Lecture 22: Choosing a Convergence Test

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Sections Covered: 10.8

### Introduction

Keep in mind, at the end of the day, these are strategies for determining what test to use, not a list of rules. There are multiple ways to handle series and there are plenty of exceptions to the rules.

**The main strategy:** Recognize the "form" the series takes, then use the test designed to handle that form.

### Test for Divergence

If it is clear that  $\lim_{n\to\infty} a_n \neq 0$ , then apply the Test for Divergence.

### Problem 1

Determine if  $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^n$  converges or diverges.

### Geometric Series

If the series can be brought into the form  $\sum_{n=1}^{\infty} ar^{n-1}$ , then it is a geometric series and converges if |r| < 1 and diverges otherwise.

#### Problem 2

Compute  $\sum_{n=1}^{\infty} \frac{1}{3} \left(-\frac{1}{12}\right)^{n-1}$ , or show it diverges.

# Telescoping Series

If the series can be brought into the form  $\sum_{n=1}^{\infty} [a_n - a_{n+1}]$ , then it is a telescoping series and converges if and only if  $\lim_{n\to\infty} a_n$  exists.

#### Problem 3

Compute  $\sum_{n=1}^{\infty} [e^{-n} - e^{-(n+1)}]$ , or show it diverges.

## *p*-Series

Special Series

If the series can be brought into the form  $\sum_{n=1}^{\infty} \frac{1}{n^p}$ , then it is a p-series and converges if p > 1 and diverges otherwise.

### Problem 4

Determine if  $\sum_{n=1}^{\infty} \frac{1}{n^{2025}}$  and  $\sum_{n=1}^{\infty} \frac{1}{n^{0.99}}$  converge or diverge.

### The Comparison Tests

If the series in similar to a p-series or a geometric series, one of the comparison tests should be considered.

#### Problem 5

Determine if  $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$  converges or diverges.

## Another Example

The Limit Comparison Test is especially useful when dealing with "algebraic functions of n" (involving roots of polynomials)

#### Problem 6

Determine if  $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{3n^3+4n^2+2}$  converges or diverges.

## The Integral Test

For  $\sum a_n = \sum f(n)$ , if f(x) can be easily integrated, then the Integral Test is useful (assuming f satisfies the requirements).

### Problem 7

Determine if  $\sum_{n=1}^{\infty} ne^{-n^2}$  converges or diverges.

# When the terms aren't always positive

When the terms are not always positive, it is a good idea to test for absolute convergence and use another method.

Or, use a test that tests for absolute convergence directly (such as the ratio and root tests).

### The Ratio Test

Series involving factorials or other products (including a constant raised to n) are good candidates for the Ratio Test.

#### Problem 8

Determine if  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$  converges or diverges.

In general

Warning: Since for any p,

$$\left| rac{(n+1)^p}{n^p} 
ight| o 1 ext{ as } n o \infty$$

It is best to avoid the Ratio Test if the terms are rational functions or algebraic functions of *n*.

In general 000000

### The Root Test

If the terms are in the form  $a_n = (b_n)^n$ , then the Root Test may be useful.

### Problem 9

Determine if  $\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$  converges or diverges.

# The Alternating Series Tests

If there is an oscillating part  $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$  and it is not absolutely convergent, test for conditional convergence with the Alternating Series Test.

#### Problem 10

Determine if  $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4+1}$  converges or diverges.

