Lubin-Tate theory gives us a hands-on way to do local (abelian) class field theory. The key is to study formal gp. laws F over $\mathcal{O}_K$ w/ "large enough" ~~torsion pts.~~ endomorphism ring (i.e., $\mathrm{End}(F) = \mathcal{O}_K$) and "enough" torsion pts. (i.e., π-torsion of F is cyclic $\mathcal{O}_K$-module).

$\mathrm{Adic}(\mathrm{CRing})$ has objects pairs (R, I) w/ R top. comm. ring and $I \trianglelefteq R$ s.t. R is I-adic.

Fully faithful $\mathrm{Adic}(\mathrm{CRing}) \hookrightarrow \mathrm{Pro}(\mathrm{CRing})$, $(R, I) \mapsto (R/I^n)_n$.

$\mathrm{Spa} : \mathrm{Adic}(\mathrm{CRing})^{op} \to \{\text{affine formal schemes}\}$ is equiv. of cat.'s, basically by definition. [We identify the two...]

$\mathrm{Spa}\, R[[x_1, \dots, x_n]]$ is formal affine n-space ($\hat{\mathbb{A}}^n_R$), where $R \in \mathrm{CRing}$. [There is "canonical" way to topologize things.]

<u>Def</u>: Let $R \in \mathrm{CRing}$. A <u>(1-dim, comm.) formal gp. law</u> over R is an ab. gp. object on $\hat{\mathbb{A}}^1_R = \hat{\mathbb{G}}_{a,R}$ in $\mathrm{Adic}(\mathrm{CRing})^{op}$.

That is, $F(x,y) \in R[[x,y]]$ satisfying

(Unity) $F(x,0) = x$, $F(0,y) = y$
(Assoc.) $F(x, F(y,z)) = F(F(x,y),z)$
(Comm.) $F(x,y) = F(y,x)$

We get inverses by an inductive process, w/ there being unique $\iota(x) \in R[[x]]$ s.t. $F(x, \iota(x)) = F(\iota(x), x) = x$.

A morphism $f : F \to G$ is $f \in tR[[t]]$ s.t. $f(F(x,y)) = G(f(x), f(y))$. Cat. $= \mathrm{FGL}_R$ [written $\iota \circ f$]

$tR[[t]]$ is ab. gp. w/ operation $f +_F g := F(f(t), g(t))$ and inverse of $f(t) \in tR[[t]]$ given by $\iota(f(t))$.

This lets us put an ab. gp. structure via $+_G$ on $\mathrm{Hom}_{\mathrm{FGL}_R}(F, G)$ and thus on $\mathrm{End}_{\mathrm{FGL}_R}(F)$ (a comm. ring structure).

It's very important here that we have both commutativity and associativity.

<u>Remark</u>: