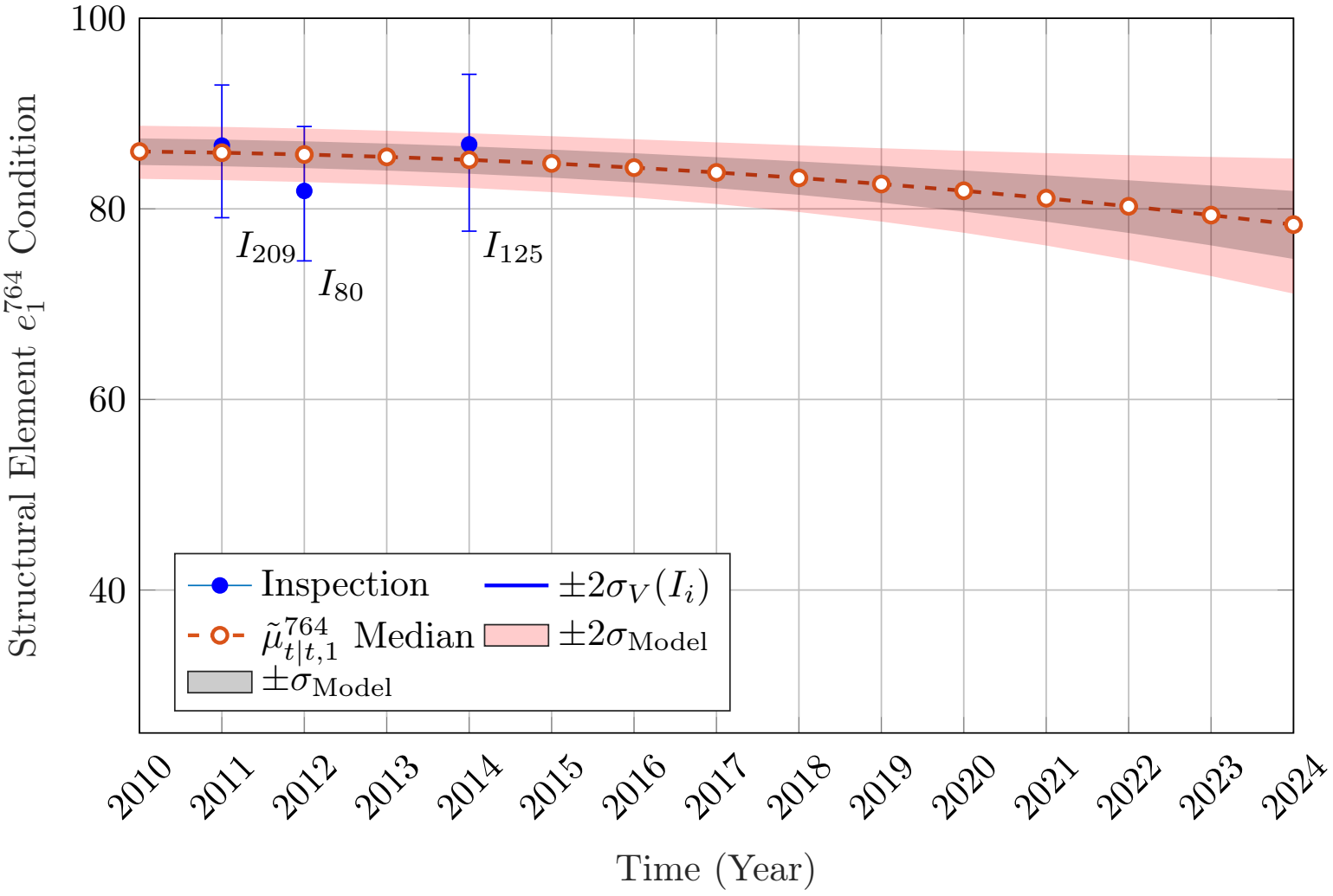

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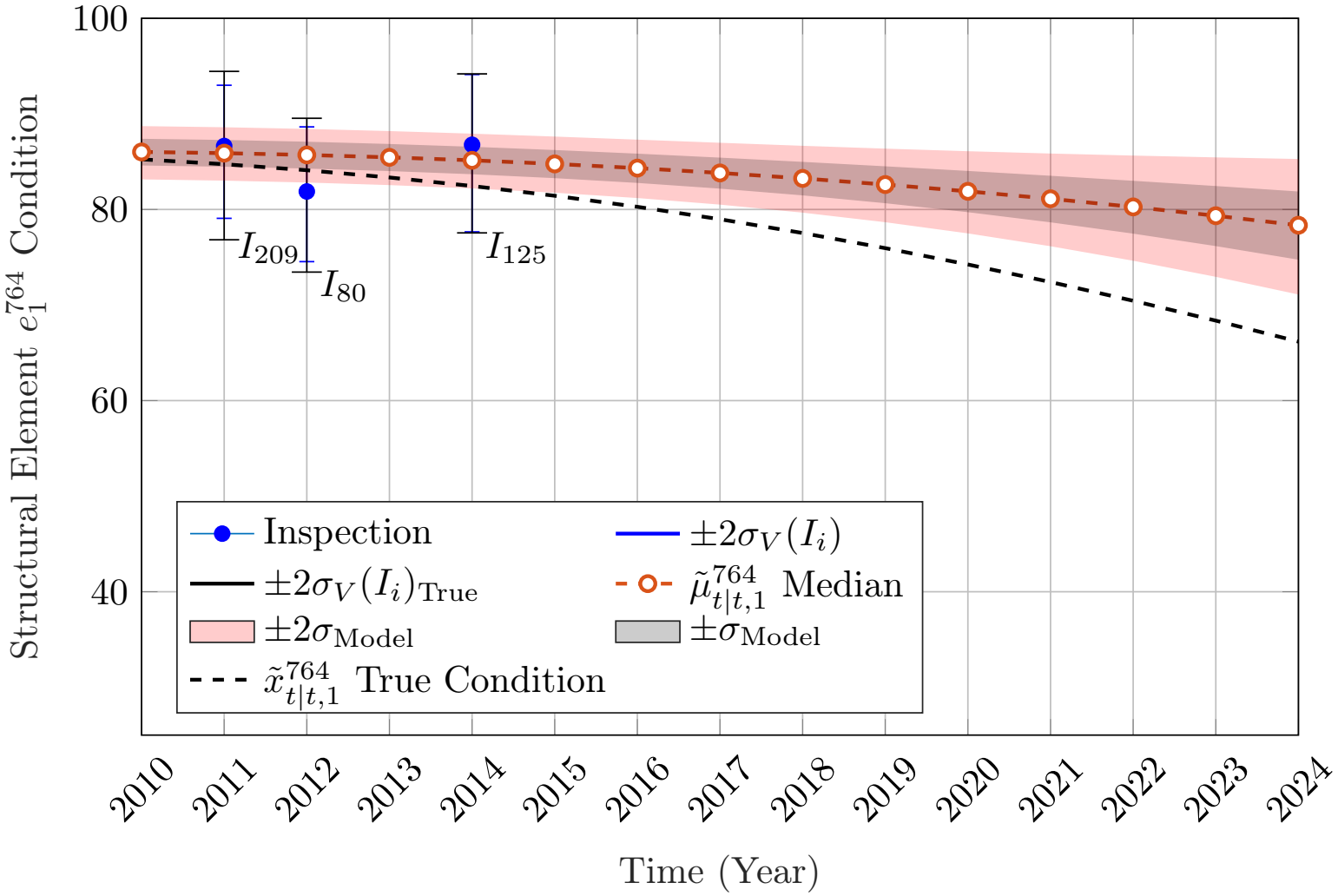

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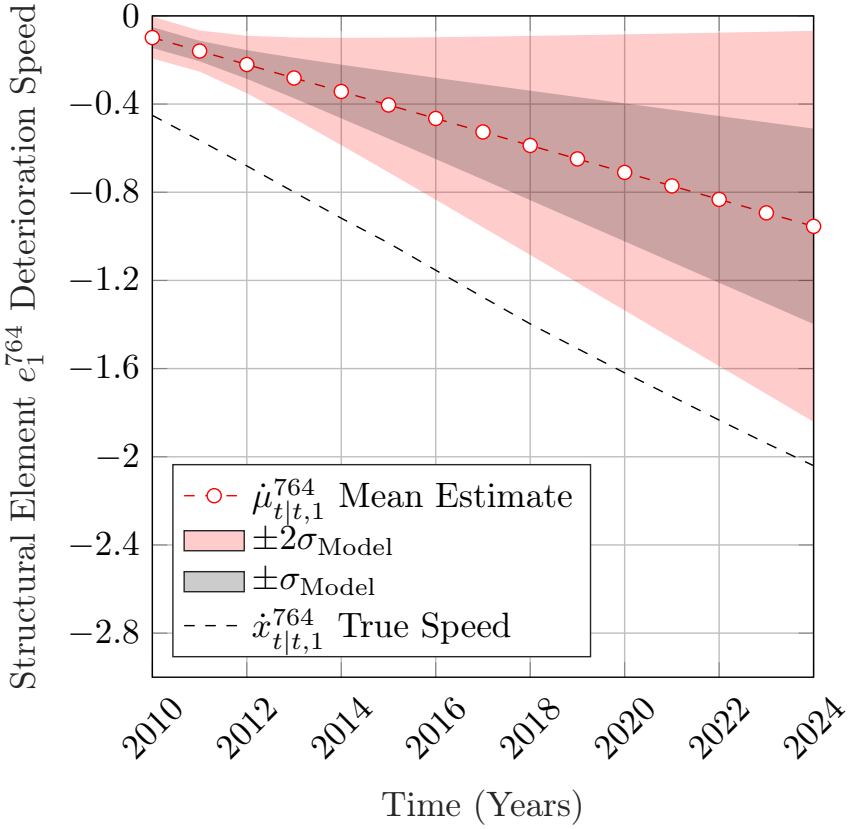
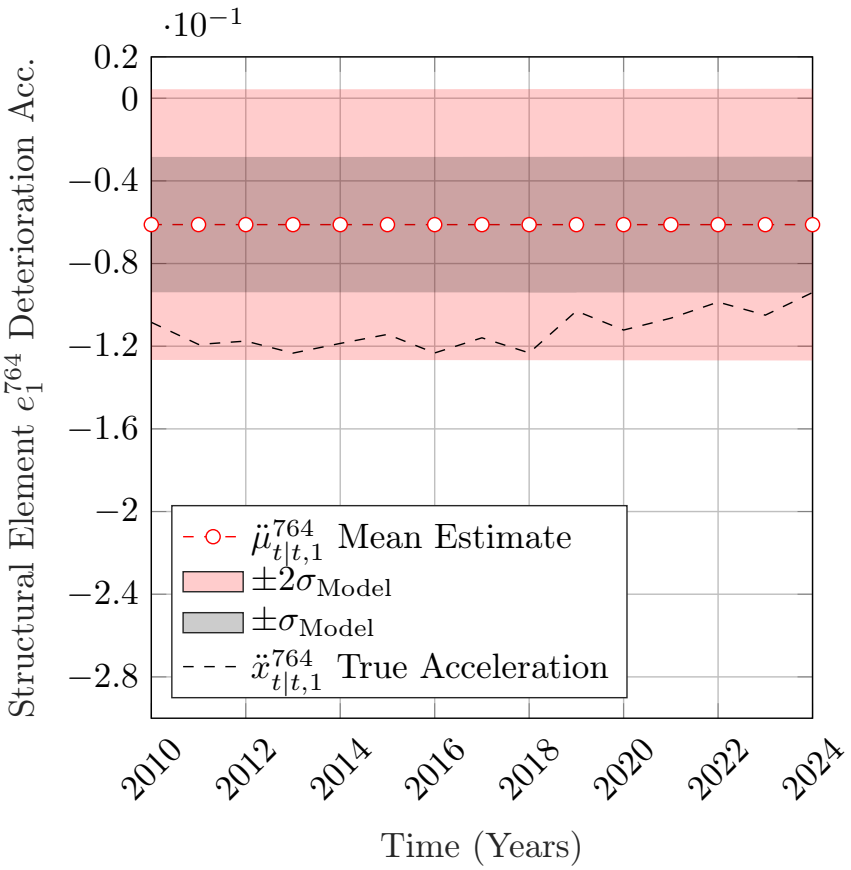
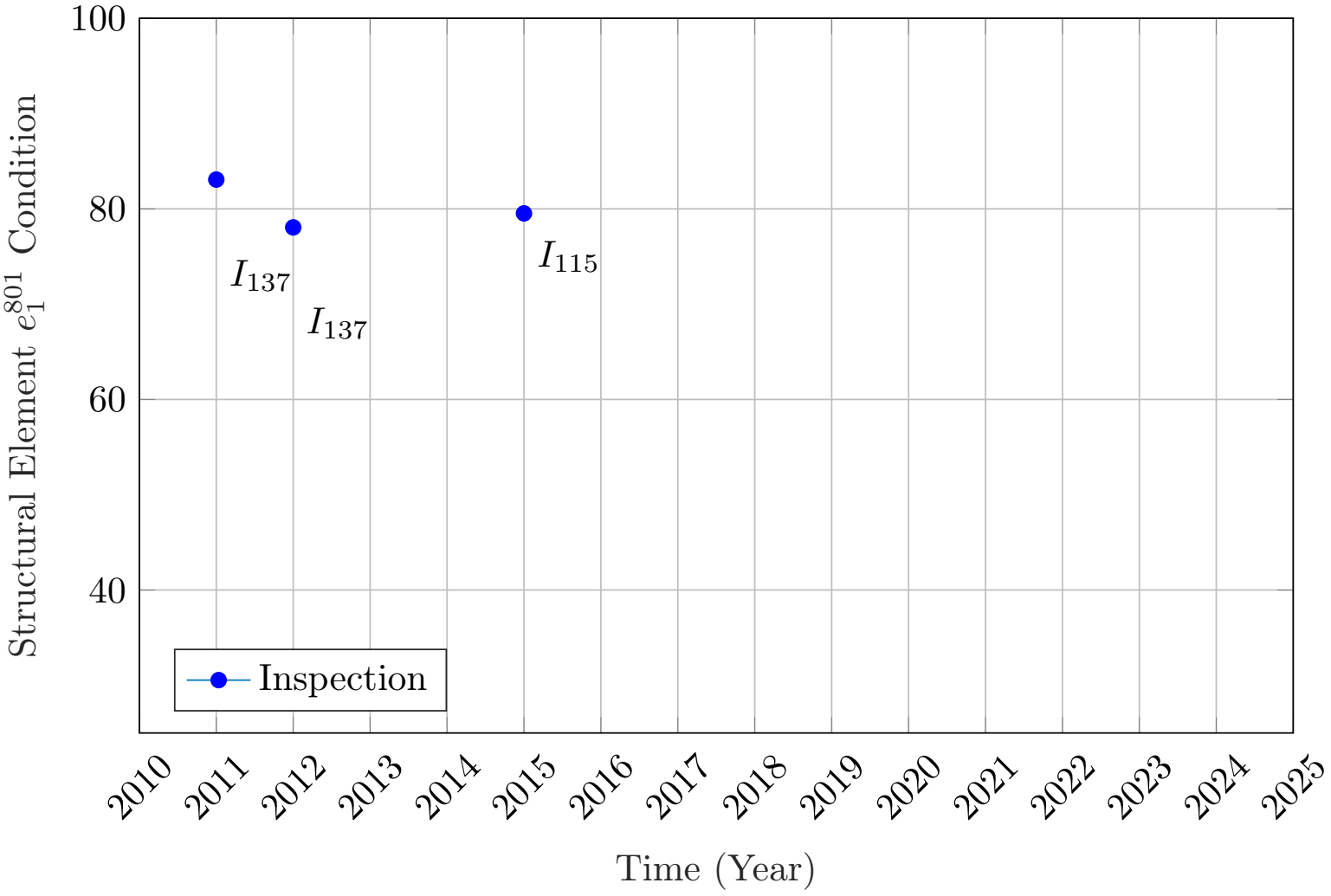




