

MIDTERM 1 - ORAL COMPONENT

You will sign up for a 20 minute Zoom meeting slot. During the meeting, I will ask you to state a definition and to state and prove a major result from Chapter 1 or 2. You will not know ahead of time which definition and result I will ask you about. They could be anything from Chapters 1 or 2. You will not be allowed to use any notes or the textbook. If you get stuck, I will give you hints.

You will write your answers in some way that I can see you writing. Options for ways to do this include:

- type in TeX and share your screen with me through Zoom, so that I can see you typing
- type in any other program, share your screen with me through Zoom
- Write on an iPad/etc, and share it with me over Zoom
- Write with a pen on paper, and have your Zoom camera oriented so that I can see you writing
- Write on a blackboard/whiteboard/etc, with Zoom camera oriented so that I can see you writing
- Any other reasonable way for me to see you write.




I do not expect you to memorize every little detail of every proof. However, I expect you to know generally how the proofs go – what steps go into them, what the general idea of the proof is. Enough that, possibly with a little prodding from me, you can reconstruct a reasonably logically coherent sketch of the proof of the fact.

To prepare, you should study/know:

- All definitions.

Statements and (sketches of) proofs of major results, including:



- Proof that $\mathcal{B}_{\mathbb{R}^n} = \otimes_1^n \mathcal{B}_{\mathbb{R}}$ 
- Basic properties of measures (Theorem 1.8).
- Carathéodory's Theorem (Theorem 1.11)
- Proof that given a premeasure on an algebra, there is a measure whose restriction to that algebra agrees with the premeasure (Theorem 1.14).
- Sums, products, limits, max, etc. of (sequences of) measurable functions are measurable.
- Measurable functions can be approximated in a strong sense (Theorem 2.10) by simple functions.
- L^1 is a metric space
- Monotone Convergence Theorem 
- Dominated Convergence Theorem
- Fatou's Lemma 
- Egoroff's Theorem
- Fubini-Tonelli theorem