# State-Space Models for Network-Scale Analysis of Bridge Inspection Data

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Department of civil engineering, geology and mines

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Funding:
Ministry of Transportation of Quebec

# Visual Inspections

Large-scale health monitoring of infrastructures (i.e. bridges) over time.

Year: 2017
Structure: 002

$$\mathbf{b}_{002} = \begin{Bmatrix} e_1^2 \\ e_2^2 \\ \vdots \\ e_{\mathrm{E}}^2 \end{Bmatrix}$$

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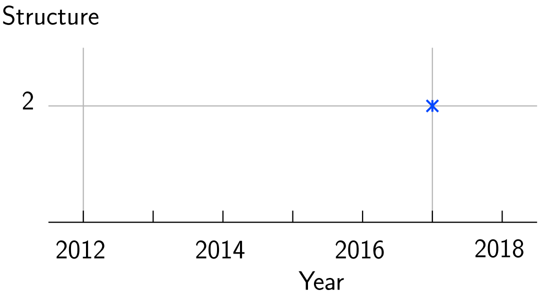


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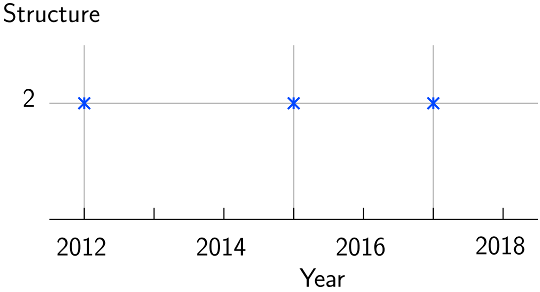


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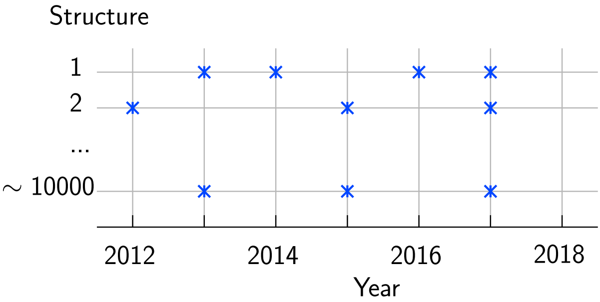


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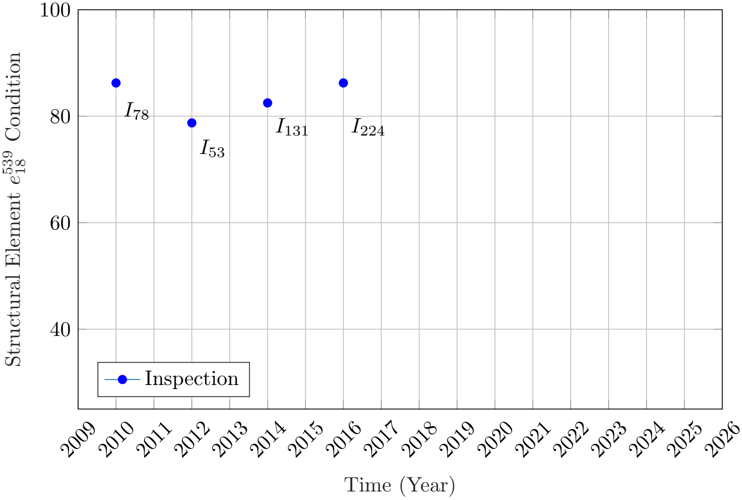
# Example of Inspection Data for a Structural Element



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  - Quantify the **uncertainty of inspections**.
  - Estimate the **deterioration rate**.

# Deterioration Behaviour Described by Kinematics

$$\overbrace{x_t \quad = \quad x_{t-1} + \dot{x}_{t-1}\Delta t + \tfrac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w \quad \text{(Condition)}}^{\text{Kinematic Equations}}$$

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$$\overbrace{\begin{array}{lcll} x_t & = & x_{t-1} + \dot{x}_{t-1}\Delta t + \frac{1}{2}\ddot{x}_{t-1}\Delta t^2 + w & \text{(Condition)} \\ \dot{x}_t & = & \dot{x}_{t-1} + \ddot{x}_{t-1}\Delta t + \dot{w} & \text{(Speed)} \end{array}}^{\text{Kinematic Equations}}$$

