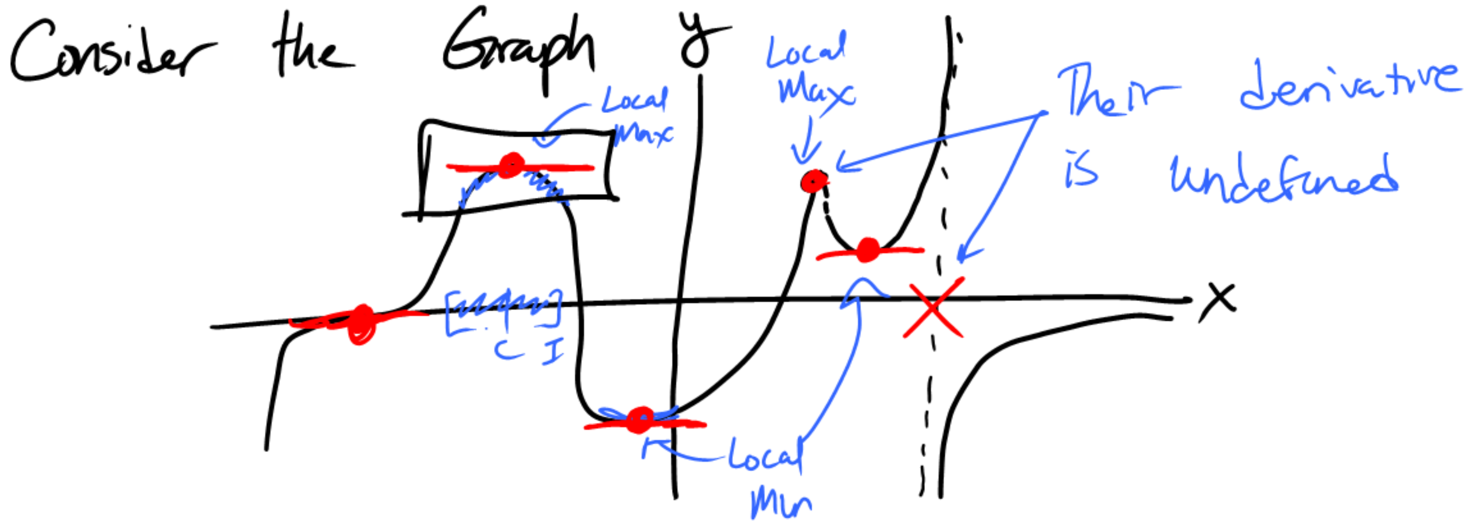
Consider the Graph

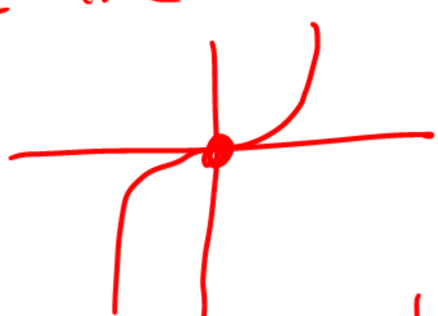


<u>Def</u> A function f has a <u>relative maximum</u> (or <u>local max</u>) at c if there is an interval I containing c such that $f(x) \leq f(c)$ for x in I.

<u>Def</u> For <u>local minimums</u> (or <u>relative min</u>) replace $f(x) \leq f(c)$ with $f(c) \leq f(x)$.

<u>Theorem</u> If f has a relative max/min at $x=c$, then either $f'(c)=0$ or $f'(c)$ is undefined.

<u>NOTE</u>: The reverse is not true



$f(x) = x^3$

$f'(x) = 3x^2$

$f'(0) = 0$, but f does not have a local max/min at 0.

<u>Def</u> If $f'(c) = 0$ or undefined, we say c is a <u>critical number</u> of f.

