Section 6 Math 2202

Functions of Two and Three Variables, Partial Derivatives and Chain Rule

1. Let's try to understand the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

   (a) What is the domain of this function?

   (b) We can't graph this in our three dimensions, because each point in the domain is already in three space, leaving no other dimension to plot the output $f(x, y, z)$.

   For a function of 3 variables, we can do the analog of level curves, called **level surfaces**. Find the level surfaces of $f(x, y, z)$ for $k = -2, -1, 0, 1, 2, 3$ and describe what each looks like as two dimensional surface. Then try sketching them all on the same xyz-axes.

Footnote: Beyond three variables, we don't usually try to visualize functions. Instead we rely on analyzing the function's definition (formula) using tools from calculus and other math to get a sense of the function. We can also draw good enough conceptual pictures, based on the intuition of 2 and 3 dimensions.

2. The function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ gives the distance to the origin of the input point (x, y, z).

   (a) Let's consider a particle moving according to $x(t) = t^2, y(t) = 3\sin t, z(t) = t + 1$. Using chain rule, compute $\frac{df}{dt}$, the derivative of the composite function $f(x(t), y(t), z(t))$ with respect to t.

   (b) This derivative of the composite function $f(x(t), y(t), z(t))$ with respect to t can be interpreted as the rate of change of $f(x, y, z)$ with respect to t as the point (x, y, z) moves along the curve C described by those parametric equations.
   In this case, recall that $f(x, y, z)$ is the distance to the origin of (x, y, z). Interpret $\frac{df}{dt}|_{t=2}$ in words.

   (c) Let's consider a particle moving according to $x(s) = 2\sin s, y(s) = 2\cos s, z(s) = 1$. Using chain rule, compute the derivative of $f(x(s), y(s), z(s))$ with respect to s. Does your answer make sense? Why or why not?