ORBITS OF THE SIX NEW SATELLITES OF NEPTUNE

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ABSTRACT

Orbital elements are presented for the six satellites discovered by *Voyager 2* at Neptune [B. A. Smith *et al.*, Science, 246, 1422 (1989)]. All six are in nearly circular direct orbits. Most have low inclinations as well, but the innermost, 1989N6, is inclined 4°7 to Neptune's equator.

1. INTRODUCTION

Voyager 2 discovered six small satellites orbiting Neptune (Smith *et al.* 1989); their IAU provisional designations, assigned in order of discovery, are 1989N1 through 1989N6. Unlike the Uranian system, in which all but one of the satellites orbit outside the outermost ring (Owen & Synnott 1987), only two of the six new satellites at Neptune are outside the ring system. Three of them (1989N3, 1989N5, and 1989N6) lie inside the 1989N2R ring, and 1989N4 is located between the 1989N1R and 1989N2R rings.

This paper presents orbital elements for these six small satellites. Section 2 describes the image analysis and Sec. 3 the orbit analysis; Sec. 4 discusses the results.

2. IMAGE ANALYSIS

Each *Voyager* carries two vidicon cameras: a wide-angle refractor with focal length 200 mm, and a narrow-angle Schmidt–Cassegrain reflector with focal length 1500 mm (Smith *et al.* 1977; Danielson *et al.* 1981). Both cameras return 800×800 pixel arrays, with 8 bits or 256 gray levels per pixel. One pixel subtends about 2″ in the narrow-angle camera or 15″ in the wide-angle camera.

The Neptune encounter produced a wealth of imaging data, much of it useful for small satellite orbit determination. In the "observatory phase," from 1989 June 5 to August 6, sequences of 46 s exposures centered on the planet revealed 1989N1 through 1989N4. The "far encounter" phase, which lasted until 1989 August 24, included a rich, diverse set of frames: 64 s ring-search mosaics, in which 1989N5 and 1989N6 were found; 15.36 s exposures centered on Neptune, in which a satellite would occasionally appear; a "ring movie" of dozens of frames targeted to the same spot in the ring plane; several satellite search mosaics; and a number of miscellaneous support images. Sprinkled throughout both phases were optical navigation frames, planned before any new satellites were found, designed expressly for improving their orbits. These were initially centered on the planet, but the later ones were retargeted to 1989N1.

Of the 9000 imaging frames returned during the entire encounter period, approximately 1100 were potential candidates for small satellite orbit determination. These were all taken through the clear filter, before closest approach to Neptune (when phase angles were low), and with sufficient exposure time to reveal the satellites. Almost all used the narrow-angle camera. Most, however, proved unusable: no satellites were in the field of view, or no stars were visible, or the images were too smeared to allow good center finding. Among the casualties were the high-resolution images of 1989N1, in which no stars were seen. The final dataset contained 387 frames, 619 satellite images, and 1425 star images, acquired over a span of almost 3 months. This is more than double the extent of the small satellite data at Uranus.

We used JPL's Optical Navigation Image Processing System (Synnott *et al.* 1986; Riedel *et al.* 1990) to determine the positions of the stars and of the centers of the satellites in each usable frame. The amount of image smear, assumed constant for all images, is estimated as part of the center finding process. Late in the data arc, when the satellite orbital motion became noticeable, the smear of each satellite image was determined separately. This software also examines the reseaux surrounding each image to determine the vidicon-induced distortions in the focal plane.

We processed most of the frames within a day in order to provide accurate retargeting of near-encounter high-resolution frames of the satellites. Frames with questionable residuals were reexamined after encounter.

3. ORBIT ANALYSIS

Centers of the satellite and star images were passed to JPL's Multimission Optical Navigation Program (Riedel *et al.* 1990) for further reduction. The heart of this program is a numerically stable variant of the Kalman (1960) filter in which the covariance matrix is expressed in factored form (Bierman 1976; Thornton 1976). We assigned uncertainties ranging from 0.5 to 2 pixels to each star measurement, depending on the amount of smear in the picture. Satellite images were often downweighted further, particularly early in the data arc when their images were dim.