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# Deterioration Behaviour Described by Kinematics

$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}}_{\boldsymbol{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \\ \ddot{x}_{t-1} \end{bmatrix}}_{\boldsymbol{x}_{t-1}} + \underbrace{\begin{bmatrix} w_t \\ \dot{w}_t \\ \ddot{w}_t \end{bmatrix}}_{\boldsymbol{w}_t}$$

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**Method: State-Space Models**

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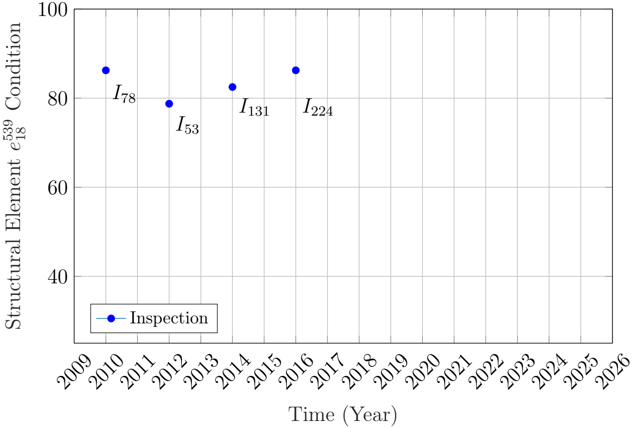
# Uncertainty of Observations (Inspectors)



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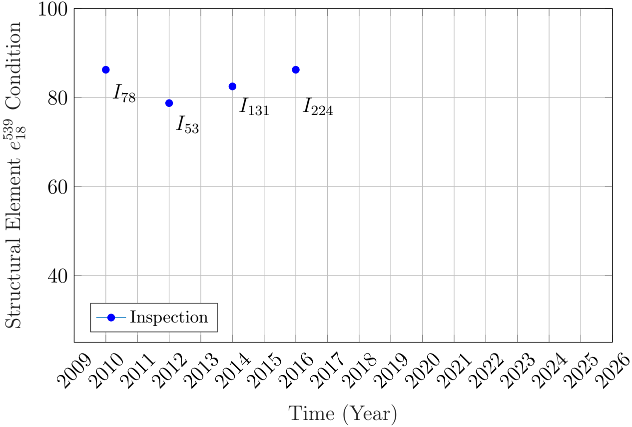


**The Uncertainty of Observations $\tilde{y}$ Depends on the Inspector Responsible for the Evaluation**

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**Proposed Solution:**

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$$\overbrace{\boldsymbol{y}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t}^{\text{observation model}},\ \underbrace{\boldsymbol{v}_t : \boldsymbol{V}(I_i) \sim \mathcal{N}(\boldsymbol{v}; \mathbf{0}, \boldsymbol{R}_t(I_i))}_{\text{observation error}}$$

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$$\underbrace{I_i \in [I_1, I_2, \ldots, I_{\mathrm{I}}] = \mathcal{I}}_{\text{inspectors}}$$

# Uncertainty of Observations (Condition)

Source: Google images

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$Y = 25$ $Y = 100$

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# Uncertainty of Observations (Condition)

$Y = 25$ $\qquad$ $Y = 100$

**The Uncertainty of Observations $\tilde{y}$ Depends on the Deterioration State**

Source: Google images

**Proposed Solution:**

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1. Uncertainty dependent on the state → **Non-linear Transformation**

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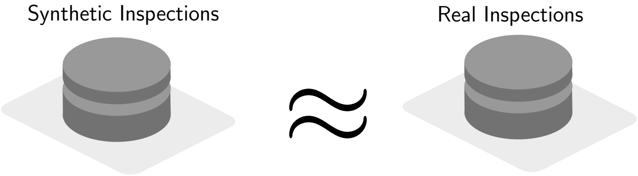
Space Transformation Function Characteristics:

1. Uncertainty dependent on the state $\rightarrow$ **Non-linear Transformation**
2. Bound the deterioration condition estimate $\tilde{x} \in [25, 100] \rightarrow$ **Step Function**

# Synthetic Inspection Data



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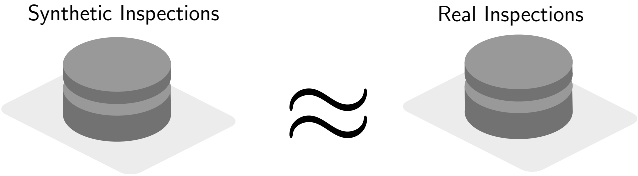


- # Structural Elements $\mathtt{E} = 10827$.

