Quiz 5: Sequences and Series (§10.2-10.4)

Score: /10

Name:____ Length: 15 minutes

Directions: Answer all the questions below in the space provided; you must show the work for full credit. Use proper notation. Clearly label the final answers.

1. (1 point) Let $\sum_{n=1}^{\infty} a_n$ be a series and suppose $a_n = f(n)$, where $f(x)$ is some function and $x \geq 1$. What requirements does $f(x)$ need to have in order to use the Integral Test? (There are 3 of them).

   Solution: f needs to be positive, continuous, and decreasing on $[1, \infty)$ to use the Integral Test.

2. (1 point) True or False: If $a_n \to 0$, then the series $\sum a_n$ converges. (No work needed, the answer is enough)

   Solution: False. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is a counterexample. $\frac{1}{n} \to 0$, however, $\sum \frac{1}{n}$ diverges.

3. (1 point) True or False: $\lim\limits_{n\to\infty} \frac{2^n}{n!} = 0$. (No work needed, the answer is enough)

   Solution: True. This can be proved using the Monotone Convergence Theorem (10.5 in the textbook). Or just recognize $2^n << n!$ when n is really large.

4. (3 points) Determine whether the series below converges or diverges (**justify your answer**). If it converges, find the value of the series.

$$\sum_{n=1}^{\infty} \frac{1}{3}\left(-\frac{1}{12}\right)^{n-1}$$

   Solution: Since $\left|-\frac{1}{12}\right| < 1$, the geometric series converges. Moreover,

$$\sum_{n=1}^{\infty} \frac{1}{3}\left(-\frac{1}{12}\right)^{n-1} = \frac{\frac{1}{3}}{1-\left(-\frac{1}{12}\right)} = \frac{\frac{1}{3}}{\frac{13}{12}} = \frac{4}{13}$$

5. For each part, determine whether each series converges or diverges. **For full points,** explain why it converges/diverges and state the test used.

(a) (2 points)

$$\sum_{n=1}^{\infty} \frac{n}{2025n+1}$$

Solution: Diverges by the Test for Divergence:

$$\lim_{n\to\infty} \frac{n}{2025n+1} = \frac{1}{2025} \neq 0$$

(b) (2 points)

$$\sum_{n=1}^{\infty} \frac{1}{n^{2025}}$$

Solution: This is a p-series where $p = 2025 > 1$, so the series converges. You can also use the Integral Test here.