

MT 3 Review

Amanda Manning

Balcony

V1	V2	V3	V4	V5	V6
U1	U2	U3	U4	U5	U6
T1	T2	T3	T4	T5	T6
S1	S2	S3	S4	S5	S6
					S7

R1	R2	R3	R4
Q1	Q2	Q3	Q4
Q5	Q6	Q7	
P1	P2	P3	P4
P5	P6	P7	
O1	O2	O3	O4
O5	O6	O7	
N1	N2	N3	N4
N5	N6	N7	
M2	M3	M4	M5
M6	M7		
L1	L2	L3	L4
L5	L6	L7	
K1	K2	K3	K4
K5	K6	K7	
J1	J2	J3	J4
J5	J6	J7	
I1	I2	I3	I4
I5	I6	I7	
H1	H2	H3	H4
H5	H6	H7	
G1	G2	G3	G4
G5	G6	G7	
F1	F2	F3	F4
F5	F6	F7	
E1	E2	E3	E4
E5	E6	E7	
D1	D2	D3	D4
D5	D6	D7	
C1	C2	C3	C4
C5	C6	C7	
B1	B2	B3	B4
B5	B6	B7	
A1	A2	A3	A4
A5	A6	A7	

V7	V8	V9	V10	V11	V12	V13	V14
U8	U9	U10	U11	U12	U13	U14	U15
U16	U17						
T8	T9	T10	T11	T12	T13	T14	T15
T16	T17						
S8	S9	S10	S11	S12	S13	S14	S15
S16	S17						

V15	V16	V17	V18	V19	V20
U18	U19	U20	U21	U22	U23
U24					
T18	T19	T20	T21	T22	T23
T24					
S18	S19	S20	S21	S22	S23
S24					

R5	R6	R7	R8
Q8	Q9	Q10	Q11
Q12	Q13	Q14	
P10	P11	P12	P13
P14	P15	P16	
O18	O19	O20	O21
O22	O23	O24	
N18	N19	N20	N21
N22	N23	N24	
M18	M19	M20	M21
M22	M23	M24	
L18	L19	L20	L21
L22	L23	L24	
K18	K19	K20	K21
K22	K23	K24	
J18	J19	J20	J21
J22	J23	J24	
I18	I19	I20	I21
I22	I23	I24	
H18	H19	H20	H21
H22	H23	H24	
G18	G19	G20	G21
G22	G23	G24	
F18	F19	F20	F21
F22	F23	F24	
E18	E19	E20	E21
E22	E23	E24	
D18	D19	D20	D21
D22	D23	D24	
C18	C19	C20	C21
C22	C23	C24	
B18	B19	B20	B21
B22	B23	B24	
A14	A15	A16	A17
A18	A19	A20	

X	A8	A9	A10	A11	A12	A13	X
Siva Somasundaram							
WTHR 200							
100 stations	144 LIV						

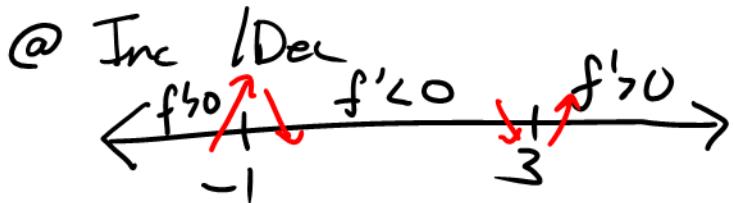
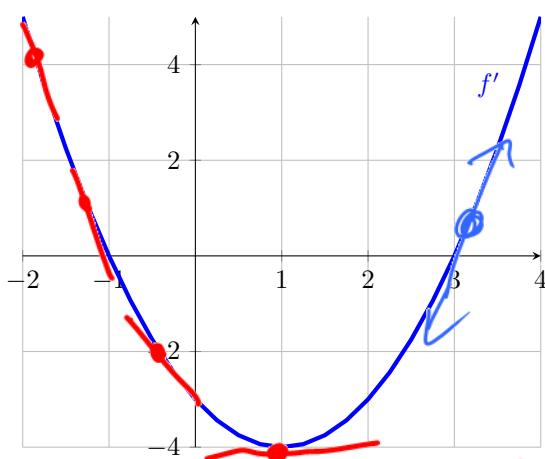
- Monday (11/10)
- 8-9 PM
- WTHR 200
 - Lectures 20-28
 - Make Sure to bring
 - Pencils / Erasers
 - PUID
 - Calculator
 - TI 30XA
 - Single line only

7:30 Section 8:30 Section

Zach Pence

Problem 1. A graph of f' is given below.