Example of Synthetic Data: Good Performance



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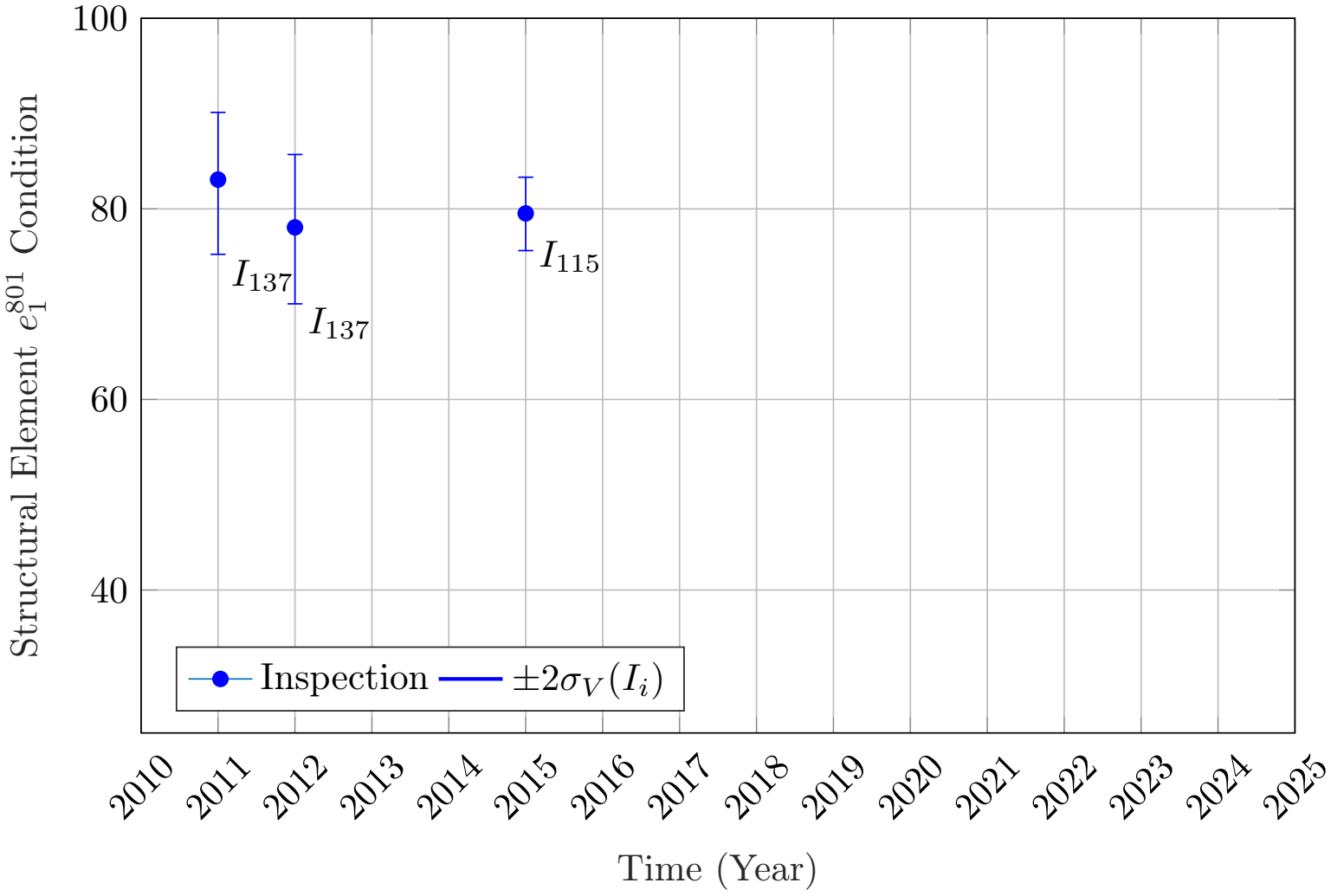

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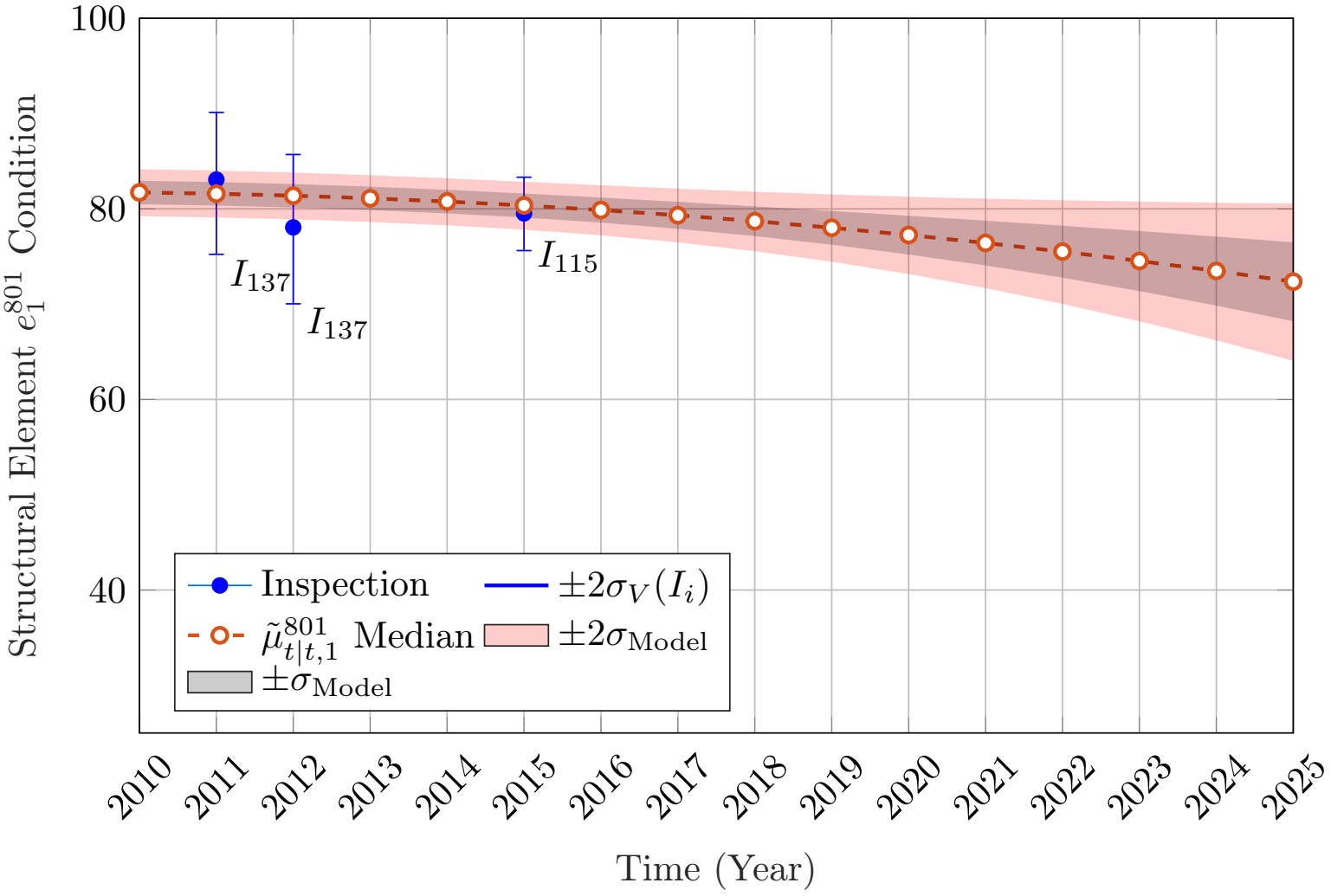

