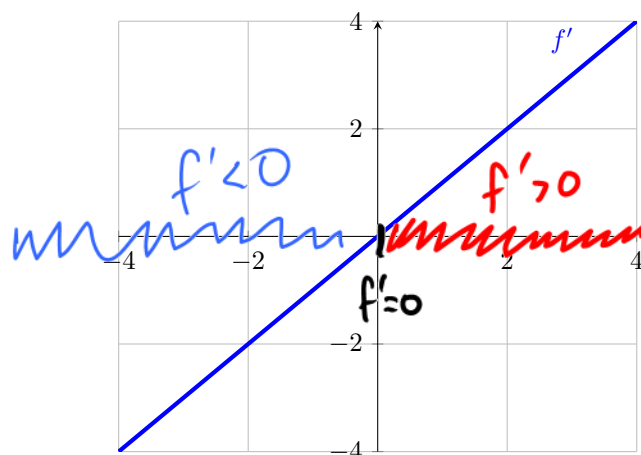


## Lecture 20: Graphical Interpretation of the Derivative

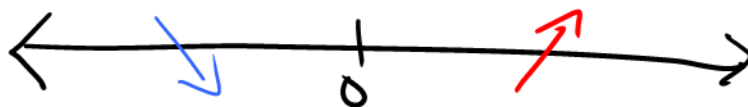
**Goal:** Recover information about a function  $f$  from the graph of  $f'$ .

### 1 Inc/Dec

**Problem 1.** Use the graph of  $f'$  to determine the following about  $f$ :



(a) Intervals of increase/decrease



Inc:  $(0, \infty)$

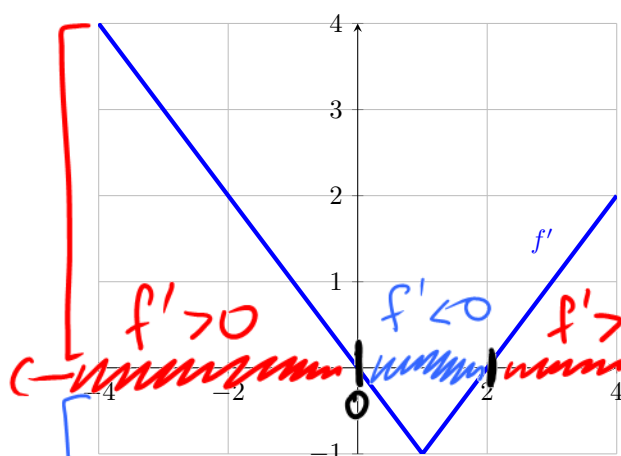
Dec:  $(-\infty, 0)$

(b) Positions of relative max/mins

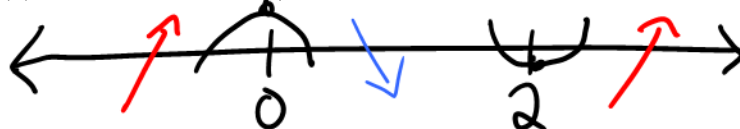
Crit Mins:  $f' = 0 \leftarrow$  when  $f'$  touches the x-axis

$f$  has a rel min at  $x=0$  by the 1<sup>st</sup> Derivative Test.

**Problem 2.** Use the graph of  $f'$  to determine the following about  $f$ :



(a) Intervals of increase/decrease



Inc:  $(-\infty, 0) \cup (2, \infty)$

Dec:  $(0, 2)$

(b) Positions of relative max/mins

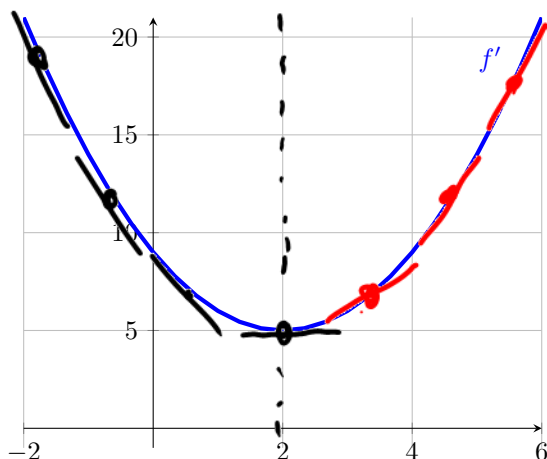
By the 1<sup>st</sup> Derivative Test

$f$  has a rel. max. at  $x=0$

$f$  has a rel. min at  $x=2$

## 2 Concavity

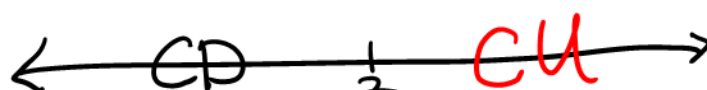
**Problem 3.** Use the graph of  $f'$  to determine the following about  $f$ :



