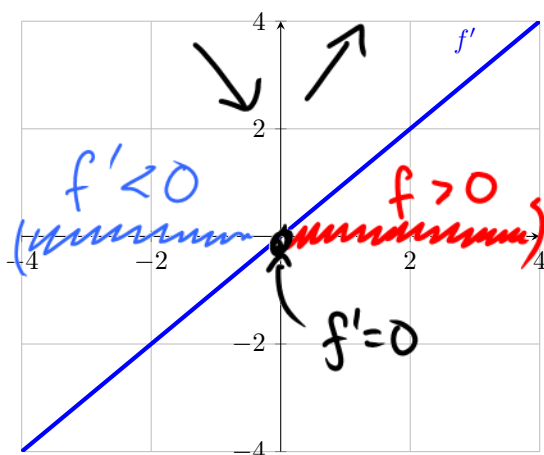


Lecture 20: Graphical Interpretation of the Derivative

Goal: Recover information about a function f from the graph of f' .

1 Inc/Dec

Problem 1. Use the graph of f' to determine the following about f :



(a) Intervals of increase/decrease

$f' > 0 \leftarrow f'$ needs to be above the x -axis

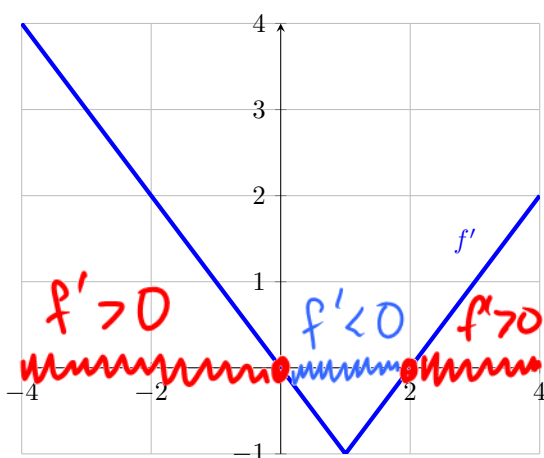
Increasing on the interval $(0, \infty)$
Dec on $(-\infty, 0)$

(b) Positions of relative max/mins

Crit Nums: $x=0$

Relative Minimum at $x=0$

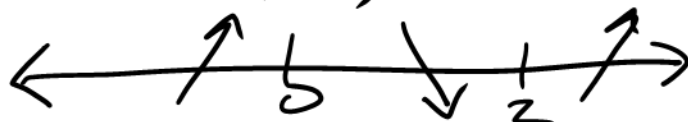
Problem 2. Use the graph of f' to determine the following about f :



(a) Intervals of increase/decrease

Inc: $(-\infty, 0) \cup (2, \infty)$

Dec: $(0, 2)$



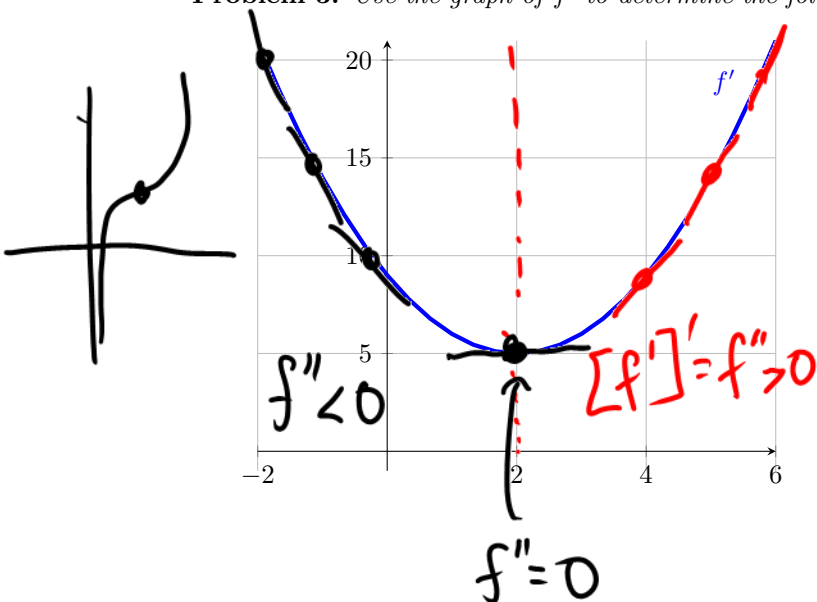
(b) Positions of relative max/mins

At $x=0$, $\nearrow \searrow$, rel. max by the 1st derivative test.

