Outline
1. Equiv. of 2 moduli interpretations
2. Shimura varieties of orthogonal type

<u>Guiding question</u> : How can isomorphism and isogeny lead to the same $\mathbb{C}$-pts., roughly speaking?

Setup the two moduli problems $(\mathcal{M}_K, \mathcal{M}'_K)$ as we did in class. ← In fact, more is true.

[Q: WTF is a quasi-isogeny?]

<u>Thm</u> : $\mathcal{M}_K \simeq \mathcal{M}'_K$ (in some appropriate sense).
can just do as gpoids.

[Need :
- Fullness
- Faithfulness
- Essential surjectivity]

None of these is obvious...

<u>Remark</u> : Quasi-isogeny classes of an ab. var. A is same data as $\hat{\mathbb{Z}}$-submodules.

[Choosing a lattice singles out a unique ab. var. for a given quasi-isogeny class.] (level structure plays a much more important role in isogeny moduli interpretation)

(L, Q) quad. space / ring R ⇝ Clifford alg. ~~[illegible]~~ $C = C(L,Q) := \underbrace{\mathcal{T}(L)}_{\text{tensor alg.}} / \langle \underbrace{x \otimes x - Q(x)}_{} \rangle$.

(Not exactly sure why this makes sense...)

C has parity-based $\mathbb{Z}/2$-grading. For Q nondeg. we get alg. grp. GSpin(L,Q) w/ (related to classical notion of spin double cover)

$$\mathrm{GSpin}(L,Q)(S) := \{ x \in (C_S^+)^\times : x L_S x^{-1} = L_S \}.$$

SES $1 \to \mathbb{G}_m \to \mathrm{GSpin} \to \mathrm{SO} \to 1$.

<u>Remark</u> : Worrying about lattice business here is really only important if we want to think about integral models.

We construct things so that we can write down a Shimura datum.

<u>Q</u>: Why do we bother moving from SO to GSpin?

This is first interesting example of a Hodge type Shimura var. (means that we have emb. into a Siegel Shimura datum) There is not a moduli interpretation in terms of ab. var.'s but Deligne still managed to construct a canonical model.

(ab. type means "very close" to Hodge type)

<u>Remark</u> : dim 22 and below can be described moduli-theoretically in terms of K3 surfaces.