

MT 3 Review

Amanda Manning

Balcony

V1	V2	V3	V4	V5	V6
U1	U2	U3	U4	U5	U6
T1	T2	T3	T4	T5	T6
S1	S2	S3	S4	S5	S6
					S7

R1	R2	R3	R4
Q1	Q2	Q3	Q4
Q5	Q6	Q7	
P1	P2	P3	P4
P5	P6	P7	
O1	O2	O3	O4
O5	O6	O7	
N1	N2	N3	N4
N5	N6	N7	
M2	M3	M4	M5
M6	M7		
L1	L2	L3	L4
L5	L6	L7	
K1	K2	K3	K4
K5	K6	K7	
J1	J2	J3	J4
J5	J6	J7	
I1	I2	I3	I4
I5	I6	I7	
H1	H2	H3	H4
H5	H6	H7	
G1	G2	G3	G4
G5	G6	G7	
F1	F2	F3	F4
F5	F6	F7	
E1	E2	E3	E4
E5	E6	E7	
D1	D2	D3	D4
D5	D6	D7	
C1	C2	C3	C4
C5	C6	C7	
B1	B2	B3	B4
B5	B6	B7	
A1	A2	A3	A4
A5	A6	A7	

V7	V8	V9	V10	V11	V12	V13	V14
U8	U9	U10	U11	U12	U13	U14	U15
U16	U17						
T8	T9	T10	T11	T12	T13	T14	T15
T16	T17						
S8	S9	S10	S11	S12	S13	S14	S15
S16	S17						

V15	V16	V17	V18	V19	V20
U18	U19	U20	U21	U22	U23
U24					
T18	T19	T20	T21	T22	T23
T24					
S18	S19	S20	S21	S22	S23
S24					

R1 R2 R3 R4

Q1	Q2	Q3	Q4	Q5	Q6	Q7
P1	P2	P3	P4	P5	P6	P7
O1	O2	O3	O4	O5	O6	O7
N1	N2	N3	N4	N5	N6	N7
M2	M3	M4	M5	M6	M7	
L1	L2	L3	L4	L5	L6	L7
K1	K2	K3	K4	K5	K6	K7
J1	J2	J3	J4	J5	J6	J7
I1	I2	I3	I4	I5	I6	I7
H1	H2	H3	H4	H5	H6	H7
G1	G2	G3	G4	G5	G6	G7
F1	F2	F3	F4	F5	F6	F7
E1	E2	E3	E4	E5	E6	E7
D1	D2	D3	D4	D5	D6	D7
C1	C2	C3	C4	C5	C6	C7
B1	B2	B3	B4	B5	B6	B7
A1	A2	A3	A4	A5	A6	A7

Main Floor

P8	X	X	X	X	P9
O8	O9	O10	O11	O12	O13
O14	O15	O16	O17	O18	O19
N8	N9	N10	N11	N12	N13
N14	N15	N16	N17	M8	M9
M10	M11	M12	M13	M14	M15
M16	M17	L8	L9	L10	L11
L12	L13	L14	L15	L16	L17
K8	K9	K10	K11	K12	K13
K14	K15	K16	K17	J8	J9
J10	J11	J12	J13	J14	J15
J16	J17	I8	I9	I10	I11
I12	I13	I14	I15	I16	I17
H8	H9	H10	H11	H12	H13
H14	H15	H16	H17	G8	G9
G10	G11	G12	G13	G14	G15
G16	G17	F8	F9	F10	F11
F12	F13	F14	F15	F16	F17
E8	E9	E10	E11	E12	E13
E14	E15	E16	E17	D8	D9
D10	D11	D12	D13	D14	D15
D16	D17	C8	C9	C10	C11
C12	C13	C14	C15	C16	C17
B8	B9	B10	B11	B12	B13
B14	B15	B16	B17	B8	B9
B10	B11	B12	B13	B14	B15
B15	B16	B17			
X	A8	A9	A10	A11	A12
A13					

