

MT 2 Review

Tomorrow (Tues. 10/21)

• 8-9 PM
• WTHR 200

Main Floor
NOT Balcony

• Lectures 11-19
Make sure to bring

• Pencils / Erasers
• PUID

• Calculator
TI 30XA
Single line only

Amanda Manning

Balcony

V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20				
U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20	U21	U22	U23	U24
T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24
S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24

R1	R2	R3	R4			
Q1	Q2	Q3	Q4	Q5	Q6	Q7
P1	P2	P3	P4	P5	P6	P7
O1	O2	O3	O4	O5	O6	O7
N1	N2	N3	N4	N5	N6	N7
M2		M3	M4	M5	M6	M7
L1	L2	L3	L4	L5	L6	L7
K1	K2	K3	K4	K5	K6	K7
J1	J2	J3	J4	J5	J6	J7
I1	I2	I3	I4	I5	I6	I7
H1	H2	H3	H4	H5	H6	H7
G1	G2	G3	G4	G5	G6	G7
F1	F2	F3	F4	F5	F6	F7
E1	E2	E3	E4	E5	E6	E7
D1	D2	D3	D4	D5	D6	D7
C1	C2	C3	C4	C5	C6	C7
B1	B2	B3	B4	B5	B6	B7
A1	A2	A3	A4	A5	A6	A7

Main Floor

P8			X	X	X	X			P9
O8	O9	O10	O11	O12	O13	O14	O15	O16	O17
N8	N9	N10	N11	N12	N13	N14	N15	N16	N17
M8	M9	M10	M11	M12	M13	M14	M15	M16	M17
L8	L9	L10	L11	L12	L13	L14	L15	L16	L17
K8	K9	K10	K11	K12	K13	K14	K15	K16	K17
J8	J9	J10	J11	J12	J13	J14	J15	J16	J17
I8	I9	I10	I11	I12	I13	I14	I15	I16	I17
H8	H9	H10	H11	H12	H13	H14	H15	H16	H17
G8	G9	G10	G11	G12	G13	G14	G15	G16	G17
F8	F9	F10	F11	F12	F13	F14	F15	F16	F17
E8	E9	E10	E11	E12	E13	E14	E15	E16	E17
D8	D9	D10	D11	D12	D13	D14	D15	D16	D17
C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
B8	B9	B10	B11	B12	B13	B14	B15	B16	B17
X	A8	A9	A10	A11	A12	A13			X

Siva Somasundaram
WTHR 200

				R5	R6	R7	R8
Q8	Q9	Q10	Q11	Q12	Q13	Q14	
P10	P11	P12	P13	P14	P15	P16	
O18	O19	O20	O21	O22	O23	O24	
N18	N19	N20	N21	N22	N23	N24	
M18	M19	M20	M21	M22	M23	M24	
L18	L19	L20	L21	L22	L23	L24	
K18	K19	K20	K21	K22	K23	K24	
J18	J19	J20	J21	J22	J23	J24	
I18	I19	I20	I21	I22	I23	I24	
H18	H19	H20	H21	H22	H23	H24	
G18	G19	G20	G21	G22	G23	G24	
F18	F19	F20	F21	F22	F23	F24	
E18	E19	E20	E21	E22	E23	E24	
D18	D19	D20	D21	D22	D23	D24	
C18	C19	C20	C21	C22	C23	C24	
B18	B19	B20	B21	B22	B23	B24	
A14	A15	A16	A17	A18	A19	A20	

