Exam 2: Math 2202

Friday, Oct 22, 2021

You have **50 minutes** for this exam. Do not spend an inordinate amount of time on any one problem.

Notes and text books are allowed. Calculators are allowed but not needed, you may leave numerical answers in terms of things like $2+\frac{1}{\sqrt{2}}$ **and** $\cos^{-1}(\frac{4}{15})$**.** Think clearly and do well!

Problem	Points	Score
1	25	
2	21	
3	21	
4	21	
5	12	
Total	100	

1. (25 pts) Consider the function

$$f(x, y) = 24xy - 12y^2 - 8x^3 + 150.$$

(a) Compute f_x.

(b) Compute f_y.

(c) Find and classify all critical points of $f(x, y)$.

We are still considering the function

$$f(x, y) = 24xy - 12y^2 - 8x^3 + 150.$$

(d) Assume $z = f(x, y)$ models the terrain of a coastal region, with z measured in feet above sea-level. Here x and y are also measured in feet.

Consider the point $(1, -1, f(1, -1))$. In which direction(s) (relative to the xy-plane) should someone travel in order to decrease their elevation most from that point? What is the slope of the terrain in that direction?

Show all relevant work.

(e) Consider the curve of intersection of the graph of $z = f(x, y)$ and the plane $x = 1$. Find the tangent line to this curve at the point where $x = 1$ and $y = -1$.

2. (21 points) Consider the function $f(x, y) = e^{x-1} \sin(x + y)$.

(a) Find a normal vector to the tangent plane of the surface $z = f(x, y)$ at the point $(1, -1, 0)$.

(b) Find the linearization $L(x, y)$ of $f(x, y)$ at the point $(x_0, y_0) = (1, -1)$.

(c) Use the linearization to estimate the value $f(0.9, -0.95)$.

3. (21 pts) Consider the vector function

$$\vec{r}(t) = \langle 5,\, \sqrt{t},\, t^3 \rangle.$$

Suppose this describes the position of a particle at time t, where time is in minutes. (Here the x, y, and z coordinates of a point are in meters.)

(a) At $t = 0$, the particle is on the sphere $x^2 + y^2 + z^2 = 25$. Will the particle stay on this sphere as time increases? Why or why not?

(b) How fast and in what direction is the particle moving at $t = 1$?

(c) Suppose $G(x, y, z) = \frac{1}{x^2+y^2+z^2}$ gives the gravitational force (in Newtons) exerted by a planet at the origin on an object at location (x, y, z) (with coordinates in meters). Calculate the gradient $\vec{\nabla} G(x, y, z)$ at the point $(x, y, z) = (5, 1, 1)$.

(d) We are still considering the particle moving along the path parametrized by the $\vec{r}(t) = \langle 5, \sqrt{t}, t^3 \rangle$. Consider the function $g(t) = G(\vec{r}(t))$, which gives the gravitational force on the particle at time t. Use the chain rule, or any other method, to determine $\frac{dg}{dt}$ at $t = 1$.

4. (21 points) Consider the surface described by

$$x^2 + 4y^2 = z^2 - 1.$$

(a) Sketch the trace of the surface on the plane $y = 1$. Be sure to label your axes and any relevant intercepts.

(b) Consider the intersection of the surface with the plane $z = 3$. Find

i. a parameterization of the curve created by this intersection

ii. the tangent vector to this curve at the point $(2, 1, 3)$.

(c) Is the tangent plane to the surface at the point $(2, 1, 3)$ parallel to the plane $z = y$? Why or why not? Show relevant work.

5. (12 pts) **True or False** If true, give a brief explanation why. If false, explain why briefly or give a *counterexample*, an example for which the statement fails.

   (a) If $f(x,y) = x^2 + y^2$, then $\vec{\nabla} f(2,3)$ is a normal vector for the tangent plane to the cylinder $x^2 + y^2 = 13$ at $(2,3,13)$.

   Circle One: **TRUE** **FALSE**

   Brief Explanation:

   (b) If $f(x,y) = e^y x$, the graph of $L(x,y) = e + e(x-1) + e(y-1)$ is a plane which is tangent to the graph of the function at $(1,1,e)$.

   Circle One: **TRUE** **FALSE**

   Brief Explanation:

   (c) If $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ parameterizes a smooth curve on the sphere $x^2+y^2+z^2 = 10$, then

$$\vec{r}\,'(t) \cdot \langle 2x(t), 2y(t), 2z(t) \rangle = 0.$$

   Circle One: **TRUE** **FALSE**

   Brief Explanation:

   (d) For $f(x,y)$ a differentiable function, if $\vec{\nabla} f(0,0)$ is zero vector, then $(0,0)$ is a local minimum or local maximum for f.

   Circle One: **TRUE** **FALSE**

   Brief Explanation: