Section 13 Math 2202

Line Integrals, Gradient/Conservative Vector Fields and Green's Theorem

1. A conservative/gradient vector field $\mathbf{F}$ is one where $\mathbf{F} = \nabla f$ for some function $f(x, y)$.

   (a) Check that $\mathbf{F}(x, y) = xy^2\mathbf{i} + x^2y\mathbf{j}$ is a conservative vector field.

   (b) Find f such that $\nabla f = \mathbf{F}$.

   (c) Find the value of $\int_C \mathbf{F}\, d\mathbf{r}$ where C is the line between $(-1, 4)$ and $(3, 5)$.
   (Remember the Fundamental Theorem of Calculus for Line Integrals: if $\mathbf{F} = \nabla f$ and is a continuous vector field and C is smooth, then the integral $\int_C \mathbf{F}\, d\mathbf{r} = ...$)

   (d) (Extra) For some extra practice, parameterize the line between $(-1, 4)$ and $(3, 5)$ and compute the line integral without FTC.

2. The following fields are conservative/gradient. Find f such that $\nabla f = \mathbf{F}$.

   (a) $\mathbf{F}(x, y) = (3x^2 - 2y^2)\mathbf{i} + (4xy + 3)\mathbf{j}$

   (b) $\mathbf{F}(x, y) = (xy\cos(xy) + \sin(xy))\mathbf{i} + (x^2\cos(xy))\mathbf{j}$

   (c) $\mathbf{F}(x, y) = 2y^{3/2}\mathbf{i} + 3x\sqrt{y}\mathbf{j}$