MA 16200: Plane Analytic Geometry and Calculus II

Lecture 29: Introduction to Polar Coordinates

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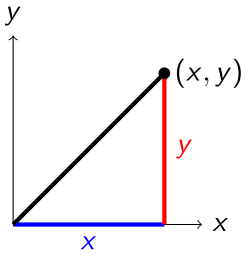
Sections Covered: 12.2 (Part I)

Cartesian Coordinates

Our usual system is called **Cartesian coordinates** (or Rectangular coordinates or Box coordinates). A point in space is described using the ordered pair

$$(x, y)$$

where x is the horizontal component and y is the vertical component.

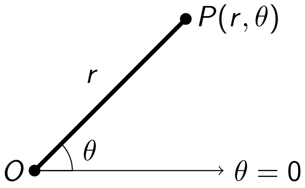


Polar Coordinates (When $r \geq 0$)

In **Polar Coordinates**, a point in space is described using the ordered pair

$$(r, \theta)$$

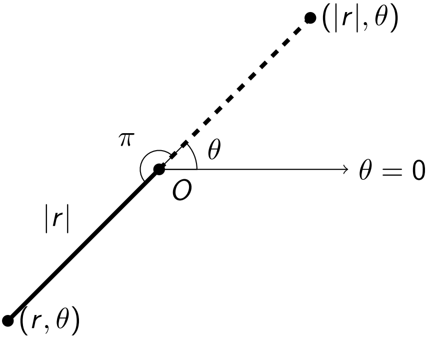
where r (called the **radial coordinate**) is the distance from the origin (called the **pole**) and θ is angle the ray $\overrightarrow{OP}$ makes with the positive x-axis (called the **polar axis**). Positive angles are measured counterclockwise.



Polar Coordinates (When $r < 0$)

When r is negative,

$$(r, \theta) \overset{def}{=} (|r|, \theta + \pi)$$

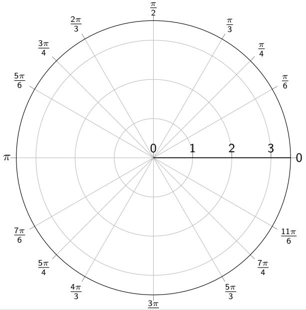


Example

Problem 1

Plot the points whose polar coordinates are given:

$$(a)\ (1, 5\pi/4) \quad (b)\ (2, 3\pi) \quad (c)\ (2, -2\pi/3) \quad (d)\ (-3, 3\pi/4)$$



Representations are not Unique

Unlike in Cartesian coordinates, there are multiple ways to describe the same point in space. Why?

In practice, we restrict $r \geq 0$ and $\theta \in (-\pi, \pi]$ to make it unique (although this is not required). This is useful in plotting points, not necessarily when plotting curves.

Example

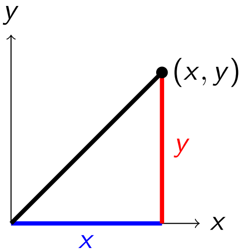
Problem 2

Give two alternative representations for each point:

$$(a)\ (1, 5\pi/4) \quad (b)\ (2, 3\pi) \quad (c)\ (2, -2\pi/3) \quad (d)\ (-3, 3\pi/4)$$

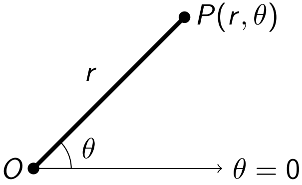
From Cartesian to Polar

How can we convert a point in Cartesian coordinates to Polar?



From Polar to Cartesian

How can we convert a point in Polar coordinates to Cartesian?



Summary of Formulas

Theorem 3 (Converting Coordinates)

A point with polar coordinates (r, θ) *has Cartesian coordinates* (x, y)*, where:*

$$x = r\cos\theta \quad y = r\sin\theta$$

A point with Cartesian coordinates (x, y) *has polar coordinates* (r, θ)*, where:*

$$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$$

Example

Problem 4

Express the point with polar coordinates $P(2, \frac{3\pi}{4})$ *in Cartesian coordinates.*

Example

Problem 5

Express the point with Cartesian coordinates $Q(1, -1)$ *in Polar Coordinates.*

Example

Problem 6

Express the point with polar coordinates $P(2, \frac{\pi}{3})$ in Cartesian coordinates.

Example

Problem 7

Express the point with Cartesian coordinates $(-4, -4\sqrt{3})$ *in Polar Coordinates.*

Functions in Polar Coordinates

We can relate r and θ like in Cartesian, these are called **polar curves**.

	Cartesian	Polar
Explicit	$y = f(x)$	$r = f(\theta)$
	$y = 3x$	$r = 1 + \sin\theta$
Implicit	$F(x, y) = 0$	$F(r, \theta) = 0$
	$x^2 + y^2 - 1 = 0$	$r \sin\theta = 4$

Example

Problem 8

What curve is represented by the polar equation $r = a$ (where a is a constant)?

Example

Problem 9

Find the polar equation for a circle whose center is the (cartesian) point (a, b) and intersects the origin.

Example

Problem 10

What curve is represented by the polar equation $\theta = \theta_0$ *(where* θ_0 *is a constant)?*

Example

Problem 11

What curve is represented by the polar equation $r = \theta$*?*

Example

Problem 12

What curve is represented by the polar equation $r \sin\theta = 10$*?*

Example

Problem 13

What curve is represented by the polar equation $r\cos\theta = 5$?

Example

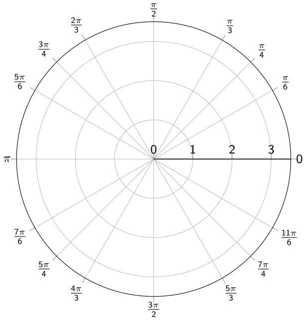
Problem 14

Convert the equation for a line $y = mx + b$ into Polar coordinates.

Annuli

Problem 15

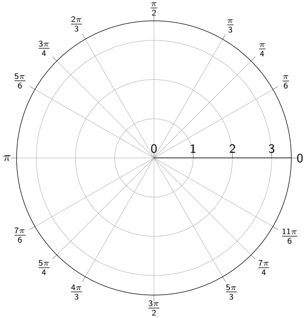
Describe the region in space: $1 \leq r \leq 2$



Sectors

Problem 16

Describe the region in space: $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$, $r < 3$



Polar Boxes

Problem 17

Describe the region in space: $1 \leq r \leq 2$, $0 \leq \theta \leq \frac{\pi}{4}$