Assignment 6

due midnight (Eastern Time), Monday, November 2, 2020

Prove all statements in the questions below. You must TeX your solutions and submit your writeup as a pdf at www.gradescope.com.
You can submit it any time until the deadline.

Question 1. (Folland 5.1.1) If $\mathcal{X}$ is a normed vector space over K ($= \mathbb{R}$ or $\mathbb{C}$), then addition and scalar multiplication are continuous from $\mathcal{X} \times \mathcal{X}$ and $K \times \mathcal{X}$ to $\mathcal{X}$. Moreover, the norm is continuous from $\mathcal{X}$ to $[0, \infty)$; in fact $\big| ||x| - ||y|| \big| \leq ||x - y||$.

Question 2. (Folland 5.1.2). $L(\mathcal{X}, \mathcal{Y})$ is a vector space and the function $|| \cdot ||$ defined by (5.3) is a norm on it. In particular, the three expressions on the right of (5.3) are always equal.

Question 3. (Folland 5.1.4) If $\mathcal{X}, \mathcal{Y}$ are normed vector spaces, the map $(T, x) \mapsto Tx$ is continuous from $L(\mathcal{X}, \mathcal{Y}) \times \mathcal{X}$ to $\mathcal{Y}$. (That is, if $T_n \to T$ and $x_n \to x$, then $T_n x_n \to Tx$.)

Question 4. (Folland 5.1.5) If $\mathcal{X}$ is a normed vector space, the closure of any subspace of $\mathcal{X}$ is a subspace.