# Analyses on the Initial State of Kalman Filter for Modeling Degredation

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Transports, Mobilité durable et Électrification des transports Québec Partenaire

# Outline

**Context**

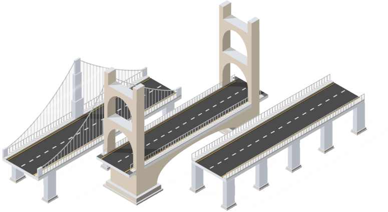
**Deterioration Model**

**Initial State Analyses**

**Results**

**Progress & Next Steps**

# Visual Inspection Data



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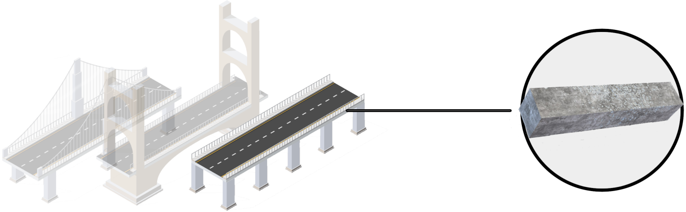

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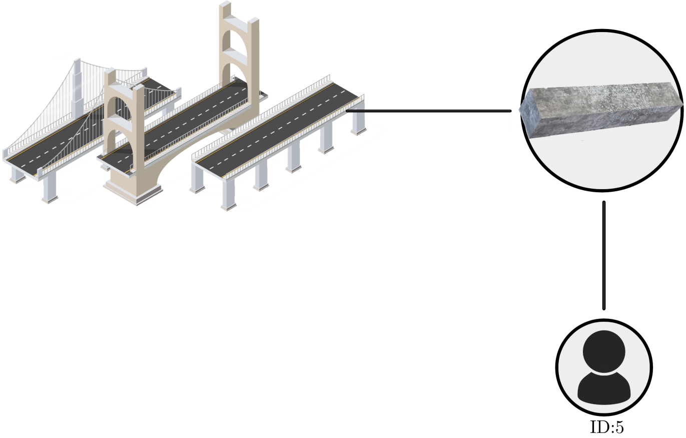

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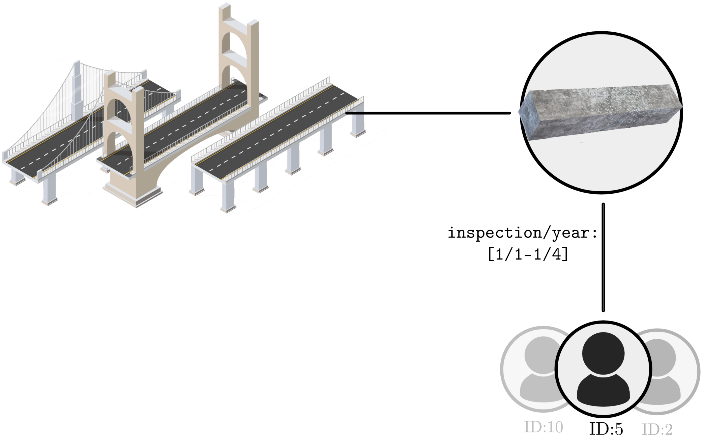

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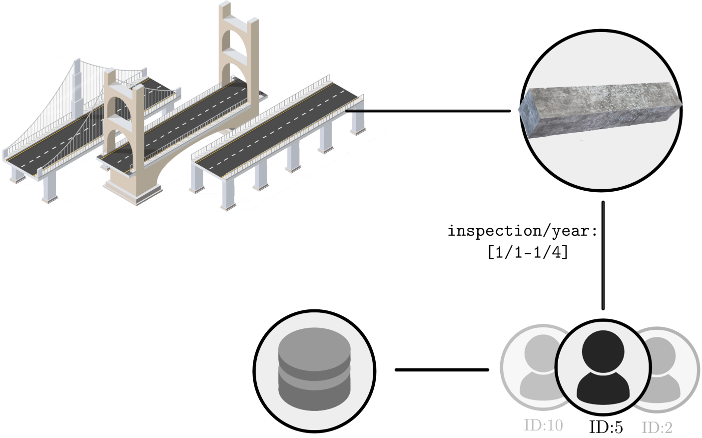

