Quiz 10 Math 2202

Guidelines

- This quiz is for you to test yourself on what we've been studying recently.
- You may and should use it when doing the online quiz later today (or tomorrow).
- You have 10 minutes. As a section, we will go over the quiz (or part of it). Solutions will be posted online as well.

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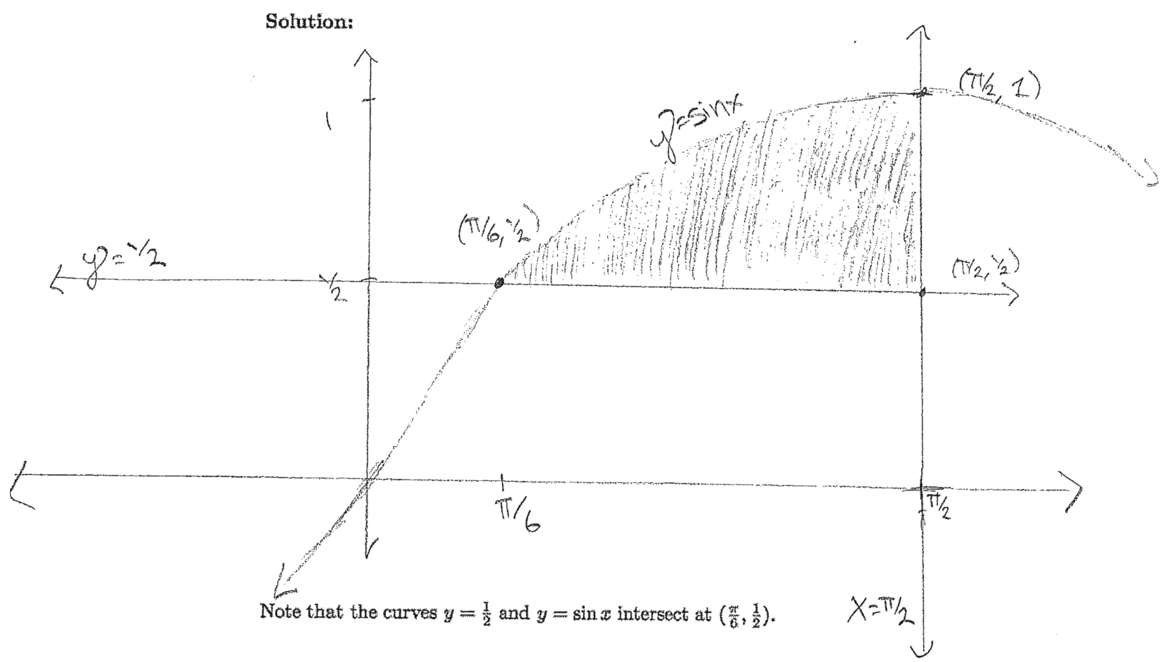
1. Consider the region D in the first quadrant which is above $y = \frac{1}{2}$, to the left of $x = \frac{\pi}{2}$, and bounded by $y = \sin x$.

   (a) Sketch the region D.

   Solution:



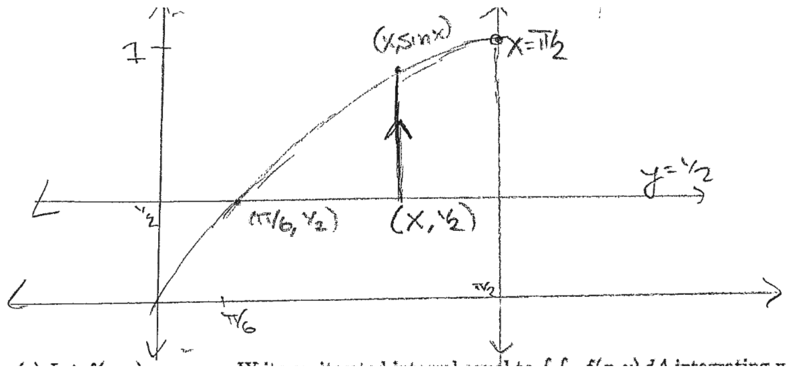
   Note that the curves $y = \frac{1}{2}$ and $y = \sin x$ intersect at $(\frac{\pi}{6}, \frac{1}{2})$.

