

Siva Somasundaram

• 8-9 PM

• LILY 1105

- Lectures 2-10

Make sure to bring:

- Pencils / Erasers

- PUID

Calculator

Section Number/Instructor's Name

• 7:30 — Sec. 19

- 8:30 — Sec. 20

Siva Somasundaram

X1	X2	X3	X4	X5	X6
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W1	W2	W3	W4	W5	W6	W7
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W8	W9	W10	W11	W12	W13
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X	V1	V2	V3	V4	V5	V6	X
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X	V7	V8	V9	V10	V11	V12	V13
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U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20
S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14	O15	O16	O17	O18	O19	O20
N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12	N13	N14	N15	N16	N17	N18	N19	N20
M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14	M15	M16	M17	M18	M19	M20
L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20
K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	K16	K17	K18	K19	K20
J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12	J13	J14	J15	J16	J17	J18	J19	J20
I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15	I16	I17	I18	I19	I20
H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13	H14	H15	H16	H17	H18	H19	H20
G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12	G13	G14	G15	G16	G17	G18	G19	G20
F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20
E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20
D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20
C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20
X	A1	A2	A3	A4	A5	A6	X	X	X	A7	A8	A9	A10	A11	A12	A13	A14		

Ethan Kessinger

Zach Pence

LULU 1105

LILY 1105
446 stations (42 LH)

7:30 Section

8:30 Section

This is an optional assignment that will be worth 2 points of extra credit. You must show work to get credit.

NOTE: These are just review problems of Lectures 2-10; these are not necessarily representative of the problems of the exam. The exam can (and most likely will) have different problems.

Directions:

1. Complete each problem on the next page, make sure to show your work. Clearly mark the question number and final answer.
2. You have two options to turn in this assignment:
 - (a) **In-person:** You can slip it under my office door located in MATH 615. Make sure your name is on it and that it is stapled together (if there are multiple pages).
 - (b) **Email:** You may email your assignment to me at pence11@purdue.edu
 - i. Scan your assignment so that it is one PDF (do not submit a bunch of images).
 - ii. In the subject line, write "EXTRA CREDIT 1 [your name]".
3. The answers will be given in the lecture after the due date (9/22). Therefore, **no late submissions will be allowed.**

Problem 1. Use the table below to compute numerically $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = 3$

$\sqrt{2}$ $\boxed{2}$ $\boxed{5}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	2.5918	2.9534	2.9955	—	3.0045	3.0455	3.4986

$\boxed{0.001} \times \boxed{3} = \boxed{0.003}$ $\boxed{2^{\text{nd}}}$ $\boxed{\text{LN}}$ $\boxed{=}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$ $\boxed{1}$ $\boxed{0.001}$ $\boxed{=}$

Problem 2. Evaluate:

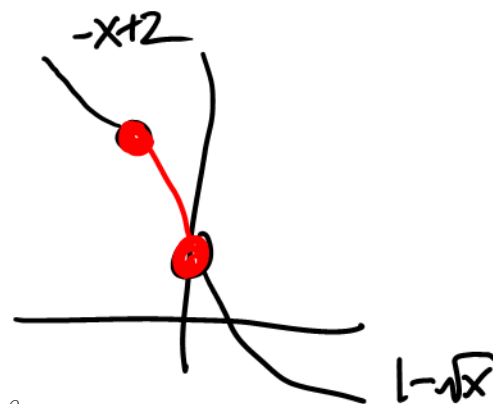
e^x

$$\lim_{x \rightarrow 0} \frac{\sin(x) - x}{4x}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x - x}{4x} &= \frac{1}{4} \lim_{x \rightarrow 0} \frac{\sin x - x}{x} = \frac{1}{4} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{x}{x} \right) \\ &= \frac{1}{4} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - 1 \right) = \frac{1}{4} [1 - 1] = 0 \end{aligned}$$

Problem 3. Let $f(x)$ be the function below:

$$f(x) = \begin{cases} -x + 2 & x < -1 \\ mx + b & -1 \leq x \leq 0 \\ 1 - \sqrt{x} & x > 0 \end{cases}$$



What values do m and b need to be to make f continuous for every value of x ?

