AwesomeMath Test A
January 4 – January 16, 2025

PLEASE READ CAREFULLY and THOROUGHLY

DEADLINE: Thursday, January 16, 2025, 5:00 PM CT

INSTRUCTIONS

- Students who wish to take Level 1 & Level 2 classes must submit solutions to Part I only.
- Students who wish to take Level 3 & Level 4 classes must submit solutions to Part II only.
- **Only one test and one part (Part I or Part II) will be graded per student**. Do NOT submit solutions to Part I and then additional solutions to Part II as a separate submission.
- Students accepted for Level 1 and Level 2 will not be able to take Level 3 and 4 classes, however students accepted for Level 3 and 4 are allowed to switch to lower-level classes if it turns out that their initial classes are too difficult.
- **This test is not timed, therefore, do not rush**. Show us your best work. Your solutions will be evaluated against mathematical experience and any math competition test scores provided on your application.
- We want you to have a positive and successful experience at our camps, therefore **your work should be yours and yours alone**. The purpose of the test is to ensure that you are placed in classes that fit your level and skills. If you consult outside resources (other people or online), then you are compromising your ability to succeed in our challenging program.
- **Do not be discouraged if you cannot solve all of the questions**. We want to see the solutions you come up with no matter how many problems you solve. Some of the problems involve complex mathematical ideas, but all of them can be solved using only elementary techniques, admittedly combined in clever ways.
- **Include all significant steps in your reasoning and computation**. We are interested in your ability to present your work, so unsupported answers will receive less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find an answer sheet. **Print out or make several copies of the blank answer sheet**. Fill out the top of each answer sheet as you go. Start each problem on a new answer sheet.
- **You may handwrite or type your solutions**. If you type your solutions, your answer sheets should still include the same information as shown on the test packet answer sheets (9-digit UIN, First and Last Name, Problem #, etc.)

SUBMISSION REQUIREMENTS

- **Your solutions must be submitted as a single pdf document**. Do NOT submit test solutions as individual pages.
- **UPLOAD your solutions file** in accordance with the directions provided on your student dashboard.
- **DO NOT email us your solutions** as it will significantly delay receipt and grading of your solutions.

AwesomeMath Admission Test

Cover Sheet

PLEASE PRINT LEGIBLY and DO NOT LEAVE ANY FIELDS BLANK

9-Digit UIN: _____

Last Name: _____ **First Name**: _____

Admission Test (check one): ( ) A ( ) B ( ) C ( ) D ( ) E

Part Completed (check one): ( ) Part I ( ) Part II

Email: _____

Important Reminders:

- **Make sure this COVER SHEET is the first page of your submission,** and that it is completely filled out**.**
- **UPLOAD your solutions file** in accordance with the directions provided on your student dashboard.
- **Your solutions must be submitted as a single pdf document**. Do NOT submit test solutions as individual pages.
- **DO NOT email us your solutions** as it will significantly delay receipt and grading of your solutions.
- Students who wish to take Level 1 & Level 2 classes must submit solutions to Part I only
- Students who wish to take Level 3 & Level 4 classes must submit solutions to Part II only
- Only one test and one part (Part I or Part II) will be graded per student. **Do NOT submit solutions to Part I and then submit additional solutions to Part II**.

AwesomeMath Admission Test Answer Sheet

9-Digit UIN:

First Name: **Last Name:**

Problem Number:

Write neatly! All work should be inside the box. Do NOT write on the back of the page!

Page Number: **out of** **total pages being submitted (including cover sheet)**

Test A

January 4 – 16, 2025

Part I

Levels 1 & 2

1. We need precisely 2025 digits to write down the numbers 1, 2, 3, ..., n on a board.

(a) Find n.

(b) Starting with the sequence $1, 2, \ldots, n$, delete all terms at odd-numbered positions. Perform this same operation repeatedly until only one number is left. What is this number?

