MT 2 Review

- Tomorrow (Tues. 10/21)
- 8-9 PM
- WTHR 200 — Main Floor NOT Balcony
- Lectures 11-19

Make sure to bring
- Pencils / Erasers
- PUID

- Calculator TI 30XA <u>Single lone only</u>

● 7:30 Section   ● 8:30 Section

Exam 2 Review

① Find crit. nums. of $y = [\ln(x+4)]^2$

Crit Nums: When y' is 0 or undefined

$$y' = 2\ln(x+4) \cdot \frac{1}{x+4}(1) = \frac{2\ln(x+4)}{x+4} \overset{\text{set}}{=} 0$$

$$2\ln(x+4) = 0$$
$$\ln(x+4) = 0$$
$$e \qquad\qquad e$$

$$x + 4 = 1$$

$$\boxed{x = -3}$$

What about $x = -4$? No (not in the domain)

Remark Critical Numbers need to be in the domain

$3\sqrt{x}$

← $x = 0$ is a c.n.

$\frac{1}{x}$

$x = 0$ is **not** a crit. num.
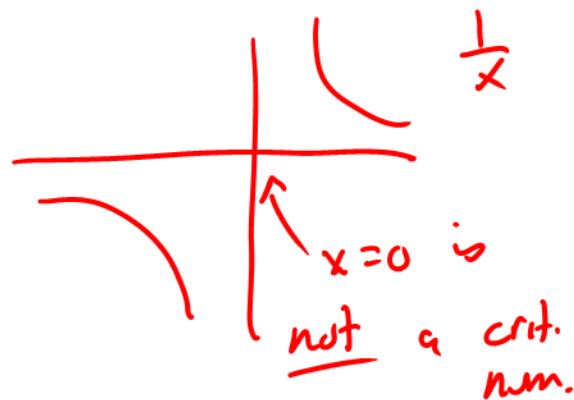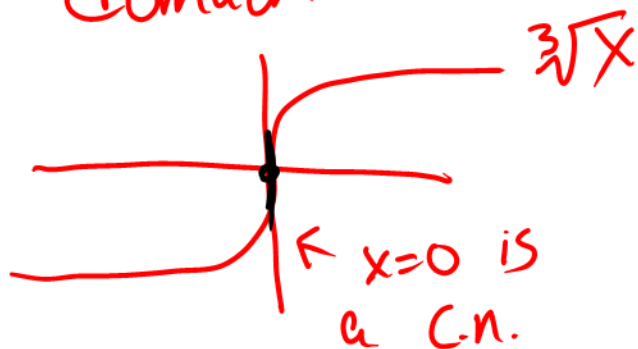
② Find $\left.\dfrac{dy}{dx}\right]_{(x,y)=(2,1)}$ given

$$\frac{d}{dx}\left(x^2 + 4xy + [y]^2 - 13 \right) = \frac{d}{dx}(0)$$

$$2x + [4x]'y + 4x[y]' + 2y\frac{dy}{dx} - 0 = 0$$

$$\underbrace{2x + 4y} + 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -(2x + 4y)$$

$$\frac{dy}{dx}(4x + 2y) = -(2x + 4y)$$

$$\frac{dy}{dx} = -\frac{2x + 4y}{4x + 2y}$$

$$\frac{dy}{dx}\Big]_{(2,1)} = -\frac{2(2) + 4(1)}{4(2) + 2(1)} = -\frac{8}{10} = \boxed{-\frac{4}{5}}$$

③
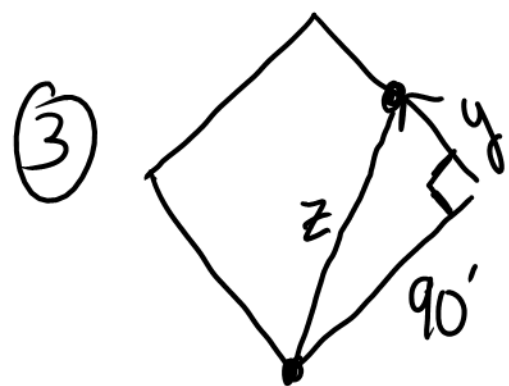
Know:
length = 90 ft
Speed of player $= \frac{dy}{dt} = 13$

Need to know: $\frac{dz}{dt}$ when $y = 45$

$$\frac{d}{dt}(90^2 + y^2) = \frac{d}{dt}(z^2)$$

$$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{\partial y}{\partial z}\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt}$$