# Summary (pg. 701 in textbook)

Table 10 & Special Series and Convergence Tests

Series or Test Form of Series Convergence Divergence Comments Geometric series $\sum_{k=0}^{\infty} a^k, \ a \neq 0 \qquad  r  < 1 \qquad  r  \ge 1 \qquad  r  \ge 1 \qquad  f /r  < 1, \text{ then } \sum_{k=1}^{\infty} a_k \qquad box box box box box box box box box box$	Table 10.4 Special Serie	es and Convergence Tests			
Geometric series $\sum_{k=0}^{\infty} a_t^k, a \neq 0 \qquad  r  < 1 \qquad  r  \ge 1 \qquad  r  \ge 1 \qquad \text{If }  r  < 1, \text{ then } \sum_{k=0}^{\infty} a_k$ Does not apply $\lim_{k \to \infty} a_k \neq 0 \qquad \text{Cannot be used to prove gence Test}$ $\sum_{k=1}^{\infty} a_k \qquad \text{Does not apply} \qquad \lim_{k \to \infty} a_k \neq 0 \qquad \text{Cannot be used to prove gence}$ Integral Test $\sum_{k=1}^{\infty} a_k \qquad \text{Does not apply} \qquad \lim_{k \to \infty} a_k \neq 0 \qquad \text{Convergence}$ The value of the integral Test $\sum_{k=1}^{\infty} \frac{1}{k^2} \qquad p > 1 \qquad p = 1 \qquad \text{Useful for comparison}$ Ratio Test $\sum_{k=1}^{\infty} a_k \qquad \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} < 1 \qquad \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad $	Ci Tt	F	Condition for	Condition for	C
Divergence Test $\sum_{k=1}^{\infty} a_k \text{ Does not apply } \lim_{k \to \infty} a_k \neq 0 \qquad \text{Cannot be used to p convergence}$ Integral Test $\sum_{k=1}^{\infty} a_k \text{ where } a_k = f(k) \qquad \int_1^{\infty} f(x)  dx \text{ convergess.} \qquad \int_1^{\infty} f(x)  dx \text{ diverges.} \qquad \text{The value of the int value of the int value of the series.}$ $positive, \text{ and decreasing}$ $p \text{-series} \qquad \sum_{k=1}^{\infty} \frac{1}{k!} \qquad p > 1 \qquad p \leq 1 \qquad \text{Useful for comparison Test}$ $\sum_{k=1}^{\infty} a_k \qquad \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  < 1 \qquad \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  < 1 \qquad \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad Inconclus$	Series or Test	rorm of Series	Convergence	Divergence	
Integral Test $\sum_{k=1}^{\infty} q_k \text{ where } a_k = f(k) \\ \text{ and } f \text{ is continuous,} \\ \text{ positive, and decreasing} \\ \text{Paseries} \qquad \sum_{k=1}^{\infty} \frac{d_k}{k^2} \\ \text{Paseries} \qquad \sum_{k=1}^{\infty} \frac{d_k}{k^2} \\ \text{Paseries} \qquad \sum_{k=1}^{\infty} \frac{d_k}{k^2} \\ \text{Ratio Test} \qquad \sum_{k=1}^{\infty} \frac{d_k}{k^2} \\ \text{Root Test} \qquad \sum_{k=1}^{\infty} \frac{d_k}{k^2} \\ \text{Emphasized} \\ \text{Imm} \left[ \frac{d_{k+1}}{d_k} \right] < 1 \\ \text{Imm} \left[ \frac{d_{k+1}}{d_k} \right] > 1 \\ \text{Inconclusive if } \lim_{k \to \infty} \sqrt{ a_k } > 1 \\ Inconclusive if $	Geometric series	$\sum_{k=0}^{\infty} ar^k, a \neq 0$	$ r  \le 1$	$ r  \ge 1$	If $ r  < 1$ , then $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}.$
walue of the series. value of the series. positive, and $f$ is continuous, positive, and decreasing value of $a$ is given; you go series $\sum_{k=1}^{\infty} \frac{1}{h^k} \qquad p>1 \qquad p \leq 1 \qquad \text{Useful for comparison Test}$ Ratio Test $\sum_{k=1}^{\infty} a_k \qquad \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} < 1 \qquad \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \frac{ a_{k+1} }{a_k} > 1 \qquad Inconclusi$	Divergence Test	$\sum_{k=1}^{\infty} a_k$	Does not apply	$\lim_{k\to\infty}a_k\neq0$	Cannot be used to prove convergence
Ratio Test $\sum_{k=1}^{\infty} a_k \qquad \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  < 1 \qquad \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1 \qquad \text{Inconclusive if } \lim_{k $	Integral Test	and f is continuous,	$\int_{1}^{\infty} f(x) \ dx \text{ converges.}$	$\int_{1}^{\infty} f(x) \ dx \ \text{diverges}.$	The value of the integral is not the value of the series.
