Quiz 6 Math 2202

Guidelines

- This quiz is for you to test yourself on what we've been studying recently.
- You have 10 minutes. As a section, we will go over the quiz (or part of it). Solutions will be posted online as well.

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1. Let $f(x, y) = \sin(-xy) + y^2$.

   (a) Compute $f_x(\frac{\pi}{4}, -1)$ and interpret it in words. (Draw a sketch in the appropriate plane parallel to one of the coordinate planes.)

   (b) Compute $f_{xx}(x, y)$. What is the sign of $f_{xx}(\frac{\pi}{4}, -1)$? What does it mean?

   Solution:

   (a) The partial derivative of $f(x, y)$, with respect to x:

$$\begin{aligned} f_x(x, y) &= \cos(-xy) \cdot (-y) \\ &= -y\cos(-xy) \end{aligned}$$

   We used the chain rule. $\frac{\partial}{\partial x} y^2 = 0$, because y is viewed as a constant when we take the partial derivative with respect to x. Then, evaluate at the point $(\frac{\pi}{4}, -1)$:

$$f_x(\frac{\pi}{4}, -1) = -(-1)\cos(-(\frac{\pi}{4})(-1)) = \frac{1}{\sqrt{2}}$$

   The partial derivative $f_x(\frac{\pi}{4}, -1)$ can be interpreted as the slope of the tangent line to the trace curve for $f(x, y)$ when $y = -1$, at the point $(x, y, z) = (\frac{\pi}{4}, -1, f(\frac{\pi}{4}, -1))$. Here is a graph of the trace curve for $y = -1$, and the tangent line:

   Note: The solution for 1(c) (not listed here) is available in the worksheet solutions.

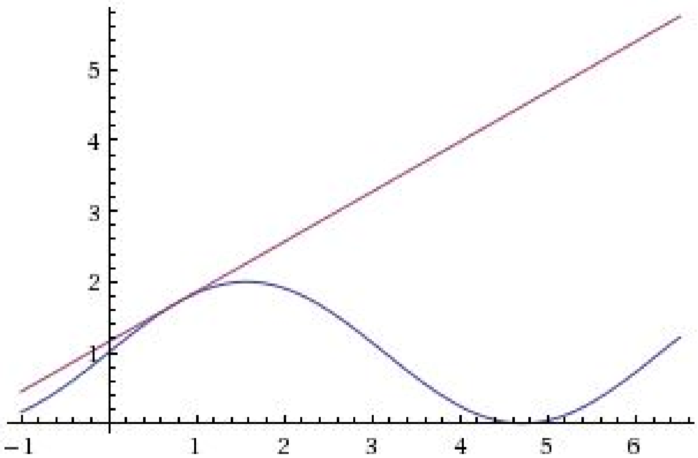

Figure 1: $y = -1,\ z = \sin(x) + 1$

(b) To compute the second partial derivative of $f(x, y)$ with respect to x, take the partial derivative of $f_x(x, y)$ with respect to x:

$$\begin{aligned} f_{xx}(x, y) &= \frac{\partial}{\partial x}(f_x(x, y)) \\ &= \frac{\partial}{\partial x}(-y\cos(-xy)) \\ &= (-y)(-y)(-\sin(-xy)) \\ &= -y^2\sin(-xy) \end{aligned}$$

Now, evaluate at $(\frac{\pi}{4}, -1)$:

$$f_{xx}(\frac{\pi}{4}, -1) = -(-1)^2\sin(-(\frac{\pi}{4})(-1)) = -\sin(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$$

Because the sign of $f_{xx}(\frac{\pi}{4}, -1)$ is negative, the graph of the trace curve for f when $y = -1$ is concave down at $(\frac{\pi}{4}, -1, f(\frac{\pi}{4}, -1, f(\frac{\pi}{4}, -1))$.

2. (a) Which of the following functions describes a two dimensional surface lying in $\mathbf{R}^3$? (In other words, which has a graph which is a 2-D surface?)
   (b) Which function, if any, describes a plane in $\mathbf{R}^3$?
   (c) Which function, if any, describes the surface of a hemisphere in $\mathbf{R}^3$?
   (d) Which function, if any, has level curves which are hyperbolas?
   (e) Which function, if any, has level curves which are lines?

   A. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$
   B. $g(x, y) = \sqrt{9 - x^2 - y^2}$
   C. $h(x, y) = 2x - 3y + 7$
   D. $k(x, y, z) = 2x - 3y + 6z + 7$ See the worksheet solutions for a table like the one we constructed in class.
   E. $m(x, y) = \sqrt{x^2 + y^2}$
   F. $n(x, y) = 3x^2 - y^2$

Solution:

A. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$

This function has three input variables, and we would need 4 dimensions to draw the graph of this function. So, this function does not describe a two-dimensional surface lying in $\mathbf{R}^3$. In particular, it does not describe a plane or a hemisphere in $\mathbf{R}^3$. So, (a), (b), and (c) are not true for this function.