At $x=2$, $\searrow \nearrow$, rel. min

2 Concavity

Problem 3. Use the graph of f' to determine the following about f :



$$\text{CU: } f'' > 0$$

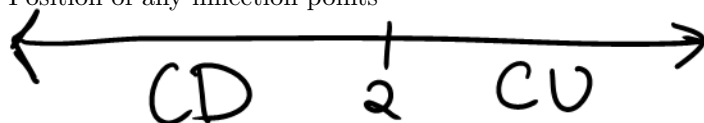
$$[f']' > 0$$

(a) When f is concave up VS. concave down

CU on $(2, \infty)$

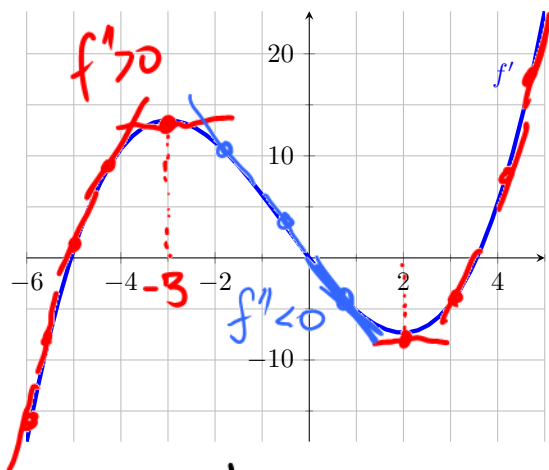
CD on $(-\infty, 2)$

(b) Position of any inflection points



Inflection Point at $x=2$

Problem 4. Use the graph of f' to determine the following about f :



(a) When f is concave up VS. concave down

CU: $(-\infty, -3) \cup (2, \infty)$

CD: $(-3, 2)$

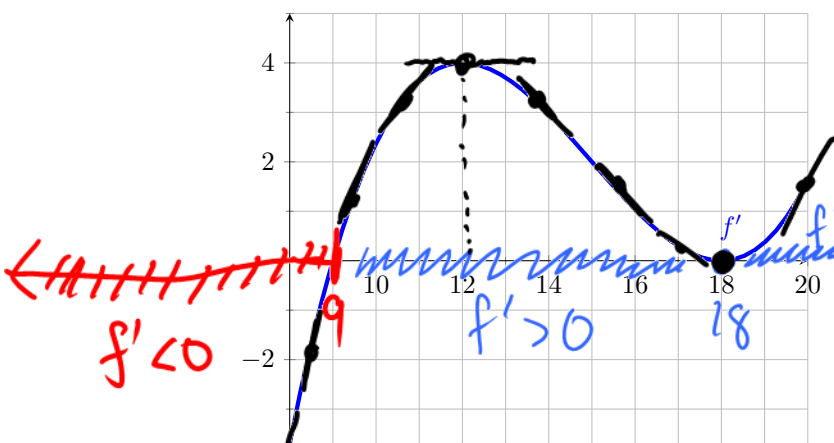


(b) Position of any inflection points

When $f'' = 0$ | Inflection Points at $x = -3, 2$

3 Putting Everything Together

Problem 5. Use the graph of f' to determine the following about f :



(a) Intervals of Increase/Decrease

Inc: $(9, \infty) \setminus \{18\}$
 $(9, 18) \cup (18, \infty)$

Dec: $(-\infty, 9)$

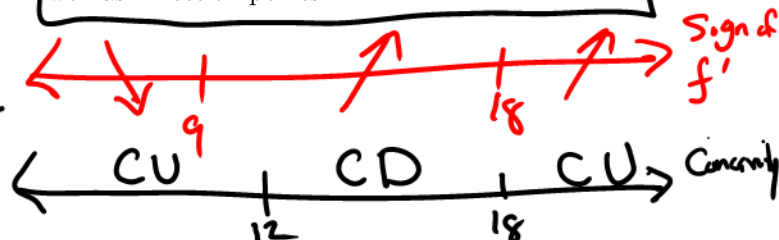
(b) When f is concave up VS concave down

CU: $(-\infty, 12) \cup (18, \infty)$

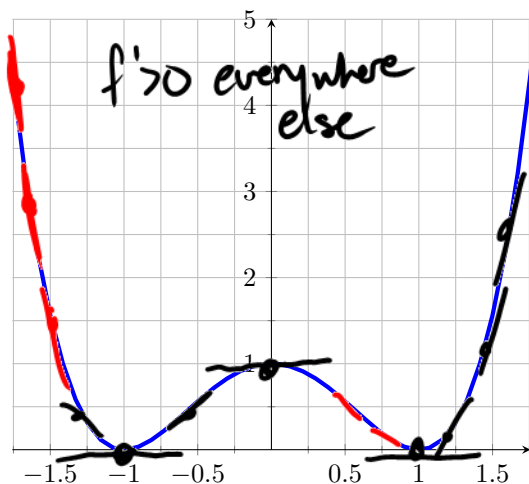
CD: $(12, 18)$

Rel. Min at $x=9$
 Inflection Points at $x=12, 18$

(c) State the locations of any relative max/mins as well as inflection points



Problem 6. Use the graph of f' to determine the following about f :



