Factoring Structural Attributes in Infrastructures' Deterioration Analysis

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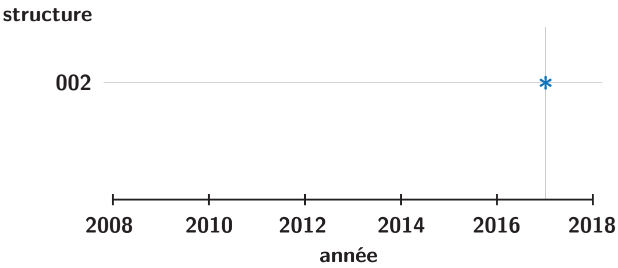
Transports, Mobilité durable et Électrification des transports Québec Partenaire

Outline

Database of Visual Inspections

Year: 2017
Structure: #002

Database of Visual Inspections



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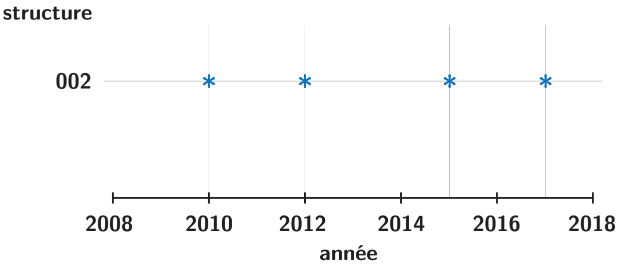

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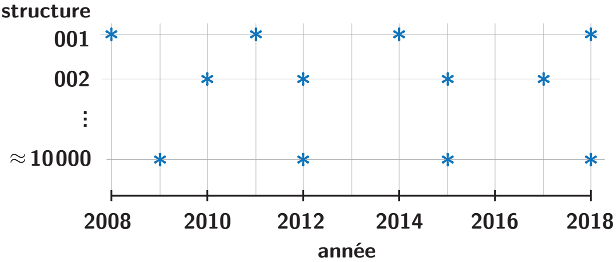

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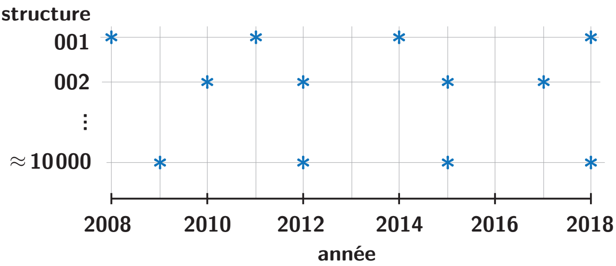
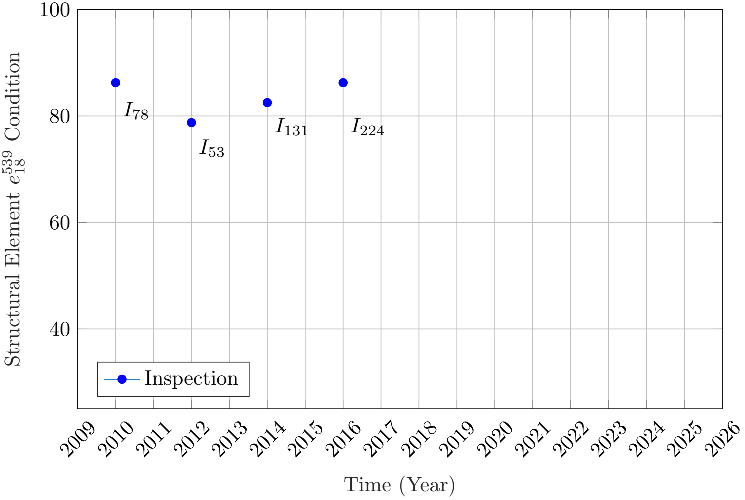