(a) When  $f$  is concave up VS. concave down

CU:  $(2, \infty)$

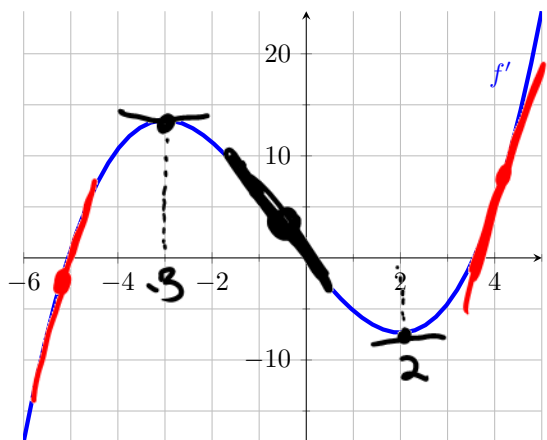
CD:  $(-\infty, 2)$



(b) Position of any inflection points

There is an inflection point at  $x=2$

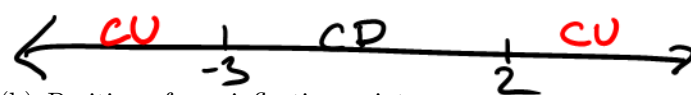
**Problem 4.** Use the graph of  $f'$  to determine the following about  $f$ :



(a) When  $f$  is concave up VS. concave down

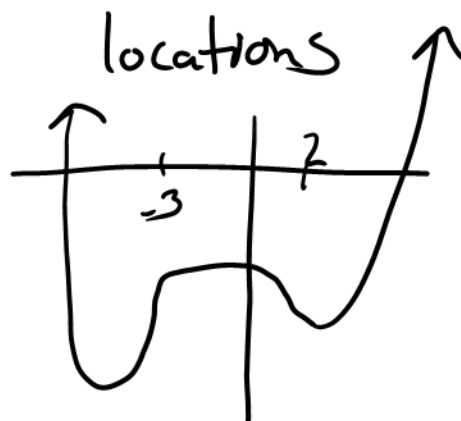
CU:  $(-\infty, -3)$   $(2, \infty)$

CD:  $(-3, 2)$



(b) Position of any inflection points

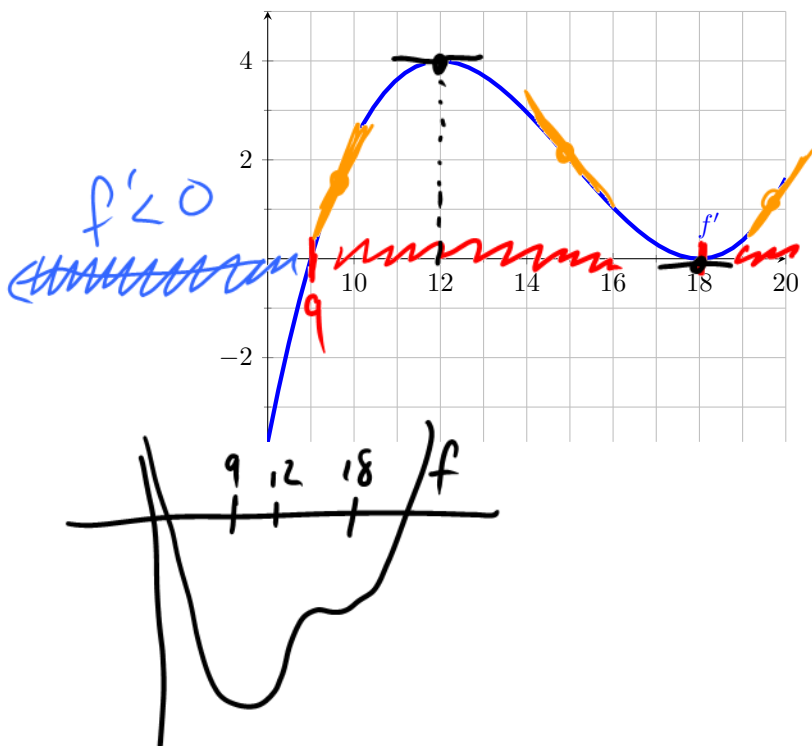
Both  $x=-3$  and  $x=2$  are locations of inflection points



