Exam 1: Math 2202

Friday, Sep 24, 2021

Name: ________________________________

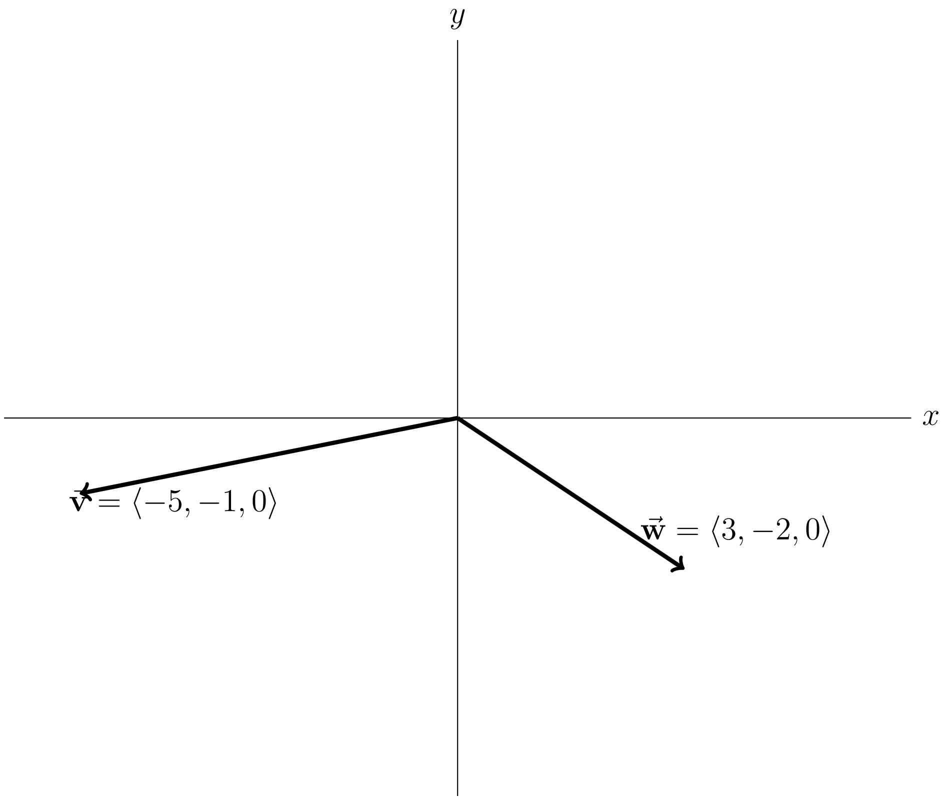
Class Time: 11 AM 12 AM

Problem	Points	Score
1	20	
2	20	
3	30	
4	15	
5	15	
Total	100	

You have **50 minutes** for this exam. Do not spend an inordinate amount of time on any one problem.

You may use your notes. A calculators is not needed, you may leave numerical answers in terms of things like $2 + \frac{1}{\sqrt{2}}$ **and** $\cos^{-1}(\frac{4}{15})$**.** Think clearly and do well!

1. (20 pts) Consider the two vectors shown, which are vectors in $\mathbf{R}^3$ lying in the xy-plane.



(a) Compute and sketch $\vec{\mathbf{v}} + \vec{\mathbf{w}}$ and $\vec{\mathbf{v}} - \vec{\mathbf{w}}$. Label clearly.

(b) Does $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ point into the page, point out of the page or lie in the page?

(c) Compute $\text{proj}_{\vec{\mathbf{w}}}\vec{\mathbf{v}}$ and sketch it on the axes above.

An airplane is flying. Suppose $\vec{\mathbf{v}}$ is the velocity vector of its charted course, and $\vec{\mathbf{w}}$ is the velocity vector of the wind. The sum $\vec{\mathbf{v}} + \vec{\mathbf{w}}$ represents the resultant velocity of the plane.

(A velocity vector gives the direction of motion and its magnitude is the speed.)

(d) What is the angle between the resultant velocity of the airplane and the velocity of its charted course?

(e) How fast is the airplane traveling? (Here units are in meters per minute.)

2. (20 points) Consider the points $P = (1, 2, 3), Q = (0, 1, -1), R = (-4, 1, 0)$.

   (a) Show that ΔPQR is a right triangle with right angle at Q.

   (b) Find the plane containing this triangle. Leave your answer in linear equation form.

3. (30 points) Consider the line L in $\mathbf{R}^3$ parallel to the vector $2\vec{\mathbf{i}} + \vec{\mathbf{k}}$ and containing the point $P = (-1, 2, 3)$, and the plane $\mathcal{P}$ of all points satisfying $x + y + z = 1$

   (a) Write parametric equations for the line L.

   (b) Find the point of intersection of the line L with the plane $\mathcal{P}$

   (c) Find the angle of intersection of L with $\mathcal{P}$

We are still considering the line L in $\mathbf{R}^3$ parallel to the vector $2\vec{\mathbf{i}} + \vec{\mathbf{k}}$ and containing the point $P = (-1, 2, 3)$.

(d) Find the distance between the line L and the origin $(0, 0, 0)$, **and** find the point on L closest to $(0, 0, 0)$.

(e) Find the line on the xz-plane that is parallel to line L and closest to L.

4. (15 points) Consider the plane $\mathcal{P}$ in $\mathbf{R}^3$ with equation $3x + 2y - z + 12 = 0$.

(a) Write the equation of the plane parallel to $\mathcal{P}$ and containing the point $A = (0, 8, 0)$.

(b) Find the distance between the two planes

(c) Consider the set of all points that have the same distance from $\mathcal{P}$ and the plane you found in (a). Describe this set with an equation or in words.

5. (15 pts) **True or False** If true, give a brief explanation why. If false, explain why briefly or give a *counterexample*, an example for which the statement fails.

(a) If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are both non-zero vectors and $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 0$, then $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are parallel.

Circle One: **TRUE** **FALSE**

Brief Explanation:

(b) Let $\vec{\mathbf{a}} = \overrightarrow{\mathbf{PQ}}$, $\vec{\mathbf{b}} = \overrightarrow{\mathbf{QR}}$, and $\vec{\mathbf{c}} = \overrightarrow{\mathbf{RP}}$ with P, Q, R, three distinct points in $\mathbf{R}^3$. Then the vectors $\vec{\mathbf{a}} \times \vec{\mathbf{b}}, \vec{\mathbf{a}} \times \vec{\mathbf{c}}$ and $\vec{\mathbf{b}} \times \vec{\mathbf{c}}$ are all parallel to each other. **Circle One:** **TRUE** **FALSE**

Brief Explanation:

(c) The equation $\langle x-2, y+4, z-8\rangle \cdot \langle 1,1,1\rangle = 0$ is the equation of a line parallel to $\langle 1,1,1\rangle$.

Circle One: **TRUE** **FALSE**

Brief Explanation:

(BONUS 2 points) Let $\vec{\mathbf{w}}$ be any non-zero vector in $\mathbf{R}^3$. Are there any vectors $\vec{\mathbf{v}}$ such that $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \vec{\mathbf{v}} + \vec{\mathbf{w}}$? If so, describe all of them completely. If not, why not?