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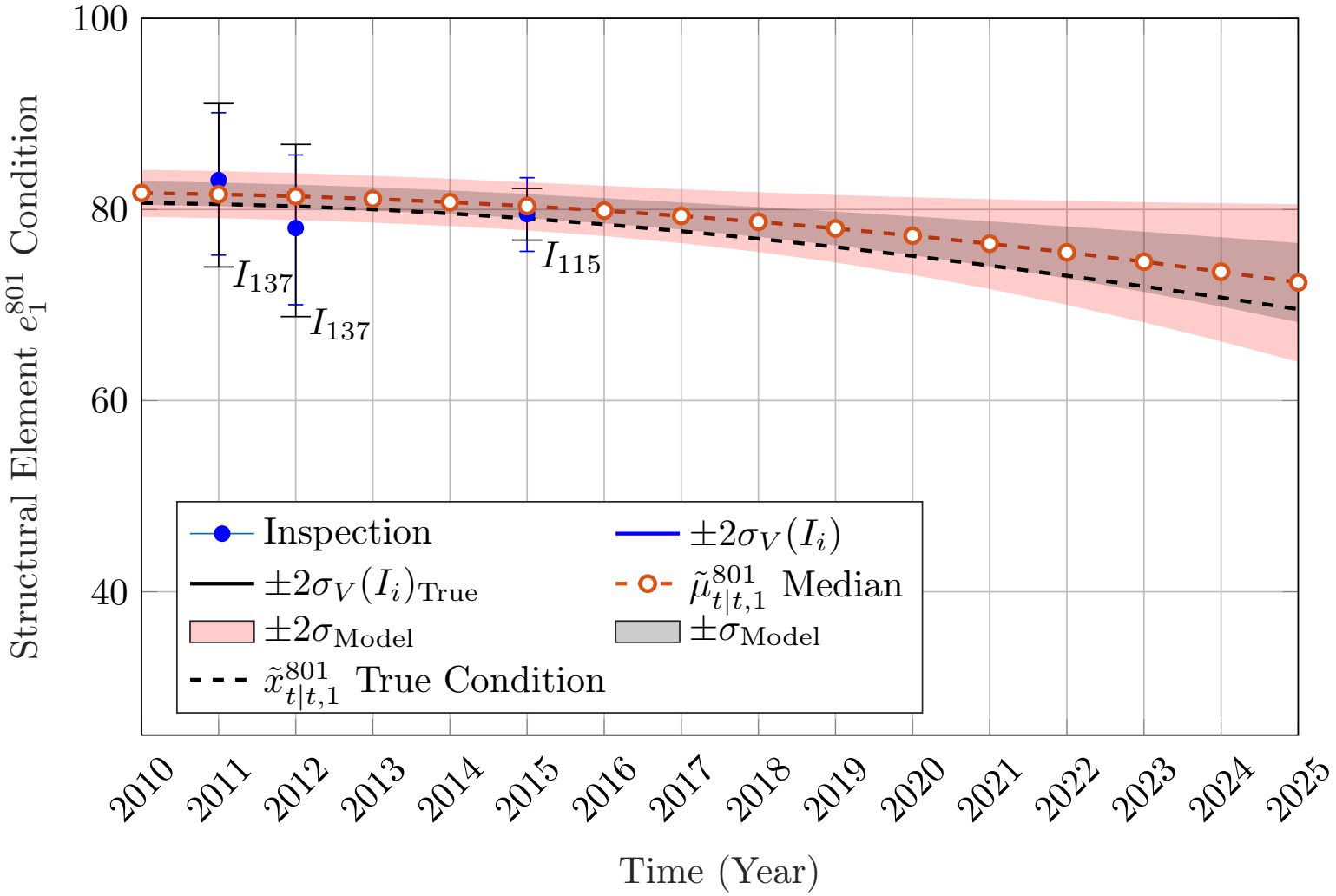

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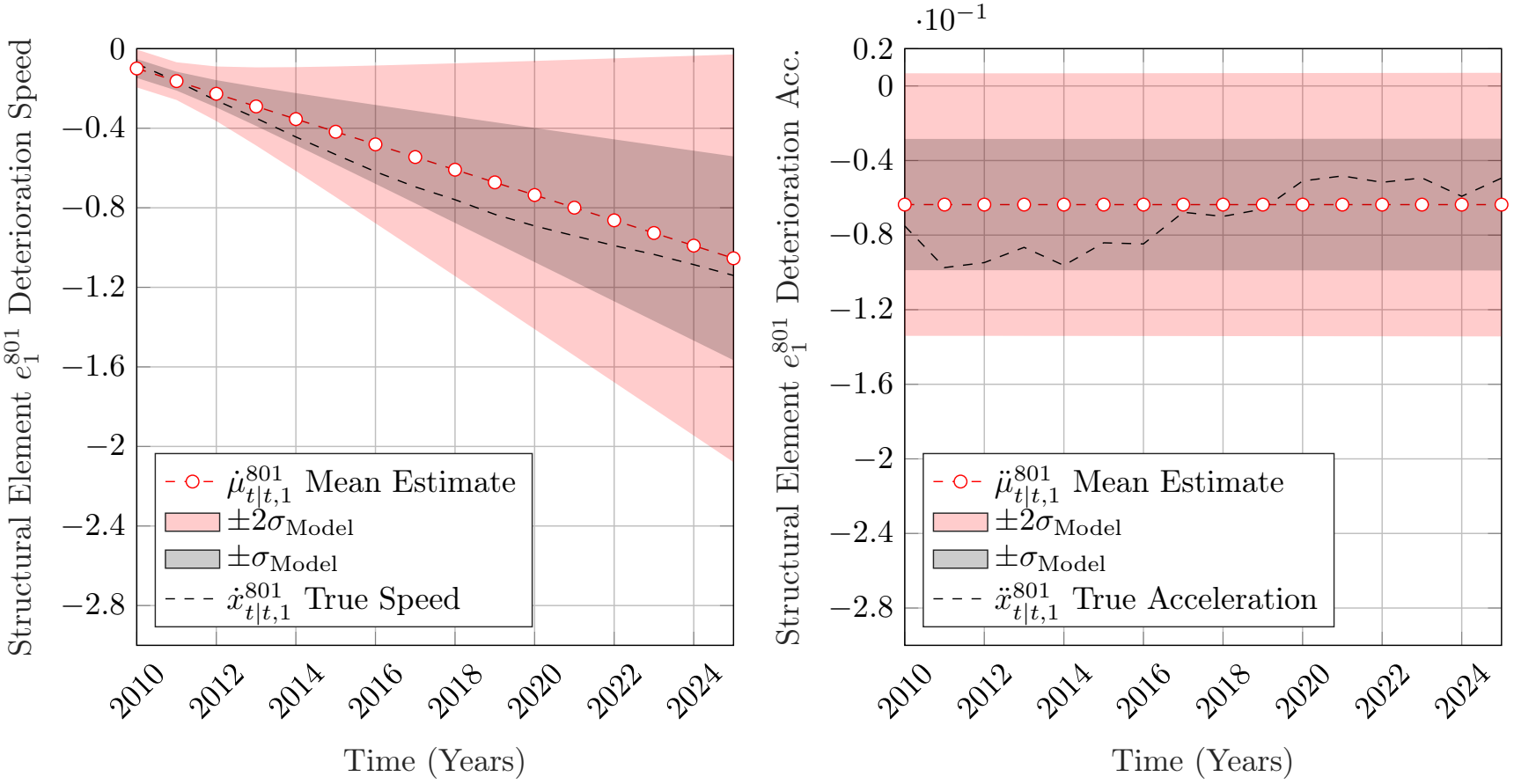
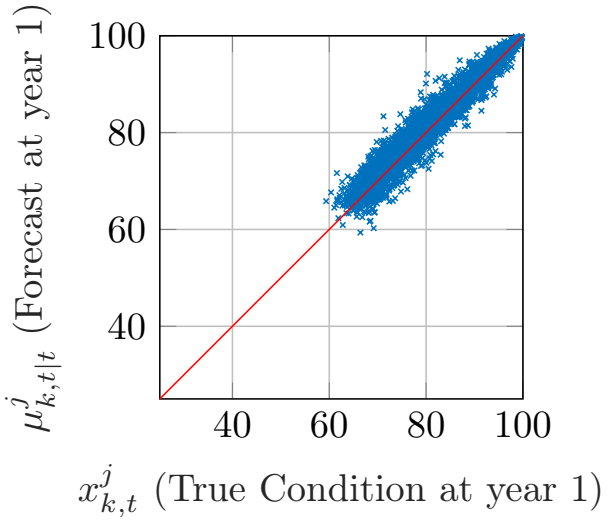


Overall Condition Prediction



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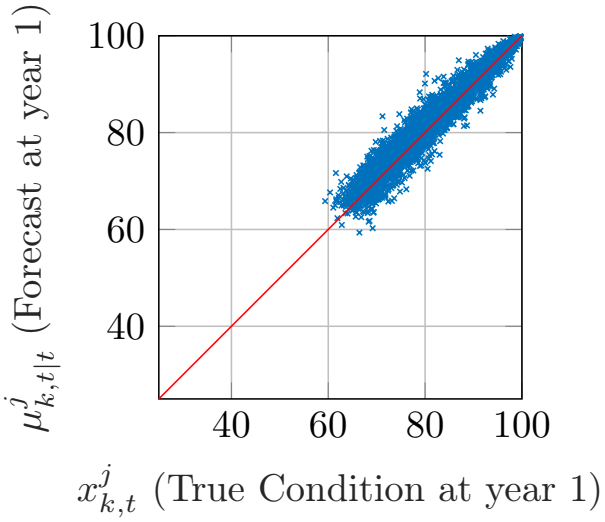

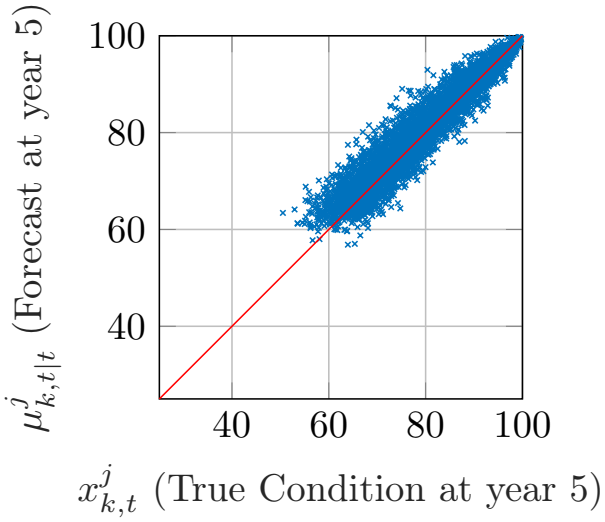

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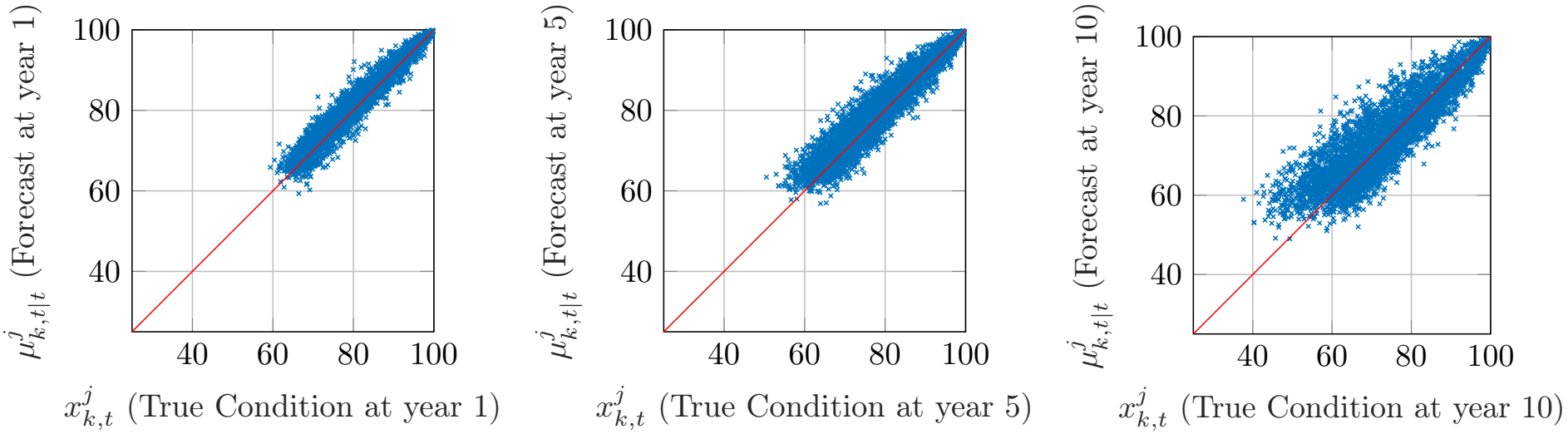