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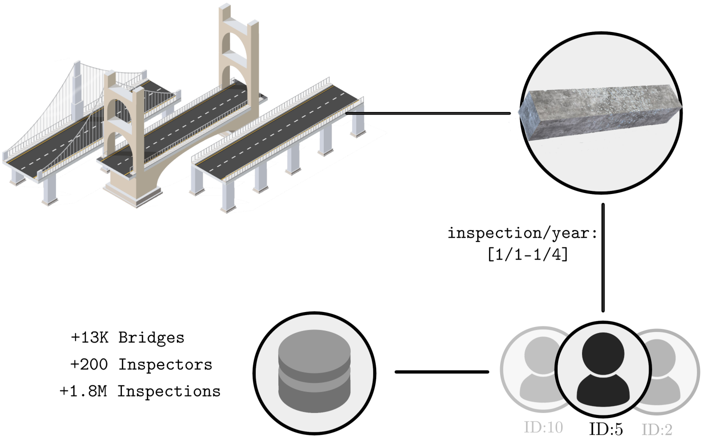
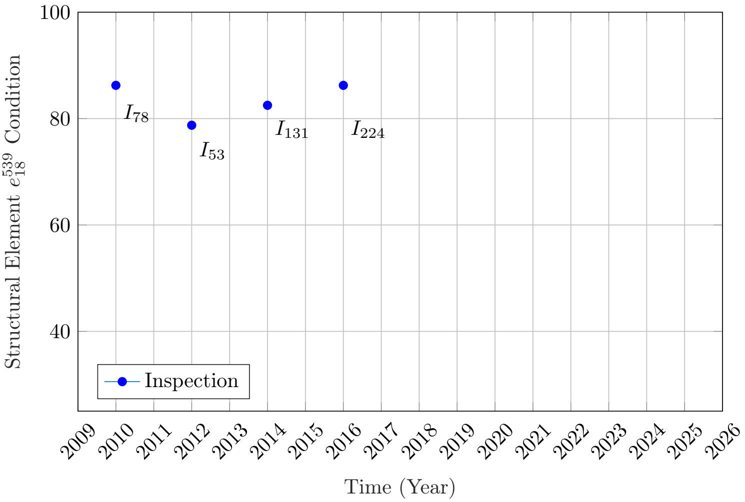


# Example: Series of Inspections on Structural Element



# Objectives

- Model the deterioration behaviour based on the data from network of bridges

# Deterioration Behaviour Described by Kinematics

$$\overbrace{x_t \quad = \quad x_{t-1} + \dot{x}_{t-1}\Delta t + \tfrac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w \quad \text{(Condition)}}^{\text{Kinematic Equations}}$$

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$$\overbrace{\begin{array}{rcll} x_t & = & x_{t-1} + \dot{x}_{t-1}\Delta t + \frac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w & \text{(Condition)} \\ \dot{x}_t & = & \dot{x}_{t-1} + \ddot{x}_{t-1}\Delta t + \dot{w} & \text{(Speed)} \end{array}}^{\text{Kinematic Equations}}$$

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$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}}_{\boldsymbol{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \\ \ddot{x}_{t-1} \end{bmatrix}}_{\boldsymbol{x}_{t-1}} + \underbrace{\begin{bmatrix} w_t \\ \dot{w}_t \\ \ddot{w}_t \end{bmatrix}}_{\boldsymbol{w}_t}$$

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**Method: State-Space Model**

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$$\overbrace{\boldsymbol{y}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t}^{\textbf{observation model}}, \; \underbrace{\boldsymbol{v}_t : \boldsymbol{V}(I_i) \sim \mathcal{N}(\boldsymbol{v}; \boldsymbol{0}, \boldsymbol{R}_t(I_i))}_{\textbf{observation error}} \; \underbrace{I_i \in [I_1, I_2, \ldots, I_{\mathrm{I}}] = \mathcal{I}}_{\textbf{inspectors}}$$