Find z when $y = 45$: $90^2 + y^2 = z^2$
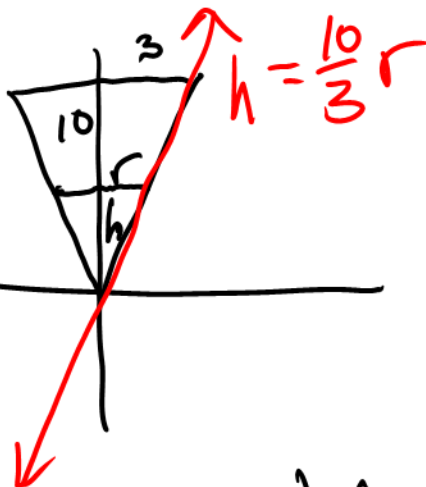
$$z = \sqrt{90^2 + y^2}. \text{ Hence, } z\Big]_{y=45} = \sqrt{90^2 + 45^2}$$

$$= \sqrt{(45 \cdot 2)^2 + (45 \cdot 1)^2} = \sqrt{45^2(2^2 + 1^2)} = 45\sqrt{5}$$

$$\frac{dz}{dt}\Big]_{\substack{y=45 \\ y'=13}} = \frac{45}{45\sqrt{5}}(13) = \frac{13}{\sqrt{5}} \text{ ft}/s$$

④ $\dfrac{dz}{dt} = \dfrac{y}{z}\dfrac{dy}{dt} = \dfrac{y}{\sqrt{\ell^2 + y^2}}\dfrac{dy}{dt}$

$$\frac{dz}{dt}\Big]_{\substack{y=\frac{1}{2}\ell \\ y'=v}} = \frac{\frac{1}{2}\ell}{\sqrt{\ell^2 + (\frac{1}{2}\ell)^2}}\,v = \frac{v}{\sqrt{5}} \text{ ft}/s$$

⑤

$h = \dfrac{10}{3}r$ Know: $\dfrac{dV}{dt} = 2 \text{ cm}^3/s$

Need to know: $\dfrac{dh}{dt}$ when $h = 5$

$$V = \frac{\pi}{3}r^2 h$$

$$V = \frac{\pi}{3}\left(\frac{3}{10}h\right)^2 h = \frac{3\pi}{100}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{100}\,3h^2\frac{dh}{dt} = \frac{9\pi}{100}h^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{100}{9\pi h^2} \frac{dV}{dt}$$

$$\left.\frac{dh}{dt}\right]_{\substack{h=5 \\ V'=2}} = \frac{100}{9\pi(5)^2}(2) = \frac{8}{9\pi} \frac{cm}{s}$$

⑥ $f(x) = x^3 - 6x^2 - 15x + 4$

Step 1 Identify Candidates for rel. extrema & IPs

$f'(x) = 3x^2 - 12x - 15 \overset{set}{=} 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0 \Rightarrow x = -1, 5$

$f''(x) = 6x - 12 \overset{set}{=} 0 \Rightarrow 6(x-2) = 0$
$\Rightarrow x = 2$

Step 2 Make Sign Chart

Test Point	-10		0		3		10
		-1		2		5	
Sign of f'	+		−		−		+

Sign of f''	$-$	$-$	$+$	$+$
Results of I/D Test	Inc ↗	↘ Dec	Dec ↘	↗ Inc
C. Test	CD	CD	CU	CU

-1
Rel. Max   2 I.P.   5 Rel. Min

⑦ Find the $\underline{\text{rel.}}$ and $\underline{\text{abs.}}$ extrema given

$$f(x) = 10 + 27x - x^3 \; ; \; x \in [0,4]$$

Crit. Nums.

$$f'(x) = 27 - 3x^2 \overset{\text{set}}{=\!=} 0$$

$$9 - x^2 = 0$$

$$(3+x)(3-x) = 0$$

$$\Rightarrow x = \pm 3 \xrightarrow{-3 \text{ not in } [0,4]} x = 3$$

Table

x	3	0	4
$f(x)$	64	10	54

$x = 0$, abs. min

$x = 3$, abs. max

$$f'(x) = 27 - 3x^2$$

$$f''(x) = -9x$$

$$f''(3) = -27 < 0$$

By 2nd $\frac{d}{dx}$ test, 3 is a rel. max