

# MT 3 Review

- Monday (11/10)
- 8-9 PM
- WTHR 200
- Lectures 20-28
- Make sure to bring
  - Pencils / Erasers
  - PUID
- Calculator
- TI 30XA
- Single lane only

Main Floor  
NOT Balcony

Amanda Manning

Balcony

V1	V2	V3	V4	V5	V6
U1	U2	U3	U4	U5	U6
T1	T2	T3	T4	T5	T6
S1	S2	S3	S4	S5	S6
R1	R2	R3	R4		
Q1	Q2	Q3	Q4	Q5	Q6
P1	P2	P3	P4	P5	P6
O1	O2	O3	O4	O5	O6
N1	N2	N3	N4	N5	N6
M2	M3	M4	M5	M6	M7
L1	L2	L3	L4	L5	L6
K1	K2	K3	K4	K5	K6
J1	J2	J3	J4	J5	J6
I1	I2	I3	I4	I5	I6
H1	H2	H3	H4	H5	H6
G1	G2	G3	G4	G5	G6
F1	F2	F3	F4	F5	F6
E1	E2	E3	E4	E5	E6
D1	D2	D3	D4	D5	D6
C1	C2	C3	C4	C5	C6
B1	B2	B3	B4	B5	B6
A1	A2	A3	A4	A5	A6

V7	V8	V9	V10	V11	V12	V13	V14
U8	U9	U10	U11	U12	U13	U14	U15
T8	T9	T10	T11	T12	T13	T14	T15
S8	S9	S10	S11	S12	S13	S14	S15

V15	V16	V17	V18	V19	V20
U18	U19	U20	U21	U22	U23
T18	T19	T20	T21	T22	T23
S18	S19	S20	S21	S22	S23

Main Floor

P8			X	X	X	X		P9
O8	O9	O10	O11	O12	O13	O14	O15	O16
N8	N9	N10	N11	N12	N13	N14	N15	N16
M8	M9	M10	M11	M12	M13	M14	M15	M16
L8	L9	L10	L11	L12	L13	L14	L15	L16
K8	K9	K10	K11	K12	K13	K14	K15	K16
J8	J9	J10	J11	J12	J13	J14	J15	J16
I8	I9	I10	I11	I12	I13	I14	I15	I16
H8	H9	H10	H11	H12	H13	H14	H15	H16
G8	G9	G10	G11	G12	G13	G14	G15	G16
F8	F9	F10	F11	F12	F13	F14	F15	F16
E8	E9	E10	E11	E12	E13	E14	E15	E16
D8	D9	D10	D11	D12	D13	D14	D15	D16
C8	C9	C10	C11	C12	C13	C14	C15	C16
B8	B9	B10	B11	B12	B13	B14	B15	B16
X	A8	A9	A10	A11	A12	A13		X

Siva Somasundaram

WTHR 200

400 stations / 444 LK

R5	R6	R7	R8
Q5	Q9	Q10	Q11
P10	P11	P12	P13
O18	O19	O20	O21
N18	N19	N20	N21
M18	M19	M20	M21
L18	L19	L20	L21
K18	K19	K20	K21
J18	J19	J20	J21
I18	I19	I20	I21
H18	H19	H20	H21
G18	G19	G20	G21
F18	F19	F20	F21
E18	E19	E20	E21
D18	D19	D20	D21
C18	C19	C20	C21
B18	B19	B20	B21
A14	A15	A16	A17