# Model Parameter Estimation

$$\mathcal{P} = \left\{ \underbrace{\sigma_v(I_1), \sigma_v(I_2), \cdots, \sigma_v(I_{\mathrm{I}})}_{\text{Inspector std.}}, \overbrace{\sigma_w}^{\text{Process error std.}}, \underbrace{n}_{\text{Transform. Param.}}, \overbrace{\dot{\mu}_0, \ddot{\mu}_0, \sigma_0^{x}, \sigma_0^{\dot{x}}, \sigma_0^{\ddot{x}}}^{\text{Initial state.}} \right\}$$

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$$\begin{aligned} \mathcal{P}^* = \arg\max_{\mathcal{P}} \quad & \mathcal{L}(\mathcal{P}), \\ \text{subject to:} \quad & \text{(parameters feasible domain)} \end{aligned}$$

# Bottleneck in the Model

$$\mathcal{P} = \left\{ \underbrace{\sigma_v(I_1), \sigma_v(I_2), \cdots, \sigma_v(I_{\mathrm{I}})}_{\text{Inspector std.}}, \overbrace{\sigma_w}^{\text{Process error std.}}, \underbrace{n}_{\text{Transform. Param.}}, \overbrace{\dot{\mu}_0, \ddot{\mu}_0, \sigma_0^{x}, \sigma_0^{\dot{x}}, \sigma_0^{\ddot{x}}}^{\text{Initial state.}} \right\}$$

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Challenges:

# Bottleneck in the Model

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Challenges:

- Very short time-series $\rightarrow$ has a significant impact.

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Challenges:

- Very short time-series $\rightarrow$ has a significant impact.
- Highly noisy data.

# Bottleneck in the Model

$$\mathcal{P} = \left\{ \underbrace{\sigma_v(I_1), \sigma_v(I_2), \cdots, \sigma_v(I_{\mathrm{I}})}_{\text{Inspector std.}}, \overbrace{\sigma_w}^{\text{Process error std.}}, \underbrace{n}_{\text{Transform. Param.}}, \overbrace{\dot{\mu}_0, \ddot{\mu}_0, \sigma_0^{x}, \sigma_0^{\dot{x}}, \sigma_0^{\ddot{x}}}^{\text{Initial state.}} \right\}$$

Challenges:

- Very short time-series $\rightarrow$ has a significant impact.
- Highly noisy data.
- Needs to be adaptive for each time-series (+10K).

# Plan de la section

**Initial State Analyses**

3.1 Initial State Parameters

3.2 Limitations with Current Approach

3.3 Initial State Toolbox

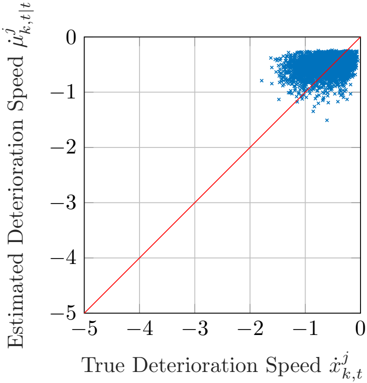
3.4 Proposed Initial State

# Initial State Parameters

$$\begin{aligned} \mu_0 &= y_1 && \| \quad \sigma_0^{\mu_0^2} = \max(p_3^2, \sigma_{Vi}^2) \\ \dot{\mu}_0 &= \mathrm{p}_1 \times (100 - \mu_{1|T}) && \| \quad \sigma_0^{\dot{\mu}_0^2} = p_1^2 \times \sigma_{1|T}^{\mu_0^2} + p_4^2 \\ \ddot{\mu}_0 &= \mathrm{p}_2 \times \dot{\mu}_0 && \| \quad \sigma_0^{\dot{\mu}_0^2} = p_2^2 \times \sigma_{1|T}^{\dot{\mu}_0^2} + p_6^2 \end{aligned}$$

