

Quiz 3: Arc Length and Surface Area; Physics (§6.5, 6.6, 6.7 [Part I])

Name: _____

Score: _____/10

Length: 15 minutes

Directions: Answer all questions below; you must show work for full credit. Use proper notation. Clearly label final answers. You may use the back of this page if you need extra space.

1. (3 points) Let \mathcal{R} be the region bounded by $y = \sin^2 x$, $x = 0$, $x = \pi/2$, and the x -axis. Set up, ***BUT DO NOT EVALUATE***, the integral for the surface area of the figure obtained when \mathcal{R} is rotated about the x -axis. Simplify completely.

Solution: Here $f(x) = \sin^2 x$, so $f'(x) = 2 \sin x \cos x = \sin 2x$. Thus, if S represents the surface area:

$$S = \int_0^{\frac{\pi}{2}} 2\pi \sin^2 x \sqrt{1 + \sin^2(2x)} \, dx$$

2. (3 points) The density of a thin, 2-meter-long object is the following:

$$\rho(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 1 - \sin \pi x & \text{if } 1 \leq x \leq 2 \end{cases}$$

Find the mass of the object (in kilograms).

Solution: If m is the mass of the object:

$$m = \int_0^2 \rho(x) \, dx = \int_0^1 1 \, dx + \int_1^2 (1 - \sin \pi x) \, dx = 2 + \left[\frac{1}{\pi} \cos \pi x \right]_1^2 = 2 + \frac{1}{\pi} (1 - (-1)) = \left(2 + \frac{2}{\pi} \right) \text{ kg}$$

3. (4 points) A particle is moved along the x -axis by a force that measures $\frac{10}{(1+x)^2}$ Newtons at a point x meters from the origin. Find the work done in moving the particle from the origin to a distance of 9 meters along the positive x -axis. Express your answer in Joules (J).

Solution: If W is the work done, then:

$$W = \int_0^9 \frac{10}{(1+x)^2} dx = -10 \left[\frac{1}{1+x} \right]_0^9 = 10 \left(1 - \frac{1}{10} \right) = 9\text{J}$$