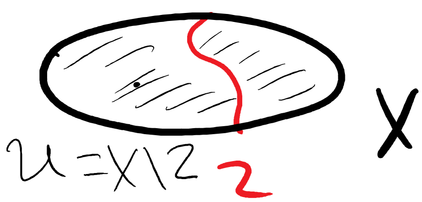
<u>Last Time</u>: We defined closed and open embeddings.

$\operatorname{Spec} A/I \hookrightarrow \operatorname{Spec} A$ induced by $A \twoheadrightarrow A/I$

In general, a map of spaces $Z \hookrightarrow X$ is a closed emb.

if, given any map $\operatorname{Spec} B \to X$,

$$\begin{tikzcd} \operatorname{Spec} B \times_X Z \arrow[r] \arrow[d, "\text{closed emb.}"'] & Z \arrow[d, hook] \\ \operatorname{Spec} B \arrow[r] & X \end{tikzcd}$$



$\mathcal{U} \hookrightarrow X$ (both affine) is an open emb. if

$\mathcal{U} = X \setminus Z$ for $Z \hookrightarrow X$ closed emb.

<u>Ex</u>: $\operatorname{Spec} A_f \hookrightarrow \operatorname{Spec} A$ is the complement of
$\operatorname{Spec} A/f \hookrightarrow \operatorname{Spec} A$.

<u>Recall</u>: $Y \to X$ map of spaces $\rightsquigarrow X \setminus Y$.

$(X \setminus Y)(A)$ is obtained by taking $x \in X(A) \cong \operatorname{Hom}(\operatorname{Spec} A, X)$

$\operatorname{Spec} 0$

s.t. $\begin{tikzcd} \operatorname{Spec} 0 = \emptyset \arrow[r] \arrow[d] \arrow[dr, phantom, "\ulcorner"] & Y \arrow[d] \\ \operatorname{Spec} A \arrow[r, "x"'] & X \end{tikzcd}$.

<u>Def</u>: $X \in \mathrm{Space}$. A <u>(Zariski) open covering</u> of X is a collection of (Zariski) open embeddings $\mathcal{U} = \{(U, i_U : U \hookrightarrow X)\}$ s.t., $\forall f \in \mathrm{Hom}(\operatorname{Spec} B, X)$ w/ $B \neq 0$, $\operatorname{Spec} B \times_X U$ is nonempty for some $U \in \mathcal{U}$.

<u>Thm</u>: Let $X = \operatorname{Spec} A$ be nonempty and $\mathcal{U} = \{(U, i_U : U \hookrightarrow X)\}$ a collection of open embeddings. TFAE:

(i) $\mathcal{U}$ is an open cov.

Spec A is quasicompact (qc).

(ii) $\exists$ finite subcollection $\mathcal{U}' \subseteq \mathcal{U}$ s.t. $\mathcal{U}'$ is a cov.

(iii) Let $x \in \mathrm{Hom}(\operatorname{Spec} k, X)$ for k a field. Then, $\exists\, U \in \mathcal{U}$ s.t. x factors through i_U. $(U = D(I_U))$

(iv) For each $U \in \mathcal{U}$, $U = X \setminus Z_U$ w/ $Z_U = \operatorname{Spec} A/I_U$. Then, $\sum_{U \in \mathcal{U}} I_U = A$.

• $\operatorname{Spec} k[x]/(x)$

Then, $\sum_{U \in \mathcal{U}} I_U = A$.

- Spec $k[x]/(x)$
- Spec $k[x]/(x^2)$

Spec $B = * \iff B$ is a field

Spec$A$ has "nice" coverings!

$f_1 + \cdots + f_n = 1_A \rightsquigarrow \{D(f_1), \ldots, D(f_n)\}$ covering

Basically, every covering of Spec$A$ looks like $\{D(I_1), \ldots, D(I_e)\}$.

Def: $X \in$ Space. A closed pt. of X is a closed emb. Spec $k \hookrightarrow X$. From this we get X°.

Ex: Play around w/ X°.

The points of X, $|X|$, is the set of Spec $k \to X$.

Ex: Given $A \in$ CRing, $|$Spec$A|$ is "equivalent" to the set of prime ideals of A.