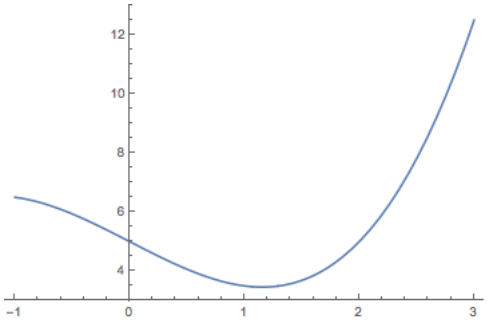
2. Integration of $y = f(x)$

   Let $f(x) = \frac{x^3}{2} - 2x + 5$.

   (a) Consider the interval $[-1, 3]$. Divide it into four equal parts and label the partition points x_0, x_1, x_2, x_3, x_4. (Note $x_0 = -1$ and $x_4 = 3$).
   What is Δx, the length of each subinterval? For each subinterval, draw a rectangle with base the length of the subinterval and height so that the left endpoint touches the curve.



   Write a Riemann sum (a sum of area of rectangles) to estimate the area under the curve and above the x-axis on the interval $[-1, 3]$ using 4 rectangles and the left endpoint rule.

   (b) Write a Riemann sum (a sum of area of rectangles) that would estimate the area under the curve and above the x-axis on the interval $[-1, 3]$ using 20 rectangles with the left endpoint rule. What is Δx? Give a formula for x_i for $i = 0, ..., 20$.

   (c) Write a limit of a Riemann sum that represents the exact area under the curve and above the x-axis on the interval $[-1, 3]$.

   (d) Write an integral that measures the exact area under the curve $f(x)$ and above the x-axis from $x = -1$ to $x = 3$. Compute it.

   (e) (Challenge) Can you write a Riemann sum that estimates the area under the curve and above the x-axis on the interval $[-1, 3]$ using n rectangles and the midpoint rule?