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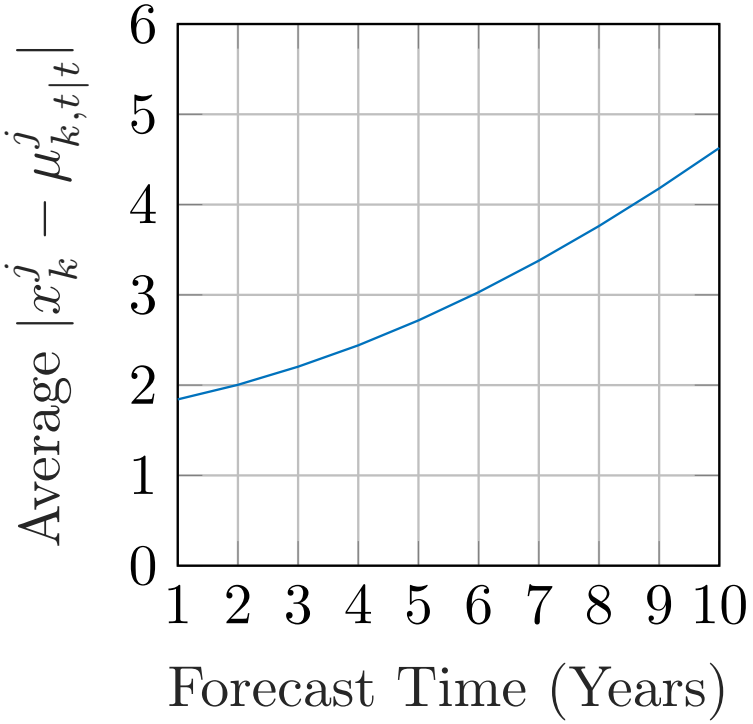
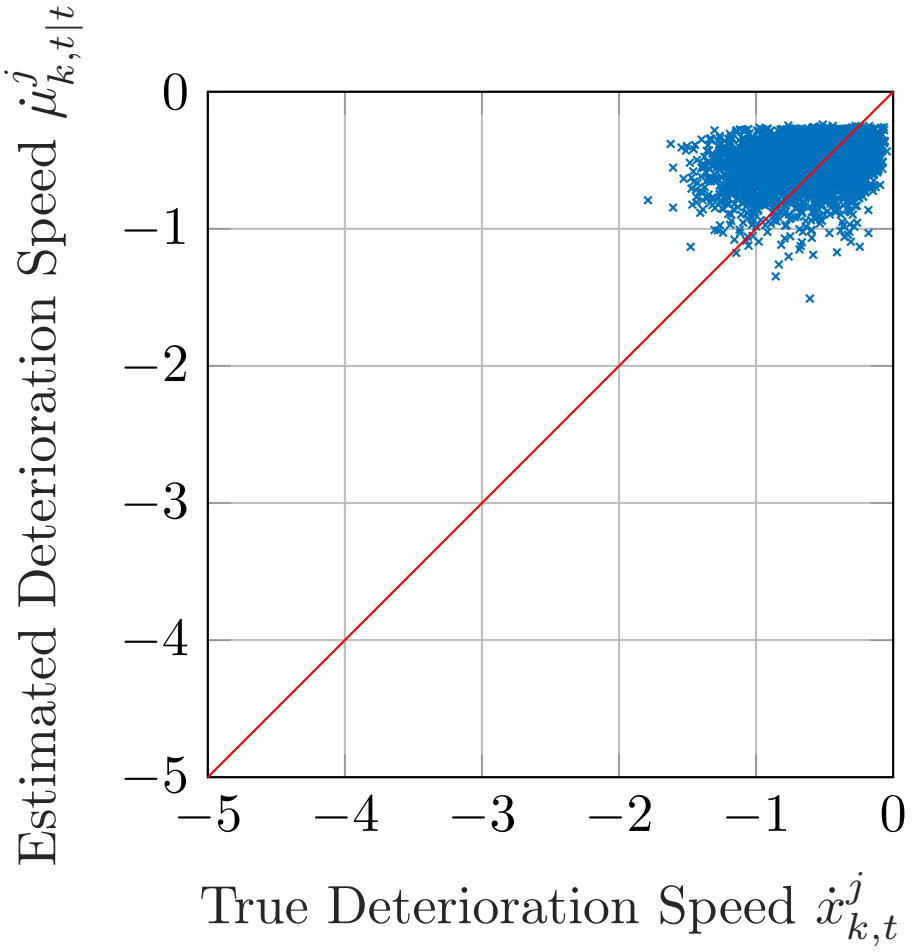
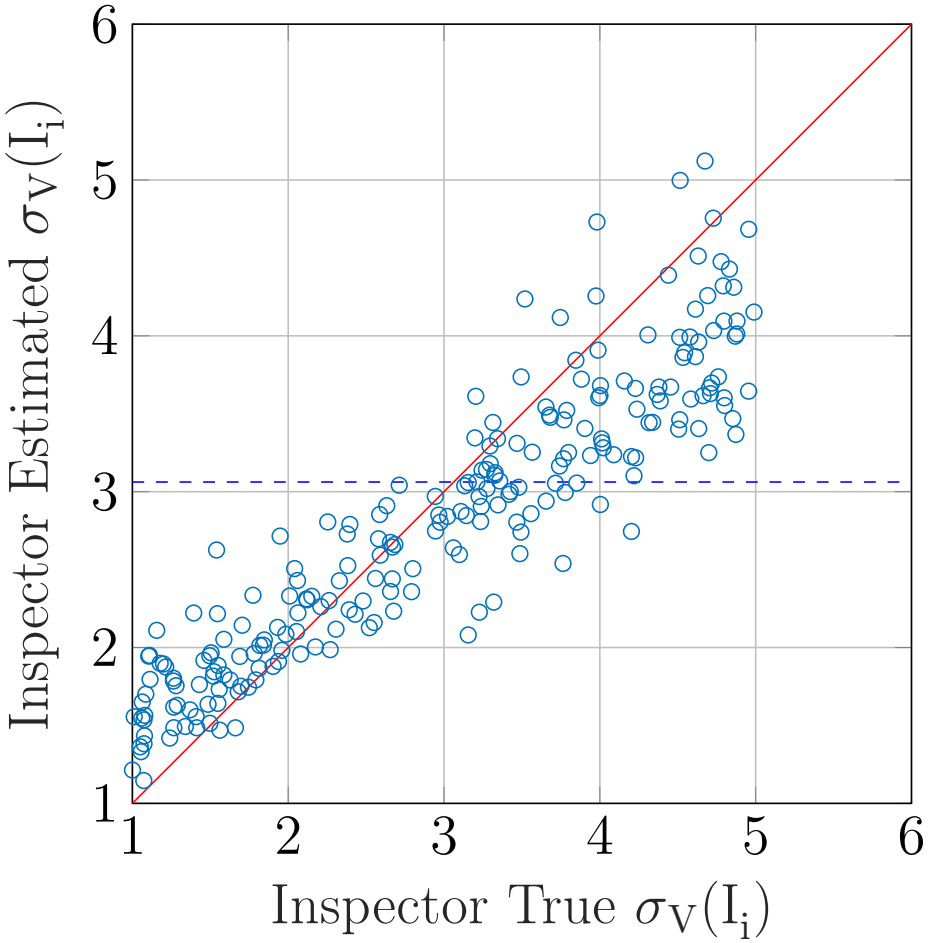


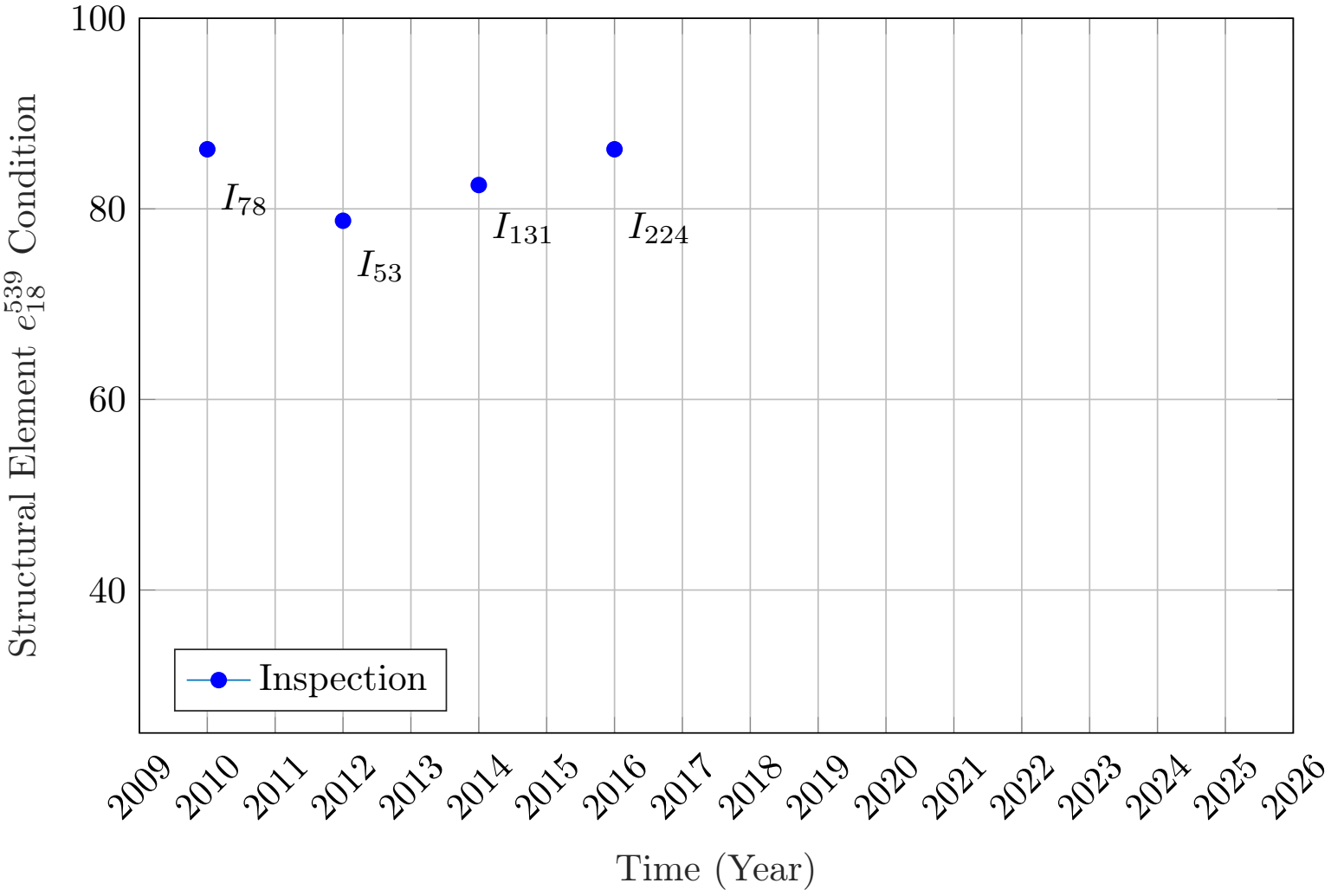
Verification: Deterioration Speed Estimate



