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Precession Matrix Based on IAU (1976) System of Astronomical Constants

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Summary. At its XVIth General Assembly held in Grenoble in 1976, the International Astronomical Union (IAU) adopted a new value for the speed of general precession in longitude, and it adopted a new basic epoch for future almanacs and ephemerides. In this paper we develop numerical values of the precession parameters and of the precession matrix which enable one to precess to and from J2000.0.

Key words: precession – astronomical constants

Introduction

At its XVIth General Assembly held in Grenoble in 1976, the IAU adopted (Duncombe et al., 1977) a new speed of general precession in longitude of $p = 5029''.0966$ per Julian century at Julian epoch J2000.0 (JED 2451545.0). This value (Lieske et al., 1977), when referred to Besselian epoch B1900.0, is $5026''.767$ per tropical century, which may be compared with the previously sanctioned IAU value (due to Newcomb) of $5025''.64$ per tropical century at B1900.0.

In addition, the IAU changed from Besselian epochs to Julian epochs. The conventional relationship between Julian epochs (designated by J plus a year) is

$$\mathrm{JE} = 2000.0 + (\mathrm{JED} - 2451545.0)/365.25 \tag{1}$$

and that for Besselian epochs is

$$\mathrm{BE} = 1900.0 + (\mathrm{JED} - 2415020.31352)/365.242198781 \tag{2}$$

where JED is the Julian Ephemeris Date. The correspondence between several Besselian and Julian epochs is given in Table 1.

The equatorial precession parameters ζ_A, z_A, θ_A defined in Fig. 1 are developed for the new IAU system of astronomical constants in the paper by Lieske et al. (1977). In this paper we will develop expressions for the precession matrix required in precessing from equinox $\mathscr{E}_A$ to equinox $\mathscr{E}_B$.

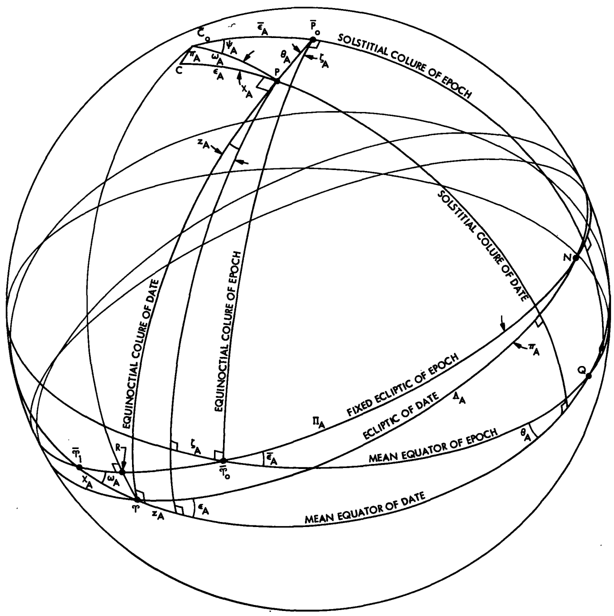

Fig. 1. Celestial sphere depicting mean ecliptics and equators at epochs $\mathscr{E}_F$ and $\mathscr{E}_D$. Ecliptic poles are represented by $\bar{C}_0$ and C with $\bar{P}_0$ and P representing the pole of the equator at the two epochs

Table 1. Correspondence between Julian and Besselian epochs

Besselian Epoch	Julian Epoch	JED
B 1899.999142	J 1900.0	2415020.0
B 1900.0	J 1900.000858	2415020.31352
B 1950.0	J 1949.999790	2433282.42345905
B 1950.000210	J 1950.0	2433282.5
B 2000.0	J 1999.998722	2451544.5333981
B 2000.001278	J 2000.0	2451545.0

Precession Angles

If $P(\alpha)$, $Q(\alpha)$, $R(\alpha)$ define vector rotations about the x_0, y_0, and z_0 axes, respectively, so that new coordinates $\boldsymbol{r}$ are related to the old coordinates $\boldsymbol{r}_0$ by

$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = P(\alpha)\boldsymbol{r}_0 = P(\alpha)\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

or by

$$\boldsymbol{r} = Q(\alpha)\boldsymbol{r}_0$$

or by

$$\boldsymbol{r} = R(\alpha)\boldsymbol{r}_0$$

for rotations about the x_0, y_0, and z_0 axes, respectively, then

$$\left.\begin{aligned} P(\alpha)&=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}\\ Q(\alpha)&=\begin{bmatrix}\cos\alpha & 0 & -\sin\alpha\\ 0 & 1 & 0\\ \sin\alpha & 0 & \cos\alpha\end{bmatrix}\\ R(\alpha)&=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\end{aligned}\right\}. \tag{3}$$