Year: 2017
Structure: #002

$\mathbf{b}_{002}$: {type, material, DJMA, Location,...}

Example: Series of Inspections on Structural Element

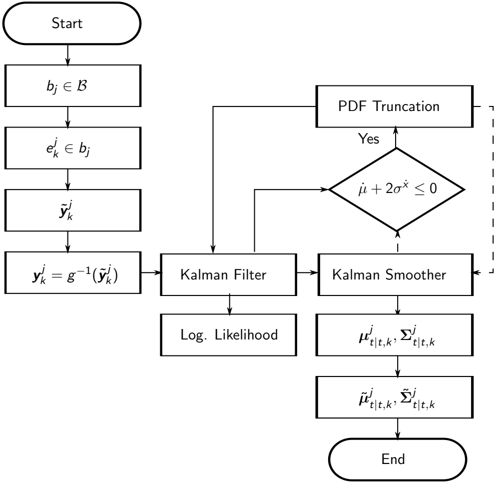


Objectives

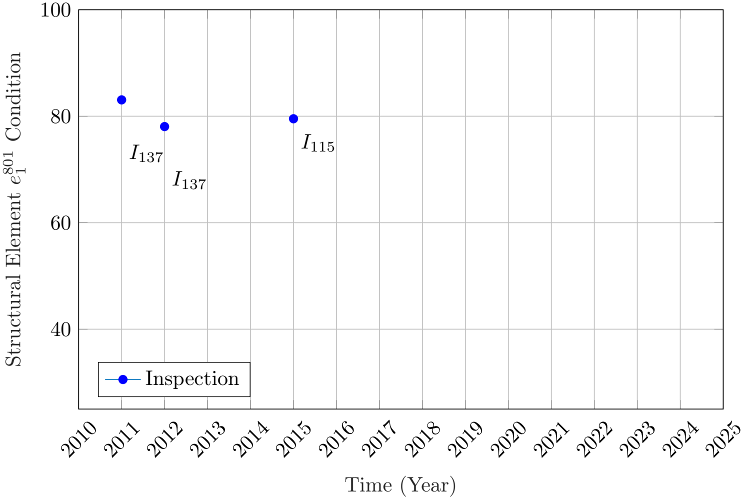
- Model the deterioration behaviour based on the data from network of bridges

Plan de la section

Deterioration Model Flowchart



Deterioration Model Performance: Condition Estimate



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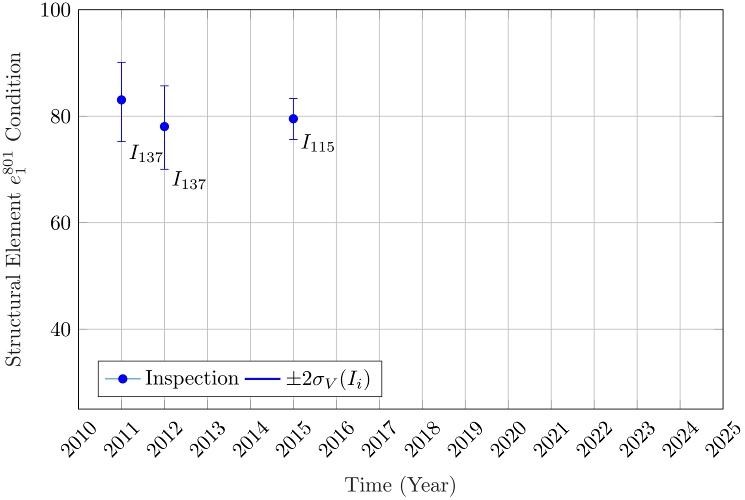

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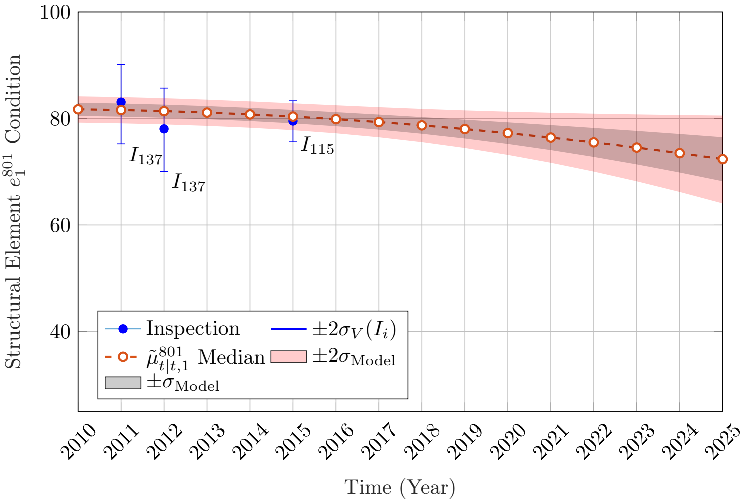

