Quadrant	x	y	xy	$2y$	Arrow Direction
1	+	+	+	+	↗
2	−	+	−	+	↖
3	−	−	+	−	↘
4	+	−	−	−	↙

Quiz 13 Math 2202

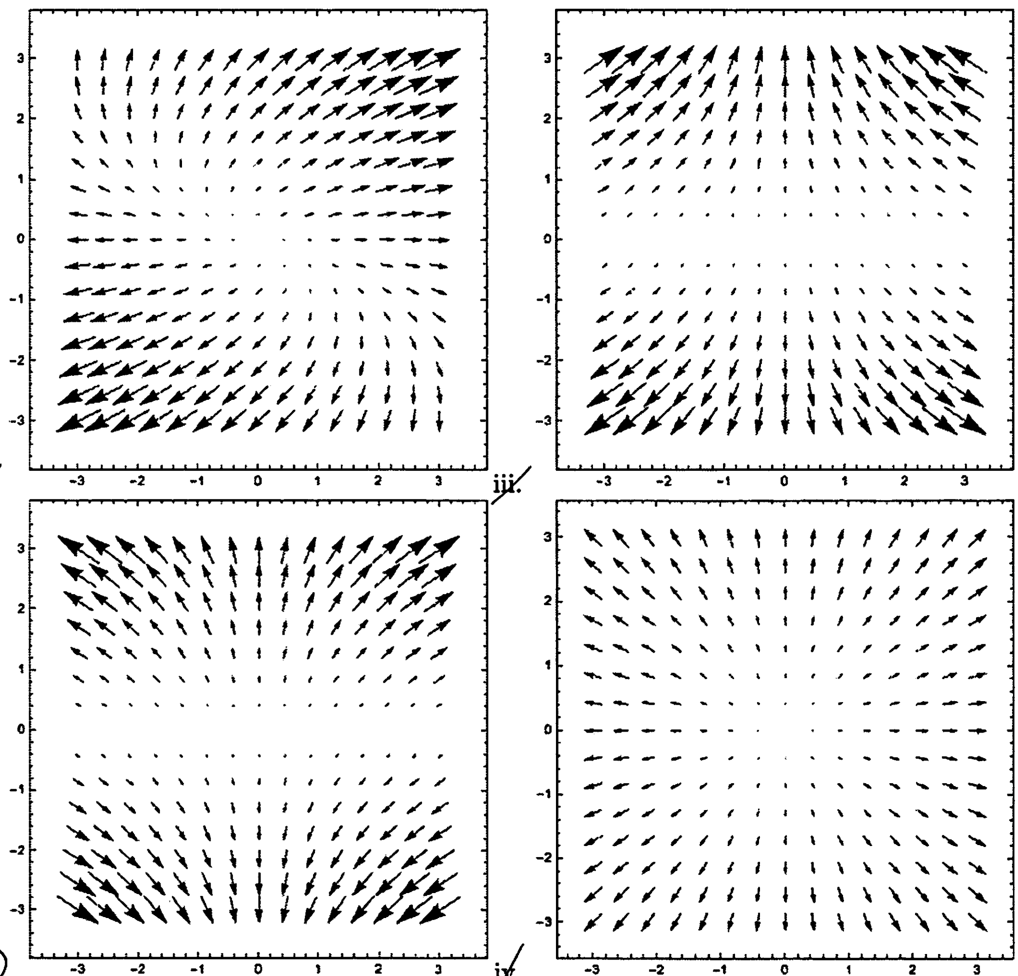
Guidelines

- This quiz is for you to test yourself on what we've been studying recently or previous material.
- You have 10 minutes. As a section, we will go over the quiz (or part of it). Solutions will be posted online as well.

---

1. Consider the vector field $\mathbf{F} = (x, y) = \langle xy, 2y\rangle$.

   (a) Which of the following could be a vector field plot of the vector field $\mathbf{F}(x, y) = \langle xy, 2y\rangle$? (Please note: the vectors in each of these fields have been scaled for easier viewing, so do not compare the lengths of vectors between two fields.)

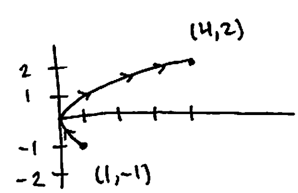


i. iii. ii. iv.

   (b) Consider the curve C parameterized by $\mathbf{r}(t) = \langle t^2, t\rangle$ with $t = -1$ to $t = 2$. Sketch the curve on the plot you chose above.

$\vec{r}(-1) = \langle 1, -1\rangle$

$\vec{r}(2) = \langle 4, 2\rangle$



[Part of a parabola]

Tangh to plot above.

$$\vec{r}(t) = \langle \underset{x(t)}{t^2}, \underset{y(t)}{t} \rangle \Rightarrow \vec{r}\,'(t) = \langle 2t, 1 \rangle$$

$$\Rightarrow \vec{F}(\vec{r}(t)) = \langle x(t)y(t), 2y(t) \rangle = \langle t^3, 2t \rangle$$

(c) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. Interpret this as the work done by the vector field $\mathbf{F}$ moving a particle from $(1, -1)$ to $(4, 2)$.

The work done is $\displaystyle\int_C \vec{F} \cdot d\vec{r} = \int_{-1}^{2} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \int_{-1}^{2} \langle t^3, 2t \rangle \cdot \langle 2t, 1 \rangle\, dt$

$$= \int_{-1}^{2} (2t^4 + 2t)\, dt = \frac{81}{5}$$

(d) Consider the straight line path C_1 from $(1, -1)$ to $(4, 2)$. Would you expect the value of $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ to be the same as the previous value. Why or why not?

We should not expect this because $\vec{F}$ is not conservative : $\frac{\partial}{\partial y}[xy] = x$ while $\frac{\partial}{\partial x}[2y] = 0$.

We can verify that the values don't match through explicit computation.

Let $\vec{p}(t) = (1-t)\langle 1, -1 \rangle + t\langle 4, 2 \rangle = \langle 1+3t, -1+3t \rangle$, $0 \le t \le 1$.

Then, $\displaystyle\int_{C_1} \vec{F} \cdot d\vec{p} = \int_0^1 \vec{F}(\vec{p}(t)) \cdot \vec{p}\,'(t)\, dt = \int_0^1 \langle (1+3t)(-1+3t), 2(-1+3t) \rangle \cdot \langle 3, 3 \rangle\, dt$

$$= 9 \neq \frac{81}{5}.$$