From the equatorial region of Fig. 1 it is seen that the coordinates $\boldsymbol{r}$ referred to the mean equinox $\mathscr{E}_D$ are related to those $\boldsymbol{r}_0$ of the fixed basic equinox $\mathscr{E}_0$ by the rotations

$$\boldsymbol{r}=\begin{pmatrix}x\\ y\\ z\end{pmatrix}=R(-90°-z_A)P(\theta_A)R(90°-\zeta_A)\begin{pmatrix}x_0\\ y_0\\ z_0\end{pmatrix}. \tag{4}$$

These are equivalent to the matrix resulting from use of the polar region of Fig. 1,

$$\boldsymbol{r}=R(-z_A)Q(\theta_A)R(-\zeta_A)\boldsymbol{r}_0. \tag{5}$$

If we denote by A the precession matrix

$$A=R(-90°-z_A)P(\theta_A)R(90°-\zeta_A)=R(-z_A)Q(\theta_A)R(-\zeta_A)$$

then

Table 2. Precession from Besselian epochs to J2000.0

YEAR	ZETA(")	Z(")	THETA(")	M(S)	N(S)
B 1850.	3456.881	3458.664	3007.246	461.036	200.483
B 1900.	2305.122	2305.914	2004.679	307.402	133.645
B 1905.	2189.918	2190.633	1904.429	292.037	126.962
B 1910.	2074.708	2075.350	1804.182	276.671	120.279
B 1915.	1959.493	1960.066	1703.936	261.304	113.596
B 1920.	1844.273	1844.781	1603.692	245.937	106.913
B 1925.	1729.048	1729.494	1503.449	230.569	100.230
B 1930.	1613.818	1614.206	1403.208	215.202	93.547
B 1935.	1498.582	1498.917	1302.969	199.833	86.865
B 1940.	1383.341	1383.626	1202.731	184.464	80.182
B 1945.	1268.094	1268.334	1102.495	169.095	73.500
B 1950.	1152.842	1153.041	1002.261	153.726	66.817
B 1955.	1037.585	1037.746	902.029	138.355	60.135
B 1960.	922.323	922.450	801.798	122.985	53.453
B 1965.	807.055	807.152	701.570	107.614	46.771
B 1970.	691.782	691.853	601.343	92.242	40.090
B 1975.	576.504	576.553	501.119	76.870	33.408
B 1980.	461.220	461.251	400.896	61.498	26.726
B 1985.	345.930	345.948	300.675	46.125	20.045
B 1990.	230.635	230.643	200.457	30.752	13.364
B 1995.	115.335	115.337	100.240	15.378	6.683

$$A=\begin{bmatrix}\cos z_A\cos\theta_A\cos\zeta_A-\sin z_A\sin\zeta_A & -\cos z_A\cos\theta_A\sin\zeta_A-\sin z_A\cos\zeta_A & -\cos z_A\sin\theta_A\\ \sin z_A\cos\theta_A\cos\zeta_A+\cos z_A\sin\zeta_A & -\sin z_A\cos\theta_A\sin\zeta_A+\cos z_A\cos\zeta_A & -\sin z_A\sin\theta_A\\ \sin\theta_A\cos\zeta_A & -\sin\theta_A\sin\zeta_A & \cos\theta_A\end{bmatrix} \tag{6}$$

where the angles ζ_A, z_A, θ_A on the new IAU system from Lieske et al. (1977) are

$$\begin{aligned}\zeta_A&=(2306''2181+1''39656\,T-0''000139\,T^2)t+(0''30188-0''000344\,T)t^2+0''017998\,t^3\\ z_A&=(2306''2181+1''39656\,T-0''000139\,T^2)t+(1''09468+0''000066\,T)t^2+0''018203\,t^3\\ \theta_A&=(2004''3109-0''85330\,T-0''000217\,T^2)t+(-0''42665-0''000217\,T)t^2-0''041833\,t^3\end{aligned} \tag{7}$$

where $\mathscr{E}_0=$ J2000.0 (JED 2451545.0) is the basic epoch, and where $\mathscr{E}_F$ and $\mathscr{E}_D$ are arbitrary epochs, or

$$\begin{aligned}T&=[\mathrm{JED}(\mathscr{E}_F)-\mathrm{JED}(\mathscr{E}_0)]/36525.\\ t&=[\mathrm{JED}(\mathscr{E}_D)-\mathrm{JED}(\mathscr{E}_F)]/36525.\end{aligned} \tag{8}$$

The parameters $\zeta_A(T,t)$, $z_A(T,t)$, $\theta_A(T,t)$ are the equatorial precession parameters required to precess from epoch $\mathscr{E}_F$ to epoch $\mathscr{E}_D$. The parameters obey a reflexive rule

$$\begin{aligned}\zeta_A(T,t)&=-z_A(T+t,-t)\\ z_A(T,t)&=-\zeta_A(T+t,-t)\\ \theta_A(T,t)&=-\theta_A(T+t,-t)\end{aligned} \tag{9}$$

for precessing from $\mathscr{E}_D$ to $\mathscr{E}_F$.