Verification: Estimation for the Uncertainty of Inspectors



Example of Real Data: Inconsistency



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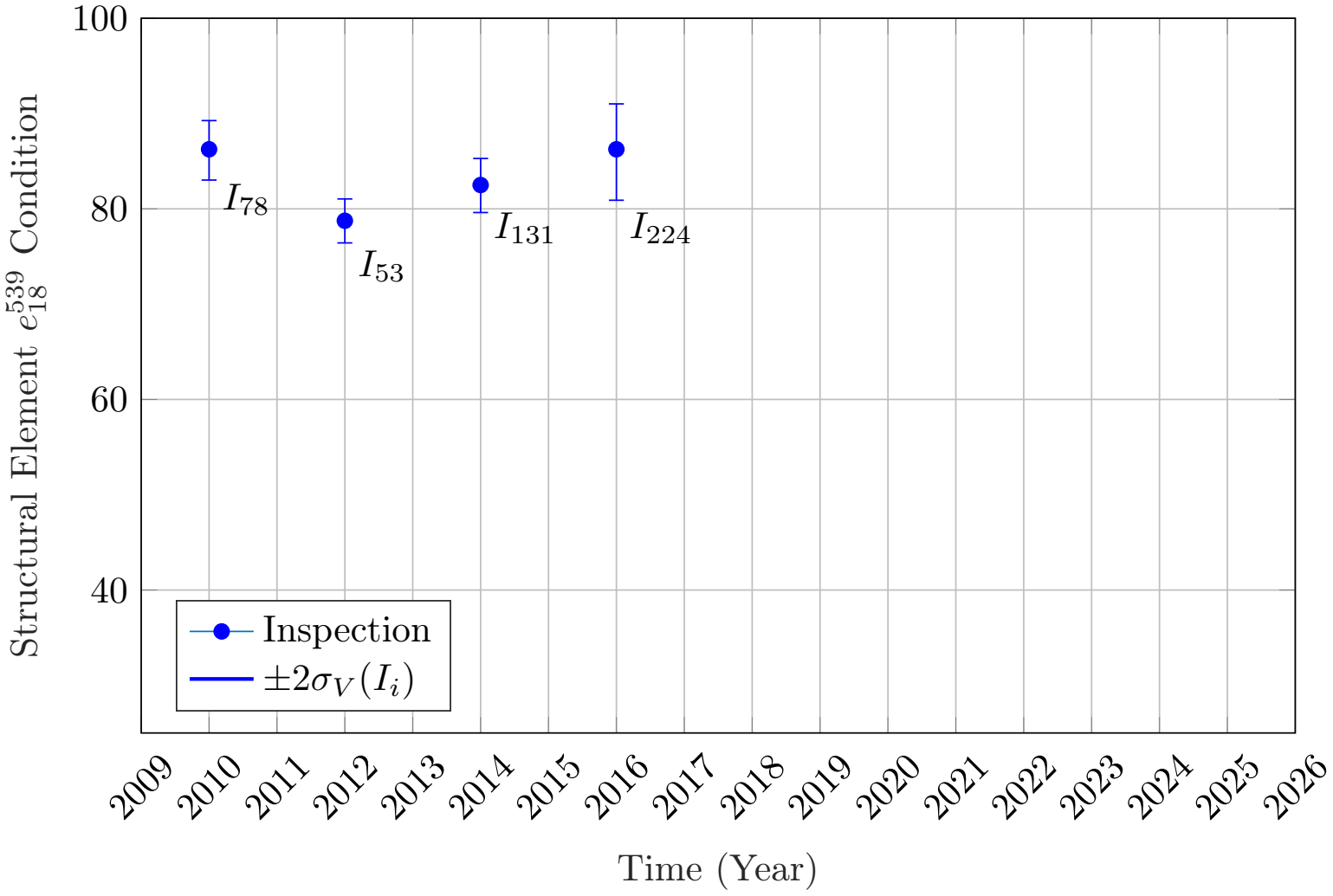

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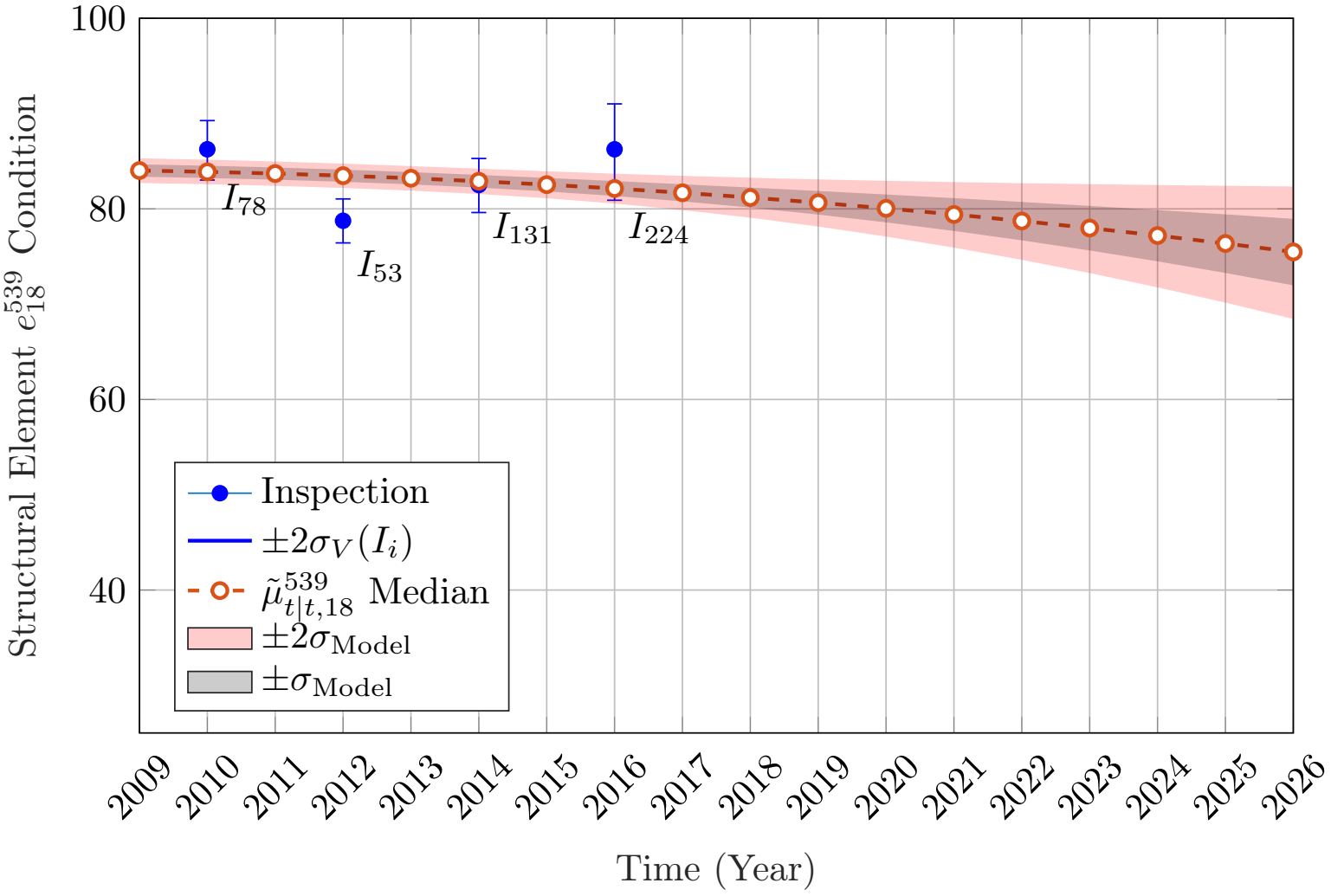

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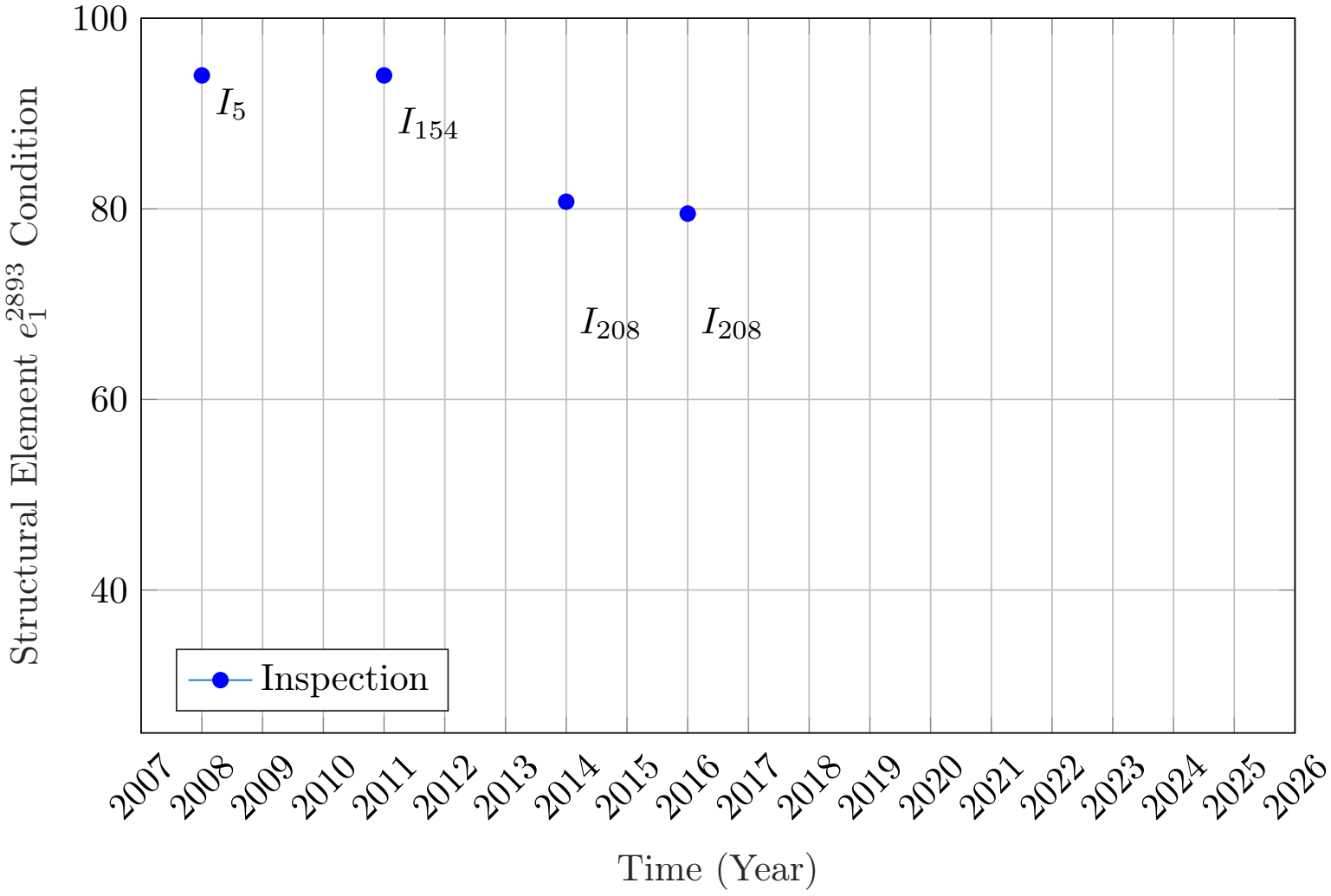