Star positions were taken from Klemola & Owen (1986); the internal errors are about 0″10 (0.05 narrow-angle pixel), below our centerfinding capability. Ephemerides for *Voyager*, Neptune, and Triton were provided by Jacobson *et al.* (1990); the standard error of a planet-relative vector is a few km at closest approach. The ephemerides also contain the adopted physical constants for the Neptune system, reproduced here with Jacobson's kind permission in Table 1.

Due to the presence of massive Triton in its retrograde orbit, a small satellite interior to Triton will have an equilibrium orbit in its "Laplacian plane" (Laplace 1966; Dobrovolskis 1980). This "plane" (actually a surface) lies between Neptune's equator and Triton's orbital plane; its inclination to Neptune's equator increases with a, and it shares its line of nodes with Triton. The Laplacian plane consequently precesses about the Neptune–Triton invariable plane at Triton's nodal rate. Over the 3 month data arc, this precession amounts to only 0.13 degree; the change in the absolute orientation of the Laplacian plane is at most 0.0010 degree and is therefore ignored.

The satellite orbits were modeled as precessing ellipses, each with a constant inclination to its Laplacian plane. We define for each satellite a planetocentric nonrotating coordinate frame whose z-axis is normal to the Laplacian plane at

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TABLE 1. Neptune system physical constants.

Neptune mass, $G\mathcal{M}_N$	6835107.0 ± 15 km^3/s^2
Neptune adopted equatorial radius, R	25225 km
Neptune second zonal coefficient, J_2	$(341.05 \pm 0.9) \times 10^{-5}$
Neptune fourth zonal coefficient, J_4	$(-3.47 \pm 0.1) \times 10^{-5}$
R.A. (1950.0) of Neptune pole	298.8575 ± 0.15 deg
Dec. (1950.0) of Neptune pole	42.8118 ± 0.05 deg
R.A. (1950.0) of normal to invar. plane	298.9474 ± 0.15 deg
Dec. (1950.0) of normal to invar. plane	43.3188 ± 0.10 deg
Triton mass, $G\mathcal{M}_T$	1427.9 ± 3.5 km^3/s^2
Triton mean orbital radius, a_T	354759.1 ± 15 km
Triton inclination to invariable plane, i_T^*	156.8342 ± 0.006 deg
Triton nodal precession rate, $d\Omega_T/dt$	0.5232 ± 0.25 deg/yr

encounter, positive north; the x-axis lies along the ascending node of the Laplacian plane on the Earth mean equator of 1950.0; and the y-axis completes the right-handed triad. In this coordinate frame, we estimate the equinoctial element set (Brouwer & Clemence 1961, Chaps. 15 and 16):

$a =$ geometric semimajor axis,

$h = e \sin \varpi$,

$k = e \cos \varpi$,

$\lambda = \varpi + M$,

$p = \tan \frac{i}{2} \sin \Omega$,

$q = \tan \frac{i}{2} \cos \Omega$,

$n_s =$ sidereal mean motion,

where Ω, ϖ, and λ are measured from the x-axis. Secular perturbations are admitted by letting Ω and ϖ be linear functions of time; their rates are computed, not estimated. We note that a is the geometric mean orbital radius, denoted as r by Greenberg (1981) and as r_0 by Borderies & Longaretti (1987), not the osculating semimajor axis or even its mean value. In our estimation process a has the role of a simple scale factor on the orbit; n_s is treated as independent from a. Similarly, the eccentricity e is geometric, not osculating.