# Estimating Speed after the First Observation:

Before:



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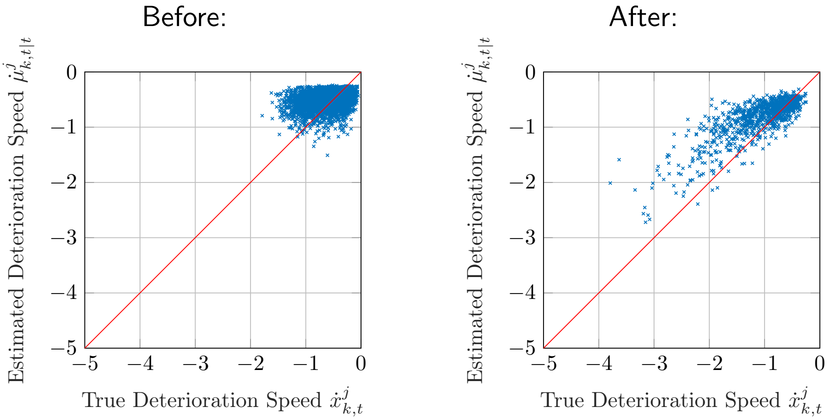
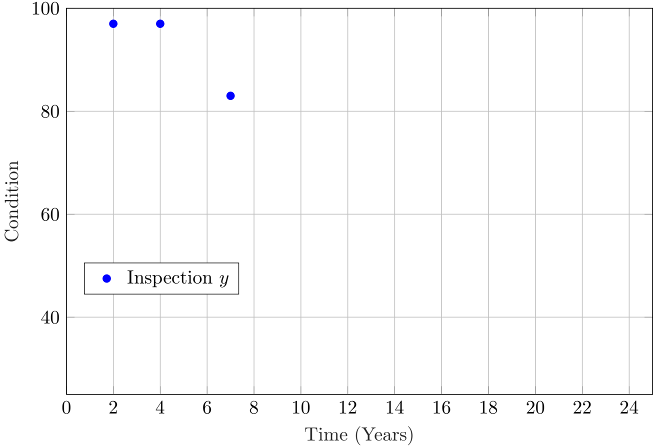


# Initial State



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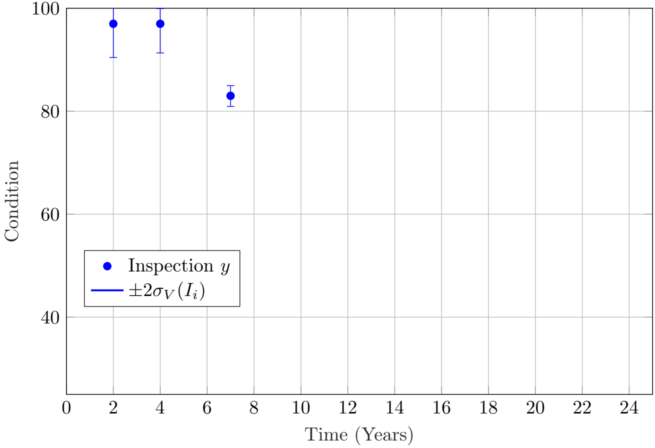

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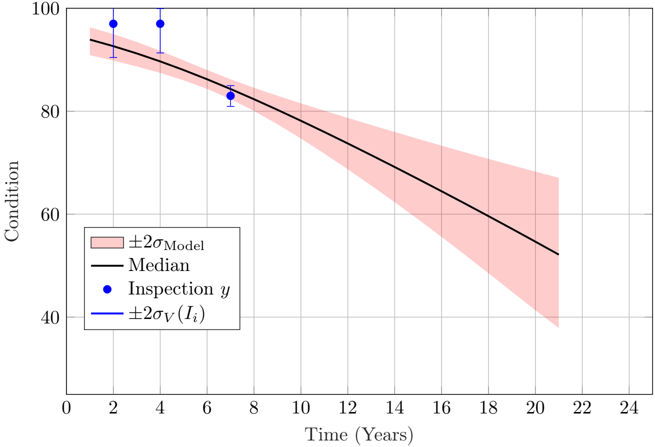