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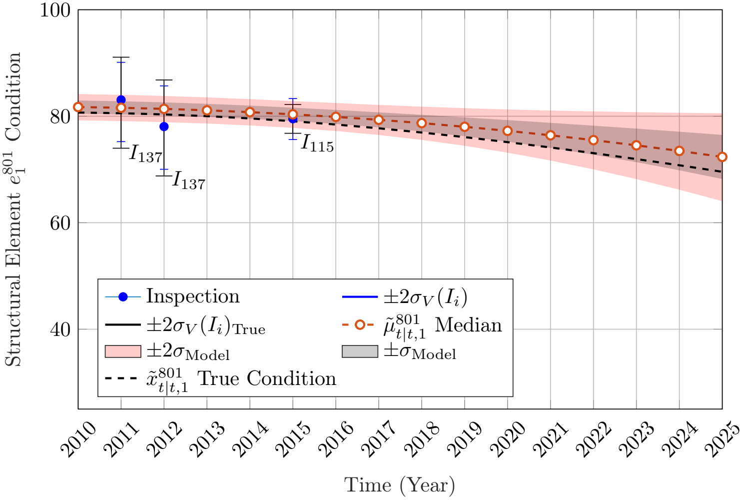
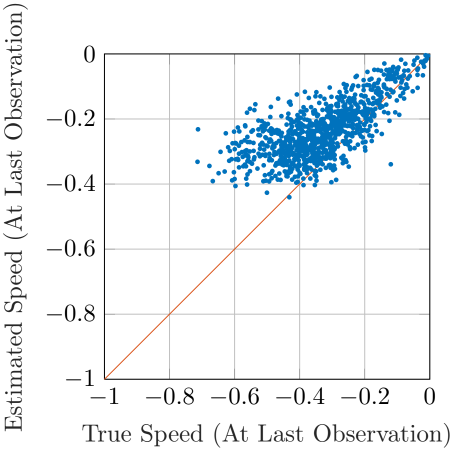


Deterioration Model Performance: Speed Estimate



Limitations & Opportunities:

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Opportunities:

- There are similarities across the structures/structural elements.

Use regression to incorporate the similarity information across the structures.

Plan de la section

Concept & Notations

For structural attributes: $\mathcal{Z} = \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{\mathrm{B}}\}$

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Define a similarity measure across structures,

$$k(\boldsymbol{z}_i, \boldsymbol{z}_j) = \exp\left(-\frac{1}{2}\sum_{p=1}\left(\frac{z_{i,p} - z_{j,p}}{l_p}\right)^2\right).$$

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$$k(\boldsymbol{z}_i, \boldsymbol{z}_j) \in [0, 1]$$

$\boldsymbol{z}_i$ is the same for all structural elements in bridge $\boldsymbol{b}_i$

Kernel Regression

$$h(z) = \frac{\sum_{i=1}^{N} k(z, z_i) x_{z,i}}{\sum_{j=1}^{N} k(z, z_j)}$$

- N: number of control point (unknown).

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- $x_{z,i}$: the i-th hidden control point (unknown).

