MA 16010: Applied Calculus I

Lecture 14: Related Rates (Geometric Relations)

Zachariah Pence

Purdue University

Sections Covered: 3.1 (Up to the Ladder Problem)

Introduction

A circle's area and radius are related by the equation:

$$A = \pi r^2$$

If A are r are changing as time advances, is there any relation between $\frac{dA}{dt}$ and $\frac{dr}{dt}$?

Plugging in Values

Problem 1

In the previous example, if $r = 2$ *and* $\frac{dr}{dt} = 3$*, then what is the value of* $\frac{dA}{dt}$*? Interpret.*

Plugging in Values (cont.)

Problem 2

In the previous example, if $r = 1$ *and* $\frac{dA}{dt} = 2\pi$*, then what is the value of* $\frac{dr}{dt}$*?*

Applying the circle example

Problem 3

The radius of a circle r is increasing at a constant rate 3 cm/min.

(1) Find the rate of change of the area of the circle (A) when the radius is 5cm.

Applying the circle example (cont.)

(2) Find the rate of change of the circumference of the circle (C) when the radius is 5cm.

Rectangular Prisms

Problem 4

The edges of a cube are shrinking at a rate of 10 *cm/s.*

(1) How fast is the volume (V) shrinking when each side length is 9cm long?

Rectangular Prisms (cont.)

(2) How fast is the surface area (A) shrinking when each side length is 9cm long?

Spheres

Problem 5

A balloon is (roughly) a sphere. The balloon deflates and its radius decreases at a rate of 2 cm/s.

(1) How fast is the volume (V) shrinking when the radius is 5cm long?

Spheres (cont.)

(2) How fast is the surface area (A) shrinking when the radius is 5cm long?

Cylinders

Problem 6

A cylindrical tank with a radius and height of 100 *cm stands upright. Water is being drained at a rate of* $7 cm^3/s$*. How fast is the water level changing when the tank is half empty.*

Cones

Problem 7

Sand pours onto a surface at $15cm^3/s$*, forming a conical pile with a base diameter that is always equal to the pile's altitude. How fast is the altitude of the pile increasing when the pile is* $8cm$ *high?*