Siva Somasundaram

WTHR 200

100 stations / 1 Lin

R5	R6	R7	R8
Q8	Q9	Q10	Q11
Q12	Q13	Q14	
P10	P11	P12	P13
P14	P15	P16	
O18	O19	O20	O21
O22	O23	O24	
N18	N19	N20	N21
N22	N23	N24	
M18	M19	M20	M21
M22	M23	M24	
L18	L19	L20	L21
L22	L23	L24	
K18	K19	K20	K21
K22	K23	K24	
J18	J19	J20	J21
J22	J23	J24	
I18	I19	I20	I21
I22	I23	I24	
H18	H19	H20	H21
H22	H23	H24	
G18	G19	G20	G21
G22	G23	G24	
F18	F19	F20	F21
F22	F23	F24	
E18	E19	E20	E21
E22	E23	E24	
D18	D19	D20	D21
D22	D23	D24	
C18	C19	C20	C21
C22	C23	C24	
B18	B19	B20	B21
B22	B23	B24	
A14	A15	A16	A17
A18	A19	A20	

Zach Pence

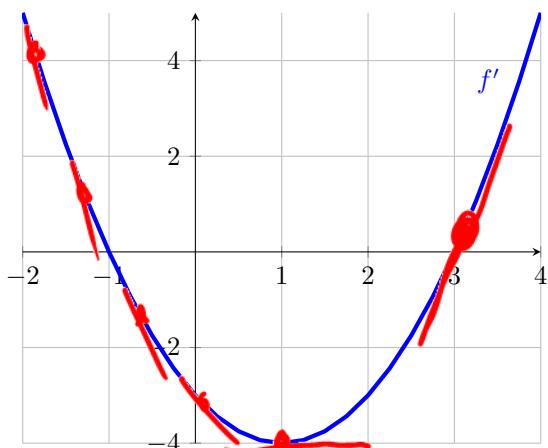
7:30 Section

8:30 Section

- Monday (11/10)
- 8-9 PM
- WTHR 200
 - Main Floor
NOT Balcony
- Lectures 20-28
- Make Sure to bring
- Pencils / Erasers
- PUID
- Calculator
- TI 30XA
- Single line only

Problem 1. A graph of f' is given below.

- Determine when f is increasing and when it is decreasing.
- Determine when f is concave up and when it is concave down.
- Locate the positions (x -coordinates) of any relative extrema and inflection points.



Incl/Dec:

$\leftarrow \begin{cases} f' > 0 \\ f' < 0 \end{cases} \rightarrow$
 1 3 $\leftarrow \begin{cases} f' > 0 \\ f' < 0 \end{cases} \rightarrow$

Inci: $(-\infty, -1) \cup (3, \infty)$

Dec: $(-1, 3)$

Rel Max @ $x = -1$

Rel Min @ $x = 3$

Concavity / I.P.s: $[f']' = f'' = 0$ @ $x = 1$

$\leftarrow f'' < 0 \Rightarrow \text{CD}$ 1 $f'' > 0 \Rightarrow \text{CU}$

CD on $(-\infty, 1)$

CU on $(1, \infty)$

I.P. at $x = 1$

Problem 2. Compute $\lim_{x \rightarrow \infty} \frac{x^2+1}{2-x}$ and $\lim_{x \rightarrow -\infty} \frac{x^2+1}{2-x}$.

$$\frac{x^2+1}{-x+2} = \frac{x(x + \frac{1}{x})}{x(-1 + \frac{2}{x})} \underset{\text{when } x \neq 0}{=} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}}$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}} = -\infty$$

$\infty \swarrow$ $\nearrow 0$
 $\searrow -1$ $\nearrow 0$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}} = \infty$$

$$\begin{array}{l} \text{deg 2} \\ \text{num: } x^2 \\ \text{denom: } x+8 \\ \hline \text{deg 1} \end{array} \quad \left| \begin{array}{l} \text{Domain: } \mathbb{R} \setminus \{-8\} \\ \text{Intercepts: } (0,0). \quad \frac{x^2}{x+8} \stackrel{\text{set}}{=} 0 \Rightarrow x^2=0 \Rightarrow x=0 \\ \text{Asymptotes: VA @ } x=-8 \\ \text{Slant: } y=x-8 \end{array} \right.$$