Kernel Regression

$$h(z) = \frac{k(z, z_1)}{\sum_{j=1}^{N} k(z, z_j)} x_{z,1} + \frac{k(z, z_2)}{\sum_{j=1}^{N} k(z, z_j)} x_{z,2} + \cdots + \frac{k(z, z_N)}{\sum_{j=1}^{N} k(z, z_j)} x_{z,N},$$

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$$h(z) = \begin{bmatrix} \lambda_1 & \lambda_2 & \ldots & \lambda_N \end{bmatrix} \times \begin{bmatrix} x_{z,1} \\ x_{z,2} \\ \cdots \\ x_{z,N} \end{bmatrix}$$

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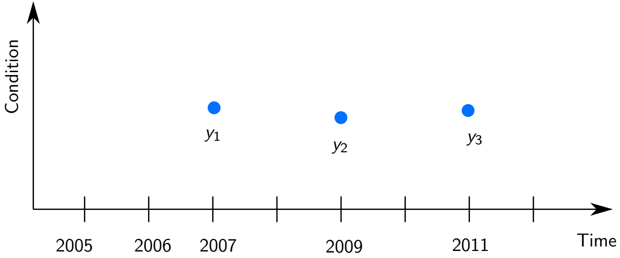
Estimating x_z: Hypotheses



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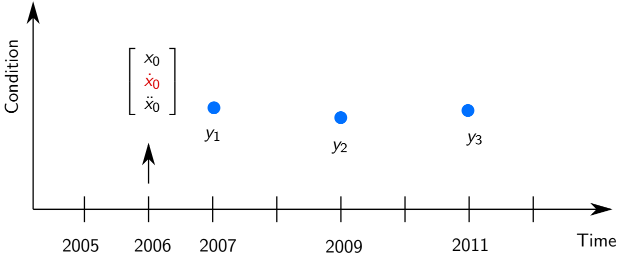
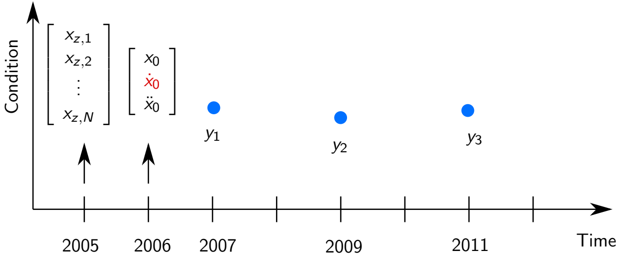


- Regression is performed for the initial time step only.

Estimating $\boldsymbol{x_z}$: Hypotheses



- Regression is performed for the initial time step only.
- No correlation between $\boldsymbol{x_z}$ and the components of $\boldsymbol{x}^{Ki}$.

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The prediction for the speed at time $(t = 0)$,

$$\boldsymbol{x}^j_{0|\boldsymbol{z},k} = \boldsymbol{A}^{\mathtt{Kr}}\boldsymbol{x}^j_{t,k} + \boldsymbol{w}^{\mathtt{Kr}} : \boldsymbol{W}^{\mathtt{Kr}} \sim \mathcal{N}(\boldsymbol{w}^{\mathtt{Kr}}; \mathbf{0}, \boldsymbol{Q}^{\mathtt{Kr}}),$$

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where:

$$\mathbf{A}^{\mathtt{Kr}} = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0}_{1\times\mathtt{N}} \\ 0 & 0 & 0 & \boldsymbol{\lambda}_{1\times\mathtt{N}} \\ 0 & 0 & 1 & \mathbf{0}_{1\times\mathtt{N}} \\ 0 & 0 & 0 & \boldsymbol{I}_{\mathtt{N}\times\mathtt{N}} \end{bmatrix}, \mathbf{Q}^{\mathtt{Kr}} = \textbf{Block diag}\left(\mathbf{0}, (\sigma_{W,0}^{1})^{2} \times \boldsymbol{I}_{\mathtt{N}\times\mathtt{N}}\right)$$

The transition model for time steps $t = 0$ to T,

$$\boldsymbol{x}^j_{t,z|(t-1,z),k} = \boldsymbol{A}^{\mathtt{Se}} \boldsymbol{x}^j_{(t-1,z)|((t-1,z),k)} + \boldsymbol{w} : \boldsymbol{W} \sim \mathcal{N}(\boldsymbol{w}; \boldsymbol{0}, \boldsymbol{Q}).$$

where,

$$\boldsymbol{A}^{\mathtt{Se}} = \textbf{Block diag}\left(\boldsymbol{A}^{\mathtt{Ki}}, \boldsymbol{I}_{\mathtt{N}\times\mathtt{N}}\right),$$

$$\boldsymbol{Q} = \textbf{Block diag}\left(\boldsymbol{Q}^{\mathtt{Ki}}, \boldsymbol{0}_{\mathtt{N}\times\mathtt{N}}\right),$$

$\boldsymbol{A}^{\mathtt{Ki}}$ is defined by,

$$\mathbf{A}^{\mathtt{Ki}} = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}.$$

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(too many structural elements).