### 3 Putting Everything Together

**Problem 5.** Use the graph of  $f'$  to determine the following about  $f$ :

In Achieve:  $(9, \infty)$

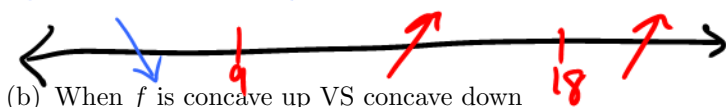


(a) Intervals of Increase/Decrease

Inc:  $(9, \infty) \setminus \{18\}$ ;  $(9, 18) \cup (18, \infty)$

Dec:  $(-\infty, 9)$

(b) When  $f$  is concave up VS concave down

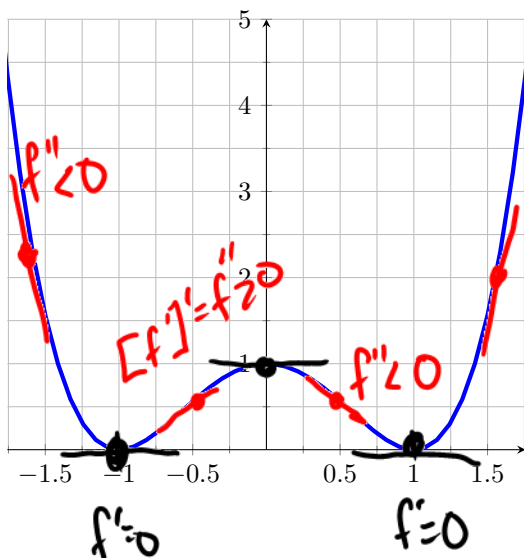


(c) State the locations of any relative max/mins as well as inflection points

Rel min at  $x=9$   
Inflection Points at  $x=12, 18$

**Problem 6.** Use the graph of  $f'$  to determine the following about  $f$ :

In achieve:  $(-\infty, \infty)$



(a) Intervals of Increase/Decrease

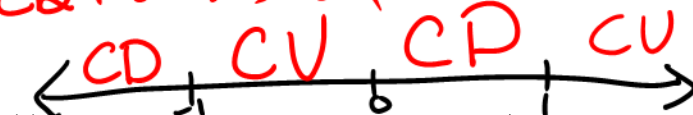
Inc:  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Dec: None

(b) When  $f$  is concave up VS concave down

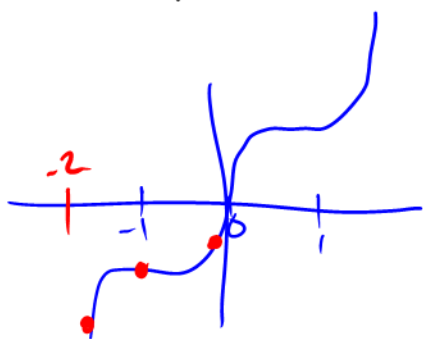
CU:  $(-1, 0) \cup (1, \infty)$

CD:  $(-\infty, -1) \cup (0, 1)$



(c) State the locations of any relative max/mins as well as inflection points

No Rel. Max/Min  
Inflection Points at  $x=-1, 0, 1$



Question 4 of 7

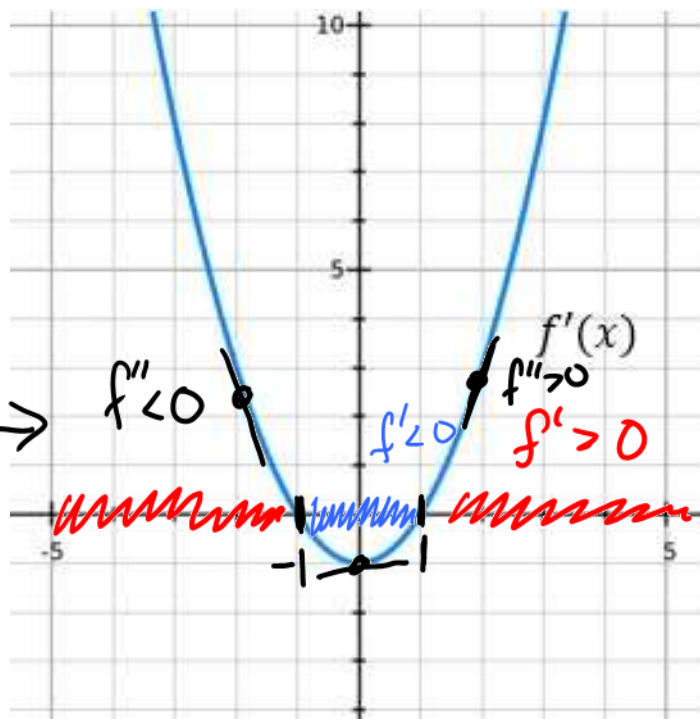
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Consider the graph of  $f'(x)$ , which represents the derivative of the function  $f(x)$ .

Answer the following questions regarding  $f(x)$ .

For (a.), (d.), (e.), and (h.), if there is more than one answer for a specific question, separate each answer from the following one using a comma. For (b.), (c.), (f.), and (g.), if there is more than one interval in an answer, separate each interval from the next by typing UNION to enter the  $\cup$  symbol.

For any part, if no answer exists at all, enter NO SOLUTION.