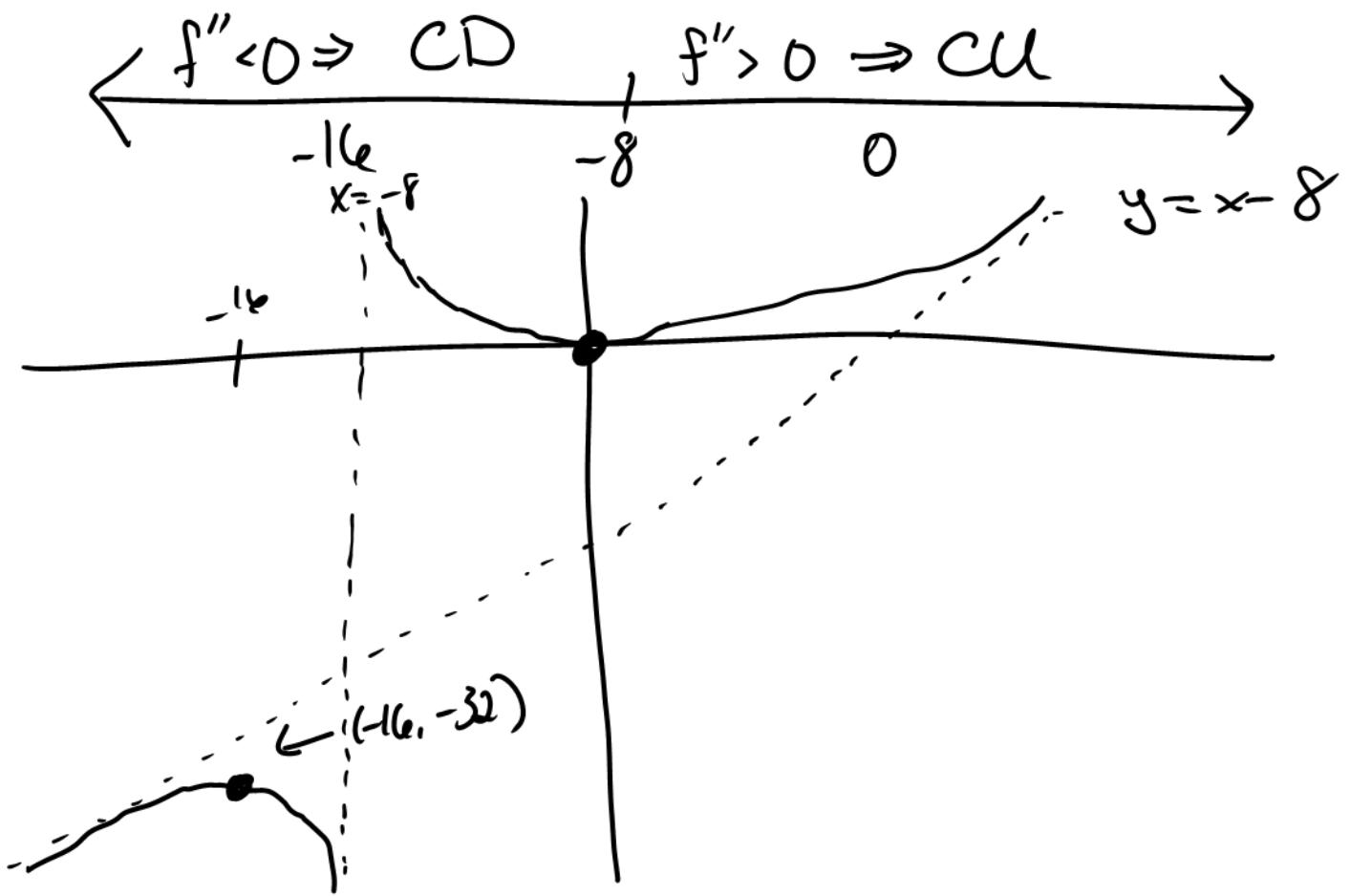
$$\begin{array}{l} x-8 \\ x+8 \overline{) x^2} \\ - (x^2 + 8x) \\ - 8x \\ - (-8x - 64) \\ \hline 64 \end{array} \quad \left| \begin{array}{l} \text{Rel Max/Min:} \\ f(x) = \frac{x^2}{x+8} \\ f'(x) = \frac{2x(x+8) - x^2}{(x+8)^2} = \frac{x^2 + 16x}{(x+8)^2} \stackrel{\text{Set}}{=} 0 \\ \Rightarrow x^2 + 16x = 0 \Rightarrow x(x+16) = 0 \Rightarrow x = -16, 0 \\ f''(x) = \frac{(2x+16)(x+8)^2 - (x^2+16x)2(x+8)}{(x+8)^4} = \frac{72(x+8)}{(x+8)^4} \end{array} \right.$$