2. Let $T_k = \dfrac{k(k+1)}{2}$ be the k-th triangular number. Prove that there is a positive integer n such that

$$T_1 - T_2 + T_3 - T_4 + T_5 - \cdots + T_n = 2025.$$

3. If a is a positive real number such that

$$a + \frac{1}{a} = 2025 + 90\sqrt{2} \quad \text{and} \quad a\sqrt{a} + \frac{1}{a\sqrt{a}} = m + n\sqrt{2},$$

for some positive integers m and n, find n.

4. The positive integer n has divisors $1 = d_1 < d_2 < \cdots < d_k = n$. Find n such that

$$d_1 d_k + d_2 d_{k-1} + \cdots + d_k d_1 = 2025.$$

5. Given that a and b are real numbers such that

$$62\sqrt{a - 2025} + 66\sqrt{b - 25} = a + b,$$

evaluate $a - b$.

6. Find all triples (p, q, r) of primes such that

$$6p^6 + 8q^6 = r^3 - 9p.$$

7. Find the maximum number of letters that can be removed from the English alphabet such that if 2025 unordered triples are randomly picked from the remaining letters, there will be at least two triples that contain all of the same elements.

8. Find all pairs (m, n) of integers such that

$$m^8 + n^8 = 187m^2n^2 - 2025.$$

9. Let $ABCD$ be a cyclic quadrilateral such that $AB = 15$, $CD = 24$, $AC + BD = 45$, and $AD \cdot BC = 140$. Find the area of $ABCD$.

10. Solve the equation

$$\frac{2025}{x} = \frac{1920}{\lfloor x \rfloor} + \frac{1}{\{x\}},$$

where $\lfloor x \rfloor$ and $\{x\}$ are the greatest integer less than or equal to x and the fractional part of x, respectively.

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Part II

Levels 3 & 4

1. Let $a_1 > 0$ and

$$a_{n+1} = a_1 a_2 \cdot \ldots \cdot a_n + 1, \quad n = 1, 2, 3, \ldots$$

Find the integer m such that

$$\sqrt{a_2 - \frac{3}{4}} + \sqrt{a_3 - \frac{3}{4}} + \cdots + \sqrt{a_{2025} - \frac{3}{4}} = a_1 + a_2 + \cdots + a_{2024} + m.$$

2. Given that

$$\prod_{n=5}^{2025} \left(\frac{n}{2} - 2 + \frac{3n+1}{n^2+n+1} \right) = \frac{k}{2^m} \cdot \frac{2023!}{2025 \cdot 2026 + 1},$$

where k and m are positive integers with k being odd, find $10k + m$.

3. Solve the equation

$$\sqrt{x^2 + 45x + 2025} - \sqrt{x^2 - 24x + 576} = 39.$$

4. Find a suitable n and a choice of the signs $+$ and $-$, with at least one minus sign, such that

$$\pm 1^3 \pm 2^3 \pm \cdots \pm n^3 = 2025.$$

5. Let a be a real number such that

$$2025a^2 + \frac{1}{45a} = 45a.$$

Evaluate

$$2025^2 a^4 + \frac{1}{45a}.$$

6. The largest real number a for which $44a^2 = 2025\lfloor a \rfloor \{a\}$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$. As usual, $\lfloor a \rfloor$ and $\{a\}$ denote the greatest integer less than or equal to a and the fractional part of a, respectively.

7. Solve in integers the system of equations

$$\begin{aligned} x^3 - 3xyz + y^3 &= 172 \\ x^2 - xy + y^2 + \frac{2025}{x+y+z} &= z(x+y-z). \end{aligned}$$

8. Consider the numbers $1, 9, 17, 25, \ldots, 2025$. We choose two numbers x and y and replace them with the number $x + y - 1$. What number remains after 253 of these operations?

9. Solve in integers the system of equations

$$x\left(y^2 - z^2\right) = 2025, \quad y\left(z^2 - x^2\right) = 341, \quad z\left(x^2 - y^2\right) = 1664.$$

10. Let $ABCD$ be a convex quadrilateral with area 2025 and let E be the intersection of AD and BC, It is known that $\angle AEB = 60^\circ$ and that AC, BD, AE, BE is an increasing arithmetic sequence with common difference $CE - DE > 0$. Find the perimeter of $ABCD$.