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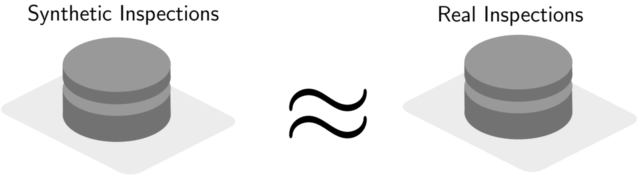
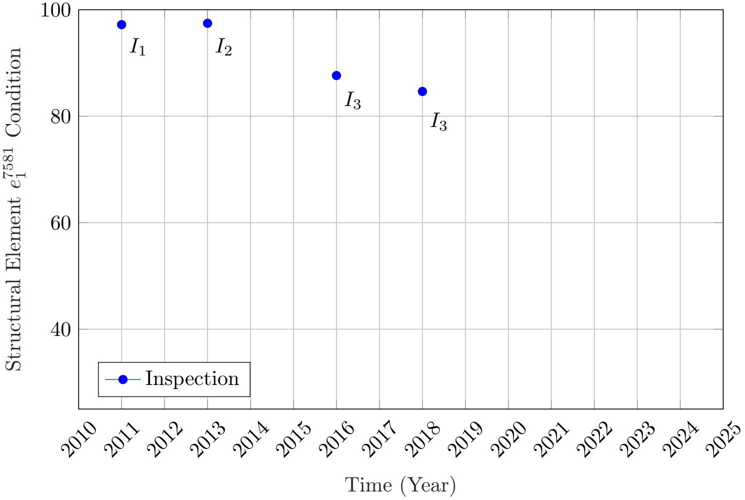



- # Structural Elements $\mathtt{E} = 10827$.
- # Inspectors $\mathtt{I} = 194$.

# Example of Synthetic Data:



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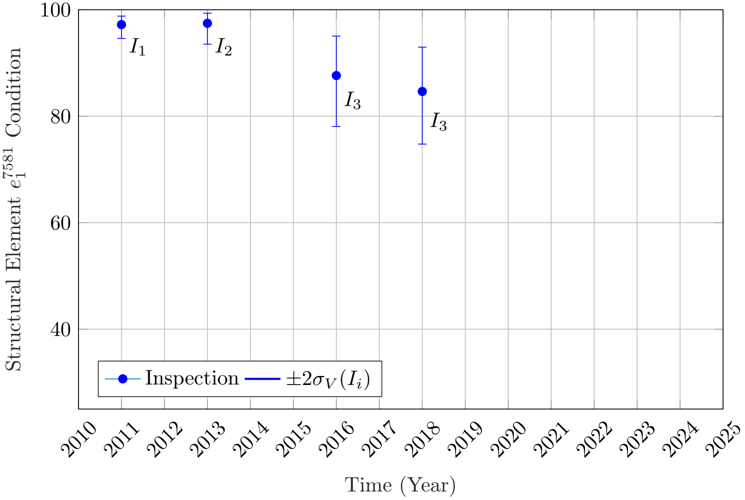

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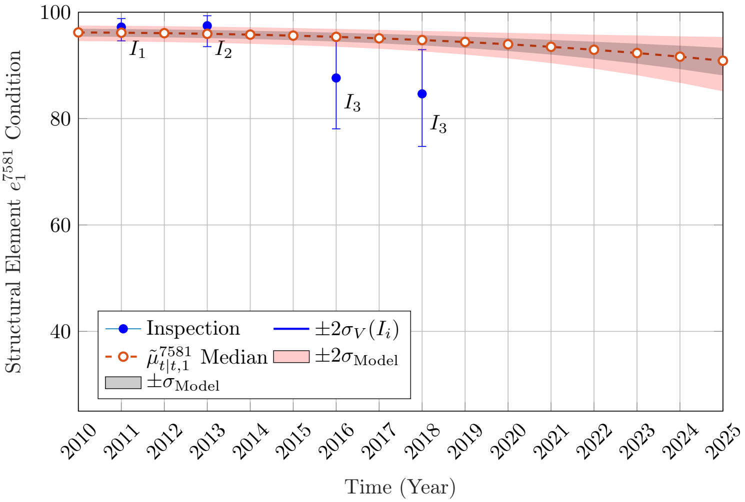

