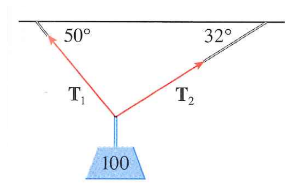
Section 2 Math 2202
Vector Addition, Dot and Cross Product

1. **Resultant Forces** (*Stewart 9.2 Example 7*) A 100-lb weight hangs from two wires as shown below



Find tensions (forces) $\mathbf{T}_1$ and $\mathbf{T}_2$ in both wires and the magnitudes of the tensions.

2. **Scalar Triple Product**

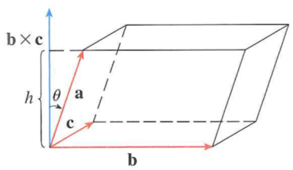


FIGURE 7

The product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is called the **scalar triple product** of the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. Its geometric significance can be seen by considering the parallelepiped determined by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. (See Figure 7.) The area of the base parallelogram is $A = |\mathbf{b} \times \mathbf{c}|$. If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b} \times \mathbf{c}$, then the height h of the parallelepiped is $h = |\mathbf{a}||\cos\theta|$. (We must use $|\cos\theta|$ instead of $\cos\theta$ in case $\theta > \pi/2$.) Thus the volume of the parallelepiped is

$$V = Ah = |\mathbf{b} \times \mathbf{c}||\mathbf{a}||\cos\theta| = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$

Therefore we have proved the following:

The volume of the parallelepiped determined by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product:

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$

Instead of thinking of the parallelepiped as having its base parallelogram determined by $\mathbf{b}$ and $\mathbf{c}$, we can think of it with base parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$. In this way, we see that

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

3. Consider the four points in $\mathbf{R}^3$, $K(1,2,3)$, $L(1,3,6)$, $M(3,8,6)$ and $N(3,7,3)$.

   (a) Show that the vectors $\overrightarrow{KL}$, $\overrightarrow{KM}$ and $\overrightarrow{KN}$ are coplanar. Explain why this means that K, L, M and N all lie in the same plane.

   (b) From part (a) we know that K, L, M and N are the vertices of a quadrilateral. Explain how you can tell that this quadrilateral is actually a parallelogram.

   (c) (*Stewart 9.4 #22*) Find the area of the parallelogram with vertices K, L, M and N.

   (d) What is the area of the triangle with vertices K, L, and M? How about the triangle with vertices L, M, N?

   (e) (*To think about...*) How many other points N' (different from N) are there such that K, L, M and N' form a parallelogram.