1st  $\frac{d}{dx}$  test:

Inc.:  $(-\infty, -1) \cup (1, \infty)$

Dec:  $(-1, 1)$

Rel Max at  $x = -1$ ; Rel Min at  $x = 1$

(a.) Critical Number(s) =

(b.) Increasing Interval(s):

(d.) Relative Maxima Occur at  $x =$

(f.) Concave Up Interval(s):

(h.) Inflection Points Occur at  $x =$

Inflection Points

Inflection Point at  $x = 0$

(c.) Decreasing Interval(s):

(e.) Relative Minima Occur at  $x =$

(g.) Concave Down Interval(s):

## Question 5 of 7

Consider the graph of  $f'(x)$ , which represents the derivative of the function  $f(x)$ .

Answer the following questions regarding  $f(x)$ .

For (a.), (d.), and (e.), if there is more than one answer for a specific question, separate each answer from the following one using a comma. For (b.) and (c.), if there is more than one interval in an answer, separate each interval from the next by typing UNION to enter the  $\cup$  symbol.

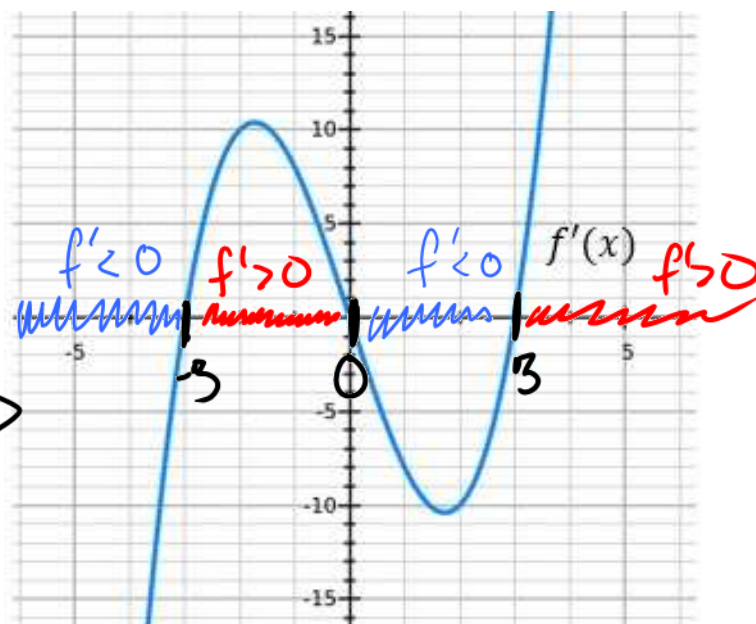
For any part, if no answer exists at all, enter NO SOLUTION.



Inc:  $(-3, 0) \cup (3, \infty)$   
Dec:  $(-\infty, -3) \cup (0, 3)$

Rel Min at  $-3, 3$

(a.) Critical Number(s) =



Rel. Max at  $x = 0$

(b.) Increasing Interval(s):

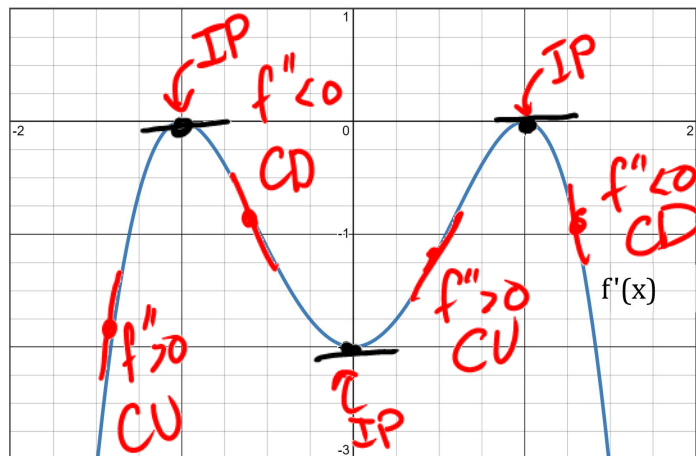
(c.) Decreasing Interval(s):

(d.) Relative Maxima Occur at  $x =$

(e.) Relative Minima Occur at  $x =$

# Question 7 of 7

The following graph shows the derivative  $f'(x)$  of the function  $f(x)$ .



Which of the following statements is true about  $f(x)$ ?

- ☐  ~~$f(x)$  is increasing on  $(-\infty, -1)$  and  $(0, 1)$ .~~
- ☐  ~~$f(x)$  is concave up on  $(-1, 1)$ .~~
- ☐  ~~$f(x)$  has one relative minimum.~~
- ☐  ~~$f(x)$  has two relative maxima.~~
- ☐  ~~$f(x)$  has one critical number.~~
- ☒  $f(x)$  has three inflection points.