Rigid Spaces

Fix G-top. T slightly finer than the very weak top. on $\mathrm{Sp}(A) = X$ (e.g., the weak top.). Tate tells us $\mathcal{O}_X$ is a sheaf (of k-alg's).

Def: A T-rigid space (over k) is a triple $(X, T_X, \mathcal{O}_X)$ w/ $X \in \mathrm{Set}$, T_X a G-top. on X, $\mathcal{O}_X$ a sheaf of k-alg's s.t. $\exists$ adm. cov. $\{X_i\}$ of X and set bijections $X_i \cong \mathrm{Sp}(A_i)$ identifying $T_X|_{X_i}$ w/ T-top. on $\mathrm{Sp}(A_i)$ and $\mathcal{O}_X|_{X_i}$ w/ $\mathcal{O}_{\mathrm{Sp}(A_i)}$.

A morphism $(X, T_X, \mathcal{O}_X) \to (Y, T_Y, \mathcal{O}_Y)$ is a function $f : X \to Y$ cont. for the G-top's together w/ k-alg. sheaf morphism $f^* : \mathcal{O}_Y \to f_*\mathcal{O}_X$.

Assume k is alg. closed. Take $\mathrm{Sp}\, k\langle T\rangle$ and $\mathrm{Sp}\, k\langle S\rangle$ and glue them in the usual way to get $\mathbb{P}^1_k$. This recovers the $\mathbb{P}^1_k$ from before, as a rigid space.

Analytification

$X \to \mathrm{Spec}\, k$ separated finite type scheme (could be non-reduced). Want to endow $X^{an} := \{\text{closed pts. of } X\}$ w/ structure of rigid space. First suppose $X = \mathrm{Spec}\, k[z_1, \dots, z_s]/(f_1, \dots, f_t)$. Given $x \in X$ closed pt., $k_x = k(z_1(x), \dots, z_s(x))$ is fin. ext. of k so has $|\cdot|$. Powerboundedness tells us naive construction will not work. We only see the closed unit polydisk, so why not look at polydisks of larger and larger radius? Fix pseudouniformizer $\pi \in k$ (so $0 < |\pi| < 1$). Given $n \geq 1$, define $X_n^{an} := \{x \in X \text{ closed} : |z_i(x)| \leq \frac{1}{|\pi|^n}\ \forall i\}$ so that

$$\bigcup_{n \geq 1} X_n^{an} = X^{an}.$$

"$z_i^{(n)} = \pi^n z_i$"

$$\begin{array}{ccl} X_n^{an} & \cong & \mathrm{Sp}\, k\langle z_1^{(n)}, \dots, z_s^{(n)}\rangle / \left(f_1\left(\frac{z_i^{(n)}}{\pi^n}\right), \dots, f_t\left(\frac{z_i^{(n)}}{\pi^n}\right)\right) \\ \cap & & \downarrow \quad z_i^{(n+1)} \mapsto \pi z_i^{(n)} \\ X_{n+1}^{an} & \cong & \mathrm{Sp}\, k\langle \cdots \rangle / (\cdots) \end{array}$$

Glue rigid structures to get rigid structure on X^{an}.