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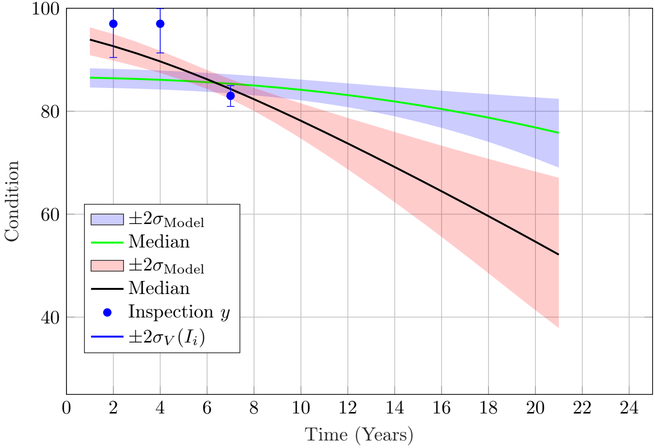
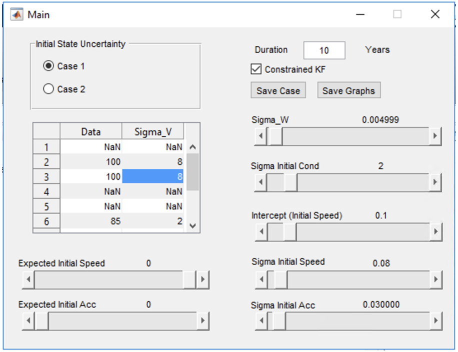


# Initial State

Problem: The parameter values that maximizes the likelihood can yield, in some cases, an unrealistic degradation behaviour.

# Assessing the Initial State

- Manually examine the parameters impact on different cases.



# Proposed Initial State

$$\mu_0 = y_1 \parallel \sigma_0^{\mu_0^2} = \max(p_3^2, \sigma_{Vi}^2)$$

$$\dot{\mu}_0 = 0 \parallel \sigma_0^{\dot{\mu}_0^2} = p_1^2 \times (100 - \mu_{1|T}) + p_2^2$$

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No Constraints at time (t=0)

Starts the Constraints at (t=1)

Constraints: $\dot{\mu} + 2\sigma^{\dot{\mu}} < 0$

# Proposed Initial State

$$\mu_0 = y_1 \;\|\; \sigma_0^{\mu_0^2} = \max(p_3^2, \sigma_{V_i}^2)$$

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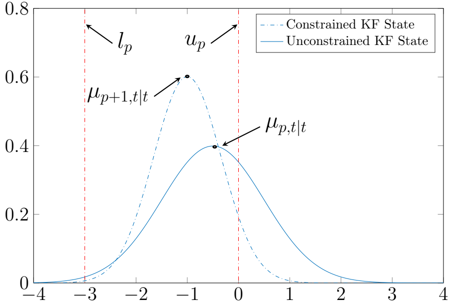
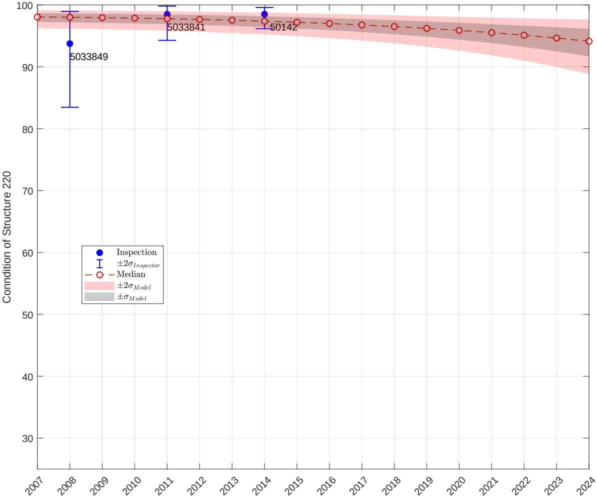


