Math 2202 Section 14
Review for Final Exam

1. (From Exam III) #5c. Consider the integral:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx.$$

Change the order of integration and write the integral in order $dx\,dy\,dz$. (Do NOT compute it.)

Hint: you may need to use multiple integrals.

2. Consider the vector field $\mathbf{F}(x, y) = \langle x^2(y-1), \frac{x^3}{3} + 1 \rangle$.

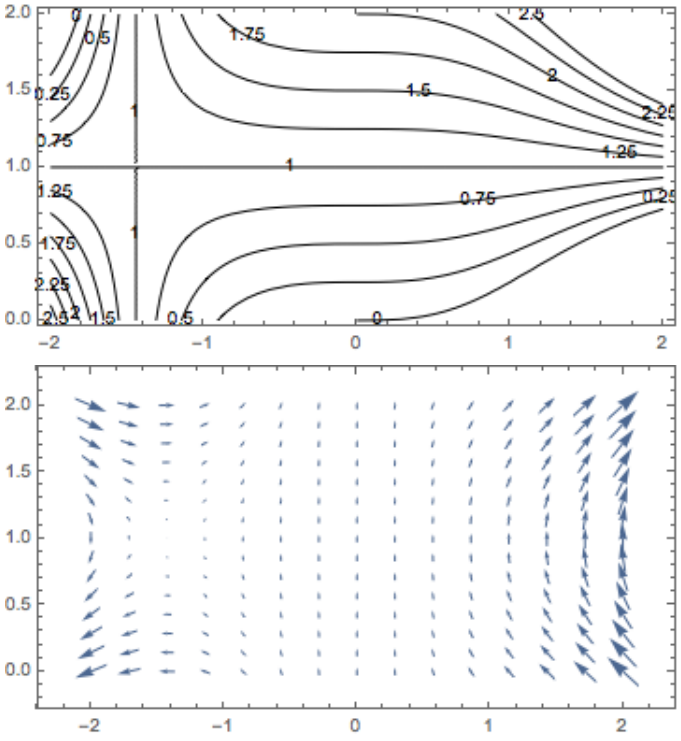
   (a) Check that $\mathbf{F}$ is a conservative/gradient vector field.

   (b) Find a function $f(x, y)$ such that $\nabla f = \mathbf{F}$. (Recall we call this the potential function of $\mathbf{F}$.)

   (c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ on the straight line path from (-1, 2) to (1, 1).

   (d) Let C be a piecewise smooth, closed and simple curve. Explain in two different ways why for you know $\oint_C \mathbf{F} \cdot d\,\mathbf{r} = 0$.

   (e) On the top is a contour plot of $f(x, y) == \frac{x^3}{3}(y-1) + y$. On the bottom is the vector plot of $\mathbf{F}$. How are the two plots related?



   (f) Find and classify all critical points of the function $f(x, y) = \frac{x^3}{3}(y-1) + y$ in $\mathbf{R}^2$. (A good check is to see if you could have guessed the approximate location and type of any of them using the contour plot on the left below.)

3. **True/False** If $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, $\text{proj}_{\mathbf{b}}\,\mathbf{a} = \text{proj}_{-\mathbf{b}}\,\mathbf{a}$.

   Give a brief explanation.

4. What is the closest point to $P = (-6, 3, 5)$ on the plane $8 = -x + 2y + 4z$?

   (a) Solve this using tools of vectors.

   (b) How could you use optimization to solve this problem? (Hint: what might you want to minimize or maximize?)

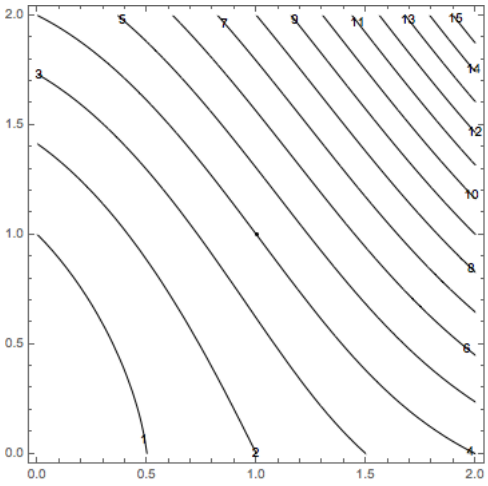
5. (From Final Exam Practice Problems) Homer Simpson falls asleep at the controls of the Springfield nuclear power station and as a result the power plant releases radiation into the surrounding area. Suppose that the radiation intensity, measured in roentgens, is given by $f(x, y) = 2x + x^2y + y^2$ where x and y are measured in miles.

   (a) Suppose you are located at $(1, 1)$. In which direction should you move to decrease the radiation intensity as quickly as possible?

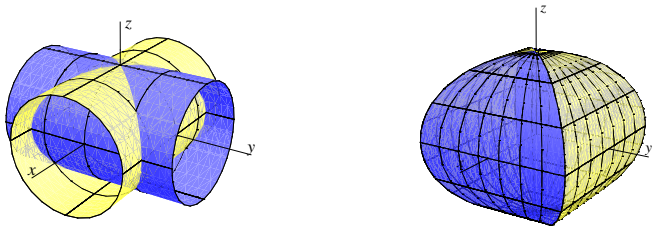
   (b) Suppose you left your position at $(1, 1)$ on a bicycle in the direction of the vector $\langle 1, 2\rangle$ at a speed of 10 miles per hour. What would be your rate of change of radiation intensity in roentgens per hour? Be sure to get the sign right; that is, would you be increasing or decreasing your exposure?

   (c) Use linear approximation to estimate the level of radiation at your neighbor's house located at $(1.1, 1.2)$.

   (d) Are your answers above consistent with the contour plot of $f(x, y)$?



6. (From Final Exam Practice Problems) Find the volume of the solid enclosed by the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$.



   Parameterize the curve where the two cylinders intersect in the first octant $(x, y \geq 0)$.