- Determine when f is increasing and when it is decreasing.
- Determine when f is concave up and when it is concave down.
- Locate the positions (x -coordinates) of any relative extrema and inflection points.



Inc: $(-\infty, -1) \cup (3, \infty)$

Dec: $(-1, 3)$

Rel Max @ $x = -1$

Rel Min @ $x = 3$



CD on $(-\infty, 1)$

CU on $(1, \infty)$

Inflection Point at $x = 1$

Problem 2. Compute $\lim_{x \rightarrow \infty} \frac{x^2+1}{2-x}$ and $\lim_{x \rightarrow -\infty} \frac{x^2+1}{2-x}$.

$$\frac{x^2+1}{-x+2} = \frac{x(x+\frac{1}{x})}{x(-1+\frac{2}{x})} = \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}}$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}} = -\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x + \frac{1}{x}}{-1 + \frac{2}{x}} = \infty$$

Problem 3. Use the techniques learned in Lecture 22 to sketch a graph of the function $y = \frac{x^2}{x+8}$.

$$\begin{array}{l} \frac{x^2 + \deg 2}{x+8} \\ \uparrow \quad \downarrow \\ \text{Domain: } \mathbb{R} \setminus \{-8\} \end{array}$$

Intercepts: $(0,0)$. Solve $\frac{x^2}{x+8} = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

Asymptote: VA @ $x = -8$

$$\begin{array}{r} x-8 \\ x+8 \overline{)x^2} \\ = (x^2 + 8x) \\ - 8x \\ \hline -(-8x - 64) \\ \hline 64 \end{array}$$

Slant Asymptote of $y = x - 8$

Rel. Max / Mins

$$f(x) = \frac{x^2}{x+8}$$

$$f'(x) = \frac{2x(x+8) - x^2}{(x+8)^2} = \frac{x^2 + 16x}{(x+8)^2}$$

Set $\equiv 0$

$$\Rightarrow x^2 + 16x = 0$$

$$x(x+16) = 0 \Rightarrow \boxed{x = -16, 0}$$

$$f'(x) = \frac{x^2 + 16x}{(x+8)^2}$$

$$f''(x) = \frac{(2x+16)(x+8)^2 - (x^2 + 16x)2(x+8)}{(x+8)^4}$$

$$f''(x) = \frac{72(x+8)}{(x+8)^4}$$

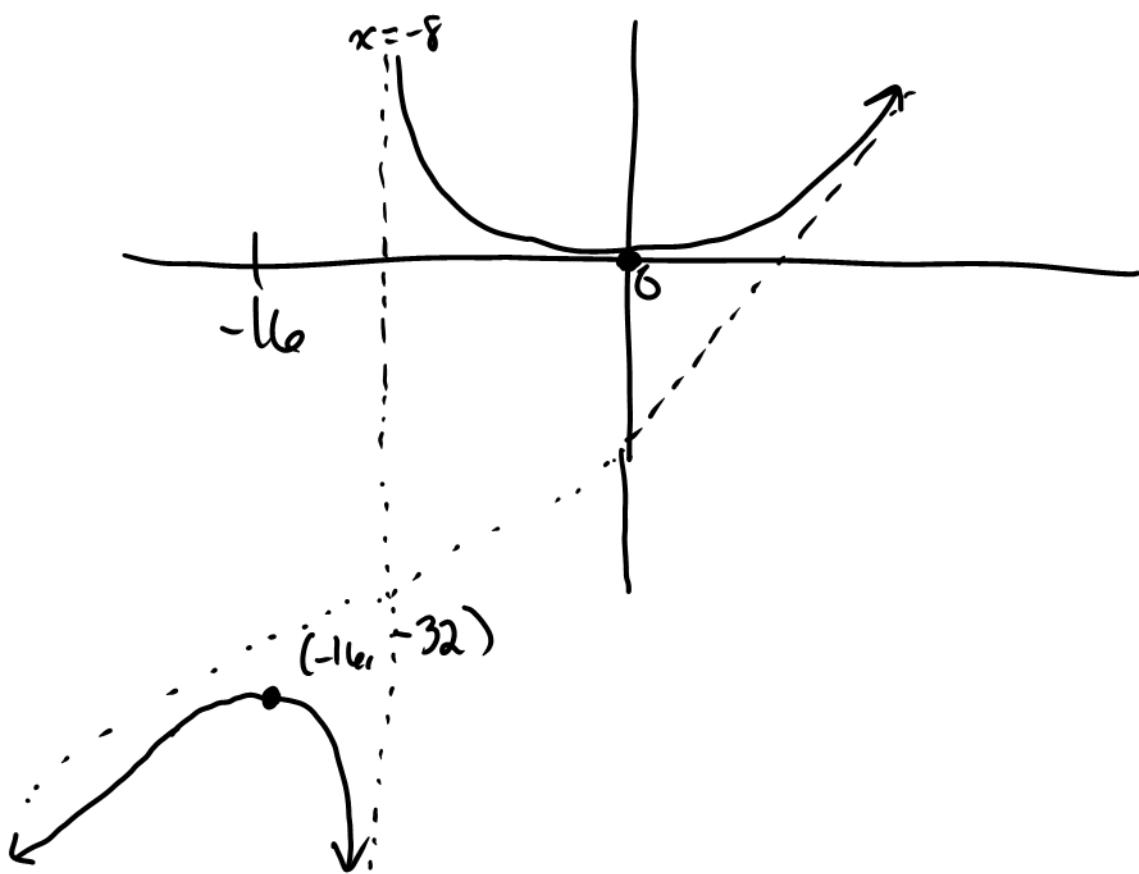
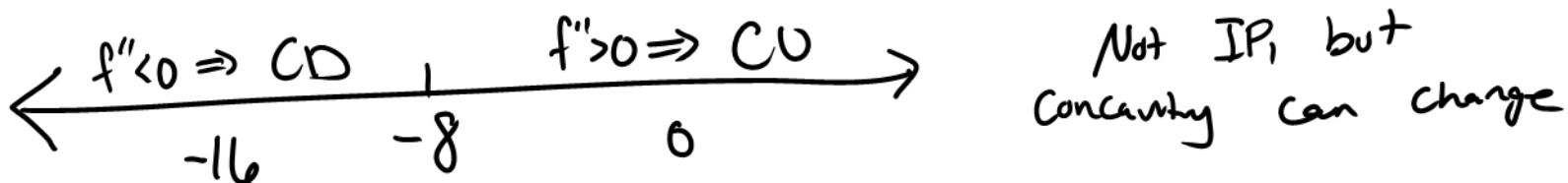
Determine Max/Min by 2nd $\frac{d}{dx}$ test:

$f''(-16) < 0 \Rightarrow$ Local Max at $x = -16$

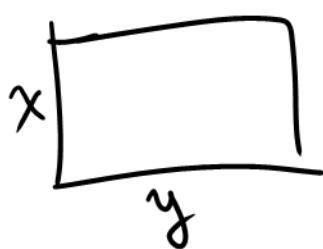
$f''(0) > 0 \Rightarrow$ Local Min at $x = 0$

Concavity / IPs:

$$f''(x) \stackrel{\text{set}}{=} 0 \Leftrightarrow 72(x+8) = 0 \Rightarrow x = -8$$



Problem 4. If a rectangle has a fixed perimeter of 40, what is its maximum area?



Obj: Max

Given:

$$A(x,y) = xy$$

$$40 = 2x + 2y$$

$$20 = x + y$$

$$y = 20 - x$$

$$A(x) = x(20-x) = 20x - x^2$$

$$A'(x) = 20 - 2x \stackrel{\text{set}}{=} 0$$

$$x = 10$$

Verify this is the location of an abs. max:

$$A''(x) = -2$$

$$A''(10) = -2 < 0 \Rightarrow \text{Local Max } @ x=10$$

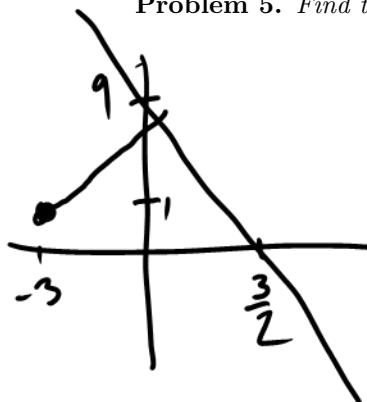
Local Max
Only 1 Crt. Num. \Rightarrow Abs. Max