$$f''(-16) < 0 \Rightarrow \text{Local Max @ } x = -16$$

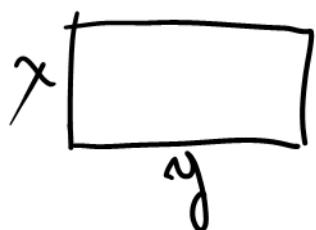
$$f''(0) > 0 \Rightarrow \text{Local Min @ } x = 0$$

$$\text{Concavity/JPs: } \frac{72(x+8)}{(x+8)^4} \stackrel{\text{Set}}{=} 0 \Rightarrow 72(x+8) = 0 \Rightarrow x = -8$$

*Not IP, but concavity
could change at an
asymptote*



Problem 4. If a rectangle has a fixed perimeter of 40, what is its maximum area?



$$\text{Obj: Maximize } A(x,y) = xy$$

$$40 = 2x + 2y$$

$$20 = x+y$$

$$y = 20-x$$

$$A(x) = x(20-x) = 20x - x^2$$

$$A'(x) = 20 - 2x \stackrel{\text{set}}{=} 0$$

$$x = 10$$

Verify it is an abs. max:

$$A''(x) = -2 < 0 \Rightarrow$$

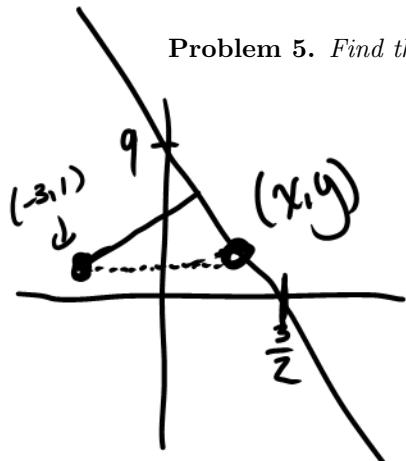
Rel. Max at $x=10$
Only 1 Crit. Num \Rightarrow Abs. Max.
at $x=10$

$$A(10) = 10(20-10) = 10^2 = 100$$

Conclusion: The max area is 100

$$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$$

Problem 5. Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.



Obj Min $s(x,y) = [d(x,y)]^2 = (x+3)^2 + (y-1)^2$

Given: $6x + y = 9$

$$y = -6x + 9$$

$$S(x) = (x+3)^2 + (-6x+9-1)^2 = (x+3)^2 + (-6x+8)^2$$

$$S(x) = 37x^2 - 90x + 73$$

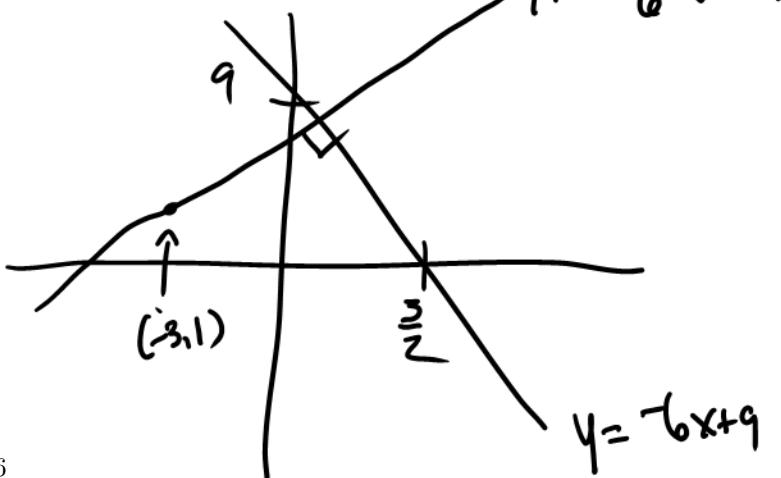
$$S'(x) = 74x - 90 \stackrel{\text{set } 0}{=} 0$$

$$x = \frac{90}{74} = \frac{45}{37}$$

Same Idea: $S''\left(\frac{45}{37}\right) = 74 > 0$ + Only 1 Cr. Min. \Rightarrow Abs. Min.