Zach Pence



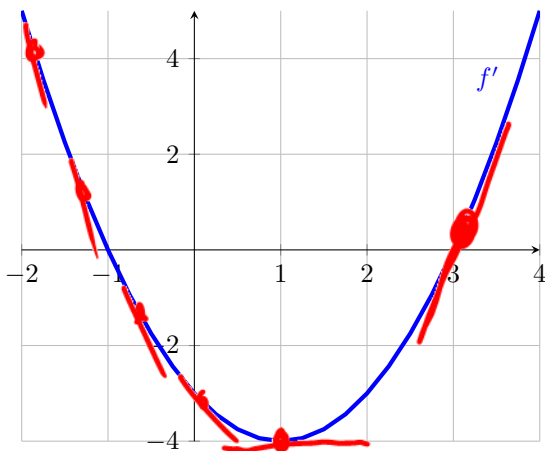
7:30 Section



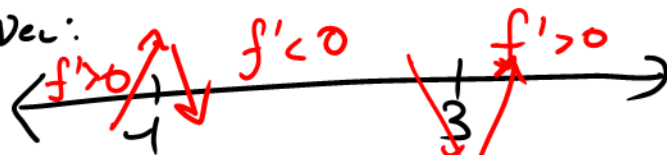
8:30 Section

**Problem 1.** A graph of  $f'$  is given below.

- Determine when  $f$  is increasing and when it is decreasing.
- Determine when  $f$  is concave up and when it is concave down.
- Locate the positions ( $x$ -coordinates) of any relative extrema and inflection points.



Inc/Dec:



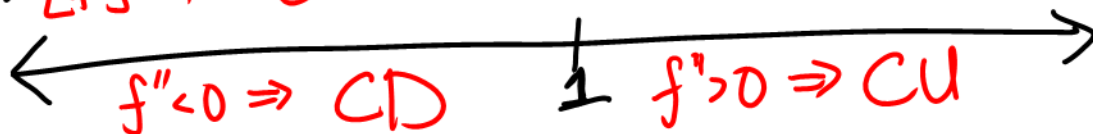
Inc:  $(-\infty, -1) \cup (3, \infty)$

Dec:  $(-1, 3)$

Rel Max @  $x = -1$

Rel Min @  $x = 3$

Concavity/IPs:  $[f']' = f'' = 0$  @  $x = 1$



CD on  $(-\infty, 1)$

CU on  $(1, \infty)$

IP at  $x = 1$

Problem 2. Compute  $\lim_{x \rightarrow \infty} \frac{x^2+1}{2-x}$  and  $\lim_{x \rightarrow -\infty} \frac{x^2+1}{2-x}$ .

$$\frac{x^2+1}{-x+2} = \frac{x(x + \frac{1}{x})}{x(-1 + \frac{2}{x})} \quad \text{when } x \neq 0 \quad \underline{\underline{=}} \quad \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}}$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}} = -\infty$$

$\begin{matrix} \nearrow \infty & & \searrow 0 \\ & \text{circled } \frac{1}{x} & \\ \downarrow -1 & & \searrow 0 \\ & \text{circled } \frac{2}{x} & \end{matrix}$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}} = \infty$$

Problem 3. Use the techniques learned in Lecture 22 to sketch a graph of the function  $y = \frac{x^2}{x+8}$ .

deg 2  
 $\sim x^2$   
 $\frac{x^2}{x+8}$   
 deg 1

Domain:  $\mathbb{R} \setminus \{-8\}$

Intercepts:  $(0,0)$ .  $\frac{x^2}{x+8} \stackrel{!}{=} 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

Asymptotes: VA @  $x = -8$

Slant:  $y = x - 8$

$$\begin{array}{r} x-8 \\ x+8 \overline{) x^2} \\ \underline{-(x^2+8x)} \\ -8x \\ \underline{-(-8x-64)} \\ 64 \end{array}$$

Rel Max/Min:

$f(x) = \frac{x^2}{x+8}$

$f'(x) = \frac{2x(x+8) - x^2}{(x+8)^2} = \frac{x^2 + 16x}{(x+8)^2} \stackrel{!}{=} 0$

$\Rightarrow x^2 + 16x = 0 \Rightarrow x(x+16) = 0 \Rightarrow x = -16, 0$

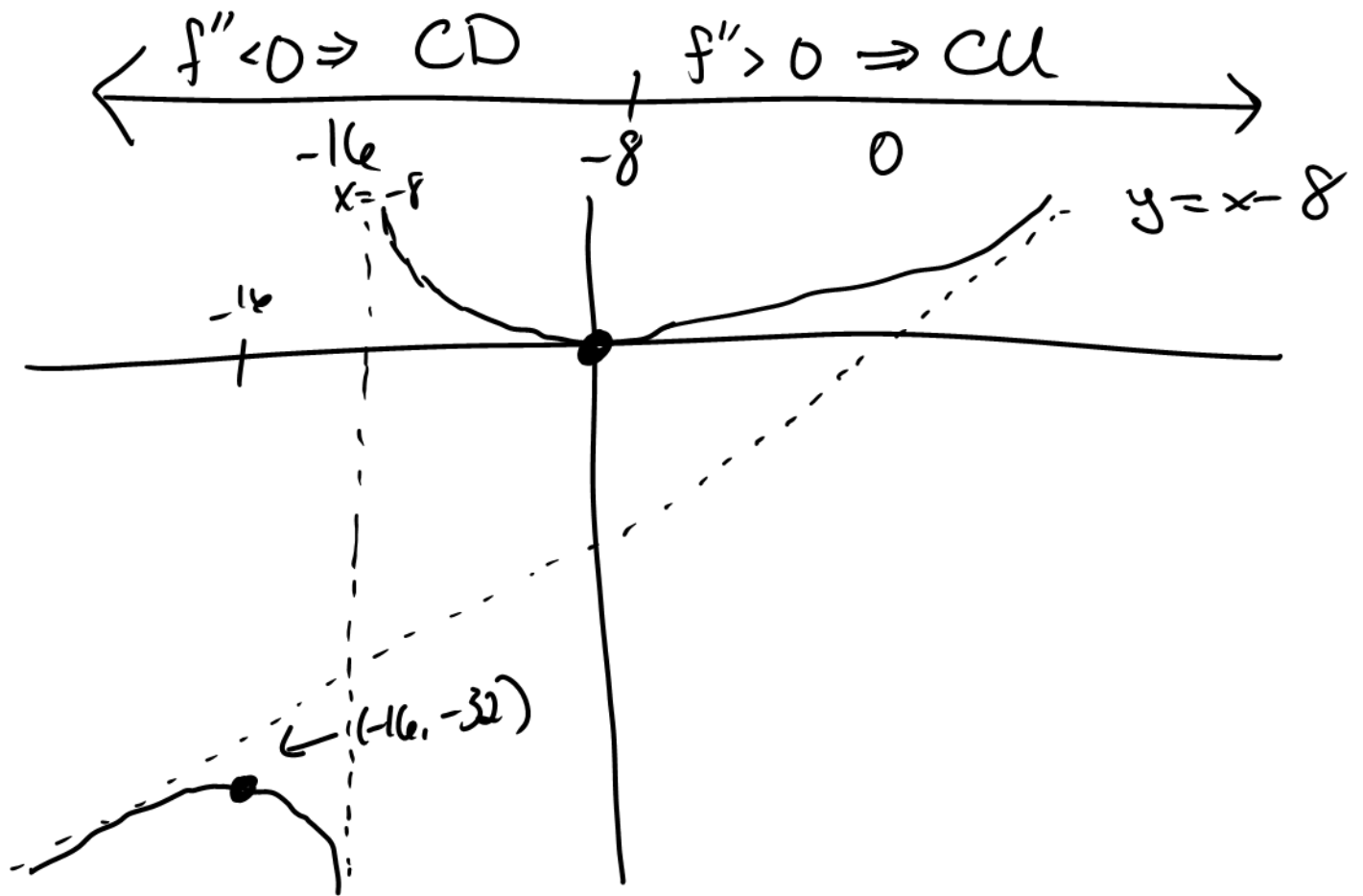
$$f''(x) = \frac{(2x+16)(x+8)^2 - (x^2+16x)2(x+8)}{(x+8)^4} = \frac{72(x+8)}{(x+8)^4}$$

$$f''(-16) < 0 \Rightarrow \text{Local Max @ } x = -16$$

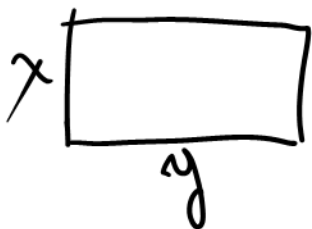
$$f''(0) > 0 \Rightarrow \text{Local Min @ } x = 0$$