Since we ignore periodic perturbations, we adopt a time-averaged Hamiltonian based on Dobrovolskis *et al.* (1989) and Owen & Porco (1991):

$$\mathcal{H} = -G\mathcal{M}_N/2a + U + V + W, \tag{1}$$

where $\mathcal{M}_N$ is the mass of Neptune. The perturbations U, V, and W arise, respectively, from Neptune's gravity harmonics, Triton's attraction, and precession about the invariable plane (Goldreich 1965). Let I denote the orbital inclination to Neptune's equator, J the inclination to Triton's orbit, and i^* the inclination to the invariable plane; these angles are related by

$$\cos J = \cos I \cos i_T + \sin I \sin i_T \cos(\Omega_N - \Omega_T), \tag{2}$$

$$\cos i^* = \cos I \cos i_0 + \sin I \sin i_0 \cos(\Omega_N - \Omega_T), \tag{3}$$

where i_T and Ω_T are the inclination and ascending node of Triton's orbit, relative to Neptune's equator; Ω_N is the longitude of the ascending node of the satellite orbit on Neptune's equator; and i_0 is the inclination of the invariable plane to Neptune's equator. (Triton's inclination to the invariable plane is $i_T^* = i_T + i_0$.) In terms of these angles, the components of the Hamiltonian are

$$\begin{aligned} U = n_s^2 a^2 \{ &\tfrac{1}{2} J_2 (R/a)^2 (1-e^2)^{-3/2} (\tfrac{3}{2} \sin^2 I - 1) + \tfrac{3}{128} J_2^2 (R/a)^4 [(1-e^2)^{-7/2}(-40 + 80 \sin^2 I - 35 \sin^4 I) \\ &+ (1-e^2)^{-3}(-16 + 48 \sin^2 I - 36 \sin^4 I) + (1-e^2)^{-5/2}(8 - 8 \sin^2 I - 5 \sin^4 I) \\ &+ (1-e^2)^{-3/2}(128 - 192 \sin^2 I)] + \tfrac{3}{128} J_4 (R/a)^4 (1-e^2)^{-7/2}(2 + 3e^2)(35 \sin^4 I - 40 \sin^2 I + 8) \}, \end{aligned} \tag{4}$$

$$V = -\frac{G\mathcal{M}_T}{a_T} \sum_{n=0}^{\infty} [P_{2n}(0)]^2 P_{2n}(\cos J) \left(\frac{a}{a_T}\right)^{2n} \sum_{k=0}^{n} \frac{(2n+1)!\, e^{2k}}{(2n-2k+1)!\, 2^{2k} (k!)^2}, \tag{5}$$

$$W = -n_s a^2 \frac{d\Omega_T}{dt} \cos i^*, \tag{6}$$

where R, J_2, and J_4 are Neptune's equatorial radius and first two even zonal harmonics; $\mathcal{M}_T$ and a_T are Triton's mass and mean orbital radius; $P_{2n}(x)$ are the Legendre polynomials; and $d\Omega_T/dt$ is Triton's nodal precession rate. Triton's orbital eccentricity is extremely small (about 0.000 15) and is ignored.

The Laplacian plane corresponds to a minimum of $\mathcal{H}$. The longitude of the ascending node of this plane is equal to that of Triton's descending node, since Triton's orbit is retrograde. We determine the inclination of each satellite's Laplacian plane to Neptune's equator, denoted i_L, by setting e to zero, Ω to $\Omega_T + 180°$, and solving $\partial\mathcal{H}/\partial I = 0$. The resulting equation for i_L is

$$\begin{aligned} &n_s^2 a^2 [\tfrac{3}{2} J_2 (R/a)^2 - \tfrac{3}{8} J_2^2 (R/a)^4 (9 + 19 \sin^2 i_L) + \tfrac{15}{16} J_4 (R/a)^4 (7 \sin^2 i_L - 4)] \sin i_L \cos i_L \\ &+ \frac{G\mathcal{M}_T}{a_T} \sum_{n=0}^{\infty} [P_{2n}(0)]^2 P_{2n}^1 [\cos(i_T + i_L)] \left(\frac{a}{a_T}\right)^{2n} \sum_{k=0}^{n} \frac{(2n+1)!\, e^{2k}}{(2n-2k+1)!\, 2^{2k}(k!)^2} + n_s a^2 \frac{d\Omega_T}{dt} \sin(i_0 + i_L) = 0, \end{aligned} \tag{7}$$

where the P_{2n}^1 are the associated Legendre polynomials of order one.