Zach Pence

7:30 Section 8:30 Section

Exam 2 Review

① Find crit. nums. of $y = [\ln(x+4)]^2$

Crit. Nums. : when $y' = 0$ or undefined

$$y' = 2 \ln(x+4) \cdot \frac{1}{x+4} \cdot (1) = \frac{2 \ln(x+4)}{x+4} \stackrel{\text{set}}{=} 0$$

$$2 \ln(x+4) = 0$$

$$\ln(x+4) = 0$$

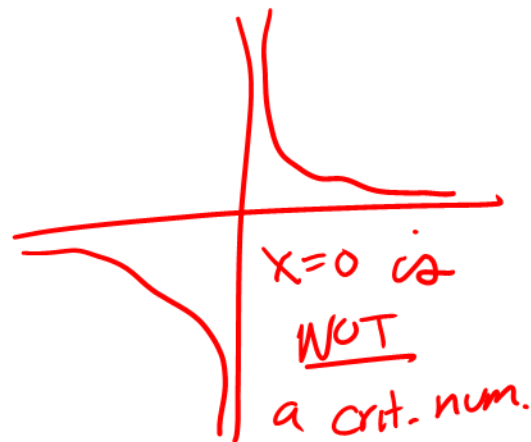
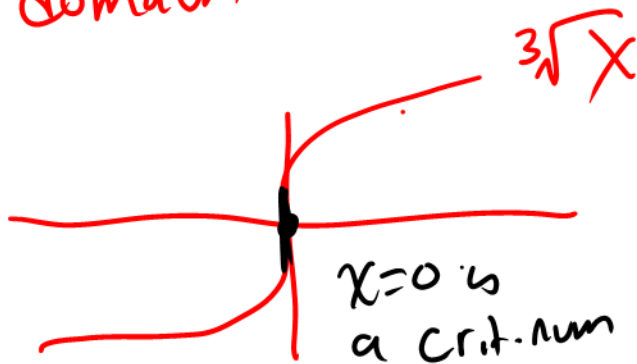
$$e$$

$$x + 4 = 1$$

$$\boxed{x = -3} \leftarrow \text{Only crit. num.}$$

$x = -4$? No

Remark Critical Numbers need to be in the domain



② Find $\frac{dy}{dx} \Big|_{(x,y)=(2,1)}$ given

$$\frac{d}{dx} (x^2 + 4xy + [y]^2 - 13) = \frac{d}{dx} (0)$$

$$2x + \frac{d}{dx}(4x)y + 4x \frac{d}{dx}(y) + 2y \frac{dy}{dx} - 0 = 0$$

$$\underbrace{2x + 4y}_{\text{bracketed}} + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$4x \frac{dy}{dx} + 2y \frac{dy}{dx} = -(2x + 4y)$$

$$\frac{d}{dx}(4x + 2y) = -(2x + 4y)$$

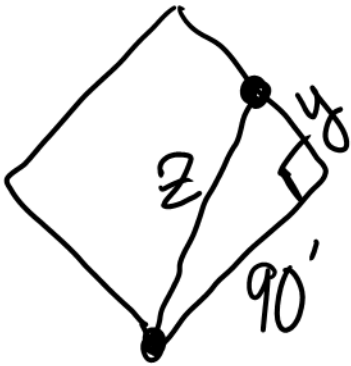
$$\frac{dy}{dx} = -\frac{2x + 4y}{4x + 2y}$$

$$\left. \frac{dy}{dx} \right|_{(2,1)} = -\frac{2(2) + 4(1)}{4(2) + 2(1)}$$

$$= -8/10 = \boxed{-4/5}$$

Recall $\ln(y) = \ln(y(x))$

$$\frac{d}{dx}(\ln y) = \frac{1}{y} \cdot \frac{dy}{dx}$$

③  Know: $\frac{dy}{dt} = 13$. Length = 90 ft

Need to know: $\frac{dz}{dt}$ when $y = 45$

$$\frac{d}{dt}(90^2 + [y(t)]^2) = \frac{d}{dt}([z(t)]^2)$$

$$2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{2y}{2z} \frac{dy}{dt} = \frac{y}{z} \frac{dy}{dt}$$

