Scalable ADMM for Convex Neural Networks on GPU

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Abstract

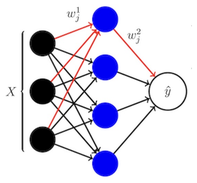
We aim to better understand the fundamental success of Neural Networks, and leverage the provable guarantees of convex optimization within a deep learning framework. Prior work has shown that training a ReLU ([2]) two-layer neural network can be reformulated as group-lasso optimization problem. **Our project investigates:**

- ADMM-style methods for solving the convex equivalent two-layer ReLU problem.
- Effect of the Nyström-ADMM Preconditioner[4] on the problem's condition number and robustness.
- Comparison of GPU accelerated methods versus CPU based methods.

ADMM Approach: ReLU Convex Formulation

Non-convex formulation of a two-layer ReLU-MLP with weights $w_j^{(1)}$, $w_j^{(2)}$:

$$f_{\text{NCVX-MLP}}(x) = \sum_{j=1}^{m} \left(x^T w_j^{(1)} \right)_+ w_j^{(2)} \qquad (1)$$



- $X \in \mathbb{R}^{n \times d}$ is a two layer ReLU-MLP, usually trained in a stochastic setting.
- Scalable in large datasets, but sensitive to hyperparameter tuning and lacks optimality guaranttees.

Equivalent convex reformulation ([3]):

$$f_{\text{CVX-MLP}}(x) = \sum_{i=1}^{P_s} D_i x^T (u_j - v_j) \qquad (2)$$

- $D_i = \mathbf{diag}(\mathbb{I}[X^T h_i \geq 0])$ are diagonal matrices used to sample P_s activation patterns of the ReLU network.
- Recent work [1] applied ADMM by introducing slack variables to arrive at a convex reformulation with mean squared error loss:

$$\min_{v,s,u} ||Fu - y||_2^2 + \beta ||v||_{2,1} + \mathbb{I}_{\geq 0}(s) \quad \text{s.t.} \quad u = v, \; Gu = s \qquad (3)$$

- Matrix $F \in \mathbb{R}^{n \times 2dP_s}$ in this formulation is block-wise constructed by $D_i X$ terms. This optimization problem yields the (simplified) ADMM updates explicitly:

$$\textbf{Primal u update}: \quad Au^{k+1} = b \quad \text{for } A = I + \frac{1}{\rho} F^T F + G^T G \qquad (4a)$$

$$\textbf{Primal v update}: \quad v^{k+1} = \mathbf{prox}_{\frac{\beta}{\rho}||\cdot||_2}(u^{k+1} + \lambda^k) \qquad (4b)$$

$$\textbf{Dual update}: \quad \lambda^{k+1} = \lambda^k + \gamma_\alpha (u^{k+1} - v^{k+1}) \qquad (4c)$$

Methods and Experiments

The problem: (4a) involves a $(2dP_s \times 2dP_s)$ linear system solve, which typically invokes an expensive Cholesky decomposition. We experiment with several approximate iterative methods to solve this in a scalable and accelerated way:

1. Randomized block coordinate descent (RBCD): Low memory compute with fast convergence.
2. Conjugate gradient (CG): Simple Baseline.
3. Nystrom-preconditioned Conjugate gradient (nysPCG): Fast convergence as a result of the "nice" condition number.

We also conduct JAX and PyTorch acceleration to benchmark against CPU performance.