The nodal and apsidal rates are obtained from Lagrange's equations (see, e.g., Brouwer & Clemence 1961, Chap. 11),

$$\begin{aligned} \frac{d\Omega}{dt} &= -\frac{1}{n_s a^2 (1-e^2)^{1/2} \sin i} \left(\frac{\partial U}{\partial I}\frac{\partial I}{\partial i} + \frac{\partial V}{\partial J}\frac{\partial J}{\partial i} \right) \\ &= n_s \{ -\tfrac{3}{2} J_2 (R/p)^2 + \tfrac{3}{32} J_2^2 (R/p)^4 [60 - 196 e^2 + 96 e^4 + (40 - 5e^2) \sin^2 I + (1-e^2)^{1/2}(-24 + 36 \sin^2 I)] \end{aligned} \tag{8}$$

$$+\tfrac{15}{32}J_4(R/p)^4(2+3e^2)(4-7\sin^2 I)\}\cos I\cos i_{\rm L}-\frac{G\mathscr{M}_{\rm T}}{n_{\rm s}a^2a_{\rm T}}\frac{\cos(i_{\rm T}+i_{\rm L})}{(1+e^2)^{1/2}}\sum_{n=0}^{\infty}[P_{2n}(0)]^2\frac{P^1_{2n}(\cos J)}{\sin J}$$

$$\times\left(\frac{a}{a_{\rm T}}\right)^{2n}\sum_{k=0}^{n}\frac{(2n+1)!\,e^{2k}}{(2n-2k+1)!\,2^{2k}(k!)^2},\tag{9}$$

$$\frac{d\varpi}{dt}=-\frac{(1-e^2)^{1/2}}{n_{\rm s}a^2e}\frac{\partial(U+V)}{\partial e}-\frac{\tan\frac{1}{2}i}{n_{\rm s}a^2(1-e^2)^{1/2}}\left(\frac{\partial U}{\partial I}\frac{\partial I}{\partial i}+\frac{\partial V}{\partial J}\frac{\partial J}{\partial i}\right)\tag{10}$$

$$=n_{\rm s}\{\tfrac{3}{2}J_2(R/p)^2(1-\tfrac{5}{4}\sin^2 I)+\tfrac{3}{128}J_2^{\,2}(R/p)^4[-144+808e^2-384e^4$$

$$+(56-1192e^2+576e^4)\sin^2 I+(270-25e^2)\sin^4 I+(1-e^2)^{1/2}(96-288\sin^2 I+216\sin^4 I)]$$

$$-\tfrac{15}{4}J_4(R/p)^4(1+\tfrac{3}{4}e^2)(1-5\sin^2 I+\tfrac{35}{8}\sin^4 I)\}+\frac{G\mathscr{M}_{\rm T}}{n_{\rm s}a^2a_{\rm T}}(1-e^2)^{1/2}\sum_{n=1}^{\infty}[P_{2n}(0)]^2P_{2n}(\cos J)$$

$$\times\left(\frac{a}{a_{\rm T}}\right)^{2n}\sum_{k=1}^{n}\frac{2k(2n+1)!\,e^{2k-2}}{(2n-2k+1)!\,2^{2k}(k!)^2}+(1-\cos i)\frac{d\Omega}{dt}.\tag{11}$$

In Eqs. (9) and (11) we use the geometric semilatus rectum, $p=a(1-e^2)$. The W term is omitted from the Hamiltonian here because we are using a nonrotating coordinate system for the orbit solution.

The solution parameter set contained the seven elements $\{a, h, k, \lambda, p, q, n_{\rm s}\}$ for each satellite, plus three stochastic pointing offset angles that give the correction to the assumed camera orientation for each picture. We relied on previous calibration frames to provide the camera focal length, pixel scale, and other camera parameters; these were then held constant for all frames.