Maximum Area?

$$A(10) = (10)(20-10) = 10^2 = 100$$

$$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$$

Problem 5. Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.



$$\text{Obj: Min } S(x,y) = d^2(x,y) = (x+3)^2 + (y-1)^2$$

$$\text{Given: } 6x + y = 9$$

$$y = -6x + 9$$

$$\begin{aligned} S(x) &= (x+3)^2 + (-6x+9-1)^2 = (x+3)^2 + (-6x+8)^2 \\ &= 37x^2 - 90x + 73 \end{aligned}$$

$$S'(x) = 74x - 90 \stackrel{\text{set}}{=} 0$$

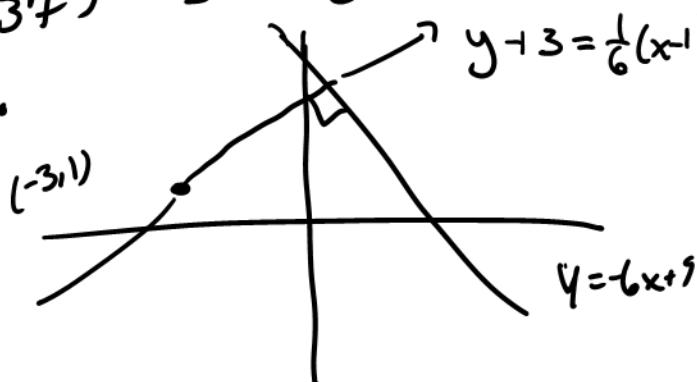
$$x = \frac{90}{74} = \frac{45}{37}$$

$x = \frac{45}{37}$ is the loc. of a rel. min

It is indeed the location of a min.

$$y = -6\left(\frac{45}{37}\right) + 9 = \frac{63}{37}$$

Conclusion: The point $\left(\frac{45}{37}, \frac{63}{37}\right)$ is closer to the point $(-3, 1)$.



Problem 6. A particle is moving through space; its acceleration function is given by $a(t) = \cos t + \sin t$ for $t \geq 0$. Find the position of the particle at time t when $s(0) = 0$ and $v(0) = 5$.

Need to solve s'' → $a(t) = \cos t + \sin t$
 the IVP $\begin{cases} s' \\ s \end{cases}$ → $\begin{cases} v(0) = 5 \\ s(0) = 0 \end{cases}$

$$a(t) = \cos t + \sin t$$

$$\int a(t) dt = \int (\cos t + \sin t) dt$$

$$v(t) = \sin t - \cos t + C$$

$$v(0) = 5 \quad s = v(0) = \sin 0 - \cos 0 + C$$

$$s = -1 + C$$

$$C = 6$$

$$v(t) = \sin t - \cos t + 6$$

$$\int v(t) dt = \int (\sin t - \cos t + 6) dt$$

$$s(t) = -\cos t - \sin t + 6t + D$$

$$0 = s(0) = -1 - 0 + 0 + D$$

$$0 = -1 + D$$

$$D = 1$$

$$s(t) = -\cos t - \sin t + 6t + 1$$

Problem 7. We estimate the area underneath the graph of $f(x) = \frac{1}{x}$ on the interval $[1, 5]$.

- Compute the left Riemann sum with 4 rectangles. Is this an overestimate or an underestimate? Explain why (The exact area is $\ln 5$, but you don't need to know that to answer the question).
- Repeat (a) using the right Riemann sum.
- Set up (but do not compute) the left Riemann sum using N rectangles (where N is a positive integer).

$$\textcircled{a} \quad \Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$$



$$L_4 = \sum_{i=0}^3 f(a+i\Delta x) \Delta x$$

$$= \sum_{i=0}^3 \frac{1}{1+i} \quad (1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$= \frac{25}{12} \leftarrow \text{Over estimate}$$

\textcircled{b} $R_4 = \sum_{i=1}^4 \frac{1}{1+i} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} \leftarrow \text{Under estimate}$

$$\textcircled{c} \quad \Delta x = \frac{5-1}{N} = \frac{4}{N}$$

$$L_N = \sum_{i=0}^{N-1} \frac{1}{1+(\frac{4}{N})i} \left(\frac{4}{N} \right)$$