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$\text{KalmanFilter}(\boldsymbol{\mu}_{t-1|(t-1,z)}, \boldsymbol{\Sigma}_{t-1|(t-1,z)}, \boldsymbol{y}_t, \boldsymbol{A}_t, \boldsymbol{Q}_t, \boldsymbol{C}_t, \boldsymbol{R}_t),$

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$$\dot{\boldsymbol{\mu}}_{0|T,\boldsymbol{z}} \in \mathbb{R}^{\mathtt{E}\times 1},\ \dot{\boldsymbol{\Sigma}}_{0|T,\boldsymbol{z}} = \textbf{Block diag}\left((\dot{\boldsymbol{\sigma}}^{1}_{0|\boldsymbol{z},1})^2, \ldots, (\dot{\boldsymbol{\sigma}}^{\mathtt{B}}_{0|\boldsymbol{z},\mathtt{E}})^2\right),$$

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$$\boldsymbol{A}^{\mathrm{Ba}} = [\boldsymbol{\lambda}_1 \ \ldots \ \boldsymbol{\lambda}_{\mathtt{E}}]^{\intercal} \in \mathbb{R}^{\mathtt{E}\times\mathtt{N}},$$

Estimating $\boldsymbol{x_z}$: Batch Method

For each element $e_k \in \mathtt{E}$:

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Estimating $\boldsymbol{x_z}$: Batch Method

Loop
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  $\rightarrow \boldsymbol{\mu}_{\boldsymbol{z}|T}$ high values.

Synthetic Structural Attribute

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$$z^1 = exp(\dot{x})$$

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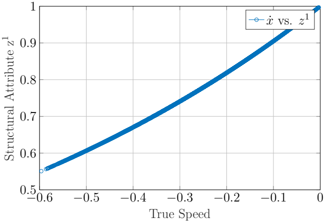

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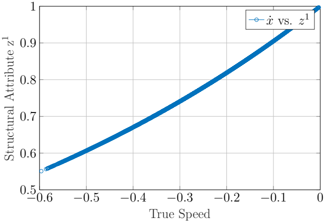

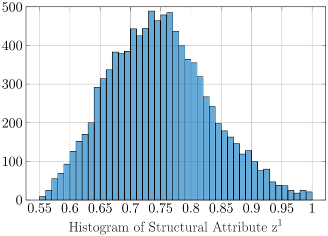

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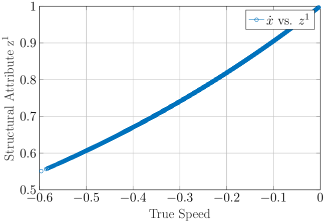
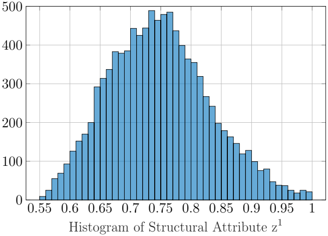
$z^2 = log(|\dot{x}|)$





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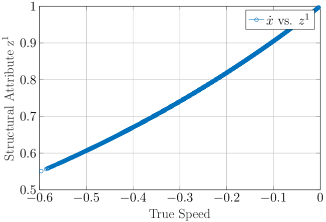

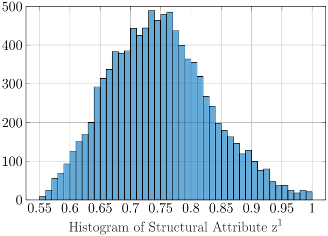

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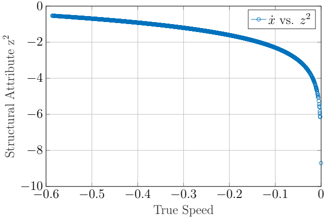

