Section 7 Math 2202

Directional Derivatives, Gradients and Local Extrema

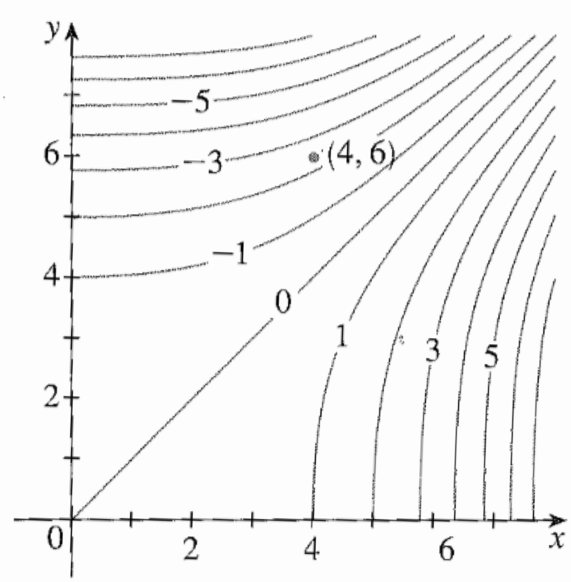
1. *Stewart 11.6 #36, modified*

   Consider a function $f(x, y)$ whose level curves are shown below.

   (a) In what direction is the gradient vector $\nabla f(4, 6)$? Sketch a vector in that direction at $(4, 6)$ and explain how you chose the direction.

   (b) Approximate the length of $\nabla f(4, 6)$. Again, explain your reasoning.

   Hint: remember that directional derivative at (x_0, y_0) in the direction $\mathbf{v}$ is the rate of change of f at (x_0, y_0) with respect to distance (that is, distance between the inputs) in the direction of $\mathbf{v}$.



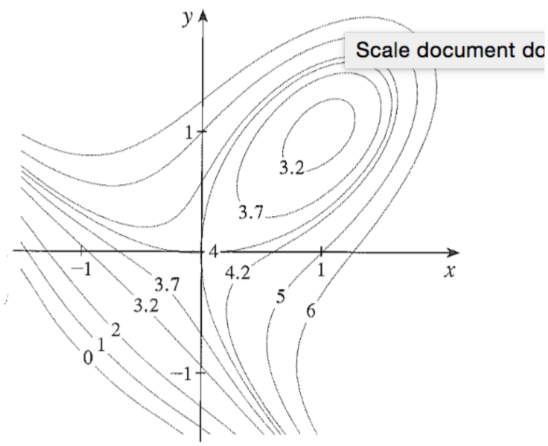
2. (*Stewart 11.7 #3*)

Let's consider

$$f(x, y) = 4 + x^3 + y^3 - 3xy$$

Use the level curves in the figures drawn below to predict the location of the critical points of f and whether f has a saddle point or a local maximum or minimum at each critical point. Explain your reasoning. Then use the Second Derivative Test to confirm your predictions.

3. $f(x, y) = 4 + x^3 + y^3 - 3xy$



3. Consider a function $f(x, y)$ which has a local minimum at $(0, 0)$ and a whole circle of local maxima at $x^2 + y^2 = 1$. (In other words, $f(x, y)$ is the same value on the circle and this is the highest value of f in a neighborhood of this circle.)

Sketch a possible contour plot of this function.

(Extra: Can you come up with a possible formula for a function with these properties? Hint: Think of a radial function, involving $x^2 + y^2$.)