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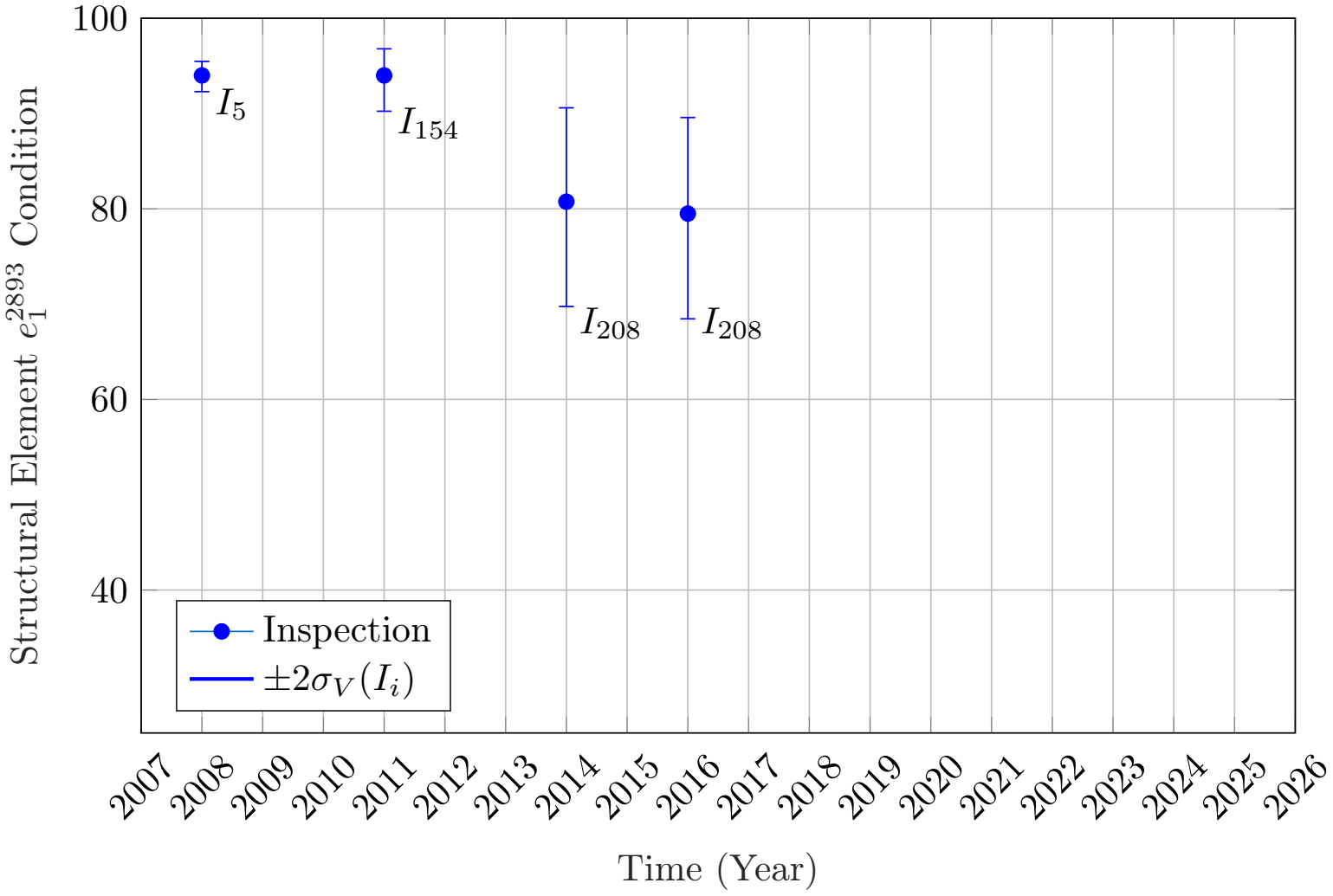

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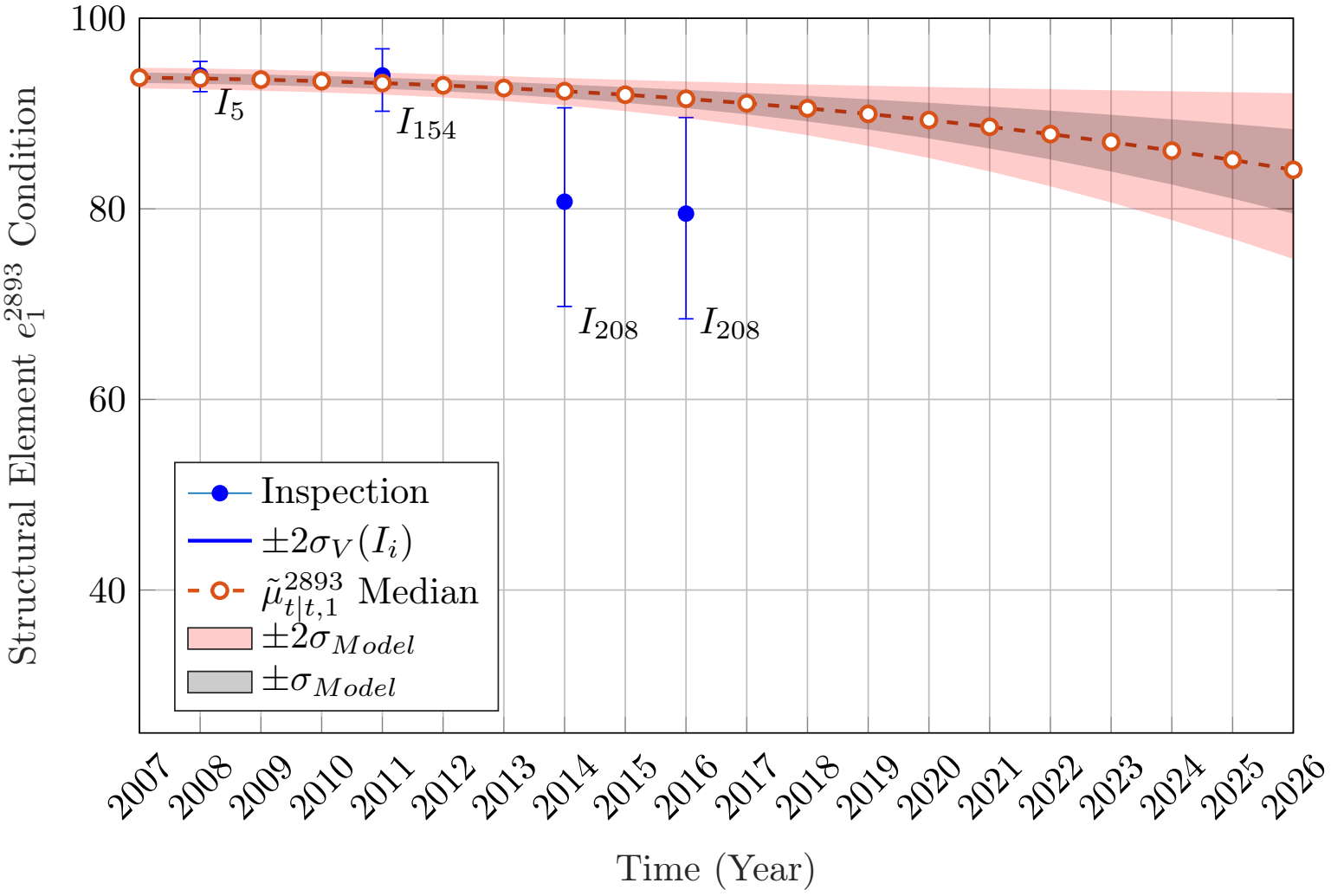

Section Outline

Next Steps & Plan

6.1 Factoring Structural Attributes

6.2 Effect of Interventions

6.3 Schedule

Sharing Information Across Structures

Source: MTQ/Google Images

$\neq$

$$\mathbf{b}_{232} = \begin{Bmatrix} \text{ponceau} \\ \text{béton} \\ 10000 \\ \text{Estrie} \end{Bmatrix}$$

$$\mathbf{b}_{154} = \begin{Bmatrix} \text{suspension bridge} \\ \text{acier} \\ 30\text{M} \\ \text{Québec} \end{Bmatrix}$$

Sharing Information Across Structures

Source: MTQ/Google Images

$\approx$

$$\mathbf{b}_{232} = \begin{Bmatrix} \text{ponceau} \\ \text{béton} \\ 10000 \\ \text{Estrie} \end{Bmatrix}$$

$$\mathbf{b}_{585} = \begin{Bmatrix} \text{ponceau} \\ \text{béton} \\ 10000 \\ \text{Estrie} \end{Bmatrix}$$

Sharing Information Across Structures

Source: MTQ/Google Images

$$\neq$$

$$\mathbf{b}_{405} = \begin{Bmatrix} \text{ponceau} \\ \text{béton} \\ 10000 \\ \text{Montreal} \end{Bmatrix}$$

$$\mathbf{b}_{776} = \begin{Bmatrix} \text{ponceau} \\ \text{béton} \\ 10000 \\ \text{Iqualuit} \end{Bmatrix}$$

Sharing Information Across Structures

Source: MTQ/Google Images

?

$$\mathbf{b}_{405} = \begin{Bmatrix} \text{ponceau} \\ \text{béton} \\ 10000 \\ \text{Montreal} \end{Bmatrix}$$

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Steps Toward Sharing Information Across Structures

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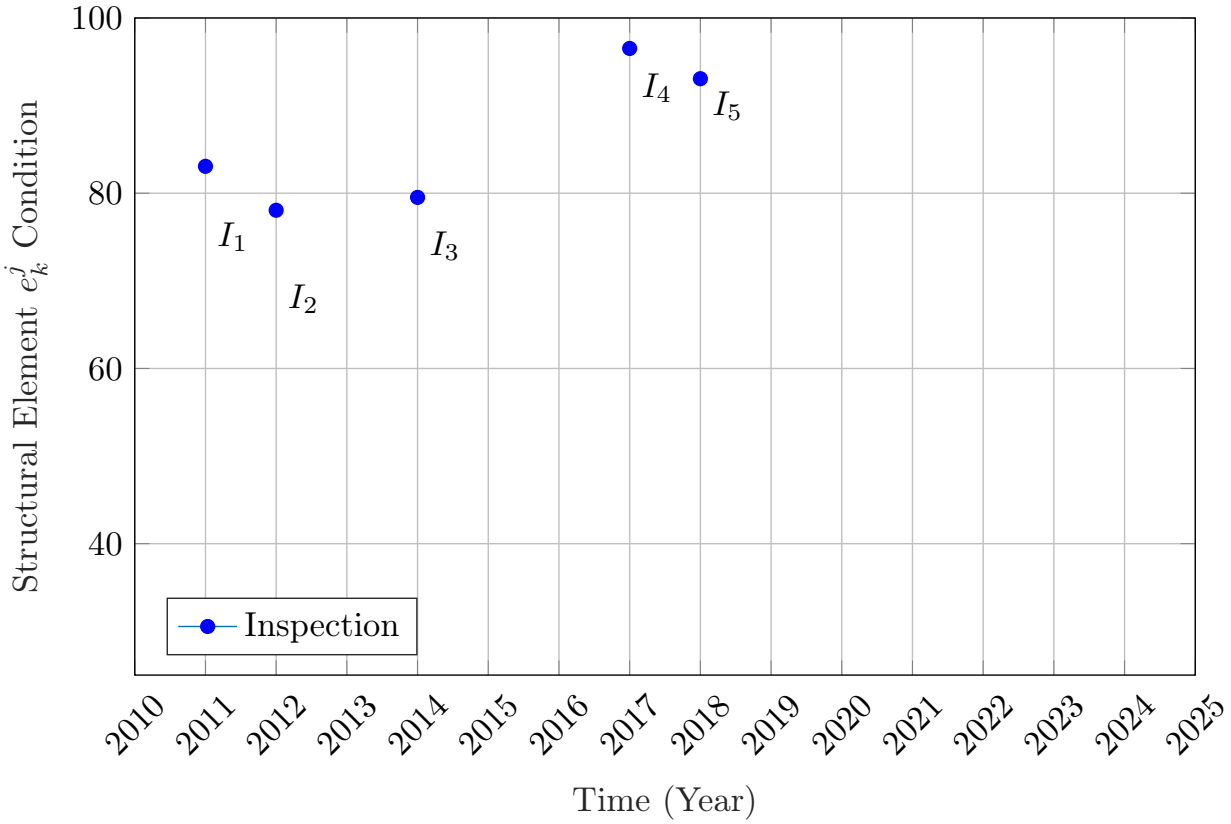
2. Fusing the developed regression model with the state-space framework