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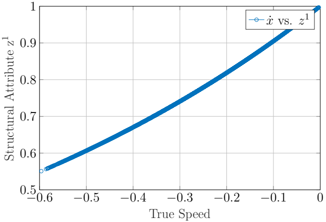

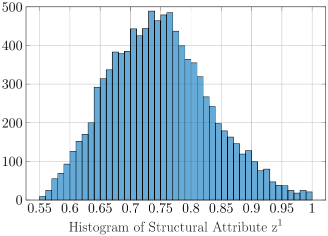

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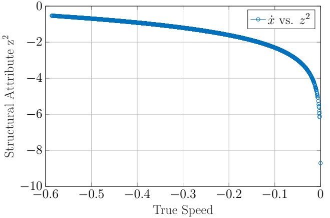

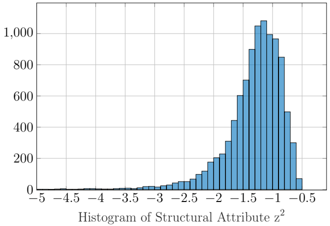

Estimating $\dot{\mu}_0$ from z^1

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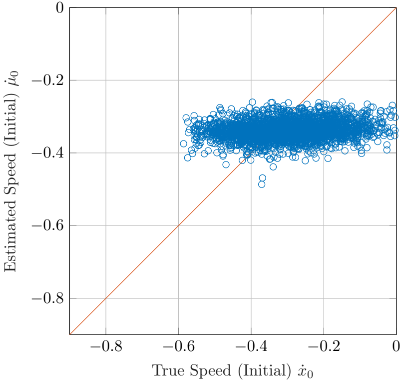

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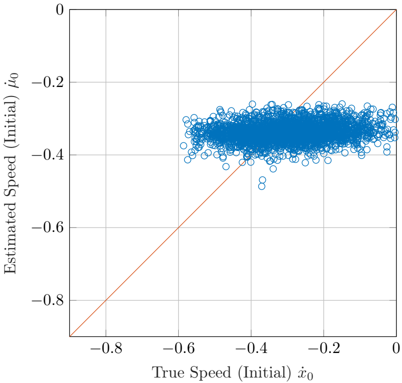

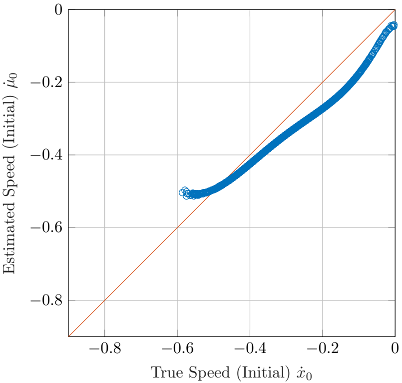

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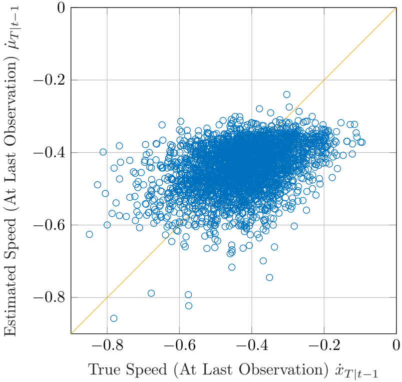