(a) Intervals of Increase/Decrease

Increasing on
 $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b) When f is concave up VS concave down

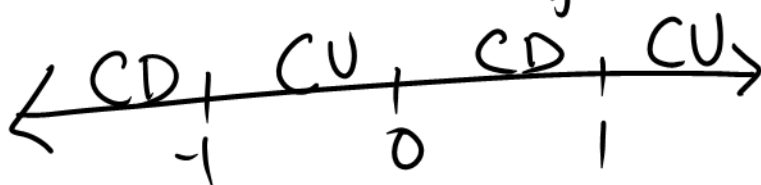
CU: $(-1, 0) \cup (1, \infty)$

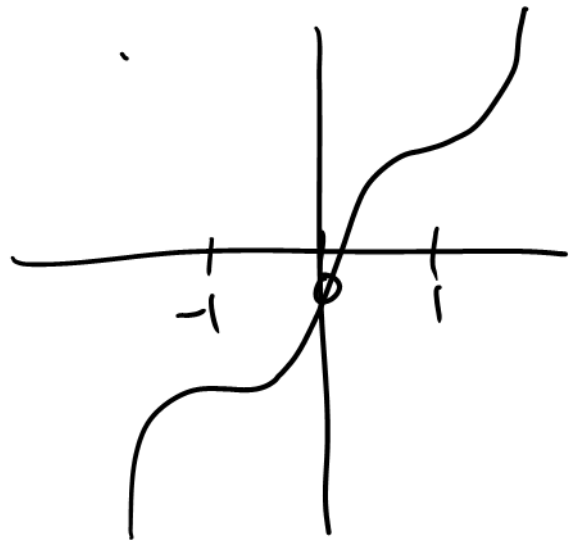
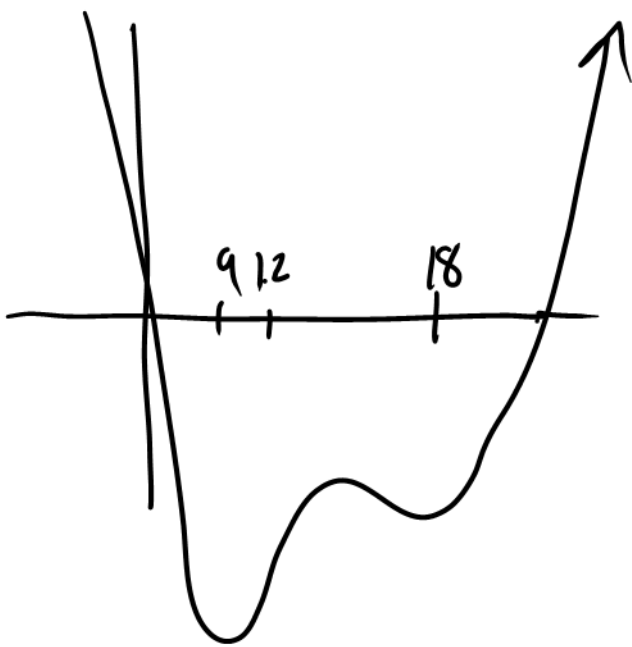
CD: $(-\infty, -1) \cup (0, 1)$

(c) State the locations of any relative max/mins as well as inflection points

No rel. max/mins

Inflection points at
 $x = -1, 0, 1$





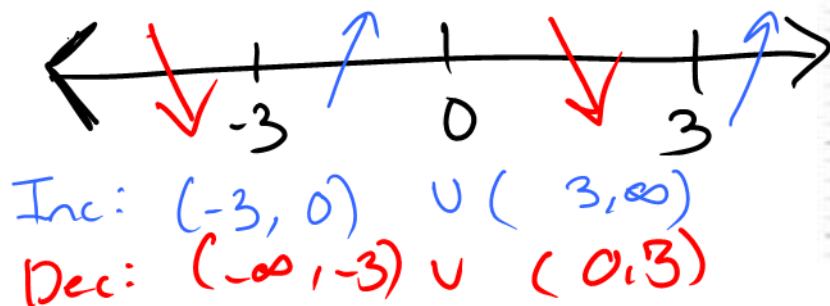
Question 5 of 7

Consider the graph of $f'(x)$, which represents the derivative of the function $f(x)$.