# Plan de la section

**Results**

4.1 Real Data Results
4.2 Synthetic Data Results

# Real Data Cases



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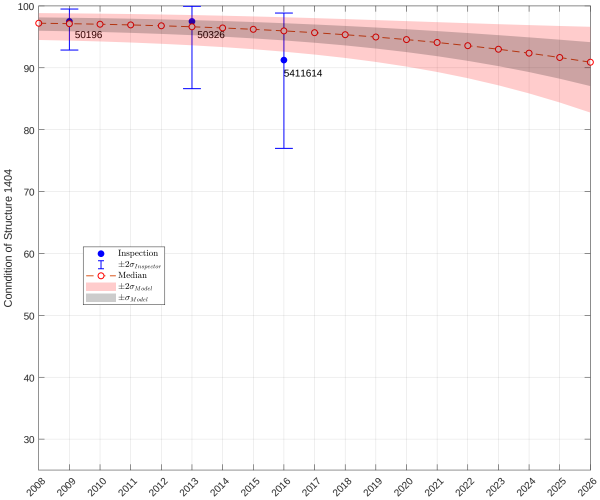

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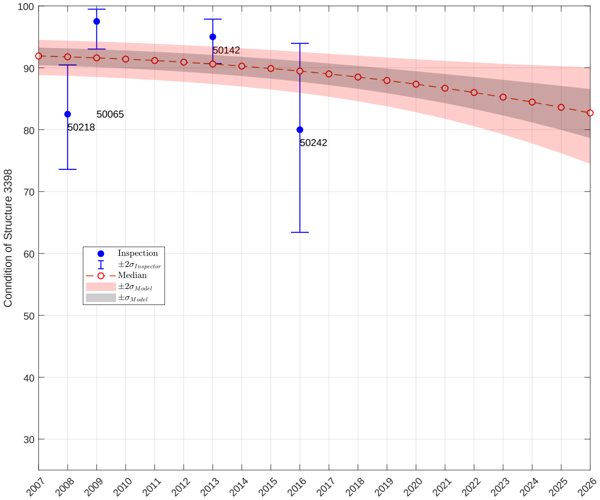

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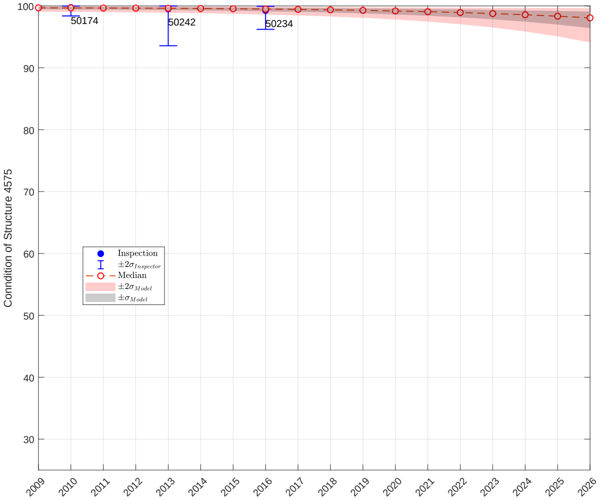

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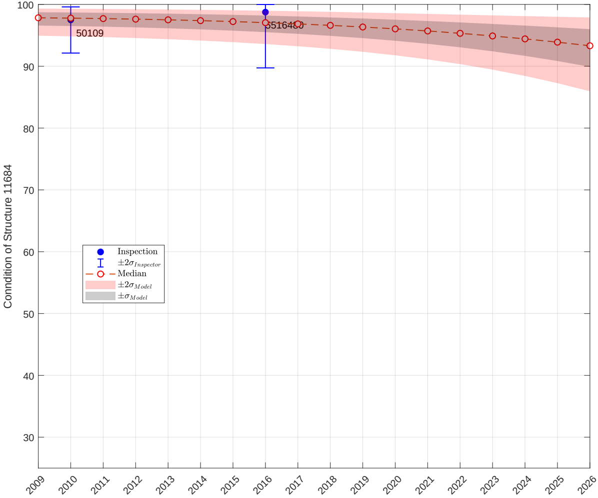
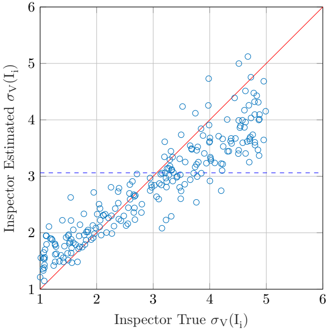