Model the Effect of Interventions



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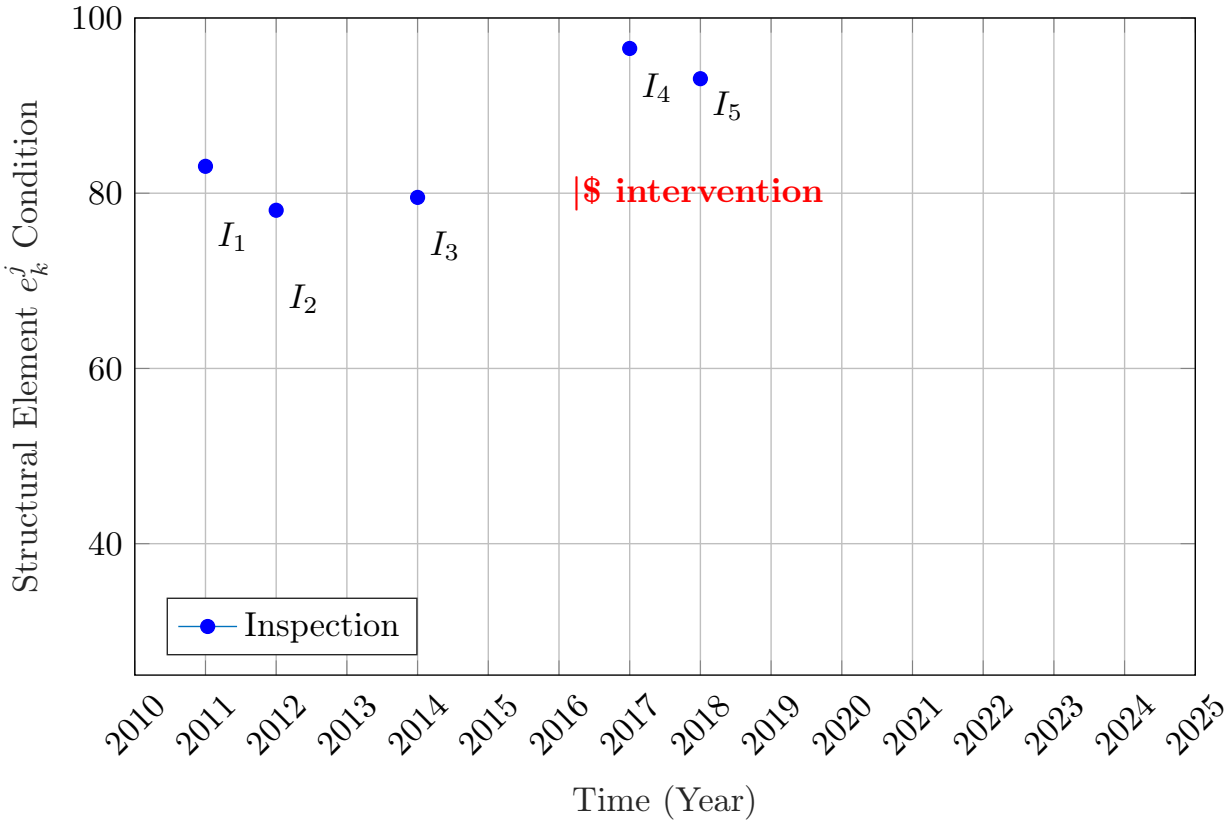
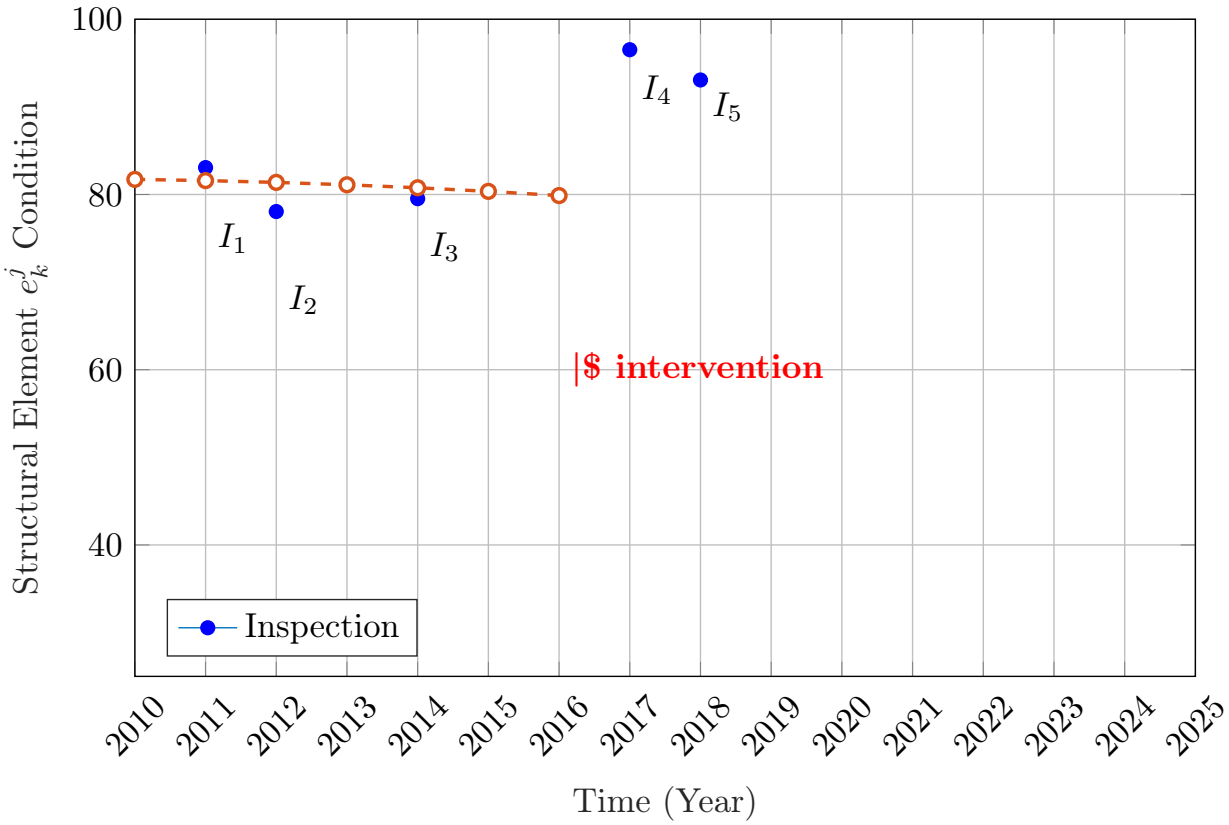


Model the Effect of Interventions

$$\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{w}_t$$



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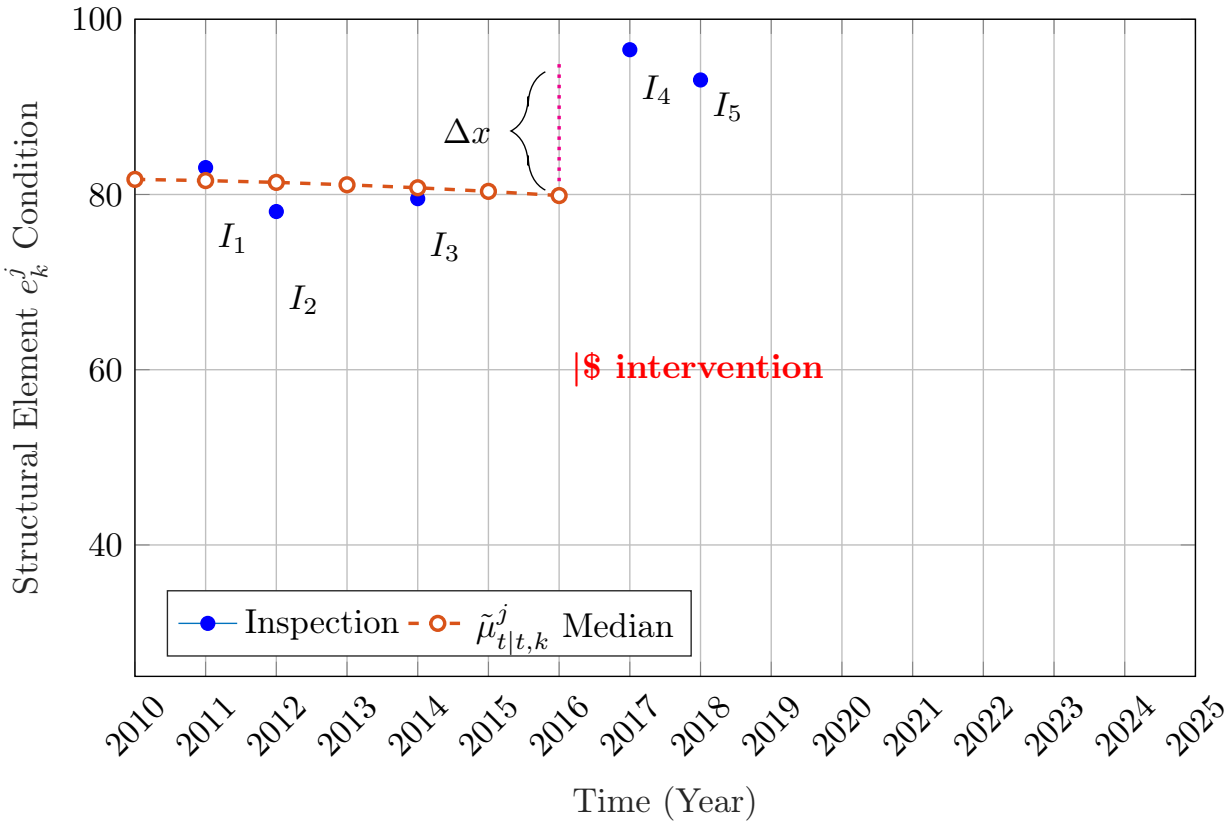
$$\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}_t\boldsymbol{u}_t + \boldsymbol{w}_t$$



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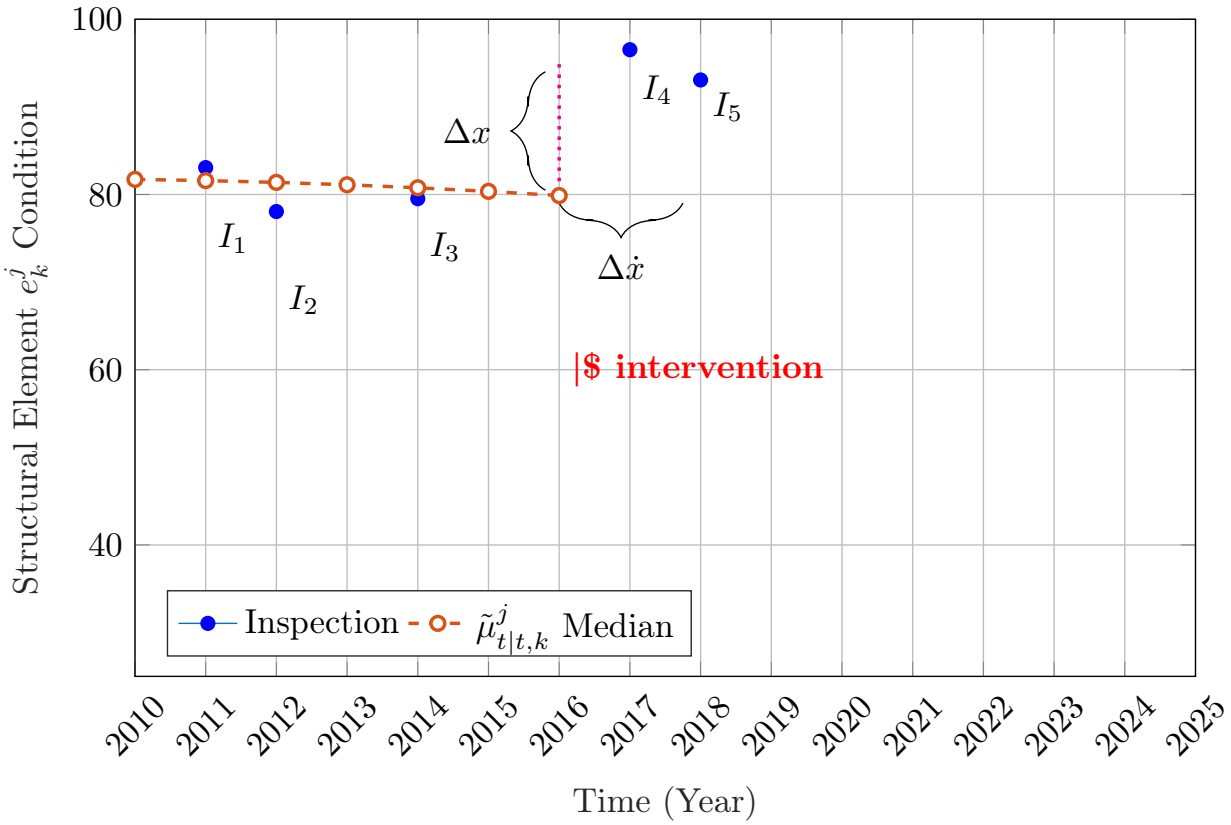

