Quiz 5 Math 2202
Solution

---

1. Consider the curve $x^2 + y^2 = 9$. Find a vector function (a parameterization) of this curve describing a particle with $\mathbf{r}(0) = \langle -3, 0\rangle$ and traveling counterclockwise.

   Solution:

   The curve $x^2 + y^2 = 9$ is a circle of radius 3. Define:

   $$\mathbf{r}(t) = \langle 3\cos(t+\pi), 3\sin(t+\pi)\rangle, \quad 0 \le t < 2\pi$$

   For any t,

   $$(3\cos(t+\pi))^2 + (3\sin(t+\pi))^2 = 9(\cos^2(t+\pi) + \sin^2(t+\pi)) = 9$$

   so $\mathbf{r}(t)$ is a point on the circle of radius 3.

   Also,

   $$\mathbf{r}(0) = \langle 3\cos(0+\pi), 3\sin(0+\pi)\rangle = \langle -3, 0\rangle$$

   This function travels counterclockwise, because the parameterization of a circle given by $\langle \cos t, \sin t\rangle$ travels counterclockwise. Alternately, because:

   $$\mathbf{r}(0) = \langle -3, 0\rangle, \quad \mathbf{r}(\pi/2) = \langle 0, -3\rangle$$

   this parameterization travels counterclockwise.

2. Consider the vector function $\mathbf{r}(t) = \langle e^{t^2}, -1, \sin t \rangle$.

   Find the tangent line to the curve at the point $P = (1, -1, 0)$. (Note: we're in $\mathbf{R}^3$, so you'll need to find parametric equations to describe this line.)

   Solution:

   To define any kind of line, we need a point and a vector. For the tangent line to the curve $\mathbf{r}(t) = \langle e^{t^2}, -1, \sin t \rangle$ at the point $P = (1, -1, 0)$, we need the point $P = (1, -1, 0)$ and the tangent vector to the curve when $\mathbf{r}(t) = P$.

   If
   $$(1, -1, 0) = P = \mathbf{r}(t) = \langle e^{t^2}, -1, \sin t \rangle,$$
   then $1 = e^{t^2}$, so $t^2 = 0$ and $t = 0$. So, we need the tangent vector $\mathbf{r}'(0)$.

   Since
   $$\mathbf{r}'(t) = \langle 2te^{t^2}, 0, \cos t \rangle, \quad \mathbf{r}'(0) = \langle 0, 0, 1 \rangle$$
   .

   The tangent line $\mathbf{s}(t)$ to the curve at the point $P = (1, -1, 0)$ is the line through P in the direction $\mathbf{r}'(0)$. So,
   $$\mathbf{s}(t) = \langle 1, -1, 0 \rangle + t\langle 0, 0, 1 \rangle = \langle 1, -1, t \rangle$$

   In parametric equations:
   $$x(t) = 1$$
   $$y(t) = -1$$
   $$z(t) = t$$

Please turn over.

3. Single Variable Calculus Derivative Recall:

(a) $\frac{d}{dx}\ln(\sqrt{x^2+y^2})$

(b) $\frac{d}{dt}\frac{e^{t^2}}{\sqrt{t}+1}$

(c) $\frac{d}{dy}\sin^2(\cos y)$

Solution:

(a) $\frac{d}{dx}\ln(\sqrt{x^2+y^2})$

$$\begin{aligned}
\frac{d}{dx}\ln(\sqrt{x^2+y^2}) &= \frac{1}{\sqrt{x^2+y^2}}\cdot\frac{d}{dx}(\sqrt{x^2+y^2}) \quad \text{(chain rule)}\\
&= \frac{1}{\sqrt{x^2+y^2}}\cdot(\frac{1}{2}(x^2+y^2)^{-1/2})\cdot\frac{d}{dx}(x^2+y^2) \quad \text{(chain rule)}\\
&= \frac{1}{\sqrt{x^2+y^2}}\cdot(\frac{1}{2}(x^2+y^2)^{-1/2})(2x+2y\frac{dy}{dx})\\
&= \frac{x+y\frac{dy}{dx}}{x^2+y^2}
\end{aligned}$$

(b) $\frac{d}{dt}\frac{e^{t^2}}{\sqrt{t}+1}$

$$\begin{aligned}
\frac{d}{dt}\frac{e^{t^2}}{\sqrt{t}+1} &= \frac{\frac{d}{dt}(e^{t^2})\cdot(\sqrt{t}+1)-(e^{t^2})\cdot\frac{d}{dt}(\sqrt{t}+1)}{(\sqrt{t}+1)^2} \quad \text{(quotient rule)}\\
&= \frac{(2te^{t^2})\cdot(\sqrt{t}+1)-(e^{t^2})\cdot(\frac{1}{2}t^{-1/2})}{(\sqrt{t}+1)^2}\\
&= \frac{(4te^{t^2})\cdot(t+\sqrt{t})-(e^{t^2})\cdot(1)}{2\sqrt{t}(\sqrt{t}+1)^2}\\
&= \frac{e^{t^2}(4t^2+4t\sqrt{t}-1)}{2\sqrt{t}(\sqrt{t}+1)^2}
\end{aligned}$$

(c) $\frac{d}{dy}\sin^2(\cos y)$

$$\begin{aligned}
\frac{d}{dy}\sin^2(\cos y) &= 2\sin(\cos y)\cdot\frac{d}{dt}(\sin(\cos y)) \quad \text{(chain rule)}\\
&= 2\sin(\cos y)\cdot(\cos(\cos y))\cdot\frac{d}{dt}(\cos y) \quad \text{(chain rule)}\\
&= 2\sin(\cos y)\cdot(\cos(\cos y))\cdot(-\sin y)\\
&= -2\sin(\cos y)\cdot(\cos(\cos y))\cdot(\sin y)
\end{aligned}$$