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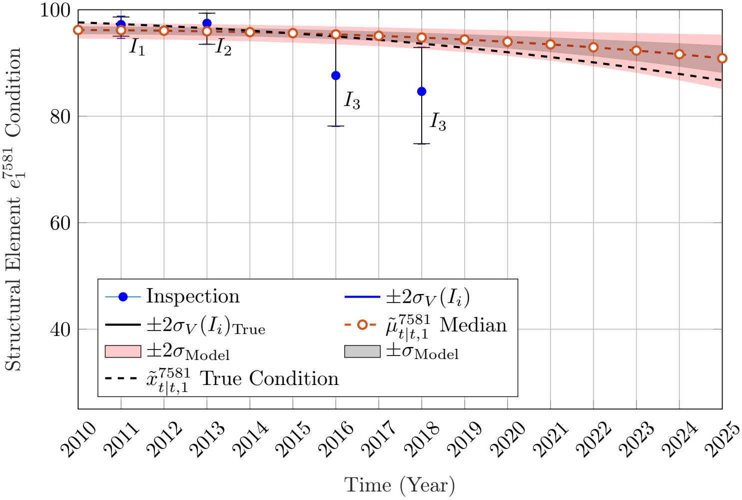
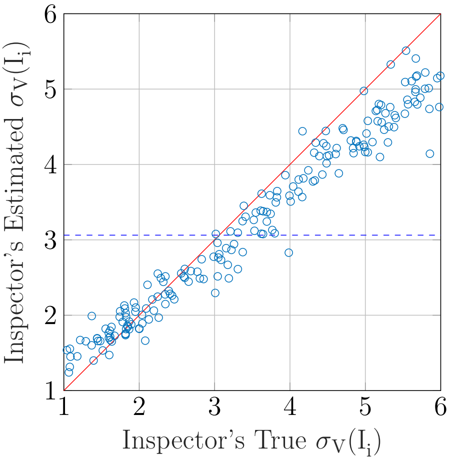
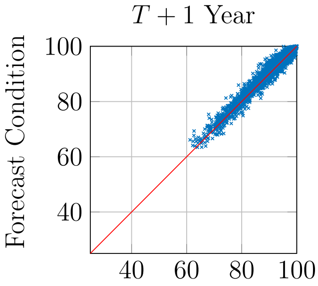