# Estimating Inspectors Uncertainties:

Before:



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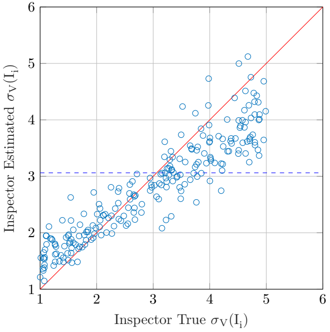
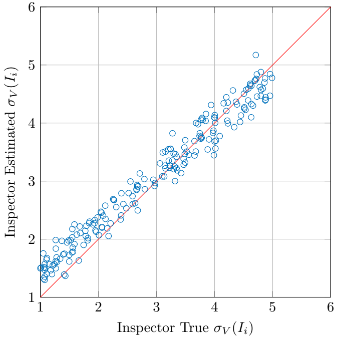
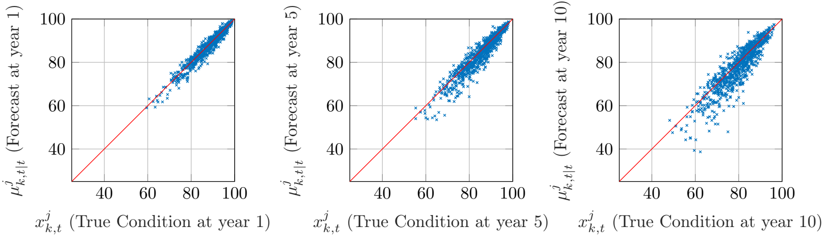



Now:



# Estimating The Condition:



# Estimating The Speed:

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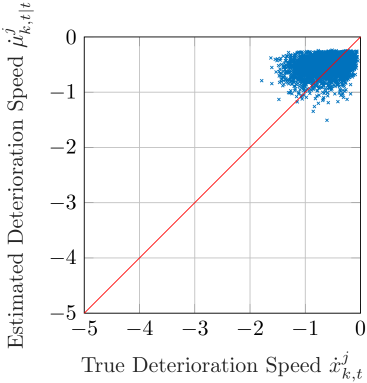

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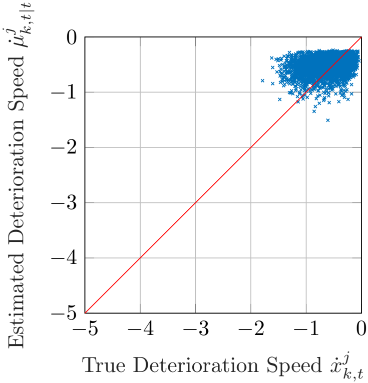


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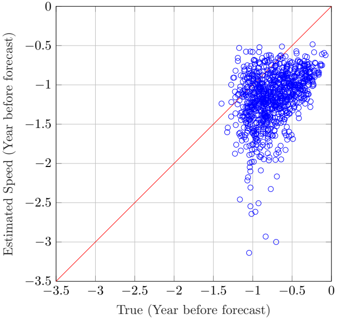

# Plan de la section

**Progress & Next Steps**
5.1 Project Progress
5.2 Next Steps

# Advancement Summary

1. Performed Initial State Analyses under different configurations.

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1. Performed Initial State Analyses under different configurations.
2. Build Initial State Analyses Toolbox.
3. Improved the estimation for the uncertainty of inspectors.
4. Improved the degradation characterization in the model.

# Next:

1. Code Optimization.

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2. Build a toolbox to investigate the best configuration for the speed.