Table 2 presents our solution for the orbits of the small satellites, in terms of the geometric classical Keplerian elements. The quoted uncertainties are standard errors and are believed to be somewhat conservative, as the standard errors assigned to the observations are typically a factor of two higher than the scatter in the postfit residuals.

4. DISCUSSION

The best fit to the data referred the orbits of all six satellites to their Laplacian planes; using either Neptune's equator or the Neptune–Triton invariable plane for a reference plane gave results that were less satisfactory. A trial solution in which the nodal and apsidal rates were estimated indicated that only 1989N1 and 1989N6 had a significantly observable precession during the time span of the observations. In the case of 1989N6, its relatively high inclination and rapid motion allow both the node and its rate to be determined quite well, and we find good agreement between the observed $d\Omega/dt$ and the computed value. For 1989N1, the long data arc shows that the node on the Laplacian plane is indeed regressing at the predicted rate to well within the uncertainty of the solution. But the observations of the other four satellites showed no sensitivity to the nodal or apsidal rates.

An additional check on the solution is provided by the 1981 May 24 occultation of a star in the vicinity of Neptune (Reitsema *et al.* 1982). Smith *et al.* (1989) report that 1989N2 caused the occultation. Using the postencounter Neptune ephemeris and the published position for the star (Mink *et al.* 1981) corrected for elliptic aberration, we find that the topocentric offset of the star from Neptune was $+1''.41$ in right ascension, $-1''.66$ in declination, with catalog uncertainties of $0''.10$ in each coordinate. The offset of 1989N2, as predicted by Table 2, was $+1''.41$ and $-1''.65$, in excellent agreement.

If the 1981 occultation is taken as an additional observation, there is no significant change in the orbit. The uncertainty in the mean motion shrinks by a factor of 2.7; the revised mean motion is $649.053\,44\pm0.000\,61$ degrees per day, corresponding to a period of $47\,922.094\pm0.045$ s. The postfit residual for the occultation remains $0''.01$.

Since the mean motions are determined to a much higher relative precision than the other elements, one can infer more accurate values of a for each satellite. We calculate a mean orbital radius by solving

$$a^3n_{\rm s}^2=G\mathscr{M}_{\rm N}[1+\tfrac{3}{2}J_2(R/a)^2(1-e^2)^{-3/2}(1-\tfrac{3}{2}\sin^2 I)-\tfrac{15}{8}J_4(R/a)^4(1-e^2)^{-7/2}(1+\tfrac{3}{2}e^2)(1-5\sin^2 I+\tfrac{35}{8}\sin^4 I)]$$

$$-G\mathscr{M}_{\rm T}\sum_{n=1}^{\infty}[P_{2n}(0)]^2P_{2n}(\cos J)\left(\frac{a}{a_{\rm T}}\right)^{2n+1}\sum_{k=0}^{n}\frac{(2n+1)!}{(2n-2k+1)!\,2^{2k}(k!)^2}e^{2k},\tag{12}$$

iteratively for a. Our software evaluated Eq. (12) first, then used the calculated a in the rate Eqs. (9) and (11). The effect of Triton (the V term) varied from 0.5 km for 1989N1 to 13 m for 1989N6. The inferred values of a are listed in Table 2 as $a_{\rm calc}$, as opposed to the solution values $a_{\rm meas}$. The uncertainties in $a_{\rm calc}$ are dominated by the uncertainties in the mean motions. The observed and calculated values for a agree to within their uncertainties.

TABLE 2. Mean orbital elements and associated data for the six small satellites of Neptune. Epoch of elements = JED 2447757.0 = 1989 August 18, 12^{h} Ephemeris time.