# Estimating Inspectors Uncertainty

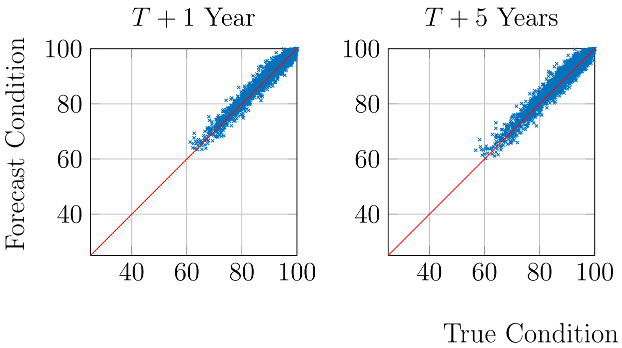


# Overall Model Performance




True Condition

# Overall Model Performance



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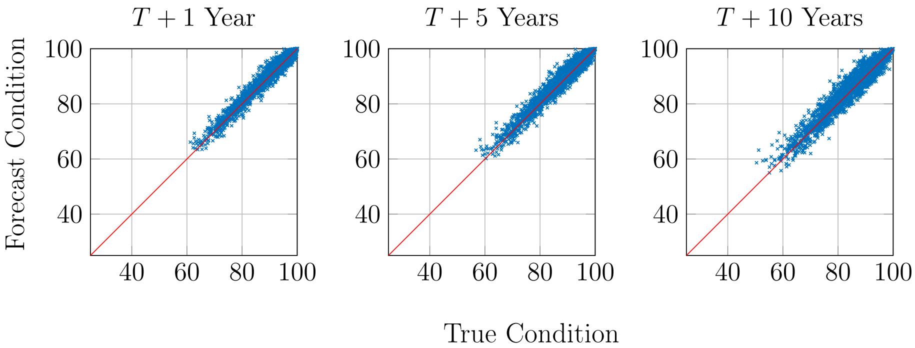
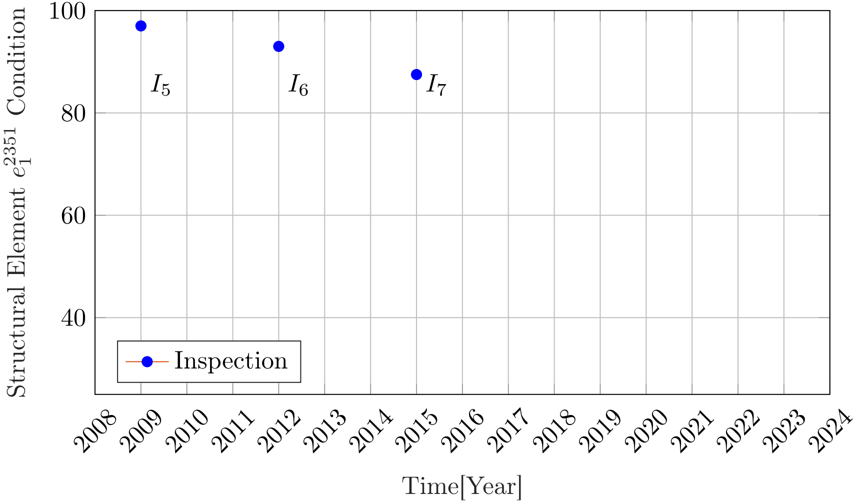


# Real Data - Case A



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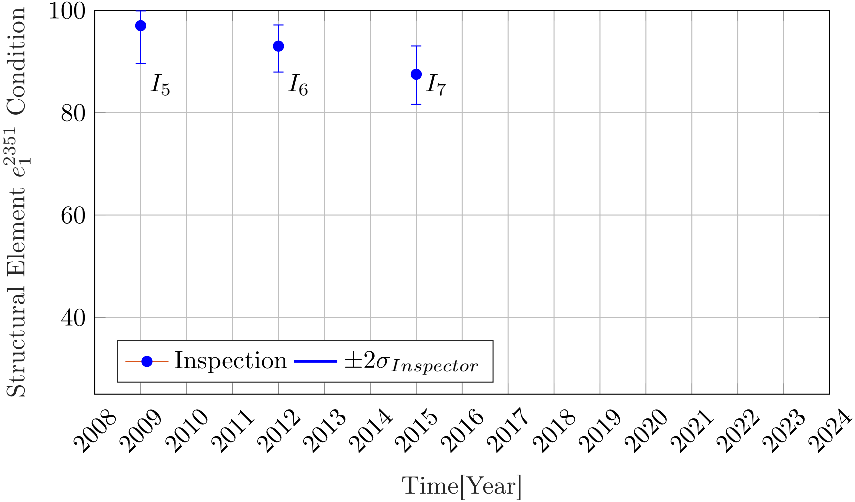

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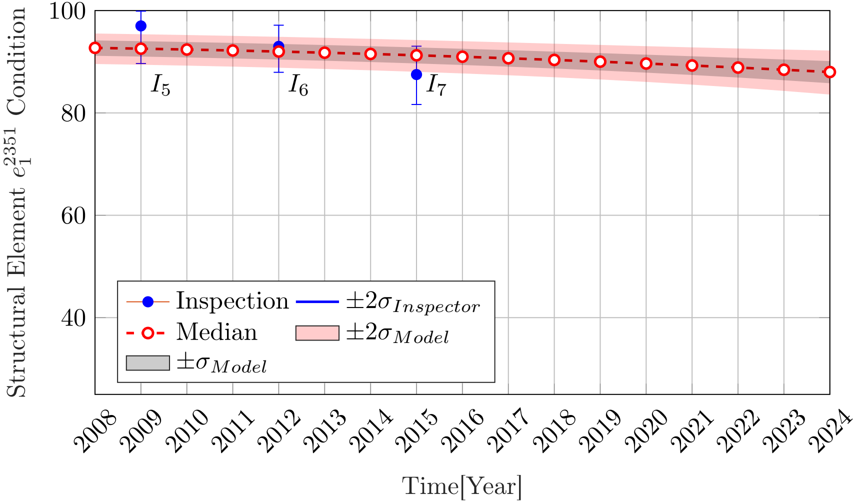