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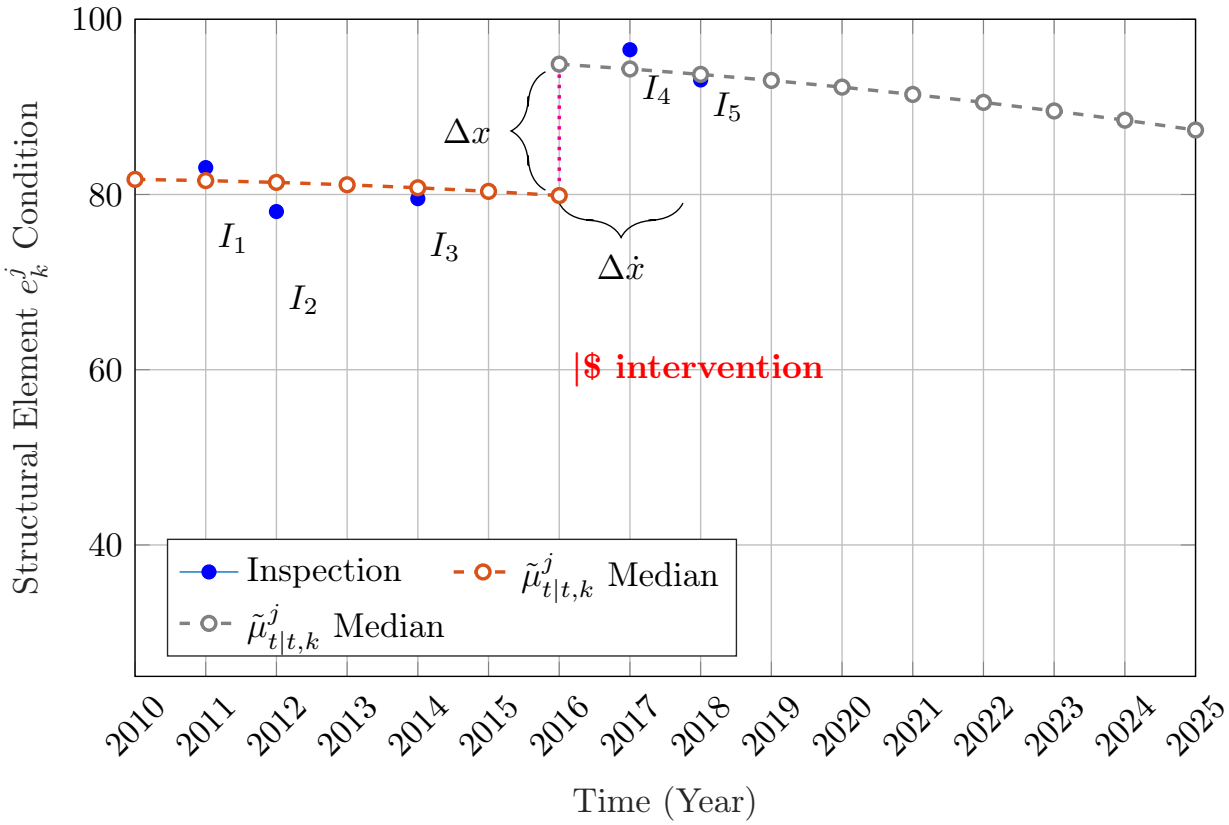
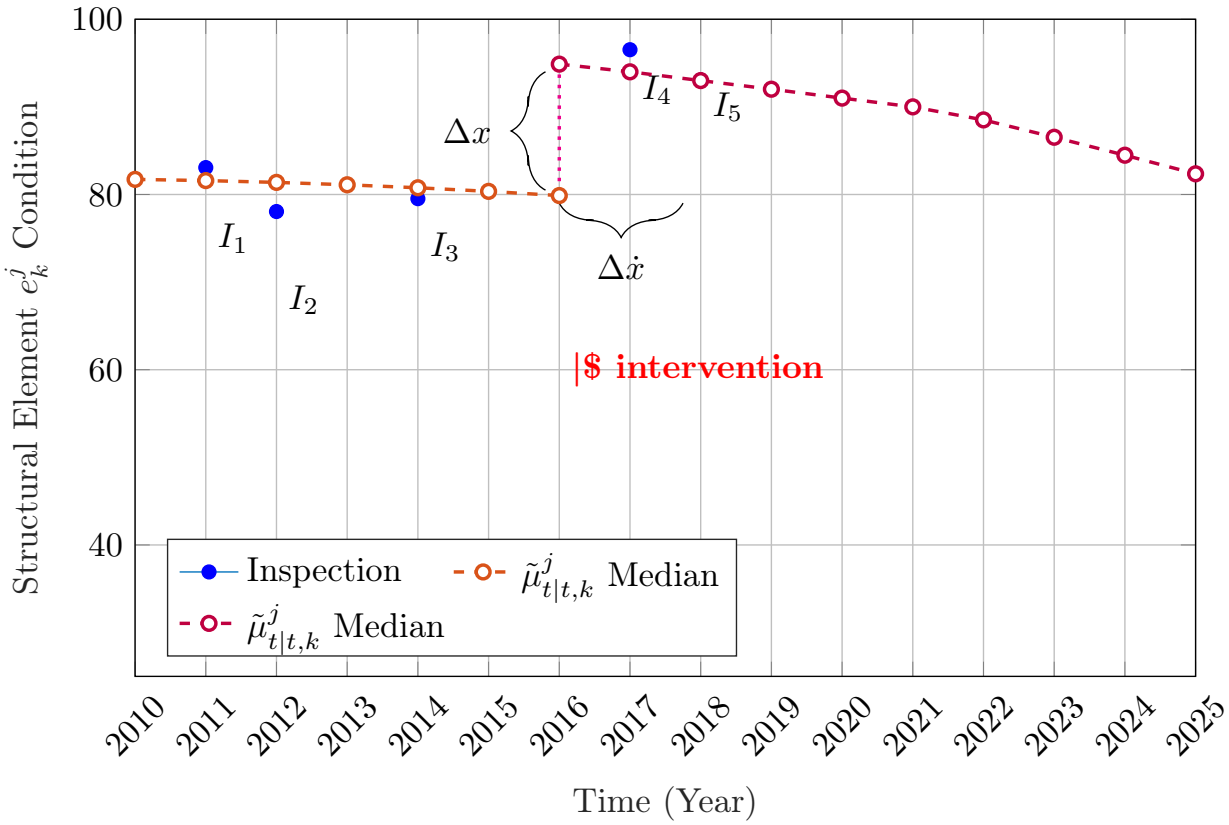


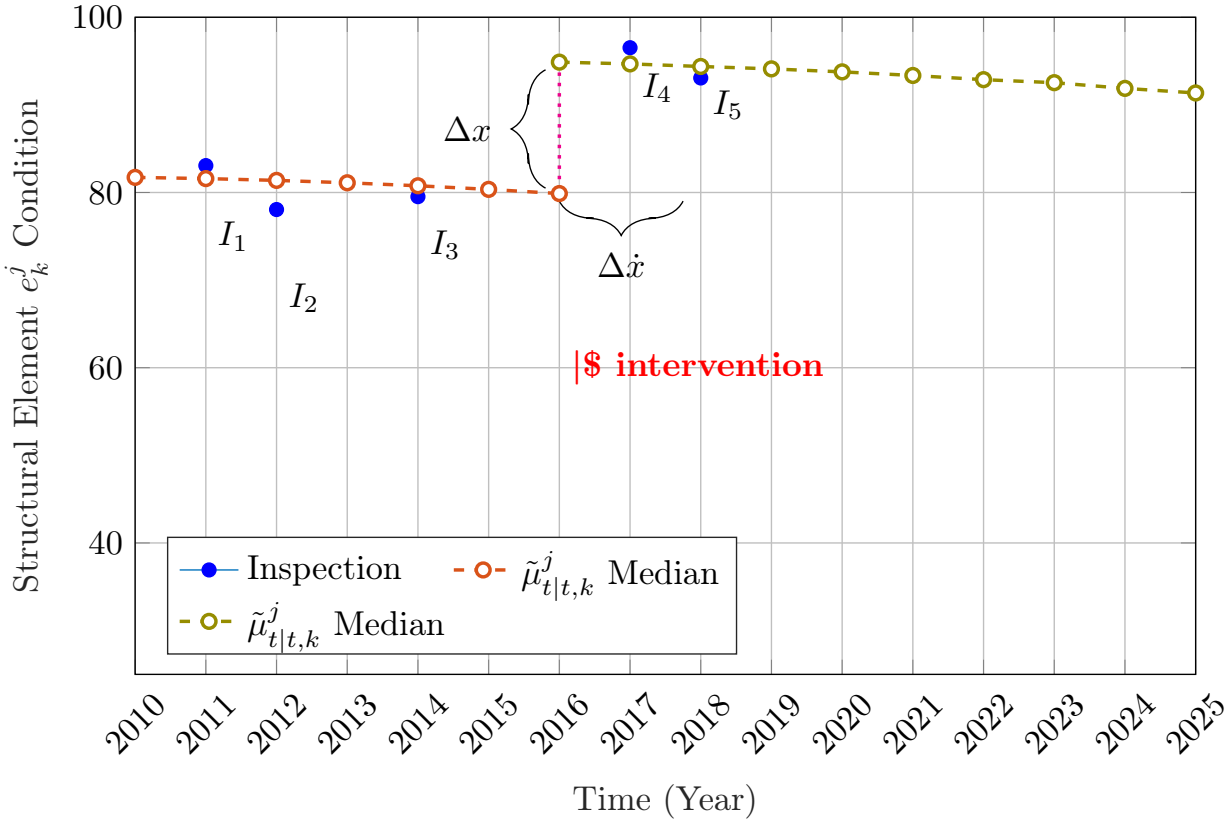
Model the Effect of Interventions

$$\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}_t\boldsymbol{u}_t + \boldsymbol{w}_t$$



Model the Effect of Interventions

$$\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}_t\boldsymbol{u}_t + \boldsymbol{w}_t$$



Schedule Summary

O	T	Description	2017 E	2017 A	2018 H	2018 E	2018 A	2019 H	2019 E	2019 A	2020 H	2020 E	2020 A	2021 H	2021 E
1	1	$\tilde{y} = g(\tilde{y}_a\%, \tilde{y}_b\%, \tilde{y}_c\%, \tilde{y}_d\%)$		✓											
	2	Empirical Model		✓											
	3	Kinematic Model	✓	✓	✓	✓									
	4	Fusing the Models													
2	1	Effect of Interventions													
	2	Empirical Model													
3	1	Notation & Formatting		✓		✓									
	2	Degradation Analyses													
	3	Intervention Analyses													