Roof Test $\sum_{k=1}^{\infty} a_k \qquad \lim_{w} \sqrt[k]{a_k} > 1 \qquad \lim_{n \to \infty} \sqrt[k]{a_k} > 1 \qquad \text{Inconclusive if } \lim_{k \to \infty} \sqrt[k]{a_k} > 1$ Comparison Test $\sum_{k=1}^{\infty} a_k \text{ where } a_k > 0 \qquad a_k \le b_k \text{ and } \sum_{k=1}^{\infty} b_k \qquad b_k \le a_k \text{ and } \sum_{k=1}^{\infty} b_k \qquad \text{inconclusive if } \lim_{k \to \infty} \sqrt[k]{a_k} > 0$ $\text{converges.} \qquad \text{diverges.} \qquad \text{diverges.}$ Limit Comparison Test $\sum_{k=1}^{\infty} a_k \text{ where} \qquad 0 \le \lim_{k \to \infty} \frac{a_k}{b_k} < \infty \text{ and } \lim_{k \to \infty} \frac{a_k}{b_k} > 0 \text{ and } \lim_{k \to \infty} \frac{a_k}{b_k} > 0 \text{ and } \lim_{k \to \infty} \frac{a_k}{b_k} > 0$ $\sum_{k=1}^{\infty} b_k \text{ diverges.}$	p-series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	p > 1	$p \le 1$	Useful for comparison tests
Comparison Test $ \sum_{k=1}^{\infty} a_{e} \text{ where } a_{k} > 0 \qquad a_{i} \leq b_{k} \text{ and } \sum_{k=1}^{\infty} b_{k} \qquad b_{k} \leq a_{k} \text{ and } \sum_{k=1}^{\infty} b_{k} \qquad \sum_{k=1}^{\infty} a_{k} \text{ is given; you} $ Limit Comparison Test $ \sum_{k=1}^{\infty} a_{e} \text{ where } \qquad 0 \leq \lim_{k \to \infty} \frac{a_{k}}{b_{k}} < \infty \text{ and } \qquad \lim_{k \to \infty} \frac{a_{k}}{b_{k}} > 0 \text{ and } \qquad \sum_{k=1}^{\infty} a_{k} \text{ is given; you} $ $ a_{k} \geq 0, b_{k} > 0 \qquad \sum_{k=1}^{\infty} b_{k} \text{ converges.} \qquad \sum_{k=1}^{\infty} b_{k} \text{ diverges.} $ Alternating Series Test $ \sum_{k=1}^{\infty} (-1)^{k} a_{k} \text{ where } \qquad \lim_{k \to \infty} a_{k} \neq 0 \qquad \text{Remainder } R_{k} \text{ satisf.} $ $ a_{k} \geq 0 \qquad 0 \leq a_{k+1} \leq a_{k} \qquad \lim_{k \to \infty} a_{k} \neq 0 \qquad \text{Remainder } R_{k} \text{ satisf.} $ $ a_{k} \geq 0 \qquad 0 \leq a_{k+1} \leq a_{k} \qquad \lim_{k \to \infty} a_{k} \neq 0 \qquad \text{Remainder } R_{k} \text{ satisf.} $	Ratio Test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty}\left \frac{a_{k+1}}{a_k}\right <1$	$\lim_{k\to\infty}\left \frac{a_{k+1}}{a_k}\right >1$	Inconclusive if $\lim_{k\to\infty}\left \frac{a_{k+1}}{a_k}\right =1$
Limit Comparison Test $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Root Test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty}\sqrt[k]{ a_k }<1$	$\lim_{k\to\infty}\sqrt[k]{ a_k }>1$	Inconclusive if $\lim_{k\to\infty} \sqrt[k]{ a_k } = 1$
$a_{k} \geq 0, b_{k} > 0$ $\sum_{k=1}^{w} b_{k} \text{ converges.}$ $\sum_{k=1}^{w} b_{k} \text{ diverges.}$ $\sum_{k=1}^{w} b_{k} \text{ diverges.}$ Alternating Series Test $\sum_{k=1}^{w} (-1)^{k} a_{k} \text{ where } \lim_{k \to \infty} a_{k} = 0 \text{ and } \lim_{k \to \infty} a_{k} \neq 0$ $0 < a_{k+1} \leq a_{k}$ $ B_{k}  \leq a_{k+1}$ $ B_{k}  \leq a_{k+1}$	Comparison Test	$\sum_{k=1}^{\infty} a_k, \text{ where } a_k \geq 0$	$a_k \le b_k$ and $\sum_{k=1}^{\infty} b_k$ converges.	$b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges.	$\sum_{k=1}^{\infty} a_k \text{ is given; you supply } \sum_{k=1}^{\infty} b_k.$
$a_{k} > 0$ $0 < a_{k+1} \le a_{k}$ $ K_{n}  \le a_{n+1}$	Limit Comparison Test				$\sum_{k=1}^{\infty} a_k \text{ is given; you supply } \sum_{k=1}^{\infty} b_k.$
Absolute Convergence $\sum_{k=0}^{\infty} a_k$ , $a_k$ arbitrary $\sum_{k=0}^{\infty}  a_k $ converges. Applies to arbitrary	Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^k a_k$ , where $a_k > 0$	$ \lim_{k \to \infty} a_k = 0 \text{ and } \\ 0 < a_{k+1} \le a_k $	$\lim_{k\to\infty}a_k\neq0$	Remainder $R_n$ satisfies $ R_n  \le a_{n+1}$
k=1	Absolute Convergence	$\sum_{k=1}^{\infty} a_k, a_k$ arbitrary	$\sum_{k=1}^{\infty}  a_k  \text{ converges.}$		Applies to arbitrary series

# Splitting Series

Sometimes splitting up the series makes it easier to understand.

### Problem 11

How would you approach testing the convergence of

$$\sum_{n=1}^{\infty} \frac{2^{n} + \cos(\pi n) \sqrt{n}}{3^{n+1}}?$$

## Algebra

Sometimes you need to manipulate the series to get it in a more recognizable form.

### Problem 12

Determine if  $\sum_{n=4}^{\infty} \frac{1}{\sqrt[4]{n^2-6n+9}}$  converges or diverges.

# Picking a series to compare with

When using the limit comparison test, looking at the end behavior of the terms  $(n \to \infty)$  is useful in figuring out what series to compare with.

#### Problem 13

Determine if  $\sum_{n=2}^{\infty} \sqrt[3]{\frac{n^2-1}{n^4+4}}$  converges or diverges.