$$y = -6\left(\frac{45}{37}\right) + 9 = \frac{63}{37}$$

Conclusion: The point $\left(\frac{45}{37}, \frac{63}{37}\right)$ is closest to the point $(-3, 1)$



Problem 6. A particle is moving through space; its acceleration function is given by $a(t) = \cos t + \sin t$ for $t \geq 0$. Find the position of the particle at time t when $s(0) = 0$ and $v(0) = 5$.

Need to solve the
IVP

$$\begin{cases} s'' \xrightarrow{a(t) = \cos t + \sin t} \\ s' \xrightarrow{v(0) = 5} \\ s(0) = 0 \end{cases}$$

$$a(t) = \cos t + \sin t$$

$$\int a(t) dt = \int (\cos t + \sin t) dt$$

$$v(t) = \sin t - \cos t + C$$

~~$v(0) = 5$~~

$$5 = v(0) = 0 - 1 + C$$

$$5 = -1 + C$$

$$C = 6$$

$$v(t) = \sin t - \cos t + 6$$

$$\int v(t) dt = \int (\sin t - \cos t + 6) dt$$

$$s(t) = -\cos t - \sin t + 6t + D$$

~~$s(0) = 0$~~

$$0 = s(0) = -1 - 0 + 0 + D$$

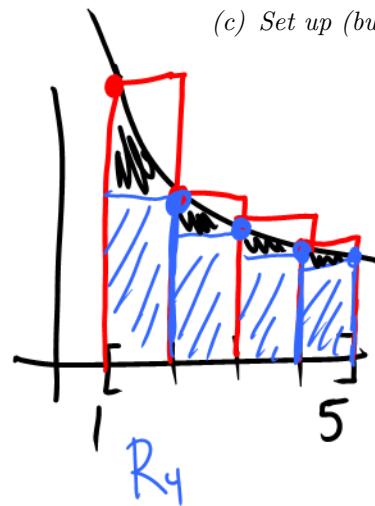
$$0 = -1 + D$$

$$D = 1$$

$$s(t) = -\cos t - \sin t + 6t + 1$$

Problem 7. We estimate the area underneath the graph of $f(x) = \frac{1}{x}$ on the interval $[1, 5]$.

- (a) Compute the left Riemann sum with 4 rectangles. Is this an overestimate or an underestimate? Explain why (The exact area is $\ln 5$, but you don't need to know that to answer the question).
 (b) Repeat (a) using the right Riemann sum.
 (c) Set up (but do not compute) the left Riemann sum using N rectangles (where N is a positive integer).



@ Compute L_4

$$\Delta X = \frac{5-1}{4} = \frac{4}{4} = 1$$

$$\Delta X = \frac{b-a}{N}$$

$$L_4 = \sum_{i=0}^3 f(1 + i(1))(1) = \sum_{i=0}^3 \frac{1}{1+i}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} \leftarrow \text{Over Estimate}$$

⑥ Compute R_4

$$R_4 = \sum_{i=1}^4 \frac{1}{1+i} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} \leftarrow \text{Under Estimate}$$

I.e., $\frac{77}{60} < \ln(5) < \frac{25}{12} \leftarrow \text{You don't need to do this}$

⑦ Find L_N

$$\Delta X = \frac{5-1}{N} = \frac{4}{N}$$

$$L_N = \sum_{i=0}^{N-1} f(a + i \Delta X) \Delta X \rightarrow L_N = \sum_{i=0}^{N-1} \frac{1}{1 + i \left(\frac{4}{N}\right)} \left(\frac{4}{N}\right)$$

\uparrow Left Endpoint