Find z when $y = 45$: $90^2 + y^2 = z^2$

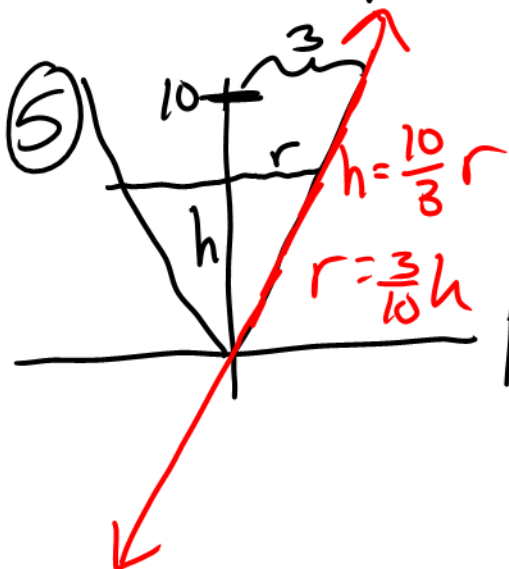
$$z = \sqrt{90^2 + y^2}. \text{ Hence, } z \Big|_{y=45} = \sqrt{90^2 + 45^2}$$

$$= \sqrt{(45 \cdot 2)^2 + (45 \cdot 1)^2} = \sqrt{45^2(2^2 + 1^2)} = 45\sqrt{5}$$

$$\left. \frac{dz}{dt} \right|_{y=45, y'=13} = \frac{45}{45\sqrt{5}} (13) = \boxed{\frac{13}{\sqrt{5}} \frac{\text{ft}}{\text{s}}}$$

$$\textcircled{4} \quad \frac{dz}{dt} = \frac{y}{z} \frac{dy}{dt} = \frac{y}{\sqrt{l^2 + y^2}} \frac{dy}{dt}$$

$$\left. \frac{dz}{dt} \right|_{y=\frac{1}{2}l, y'=v} = \frac{\frac{1}{2}l}{\sqrt{l^2 + \frac{1}{4}l^2}} v = \dots = \frac{v}{\sqrt{5}} \text{ ft/s}$$



Known: $\frac{dV}{dt} = 2 \text{ cm}^3/\text{s}$

NTK: $\frac{dh}{dt}$ when $h = 5$

$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3} \left(\frac{3}{10}h \right)^2 h = \frac{3\pi}{100} h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{100} h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{100}{9\pi h^2} \frac{dV}{dt}$$

$$\left. \frac{dh}{dt} \right|_{h=5} = \frac{100}{9\pi(5)^2} 2 = \boxed{\frac{8}{9\pi} \text{ cm/s}}$$

(6) $f(x) = x^3 - 6x^2 - 15x + 4$

Step 1 Find candidates for rel. max/mins/ I.P.s

$$f'(x) = 3x^2 - 12x - 15 \stackrel{\text{set}}{=} 0$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \Rightarrow x = -1, 5$$

$$f''(x) = 6x - 12$$

$$6(x-2) \stackrel{\text{set}}{=} 0 \Rightarrow x = 2$$

Step 2 Make Sign Chart

	<div> <div>←</div> <div>0</div> <div>→</div> </div>						
Test Points	-10	-1	0	2	3	5	10
Sign of f'	+		-		-		+
Sign of f''	-		-		+		+
Result of I/D Test	Inc ↗	↘ Dec			Dec ↘	↗ Inc	
Result of C Test	CD	CD			CU	CU	
		-1		2		5	
		Rel. Max		I.P.		Rel. Min	

⑦ Find abs. max/min given $f(x) = 10 + 27x - x^3$ on $[0, 4]$

$$f'(x) = 27 - 3x^2 \stackrel{!}{=} 0$$

$$9 - x^2 = 0$$

$$(3+x)(3-x) = 0$$

$$\Rightarrow x = \pm 3 \quad \xrightarrow{\text{3 is not in } [0, 4]} \quad x = 3$$

x	3	0	4
$f(x)$	64	10	54

$x=0$ is abs. min

$x=3$ is abs. max

Unnecessary here, but use the 2nd $\frac{1}{dx}$ test to verify $x=3$ is a rel. max.

$$f'(x) = 27 - 3x^2$$

$$f''(x) = -6x$$

$$f''(3) = -6(3) = -18 < 0$$

$f'' < 0 \Rightarrow f$ is CD near $3 \Rightarrow x=3$ is the loc. of a rel. max