As we explored in discussion, this function has level *surfaces* instead of level *curves*. To find the level surfaces, set $f(x, y, z) = k$ for some constant k:

$$k = \sqrt{x^2 + y^2 + z^2}$$

For $k \geq 0$ this describes the set of points distance k from the origin in $\mathbf{R}^3$.

Since this function has level surfaces instead of level curves, (d) and (e) are also false.

In total: none of (a)-(e) are true for this function.

B. $g(x, y) = \sqrt{9 - x^2 - y^2}$

This function has two input variables, and does describe a two dimensional surface lying in $\mathbf{R}^3$ - (a) is true. If

$$g(x, y) = z = \sqrt{9 - x^2 - y^2},$$

Then,

$$z^2 = 9 - x^2 - y^2$$

And,

$$x^2 + y^2 + z^2 = 9$$

So, every point on the surface $z = g(x, y)$ is also on the sphere of radius three centered at the origin. Since, in our original equation $z = \sqrt{9 - x^2 - y^2}$ the variable z is always nonnegative, the graph of our surface is just the top half of the sphere in $\mathbf{R}^3$ - a hemisphere.

Then, (c) is true but (b) is not.

By setting $z = g(x, y) = k$ for some constant k,

$$k = g(x, y) = \sqrt{9 - x^2 - y^2}$$

,

$$x^2 + y^2 = k^2 - 9$$

The level curves of this function are circles, and so (d) and (e) are false.

In total: (a) and (c) are true for this function.

C. $h(x, y) = 2x - 3y + 7$

This function has two input variables, and does describe a two dimensional surface lying in $\mathbf{R}^3$ - (a) is true. If

$$h(x, y) = z = 2x - 3y + 7,$$

Then,

$$z - 2x + 37 = 7$$

Which is the equation for a plane. So, (b) is true and (c) is not.

By setting $z = h(x, y) = k$ for some constant k,

$$k = h(x, y) = 2x - 3y + 7$$

,

$$y = \frac{2}{3}x + \frac{7 - k}{3}$$

The level curves of this function are lines, and so (e) is true and (d) is false.

In total: (a) and (e) are true for this function.

D. $k(x, y, z) = 2x - 3y + 6z + 7$

This function has three input variables, and we would need 4 dimensions to draw the graph of this function. So, this function does not describe a two-dimensional surface lying in $\mathbf{R}^3$. In particular, it does not describe a plane or a hemisphere in $\mathbf{R}^3$. So, (a), (b), and (c) are not true for this function.

Similar to the function we explored in discussion, this function has level *surfaces* instead of level *curves*. To find the level surfaces, set $k(x, y, z) = c$ for some constant c:

$$c = 2x - 3y + 6z + 7$$

$$2x - 3y = 6z = c - 7$$

So, this function has level surfaces which are planes.

Since this function has level surfaces instead of level curves, (d) and (e) are also false.

In total: none of (a)-(e) are true for this function.

E. $m(x, y) = \sqrt{x^2 + y^2}$

This function has two input variables, and does describe a two dimensional surface lying in $\mathbf{R}^3$ - (a) is true. In fact,

$$m(x, y) = z = \sqrt{x^2 + y^2}$$

is the equation of a cone in $\mathbf{R}^3$. So, (b) and (c) are false.

B setting $m(x, y) = k$ for some constant k,

$$k = m(x, y) = \sqrt{x^2 + y^2}$$

,

$$k^2 = x^2 + y^2$$

So, this function has level curves which are circles, and (d) and (e) are false.

In total: (a) is true for this function.

F. $n(x, y) = 3x^2 - y^2$

This function has two input variables, and does describe a two dimensional surface lying in $\mathbf{R}^3$ - (a) is true. In fact,

$$n(x, y) = z = 3x^2 - y^2$$

is the equation of a hyperbolic paraboloid in $\mathbf{R}^3$. So, (b) and (c) are false.

B setting $n(x, y) = k$ for some constant k,

$$k = n(x, y) = 3x^2 - y^2$$

,

So, this function has level curves which are hyperbolas. Then, (d) is true and (e) is false.

In total: (a) and (d) are true for this function.

Think about it... For the functions above which do *not* describe a two-dimensional surface in $\mathbf{R}^3$, how can you think about visualizing them? (Hint: If they are 3-dimensional spaces, use the idea of level surfaces.)