In order to guarantee numerical reflexive properties, the coefficient of Tt^2 in ζ_A has been changed from the $-0''000345\,Tt^2$ of the original paper by Lieske et al. (1977). If one desires to precess to and from equinox J2000.0 one merely needs to evaluate the series Eq. (7) with $T=0$ and $t=(\mathrm{JED}-2451545.0)/36525$. The angles ζ_A, z_A, θ_A then are the required ones for precession from J2000.0 to equinox JED. If one desires the precess from equinox JED to J2000.0, then by use of Eq. (9) one first calculates $\zeta_A(0,t)$, $z_A(0,t)$, and $\theta_A(0,t)$ for precession from J2000 to JED and then replaces ζ_A by $-z_A$, z_A by $-\zeta_A$ and θ_A by $-\theta_A$ in order to obtain $\zeta_A(T+t,-t)$, $z_A(T+t,-t)$, and $\theta_A(T+t,-t)$ for precessing from J2000.0 to equinox JED: $\zeta_A(t,-t)=-z_A(0,t)$, $z_A(t,-t)=-\zeta_A(0,t)$, and $\theta_A(t,-t)=-\theta_A(0,t)$.

In Table 2 are presented the equatorial precession angles ζ_A, z_A, θ_A (along with the quantities $m=\zeta_A+z_A$ and $n=\theta_A$ which are employed for lower order precession of right ascension and declination) for precessing from Besselian epoch in the left-hand column to Julian epoch J2000.0. In Table 3 similar quantities are given for precessing from Julian epoch J2000.0 to Julian epoch given in the left-hand column. The IAU conventions given in Eq. (1) and (2) are employed in constructing the table.

Precession Matrix

In precessing positions from equinox $\mathscr{E}_a$ to equinox $\mathscr{E}_b$ and vice-versa one requires the matrices $A(a,b)$ or $A(b,a)$, defined in the sense

$$r_b = A(a,b)\, r_a \quad \text{and} \qquad r_a = A(b,a)\, r_b, \tag{10}$$

with a and b representing $\mathscr{E}_F$ and $\mathscr{E}_D$ in Eq. (8).

Because of the orthonormal nature of the matrices, $A(a,b) = A^T(b,a)$ so that we also have

$$r_a = A^T(a,b)\, r_b. \tag{11}$$

Hence in order to precess to or from $\mathscr{E}_a$ we need only $A(a,b)$.

In general one will calculate the matrix A by using Eq. (7) and (6). However, for persons who desire a more compact numerical expression for Eq. (6) we have expanded the matrix formulation of Eq. (6) by use of Broucke's (1969) Poisson series manipulation software and obtain

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\left.\begin{aligned}
10^{12}(a_{11}-1) &= (-297{,}235{,}029 - 262{,}610\,T - 60\,T^2)\,t^2 \\
&\quad + (-131{,}305 - 60\,T)\,t^3 \\
&\quad + (14{,}713 + 24\,T)\,t^4 \\
&\quad + 12\,t^5 \\
10^{12}(a_{22}-1) &= (-250{,}023{,}299 - 302{,}810\,T - 62\,T^2)\,t^2 \\
&\quad + (-151{,}405 - 62\,T)\,t^3 \\
&\quad + (12{,}373 + 24\,T)\,t^4 \\
&\quad + 12\,t^5 \\
10^{12}(a_{33}-1) &= (-47{,}211{,}730 + 40{,}198\,T + 2\,T^2)\,t^2 \\
&\quad + (20{,}099 + 2\,T)\,t^3 \\
&\quad + (2340 - 2\,T)\,t^4 \\
&\quad - t^5 \\
10^{12}\,a_{12} &= (-22{,}361{,}721{,}730 - 13{,}541{,}428\,T + 1348\,T^2)\,t \\
&\quad + (-6{,}770{,}714 + 1348\,T)\,t^2 \\
&\quad + (2{,}216{,}011 + 3256\,T + T^2)\,t^3 \\
&\quad + (1537 + T)\,t^4 - 68\,t^5 \\
10^{12}\,a_{21} &= -10^{12}\,a_{12} - 182\,t^4 \\
10^{12}\,a_{13} &= (-9{,}717{,}173{,}455 + 4{,}136{,}914\,T + 1052\,T^2)\,t \\
&\quad + (2{,}068{,}457 + 1052\,T)\,t^2 \\
&\quad + (963{,}114 + 282\,T)\,t^3 + 350\,t^4 - 29\,t^5 \\
10^{12}\,a_{31} &= -10^{12}\,a_{13} + 418\,t^4 \\
10^{12}\,a_{23} &= (-108{,}646{,}364 - 19{,}538\,T + 46\,T^2)\,t^2 \\
&\quad + (-28{,}443 + 44\,T)\,t^3 \\
&\quad + (5394 + 4\,T)\,t^4 + 4\,t^5 \\
10^{12}\,a_{32} &= 10^{12}\,a_{23} + (37{,}348 + 4\,T)\,t^3 + 2\,t^4 - 4\,t^5.
\end{aligned}\right\} \tag{12}$$