(b) Let $f(x, y) = y \cos x$. Write an iterated integral equal to $\int\int_D f(x, y)\, dA$ integrating with respect to x first. Indicate how you are slicing in the region D.

$$\int\int_D f(x,y)dA = \int_{\frac{1}{2}}^{1} \int_{\arcsin y}^{\frac{\pi}{2}} y \cos x \; dx \; dy$$

We have these bounds because the region has y-values between $\frac{1}{2}$ and 1, and for each fixed y-value in the region, the x-values of points in the region vary from $\arcsin y$ to $\frac{\pi}{2}$.

We are slicing like this:

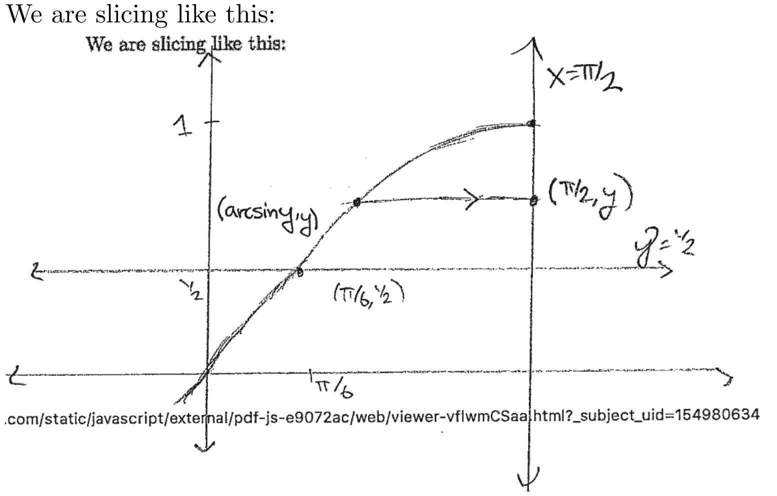


(c) Let $f(x, y) = y \cos x$. Write an iterated integral equal to $\int\int_D f(x, y)\, dA$ integrating with respect to y first. Indicate how you are slicing in the region D.

$$\int\int_D f(x,y)dA = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{\frac{1}{2}}^{\sin x} y \cos x \; dy \; dx$$

We have these bounds, because the region has x-values between $\frac{\pi}{6}$ and $\frac{\pi}{2}$, and for each fixed x-value in the region, the y-values of points in the region vary from $\frac{1}{2}$ to $\sin x$.

We are slicing like this:



(d) Let $f(x,y) = y\cos x$. Compute $\int\int_D f(x,y)\,dA$ using whichever iterated integral seems easier.

For the first integral:

$$\begin{aligned}
\int\int_D f(x,y)dA &= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_{\frac{1}{2}}^{\sin x} y\cos x \ dy \ dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}y^2\cos x|_{y=\frac{1}{2}}^{y=\sin x} \ dx \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [\frac{1}{2}(\sin x)^2\cos x - \frac{1}{2}(\frac{1}{2})^2\cos x] \ dx \\
&= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x)^2\cos x \ dx - \frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \ dx \\
&= \frac{1}{2}[\frac{1}{3}(\sin x)^3]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - \frac{1}{8}[\sin x]]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\
&= \frac{1}{6}[(\sin(\frac{\pi}{2}))^3 - (\sin(\frac{\pi}{6}))^3] - \frac{1}{8}[\sin(\frac{\pi}{2}) - \sin(\frac{\pi}{6})] \\
&= \frac{1}{12}
\end{aligned}$$

For the second integral:

$$\begin{aligned}
\int\int_D f(x,y)dA &= \int_{\frac{1}{2}}^{1}\int_{\arcsin y}^{\frac{\pi}{2}} y\cos x \ dx \ dy \\
&= \int_{\frac{1}{2}}^{1} y\sin x|_{x=\arcsin y}^{x=\frac{\pi}{2}} \ dy \\
&= \int_{\frac{1}{2}}^{1} y\sin(\frac{\pi}{2}) - y\sin(\arcsin y) \ dy \\
&= \int_{\frac{1}{2}}^{1} y - y^2 \ dy \\
&= [\frac{1}{2}y^2 - \frac{1}{3}y^3]_{\frac{1}{2}}^{1} \\
&= [\frac{1}{2} - \frac{1}{3}] - [\frac{1}{8} - \frac{1}{24}] \\
&= \frac{1}{12}
\end{aligned}$$

(e) (*Think about it...*) Write an iterated integral which represents the area of D.