We will be interested in stacks of the following form.

- $\mathcal{M}$ regular of dim 2, proper flat over $\operatorname{Spec}\mathbb{Z} = S$ and relative complete intersection. $\mathcal{M}$ also connected.
- $M := \mathcal{M}_{\mathbb{C}}$ has orbifold presentation $[\Gamma \backslash X]$ for X compact Riemann surface and Γ finite grp.

We start simple w/ the 1-dim case. Let $\mathcal{Z}$ be DM stack reduced and proper of rel. dim 1 / alg. closed field k. $\mathcal{L}$ line bundle / $\mathcal{Z}$, x closed geom. pt. of $\mathcal{Z}$, $\tilde{\mathcal{O}}_{\mathcal{Z},x}$ strictly Henselian local ring

$$\tilde{\mathcal{O}}_{\mathcal{Z},x} = \bigoplus_i \underbrace{\mathcal{O}_i}_{\text{int. dom.}}$$, decomposition based on formal branches of $\mathcal{Z}$ through x.

s rational section of $\mathcal{L} \rightsquigarrow \deg_x(s) := \sum_i \operatorname{ord}_{\mathcal{O}_i}(s_i)$ for s_i image of s in $\mathcal{L} \otimes_{\mathcal{O}_{\mathcal{Z},x}} \mathcal{O}_i$.

$$\deg(\mathcal{L}) = \deg(\mathcal{Z},\mathcal{L}) := \sum_{x \in \mathcal{Z}(k)} \frac{1}{|\operatorname{Aut}(x)|} \deg_x(s).$$ [need to base change to $\bar{k}$ for k not alg. closed]

<u>Properties</u>: (i) $\deg(\mathcal{L} \otimes \mathcal{L}') = \deg(\mathcal{L}) + \deg(\mathcal{L}')$.

(ii) $f: \mathcal{Z}' \to \mathcal{Z}$ finite flat of constant degree $\Rightarrow \deg(f^*\mathcal{L}) = \deg(f)\deg(\mathcal{L})$.

Latter is useful in part because, letting $\pi: \tilde{\mathcal{Z}} \to \mathcal{Z}$ be normalization, $\deg(\mathcal{Z},\mathcal{L}) = \deg(\tilde{\mathcal{Z}}, \pi^*(\mathcal{L}))$.

Normality tells us $\tilde{\mathcal{O}}_{\tilde{\mathcal{Z}},x}$ is DVR.

$$\mathcal{Z} \text{ stack} / \mathbb{F}_q,\ \Gamma(\mathcal{Z},\mathcal{O}) = \mathbb{F}_q \rightsquigarrow \widehat{\deg}(\mathcal{Z},\mathcal{L}) := \deg(\mathcal{Z},\mathcal{L}) \overbrace{\log q}^{\text{(What role does this play?)}}.$$