Only Discontinuity that could occur are at $x = -1, 0$

At $x = -1$,
 $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (-x + 2) = 3 \xrightarrow{\text{WANT}} f(-1) = \lim_{x \rightarrow -1^+} f(x)$

Our line needs to contain the point $(-1, 3)$

At $x = 0$,
 $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (1 - \sqrt{x}) = 1 \xrightarrow{\text{WANT}} f(0) = \lim_{x \rightarrow 0^-} f(x)$

Want the point $(0, 1)$

$$m = \frac{3-1}{-1-0} = \frac{2}{-1} = -2$$

$$b = 1$$

Def of Continuity

$$\lim_{x \rightarrow a} f(x) = f(a)$$

① "Elementary Fns" are continuous on their domain

② $\frac{p(x)}{q(x)}$ [p, q polynomials] continuous when $q(x) \neq 0$

$$\frac{x+1}{x^2+3x+2}$$

Problem 4. Use the definition of the derivative to compute $f'(x)$ if $f(x) = x^2 - 1$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 [x^2 - 1]' &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 1 - [x^2 - 1]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 1 - x^2 + 1}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\
 &= \lim_{h \rightarrow 0} (2x + h) = 2x
 \end{aligned}$$

$$\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = e^x$$

Problem 5. Find the equation of the line tangent to $f(x) = \sqrt{x}$ at $x = 4$

Slope: $f'(4)$

Power Rule: $[x^p]' = p x^{p-1}$

$$\begin{aligned}
 f'(x) &= [\sqrt{x}]' = [x^{\frac{1}{2}}]' = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} \\
 &= \frac{1}{2\sqrt{x}} \\
 f'(4) &= \frac{1}{2\sqrt{4}} = \frac{1}{4}
 \end{aligned}$$

Point: $(4, f(4)) = (4, \sqrt{4}) = (4, 2)$

Equation:

$$\begin{aligned}
 y - 2 &= \frac{1}{4}(x - 4) \\
 y &= \frac{1}{4}(x - 4) + 2
 \end{aligned}
 \quad \rightarrow \quad
 \begin{aligned}
 y &= \frac{1}{4}x - 1 + 2 \\
 y &= \frac{1}{4}x + 1
 \end{aligned}$$

Problem 6. Differentiate the following functions:

(i) $f(x) = 3x^4 + 7x^2 - 5x + 9$

$$\begin{aligned} f'(x) &= 3[x^4]' + 7[x^2]' - 5[x] + [9]' \\ &= 3(4x^3) + 7(2x) - 5 \cdot 1 + 0 \\ &= 12x^3 + 14x - 5 \end{aligned}$$

(ii) $g(x) = e^x(x^3 + 1)$

Product Rule: $(fg)' = f'g + fg'$

$$\begin{aligned} f'(x) &= [e^x]'(x^3 + 1) + e^x[x^3 + 1]' = e^x(x^3 + 1) + e^x(3x^2) \\ &= e^x(x^3 + 3x^2 + 1) \end{aligned}$$

(iii) $h(x) = \frac{\ln(x)}{\sin(x)}$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$\left[\frac{\ln x}{\sin x}\right]' = \frac{[\ln x]' \sin x - [\ln x][\sin x]'}{[\sin x]^2} = \frac{\frac{\sin x}{x} - \ln x \cos x}{\sin^2 x}$$

(iv) $s(x) = \tan(x^2 + 3x)$

Chain Rule: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$$\begin{aligned} [\tan(x^2 + 3x)]' &= \underbrace{[\tan(x^2 + 3x)]'}_{\text{w.r.t. } "x^2 + 3x"} \cdot \underbrace{[x^2 + 3x]'}_{\text{w.r.t. } x} \\ &= \sec^2(x^2 + 3x) \cdot (2x + 3) \end{aligned}$$

Problem 7. The position of a dampened pendulum is measured from the angle (in radians) from its resting point. Its position after t seconds is given by the equation:

$$s(t) = e^{-t} \cos\left(\frac{\pi}{2}t\right)$$

(i) What is the velocity function $v(t)$ (in radians per second)?

(ii) What is the velocity after 1 second?

a

$$\begin{aligned} v(t) = s'(t) &= [e^{-t}]' \cos \frac{\pi}{2}t + e^{-t} [\cos \frac{\pi}{2}t]' \\ &= (-1)e^{-t} \cos \frac{\pi}{2}t + e^{-t} \left[\frac{\pi}{2} - \sin \frac{\pi}{2}t \right] \\ &= -e^{-t} \left[\cos \frac{\pi}{2}t + \frac{\pi}{2} \sin \frac{\pi}{2}t \right] \end{aligned}$$

b

$$v(1) = -e^{-1} \left(0 + \frac{\pi}{2} \right) = -\frac{\pi}{2e} \quad \text{radians/sec}$$

Problem 8. Find $\frac{dy}{dx}$ if:

$$y = (x^2 + 1)^x$$

$$[\ln(y)]' = [x \ln(x^2 + 1)]'$$

$$\frac{y'}{y} = \ln(x^2 + 1) + x \cdot \frac{1}{x^2 + 1} (2x)$$

$$y' = (x^2 + 1)^x \left[\ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right]$$