Main Results

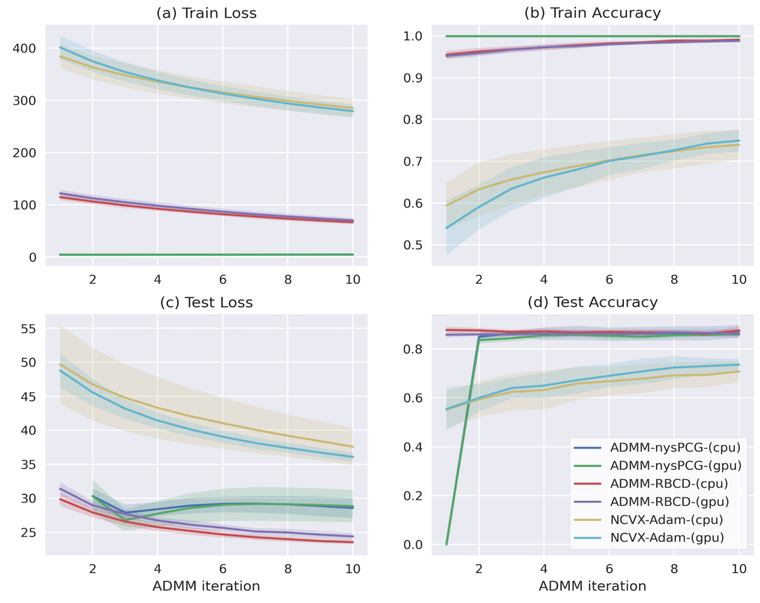


Figure 1. Inexact ADMM Performance on CIFAR-10

- Figure (1) show the performance of our solvers on CIFAR-10 ($n = 10000$, $d = 3072$, $P_s = 50$) after 10 ADMM steps, benchmarked against a two-layer ReLU network trained with non-convex backprop (Adam optimizer with learning rate 10^{-4}).
- Figure (3) show the corresponding solve times for these results.

GPU and Condition Numbers

- Figure (2) shows initial results of preconditioners on the condition number κ.
- Nystrom Sketch Preconditioner had the best effect and increased robustness against hyperparameter tuning!

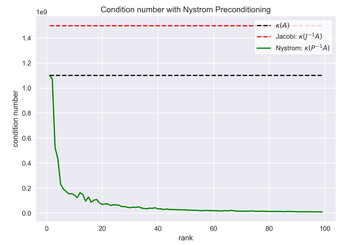


Figure 2. Condition number vs. Rank(A)

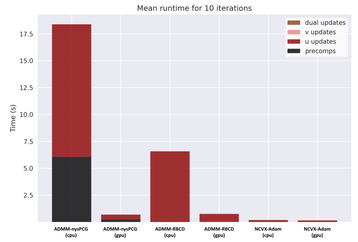


Figure 3. Solve time breakdowns.

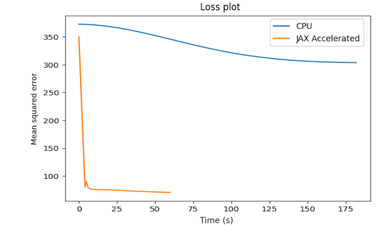


Figure 4. JAX on RBCD method.

- Figure (4): shows the first algorithm, RBCD, to leverage JAX acceleration on A100 GPU with Just In Time compilation against standard CPU usage.
- So far: RBCD methods, initial point methods, and truncated newton show better efficiency in JAX but more experiments needed

Conclusions and Next Steps

- Promising results of Nyström Preconditioning can be applied to RBCD, which has the best performance thus far.
- Package all methods with JAX acceleration backend for easy pip install.
- Provide provable guarantees on networks satisfying desirable input-output properties or specifications.
- Code at: `https://github.com/zachary-shah/admmNN`

References

[1] Yatong Bai, Tanmay Gautam, and Somayeh Sojoudi. Efficient global optimization of two-layer ReLU networks: Quadratic-time algorithms and adversarial training. January 2022.

[2] Aaron Mishkin, Arda Sahiner, and Mert Pilanci. Fast convex optimization for two-layer relu networks: Equivalent model classes and cone decompositions, 2022.

[3] Mert Pilanci and Tolga Ergen. Neural networks are convex regularizers. In Hal Daumé III and Aarti Singh, editors, *Proceedings of the 37th International Conference on Machine Learning*, volume 119 of *Proceedings of Machine Learning Research*, pages 7695–7705. PMLR, 13–18 Jul 2020.

[4] Shipu Zhao, Zachary Frangella, and Madeleine Udell. Nysadmm: faster composite convex optimization via low-rank approximation. In *International Conference on Machine Learning*, pages 26824–26840. PMLR, 2022.