Section 6 Math 2202

Functions of Two and Three Variables, Partial Derivatives and Chain Rule

Comments for Facilitator:

- The main goals of today are
  - to help students get better at recognizing the type of object described by a multivariable function, both in terms of dimension (is it a surface or something 3D) and then in the case of surfaces, what type of surface it is, if applicable.
  - help them interpret what the chain rule for the case of all input functions of t can represent.
- Do Quiz (10-12 min). Offer them 2 more minutes at the 10 minute mark.
- Go over quiz (15 min) #1 and #2 (the functions of two variables) Take time here to really make sure students are sketching the trace curves and making sense of the partial derivatives.
- Theme of Today: Functions of Three Variables, Linearization and Chain Rule
  - First go over the example of the plane (D) from the quiz, and introduce the concept of level surfaces to visualize functions of three variables.
  - Then do the problem #1: understand $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ as a three-dimensional space whose level surfaces $w = k$ are spheres of radius k.
  - Think about composite functions. Give them the chain rule for $f(x, y, z) = w$ where x, y, z are differentiable funcitons of t. Mention this can be extended to more variables naturally.
  - Open Questions

1. Let's try to understand the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

   (a) What is the domain of this function?

   (b) We can't graph this in our three dimensions, because each point in the domain is already in three space, leaving no other dimension to plot the output $f(x, y, z)$.

   For a function of 3 variables, we can do the analog of level curves, called **level surfaces**. Find the level surfaces of $f(x, y, z)$ for $k = -2, -1, 0, 1, 2, 3$ and describe what each looks like as two dimensional surface. Then try sketching them all on the same xyz-axes.

2. The function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ gives the distance to the origin of the input point (x, y, z).

(a) Let's consider a particle moving according to $x(t) = t^2, y(t) = 3\sin t, z(t) = t + 1$. Using chain rule, compute $\frac{df}{dt}$, the derivative of the composite function $f(x(t), y(t), z(t))$ with respect to t.

(b) This derivative of the composite function $f(x(t), y(t), z(t))$ with respect to t can be interpreted as the rate of change of $f(x, y, z)$ with respect to t as the point (x, y, z) moves along the curve C described by those parametric equations.

In this case, recall that $f(x, y, z)$ is the distance to the origin of (x, y, z). Interpret $\frac{df}{dt}|_{t=2}$ in words.

(c) Let's consider a particle moving according to $x(s) = 2\sin s, y(s) = 2\cos s, z(s) = 1$. Using chain rule, compute the derivative of $f(x(s), y(s), z(s))$ with respect to s. Does your answer make sense? Why or why not?