Name	1989N1	1989N2	1989N3
Incl. of Laplacian plane to Neptune equator (°)	0.5475	0.0479	0.0084
R.A. of normal to Laplacian plane (°)	298.7578	298.8487	298.8559
Dec. of normal to Laplacian plane (°)	42.2692	42.7643	42.8035
a_{meas} (km)	117635.0 ± 16.8	73545.7 ± 6.9	52531.3 ± 8.5
a_{calc} (km)	117647.11 ± 0.23	73548.33 ± 0.14	52525.95 ± 0.10
e (x10^3)	0.438 ± 0.107	1.386 ± 0.088	0.139 ± 0.166
ϖ (°)	99.48 ± 7.42	149.92 ± 3.82	126. ± 64.
i (°)	0.0392 ± 0.0083	0.2008 ± 0.0098	0.0655 ± 0.0153
Ω (°)	150.00 ± 12.47	10.00 ± 2.54	154.59 ± 11.26
λ (°)	213.6694 ± 0.0066	184.8281 ± 0.0093	85.2720 ± 0.0139
n_s (°/day)	320.7654 ± 0.0009	649.0534 ± 0.0016	1075.7342 ± 0.0028
P (s)	96968.06 ± 0.26	47922.09 ± 0.12	28914.21 ± 0.08
$d\varpi/dt$ (°/day)[a]	0.078914	0.393133	1.276761
$d\Omega/dt$ (°/day)	−0.077334	−0.392209	−1.274866
R.A. of normal to orbit plane (°)	298.7844	298.8960	298.8943
Dec. of normal to orbit plane (°)	42.3032	42.5665	42.8626
Number of Observations	183	144	109
First Observation	7 June 03:12	25 Jul 23:49	25 Jul 23:49
Last Observation	23 Aug 08:54	24 Aug 21:08	24 Aug 21:32
Data Arc (days)	77.237	29.888	29.905
(revs)	68.820	53.886	89.361
RMS Residual (pixels)	0.31	0.33	0.43
(lines)	0.22	0.27	0.28

Name	1989N4	1989N5	1989N6
Incl. of Laplacian plane to Neptune equator (°)	0.0197	0.0066	0.0054
R.A. of normal to Laplacian plane (°)	298.8539	298.8563	298.8565
Dec. of normal to Laplacian plane (°)	42.7922	42.8053	42.8064
a_{meas} (km)	61945.1 ± 15.2	50069.2 ± 13.3	48233.1 ± 16.4
a_{calc} (km)	61952.67 ± 0.13	50074.55 ± 0.30	48227.30 ± 0.36
e (x10^3)	0.120 ± 0.149	0.156 ± 0.250	0.328 ± 0.353
ϖ (°)	220. ± 64.	46. ± 85.	85. ± 42.
i (°)	0.0544 ± 0.0132	0.2054 ± 0.0217	4.7382 ± 0.0338
Ω (°)	112.2 ± 15.8	89.85 ± 7.70	48.66 ± 0.46
λ (°)	46.6443 ± 0.0111	239.7371 ± 0.0275	60.2604 ± 0.0418
n_s (°/day)	839.6598 ± 0.0025	1155.7556 ± 0.0101	1222.8441 ± 0.0138
P (s)	37043.58 ± 0.11	26912.27 ± 0.24	25435.78 ± 0.29
$d\varpi/dt$ (°/day)	0.715961	1.509866	1.699098
$d\Omega/dt$ (°/day)	−0.714836	−1.507575	−1.714075
R.A. of normal to orbit plane (°)	298.9226	299.1362	303.4720
Dec. of normal to orbit plane (°)	42.8127	42.8044	39.5813
Number of Observations	107	46	30
First Observation	28 Jul 06:20	8 Aug 07:39	8 Aug 04:06
Last Observation	23 Aug 20:26	22 Aug 11:34	23 Aug 21:23
Data Arc (days)	26.587	14.163	15.721
(revs)	62.012	45.470	53.399
RMS Residual (pixels)	0.46	0.39	0.43
(lines)	0.32	0.34	0.30

[a]The orientation of the Laplacian plane, $d\varpi/dt$, and $d\Omega/dt$ were calculated as described in the text and held fixed in the estimation process.

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