Section 1 Math 2202
The Next Dimension: Distance and Equations in $\mathbf{R}^3$

1. This problem is meant to help you explore what equations describe in the plane $\mathbf{R}^2$ compared to in 3-dimensional space $\mathbf{R}^3$.

   (a) What does the equation $x = 4$ represent in the xy-plane, $\mathbf{R}^2$? What does it represent in xyz-space, $\mathbf{R}^3$? Illustrate with sketches.

   (b) What does the equation $y = 3$ represent in $\mathbf{R}^3$? What does $z = 5$ represent? What does the pair of equations $\{y = 3, z = 5\}$ represent? In other words, describe the set of points (x, y, z) such that we have both $y = 3$ and $z = 5$. Illustrate with a sketch.

   (c) What does the set of three equations $\{x = 4, y = 5, z = 3\}$ represent in $\mathbf{R}^3$? In other words, describe the set of points (x, y, z) such that $x = 4$ and $y = 5$ and $z = 3$. Illustrate with a sketch.

   (d) Points are considered to be "0-dimensional", lines are "1-dimensional", planes are "2-dimensional", and $\mathbf{R}^3$ is "3-dimensional". Describe anything you notice about the relationship between the number of equations in x, y and z and the dimension of the shape they represent in $\mathbf{R}^3$. Do the same thing for $\mathbf{R}^2$.

2. In this problem, you'll explore the set of points equidistant from two given points in $\mathbf{R}^3$. As you do, reflect back to what the set of points equidistant from two points in $\mathbf{R}^2$ is (Hint: high school geometry).

   (a) Plot the points $(2, -1, 5)$ and $(2, 6, 5)$. Write an equation for the set of points equidistant from $(2, -1, 5)$ and $(2, 6, 5)$ and simplify it as much as possible. Describe the set as specifically as possible. (Hint: Start with a generic point (x, y, z). What would it mean for this point to be equidistant from the two given points? Write an equation to describe this.)

   (b) Plot the points $(2, -1, 5)$ and $(3, 6, 0)$. Write an equation for the set of points equidistant from $(2, -1, 5)$ and $(3, 6, 0)$ and simplify it as much as possible. Describe the set as specifically as possible.

3. **What region is this in $\mathbf{R}^3$?** For each of the following, describe as fully as possible the region represented by the equation or set of equations.

   Here are some things to address and tips for thinking and visualizing:

   - Is it a familiar geometric object?
   - What *dimension* is it? Is it a line or curve (one-dimensional/1D)? Is it two-dimensional like a plane? For example, if you think "cone", is it the surface of the cone (2D) or a solid cone (3D)?
   - If the equation doesn't involve one of the variables (for example, there is no z in the equation), what does that mean about the possible values of z for points in the region? (A variable which does not have any restriction on it is sometimes called a *free variable* or *independent variable.*)
   - How does the region relate to the coordinate planes or axes? Is is parallel or perpendicular to any of them?
   - For some, it may be useful to think about whether the object intersects the coordinate planes. To test intersection with coordinate plane (eg: xy-plane) we use equation for that plane (eg: $z = 0$) and solve those equations together. We call such an intersection a *trace* of the region. However, we still have to consider the equation in $\mathbf{R}^3$ - what's the role of the other variable (eg: z)?

   (a) $x = 3, y = -7$
   (b) $y = 2x + 3$
   (c) $y = 2x + 3, z = 5$
   (d) $y = 3x + 1, z = -2, x = y$
   (e) $x^2 = 3$
   (f) $x^2 + y^2 + z^2 = 3$
   (g) $y = x^2 + 3$
   (h) $x^2 + z^2 = 3$
   (i) $y^2 + 2z^2 = 3$

4. (Exploratory) Another way to think about describing regions in $\mathbf{R}^2$, $\mathbf{R}^3$ and higher dimensions is in terms of a *parameteric equation.* For example, in $\mathbf{R}^2$, consider all points

   $$\langle \cos t, \sin t \rangle$$

   where t is any real value. What is this object? What about the following?

   $$\langle 2\cos t, \sin t \rangle$$