Exam 3

Math 2202

Friday, Nov 19, 2021

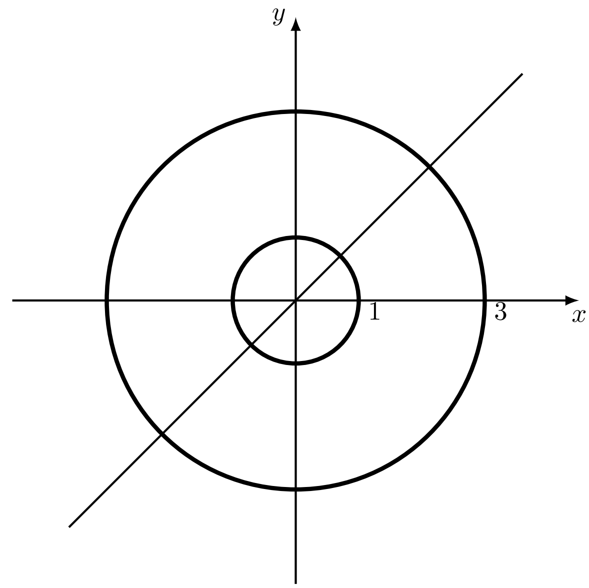
Name: ________________________________

Problem	Points	Score
1	20	
2	20	
3	24	
4	16	
5	20	
Total	100	

You have 50 minutes for this exam. Do not spend an inordinate amount of time on any one problem. **Notes and books are allowed. A calculator is not needed, you may leave numerical answers in terms of things like $\sqrt{2}$ and $\cos^{-1}(\frac{4}{15})$.** Think clearly and do well!

1. (20 pts)

(a) Consider the region R in the first quadrant above the line $y = x$ and between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. (All are graphed below.) Shade the appropriate region below.



(b) Compute the area of this region using an iterated integral.

(c) Suppose the region R represents a garden's surface, where x, y are measured in meters and the density of nitrogen in the surface soil is given by $N(x, y) = x^2y^2$ grams per square meter.

Set up, but **do not** evaluate, an integral which represents the average density of nitrogen in the garden's surface (the region R).

NOTE:

- You should specify the value of any constants.
- DO NOT EVALUATE the integral.

2. (20 pts) Consider the double integral

$$\int_0^9 \int_{\sqrt{x}}^3 e^{y^3}\, dy\, dx$$

(a) Sketch the region of integration for the integral. Indicate in your sketch how the region is being sliced in the order of integration given (vertically or horizontally). In other words, sketch a line representing a sample slice.

(b) Switch the order of integration to $dx\, dy$.

(c) Evaluate the integral in which ever order of integration you choose. *Show all work for partial credit.*

3. (24 pts) Consider the solid E bounded by
   - the paraboloid $z = x^2 + y^2 + 7$
   - the cylinder $x^2 + y^2 = 4$
   - and the plane $2z - y = 6$

   (a) Give a rough sketch of the solid E.

   (b) Set up an integral which measures the volume of this solid E in rectangular coordinates.
   You may find it useful to first describe the set of points of E in rectangular coordinates:

   $$E = \{(x, y, z) | \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \}.$$

   (c) Set up an integral which measures the volume of this solid E in cylindrical coordinates.
   You may find it useful to first describe the set of points of E in cylindrical coordinates:

   $$E = \{(r\cos(\theta), r\sin(\theta), z) | \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \}.$$

(d) Compute the volume of the solid E by a method of your choice.

4. (16 points) Consider the region R of the xy-plane bounded by $x = 0, y = 1$ and $x = 2y^2$.

Consider the transformation $T(u, v) = (x, y)$ of the uv-plane given by

$$x = 2u^2,\ y = u + v$$

with inverse transformation $u = \sqrt{\frac{x}{2}}$ and $v = y - \sqrt{\frac{x}{2}}$.

(a) Which of the following is the region **in the first quadrant** of the uv-plane which is transformed by T into the region R?

(Circle one. For partial credit, show any work below.)

i. The region bounded by $v = 1, u = 1, u + v = 1$

ii. The region bounded by $v = 0, u = 1, u + v = 1$

iii. The region bounded by $v = 0, u = 0, u + v = 1$

iv. The region bounded by $v = 1, u = 0, u = v$

(b) Consider the integral

$$\int_0^1 \int_0^{2y^2} \sqrt{x}\, dx\, dy.$$

Perform a change of coordinates to get an iterated integral with respect to u and v. Your answer should only involve u and v. (Do NOT compute it.)

5. (20 points) Consider the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\,dz\,dy\,dx.$$

(a) Sketch the region of integration, and label the bounding surfaces. *You may accompany your picture with words.*

(b) Write the integral you get when you change to spherical coordinates. (Do NOT compute it.)

(c) Consider the original integral again.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\, dy\, dx.$$

Change the order of integration and write the integral in order $dx\, dy\, dz$. (Do NOT compute it.) *Hint: you may need to use multiple integrals.*