Concavity/IPs:  $\frac{72(x+8)}{(x+8)^4} \stackrel{!}{=} 0 \Rightarrow 72(x+8) = 0 \Rightarrow x = -8$

Not IP, but concavity could change at an asymptote



Problem 4. If a rectangle has a fixed perimeter of 40, what is its maximum area?



Obj: Maximize

$$A(x,y) = xy$$

$$40 = 2x + 2y$$

$$20 = x + y$$

$$y = 20 - x$$

$$A_{xx} = x(20 - x) = 20x - x^2$$

$$A'(x) = 20 - 2x \stackrel{\text{set}}{=} 0$$

$$x = 10$$

Verify it is an abs. max:

$$A''(x) = -2 < 0 \Rightarrow$$

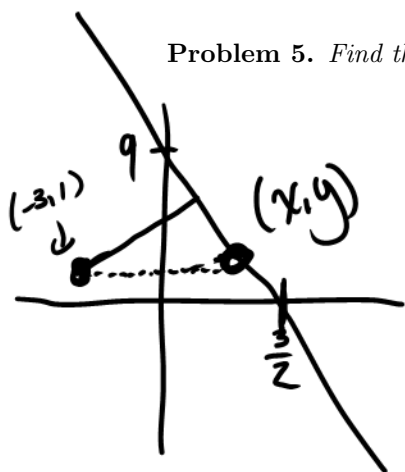
Rel. Max at  $x=10$   
 +  
 Only 1 Crit. num  $\Rightarrow$  Abs. Max.  
 at  $x=10$

$$A_{(10)} = 10(20 - 10) = 10^2 = 100$$

Conclusion: The max area is 100

$$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$$

Problem 5. Find the point on the line  $6x + y = 9$  that is closest to the point  $(-3, 1)$ .



Obj Min  $s(x, y) = [d(x, y)]^2 = (x+3)^2 + (y-1)^2$

Given:  $6x + y = 9$   
 $y = -6x + 9$

$$S(x) = (x+3)^2 + (-6x+9-1)^2 = (x+3)^2 + (-6x+8)^2$$

$$S(x) = 37x^2 - 90x + 73$$

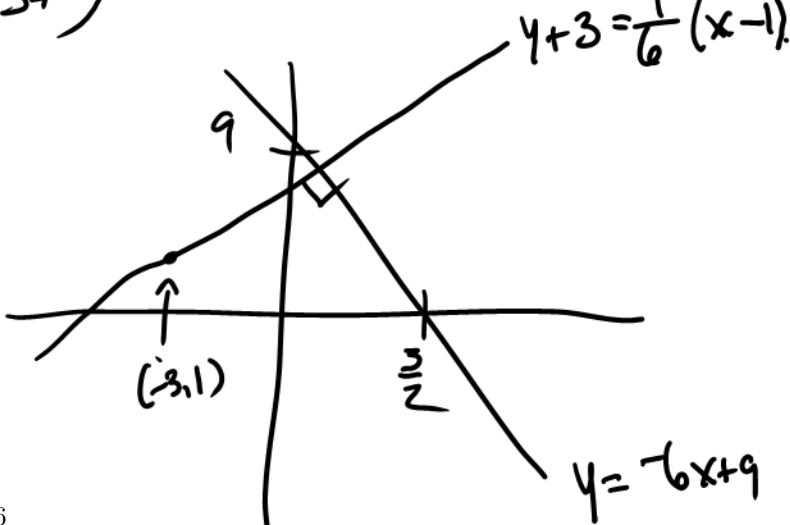
$$S'(x) = 74x - 90 \stackrel{\text{set}}{=} 0$$

$$x = \frac{90}{74} = \frac{45}{37}$$

Same Idea:  $S''(\frac{45}{37}) = 74 > 0$  + Only 1  $\Rightarrow$  Abs. Min.  
 Crit. Min

$$y = -6\left(\frac{45}{37}\right) + 9 = \frac{63}{37}$$

Conclusion: The point  $\left(\frac{45}{37}, \frac{63}{37}\right)$  is closest to the point  $(-3, 1)$