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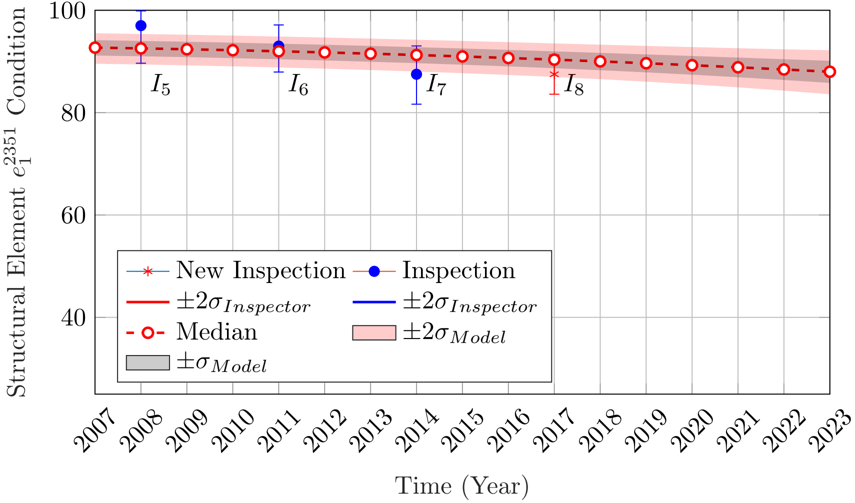
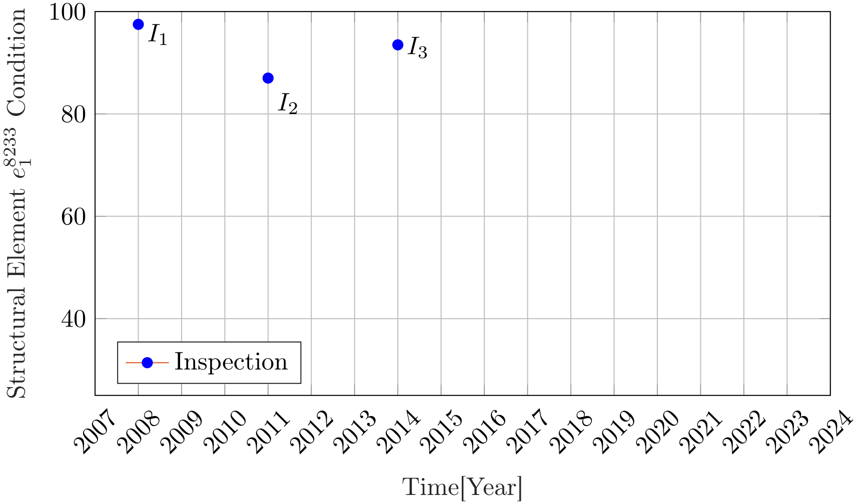


# Real Data - Case B



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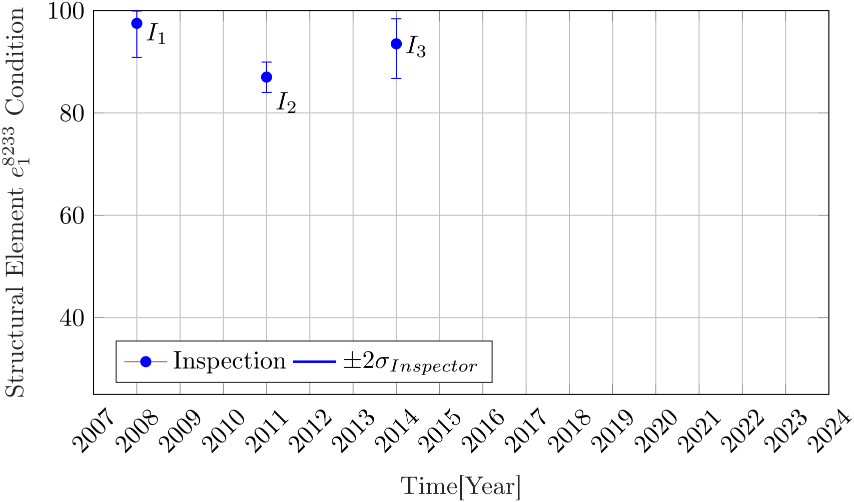