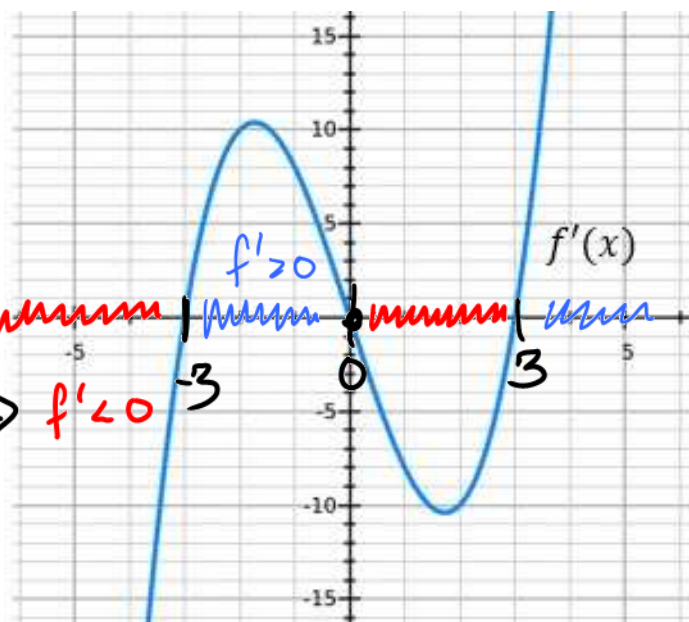
Answer the following questions regarding $f(x)$.

For (a.), (d.), and (e.), if there is more than one answer for a specific question, separate each answer from the following one using a comma. For (b.) and (c.), if there is more than one interval in an answer, separate each interval from the next by typing UNION to enter the \cup symbol.

For any part, if no answer exists at all, enter NO SOLUTION.



Rel Min: $-3, 3$ Rel Max: 0



(a.) Critical Number(s) =

(b.) Increasing Interval(s):

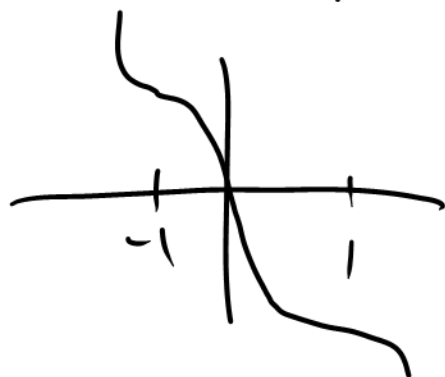
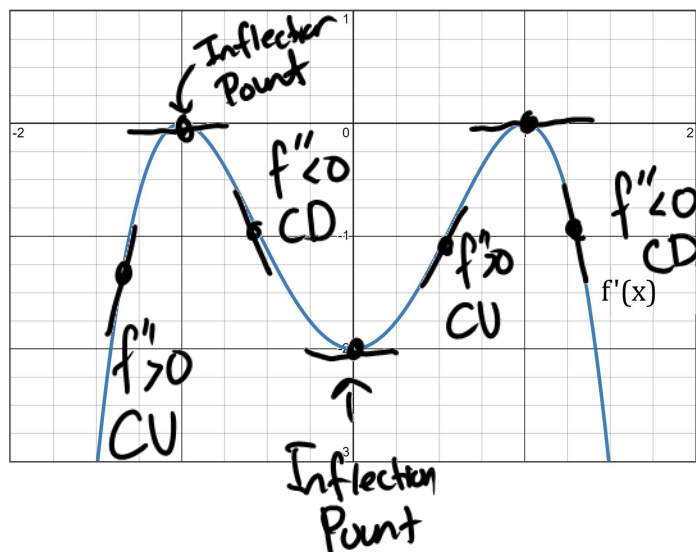
(c.) Decreasing Interval(s):

(d.) Relative Maxima Occur at $x =$

(e.) Relative Minima Occur at $x =$

Question 7 of 7

The following graph shows the derivative $f'(x)$ of the function $f(x)$.



Which of the following statements is true about $f(x)$?

- ☐ ~~$f(x)$ is increasing on $(-\infty, -1)$ and $(0, 1)$.~~
- ☐ ~~$f(x)$ is concave up on $(-1, 1)$.~~
- ☐ ~~$f(x)$ has one relative minimum.~~
- ☐ ~~$f(x)$ has two relative maxima.~~
- ☐ ~~$f(x)$ has one critical number.~~
- ☒ $f(x)$ has three inflection points.