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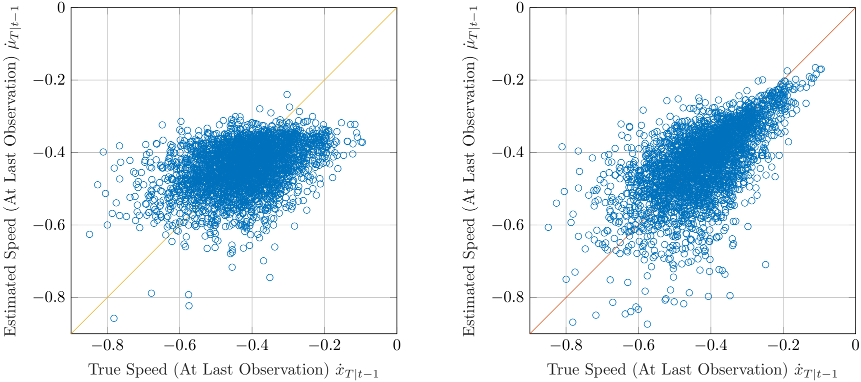

Estimating $\dot{\mu}_0$ from z^2

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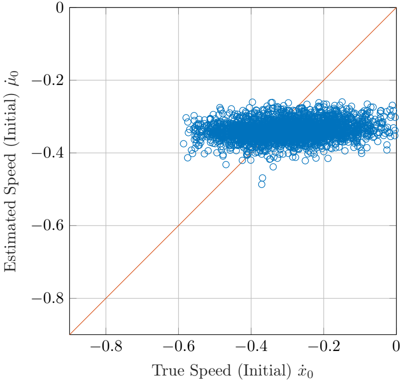

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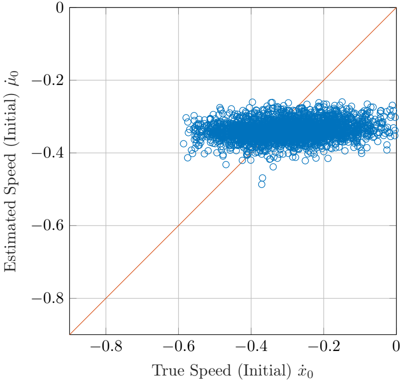

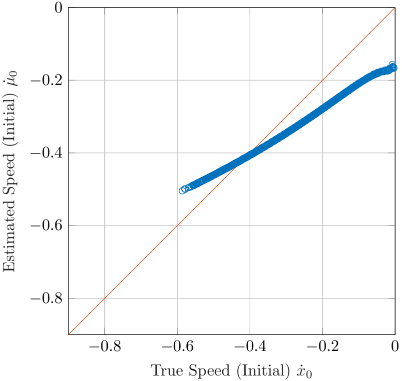

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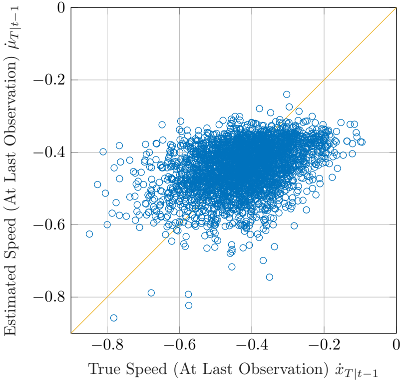

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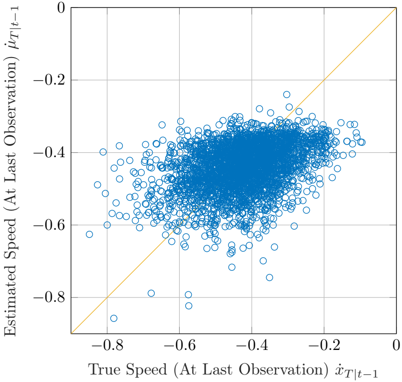

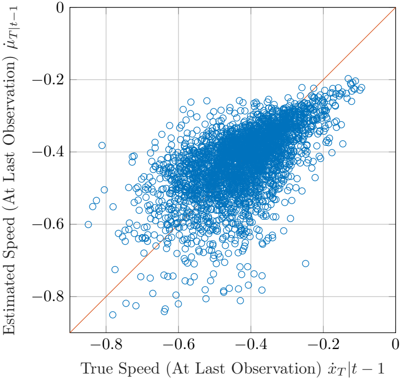

Next:

1. Assess the performance of Kernel Regression with noisy functions.

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1. Assess the performance of Kernel Regression with noisy functions.
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3. Perform analyses on Real Data.