**Problem 6.** A particle is moving through space; its acceleration function is given by  $a(t) = \cos t + \sin t$  for  $t \geq 0$ . Find the position of the particle at time  $t$  when  $s(0) = 0$  and  $v(0) = 5$ .

Need to solve the  
IVP

$$\begin{aligned} s'' &\rightarrow a(t) = \cos t + \sin t \\ s' &\rightarrow \begin{cases} v(0) = 5 \\ s(0) = 0 \end{cases} \end{aligned}$$

$$a(t) = \cos t + \sin t$$

$$\int a(t) dt = \int (\cos t + \sin t) dt$$

$$v(t) = \sin t - \cos t + C$$

$$\underline{v(0) = 5:}$$

$$5 = v(0) = 0 - 1 + C$$

$$5 = -1 + C$$

$$C = 6$$

$$v(t) = \sin t - \cos t + 6$$

$$\int v(t) dt = \int (\sin t - \cos t + 6) dt$$

$$s(t) = -\cos t - \sin t + 6t + D$$

$$\underline{s(0) = 0:}$$

$$0 = s(0) = -1 - 0 + 0 + D$$

$$0 = -1 + D$$

$$D = 1$$

$$\boxed{s(t) = -\cos t - \sin t + 6t + 1}$$

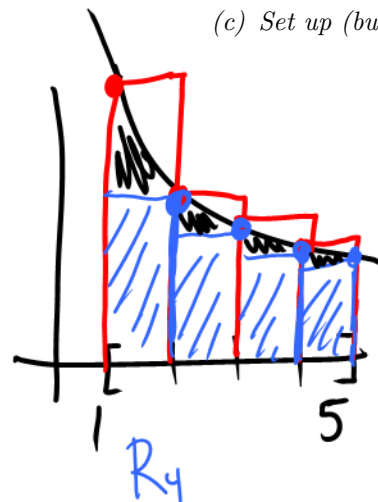


**Problem 7.** We estimate the area underneath the graph of  $f(x) = \frac{1}{x}$  on the interval  $[1, 5]$ .

(a) Compute the left Riemann sum with 4 rectangles. Is this an overestimate or an underestimate? Explain why (The exact area is  $\ln 5$ , but you don't need to know that to answer the question).

(b) Repeat (a) using the right Riemann sum.

(c) Set up (but do not compute) the left Riemann sum using  $N$  rectangles (where  $N$  is a positive integer).



@ Compute  $L_4$   
 $\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$

$$\Delta x = \frac{b-a}{N}$$

$$L_4 = \sum_{i=0}^3 f(1 + i(1))(1) = \sum_{i=0}^3 \frac{1}{1+i}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} \leftarrow \text{Over Estimate}$$

⑥ Compute  $R_4$

$$R_4 = \sum_{i=1}^4 \frac{1}{1+i} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} \leftarrow \text{Under Estimate}$$

Yes

$$\frac{77}{60} < \ln(5) < \frac{25}{12} \leftarrow \text{You don't need to do this}$$

⑦ Find  $L_N$

$$\Delta x = \frac{5-1}{N} = \frac{4}{N}$$

$$L_N = \sum_{i=0}^{N-1} f(a + i \Delta x) \Delta x \Rightarrow L_N = \sum_{i=0}^{N-1} \frac{1}{1 + i \left(\frac{4}{N}\right)} \left(\frac{4}{N}\right)$$

$\uparrow$  Left Endpoint