Ex1/ Is $x = \pi$ a critical number of $f(x) = 8\sin\left(x - \frac{\pi}{2}\right)$

$f'(x) = 8\cos\left(x - \frac{\pi}{2}\right)$

$f'(\pi) = 8\cos\left(\pi - \frac{\pi}{2}\right) = 8\cos\left(\frac{\pi}{2}\right) = 0$ ✓ Yes

Ex2/ Is $x = \frac{\pi}{2}$ a critical number for $f(x) = 8\sin\left(x - \frac{\pi}{2}\right)$

$f'(x) = 8\cos\left(x - \frac{\pi}{2}\right)$

$f'\left(\frac{\pi}{2}\right) = 8\cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 8\cos 0 = 8 \neq 0$ ✗ No

Ex3/ Find all critical numbers of $f(x) = x^3 - 75x$

$f'(x) = 3x^2 - 75$

$f'(x)$ undefined: Never

$f'(x) = 0$: $3x^2 - 75 \overset{\text{set}}{=} 0$

$3(x^2 - 25) = 0$

$3(x+5)(x-5) = 0$

$\Rightarrow \boxed{x = -5, 5}$

Ex4/ Find all critical numbers of $f(x) = \tan x$ on the interval $[-\pi, \pi)$

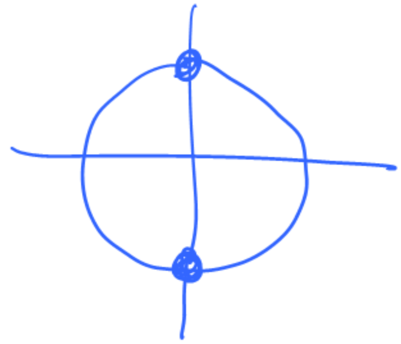
$f'(x) = \sec^2 x = \dfrac{1}{\cos^2 x}$

$f'(x) = 0$: Never

$f'(x)$ undefined:

$$\cos^2 x = 0$$

$$\cos x = 0$$



On the interval $[-\pi, \pi)$:

$$\boxed{x = -\frac{\pi}{2}, \frac{\pi}{2}}$$

Ex 5/ Find all critical numbers of $f(x) = \frac{\ln x}{x}$

$$f'(x) = \frac{\left(\frac{1}{x}\right)\cdot x - \ln(x)\cdot 1}{x^2} = \frac{1-\ln x}{x^2}$$

$f'(x)$ undefined:

$$x^2 \overset{\text{set}}{=} 0 \rightarrow \boxed{x = 0}$$

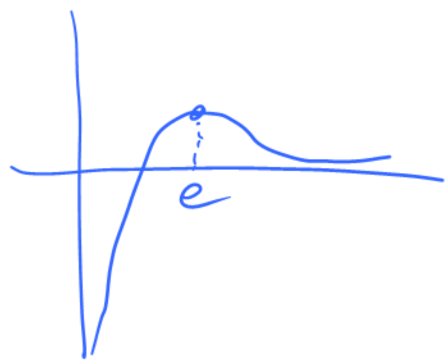
$f'(x) = 0$:

$$1 - \ln x \overset{\text{set}}{=} 0$$

$$\ln x = 1$$

$$e^{\ln x} = e^1$$

$$\boxed{x = e}$$



Ex 6/ Find all critical numbers of $f(x) = xe^x$

$$f'(x) = 1\cdot e^x + x\cdot e^x = e^x + xe^x = (1+x)e^x$$

$$(1+x)e^x \overset{\text{set}}{=} 0$$

$\Rightarrow$ Either $1+x=0$ OR $e^x=0$

$\boxed{x=-1}$ Never

Ex 7/ $f(x)=(x^2-1)^3$

$f'(x)=3(x^2-1)^2(2x) \overset{set}{=} 0$

Either, $6x=0$ OR $(x^2-1)^2=0$

$\boxed{x=0}$

$x^2-1=0$

$\boxed{x=-1,1}$

Ex 9/ $f(x)=2x^3-3x^2-12x+1$

$f'(x)=6x^2-6x-12 \overset{set}{=} 0$

$6(x^2-x-2)=0$

$6(x-2)(x+1)=0$

$\Rightarrow$ Either $\cancel{6=0}$ OR $x-2=0$ OR $x+1=0$

$\boxed{x=2}$ $\boxed{x=-1}$

Quadratic Formula:

$ax^2+bx+c=0 \Longrightarrow x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

~~Ex 10~~ $f(x) = x^3 + 5x^2 + 7x$

$$f'(x) = \underbrace{3}_{a}x^2 + \underbrace{10}_{b}x + \underbrace{7}_{c}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(7)}}{2(3)} = \frac{-10 \pm \sqrt{100 - 84}}{6} = \frac{-10 \pm \sqrt{16}}{6}$$

$$= \frac{-10 \pm 4}{6} \Longrightarrow x = \frac{-6}{6}, \frac{-14}{6} \Longrightarrow \boxed{x = -1, -\frac{7}{3}}$$