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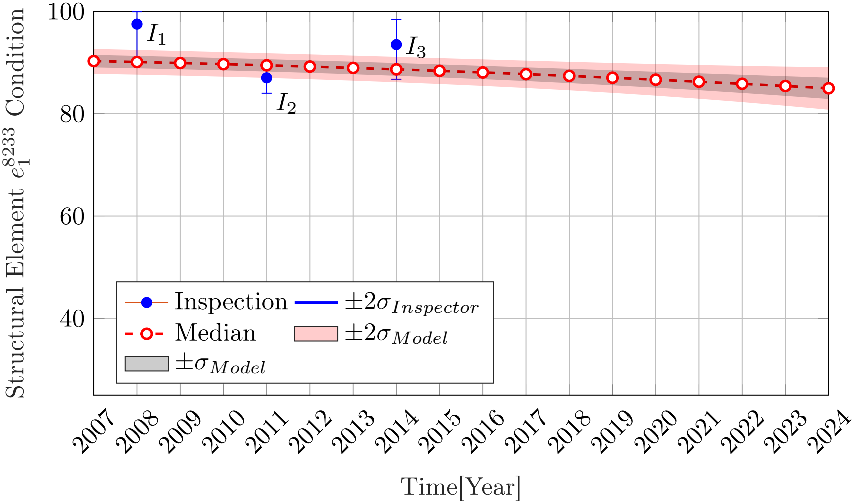

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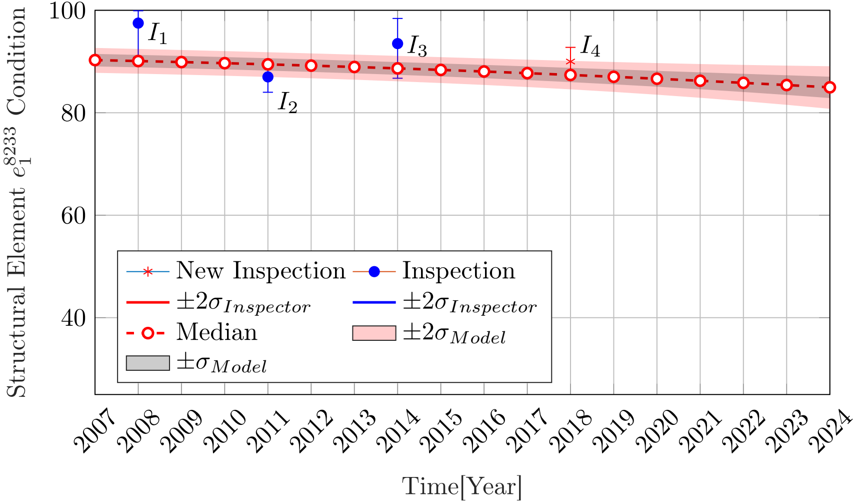
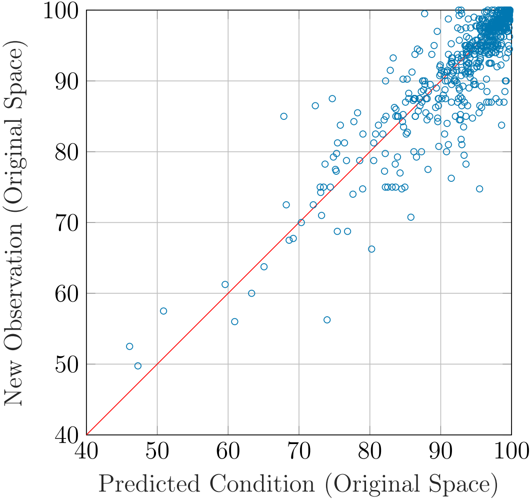


# Condition Validation - Uncertainty



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2. The uncertainty of visual inspections is quantified according to:
    a. The inspector performing the evaluations.
    b. The deterioration state of the structural element.
3. The predictive capacity for the condition and speed is verified & validated.