The above results are valid to 10^{-12}, with T and t defined as in Eq. (8), so that $T=0$ if one is precessing from (or to) J2000.0.

For precessing from B1950.0 to J2000.0 one may employ the entries in Table 2 for ζ_A, z_A, θ_A and Eq. (6), or one may employ Eq. (12) with $T=0$, $t = -0.5000020955\,77002$ and use the reflexive property of Eq. (10), or one may directly evaluate Eq. (7) and Eq. (6) to obtain

$$A(\mathrm{B}1950.0,\ \mathrm{J}2000.0) = \begin{bmatrix} 0.9999\ 2570\ 7952\ 3629 & -0.0111\ 7893\ 8137\ 7700 & -0.0048\ 5900\ 3815\ 3592 \\ 0.0111\ 7893\ 8126\ 4276 & 0.9999\ 3751\ 3349\ 9888 & -0.0000\ 2716\ 2594\ 7142 \\ 0.0048\ 5900\ 3841\ 4544 & -0.0000\ 2715\ 7926\ 2585 & 0.9999\ 8819\ 4602\ 3742 \end{bmatrix} \tag{13}$$

in the sense

$$r_{\mathrm{J}2000.0} = A(B1950.0,\ \mathrm{J}2000.0)\, r_{\mathrm{B}1950.0}$$

Table 3. Precession from J2000.0 to Julian epochs

YEAR	ZETA(")	Z(")	THETA(")	M(S)	N(S)
J 1950.	-1153.036	-1152.838	-1002.257	-153.725	-66.817
J 1955.	-1037.739	-1037.578	-902.022	-138.354	-60.135
J 1960.	-922.440	-922.313	-801.790	-122.984	-53.453
J 1965.	-807.140	-807.043	-701.559	-107.612	-46.771
J 1970.	-691.839	-691.767	-601.331	-92.240	-40.089
J 1975.	-576.536	-576.486	-501.104	-76.868	-33.407
J 1980.	-461.232	-461.200	-400.879	-61.495	-26.725
J 1985.	-345.926	-345.908	-300.656	-46.122	-20.044
J 1990.	-230.619	-230.611	-200.435	-30.749	-13.362
J 1995.	-115.310	-115.308	-100.217	-15.375	-6.681
J 2000.	.000	.000	.000	.000	.000
J 2005.	115.312	115.314	100.214	15.375	6.681
J 2010.	230.625	230.633	200.427	30.751	13.362
J 2015.	345.940	345.957	300.637	46.126	20.042
J 2020.	461.256	461.288	400.845	61.503	26.723
J 2025.	576.574	576.623	501.050	76.880	33.403
J 2030.	691.893	691.964	601.254	92.257	40.084
J 2035.	807.214	807.311	701.455	107.635	46.764
J 2040.	922.537	922.664	801.653	123.013	53.444
J 2045.	1037.861	1038.021	901.850	138.392	60.123
J 2050.	1153.187	1153.385	1002.044	153.771	66.803

Summary

Numerical values of the precession angles ζ_A, z_A, θ_A are given in Tables 2 and 3, calculated from Eq. (7). Values for the precession matrix Eq. (6) are given in Eq. (13) for precessing from B1950.0 to J2000.0, and an expansion is given in Eq. (12). By means of these tables one can readily convert to the new IAU equinox with the new value for the speed of general precession. Fortran programs are available from the author on request.

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