Def: Let T, T' be G-topologies on $X \in \mathrm{Set}$.

- T' is <u>finer</u> than T if every T-open is T'-open and T-cover is a T'-cover.
- T' is <u>slightly finer</u> than T if it is finer than T and
  - every T'-~~cove~~ open has T'-cover by T-opens;
  - every T'-cover of T-open can be refined to T-cover.

Point: T' slightly finer than $T \Rightarrow$ every T-sheaf extends uniquely to T'-sheaf. This also does not affect cohom.

Fact: There is a slightly finest G-topology. $T \rightsquigarrow T^*$

- $\mathcal{U} \subseteq T$ is <u>T^*-open</u> if $\exists$ naïve covering $\mathcal{U} = \bigcup_i \mathcal{U}_i$ w/ $\mathcal{U}_i$ T-open s.t. $\forall$ T-open $V \subseteq \mathcal{U}$ the naïve cov. $V = \bigcup_i (V \cap \mathcal{U}_i)$ can be refined to T-cov.
- <u>Naïve cov.</u> of T^*-open $\mathcal{U}$, of form $\mathcal{U} = \bigcup_i \mathcal{U}_i$, is <u>$T^*$-cov.</u> if all $\mathcal{U}_i$ are T^*-open and $\forall$ T-open $V \subseteq \mathcal{U}$, the naïve cov. $V = \bigcup_i (V \cap \mathcal{U}_i)$ can be refined to T-cov.

Def: The <u>weak G-top.</u> on $\mathbb{P}$ is:

- <u>admissible opens</u> are $\mathbb{P}$ and affinoid subsets of $\mathbb{P}$
- <u>admissible cov.</u> of admissible open $\mathcal{U}$ is naïve cov. $\mathcal{U} = \bigcup_i \mathcal{U}_i$ s.t. $\mathcal{U}$ is already covered by finitely many $\mathcal{U}_i$.
- <u>strong G-top.</u> = (weak G-top.)*.

Prop: Admissible opens for strong top. on $\mathbb{P}$ are open sets for canonical (=metric) top. A cov. $\mathcal{U} = \bigcup_{i \in I} \mathcal{U}_i$ of admissible open by admissible opens is <u>admissible</u> iff $\forall$ affinoid $F \subseteq \mathcal{U}$ one can find a finite subset $J \subseteq I$ and $\forall j \in J$ some $F_j \subseteq \mathcal{U}_j$ affinoid s.t. $F \subseteq \bigcup_{j \in J} F_j$ (probably same as equality...).

Ex: In strong (G-)top., $\overbrace{\{z \in \mathbb{P} : |z| < 1\}}^{= D}$ is admissible open w/ admissible covers $\bigcup_{0<r<1} \{z \in \mathbb{P} : |z| < r\}$ ~~or~~ and $\bigcup_{0<r<1} \{z \in \mathbb{P} : |z| \leq r\}$.

Ex: Closed disk $\overline{D} = \{z \in \mathbb{P} : |z| \leq 1\}$ is open for strong top. but $\overline{D} = \{z \in \mathbb{P} : |z| = 1\} \cup D$ is not admissible cov. But, $\{z \in \mathbb{P} : |z| \leq r\} \cup \{z \in \mathbb{P} : |z| \geq r\}$ is admissible $\forall\ 0 < r < 1$.

Def: X set w/ G-top. Seq. of sheaves $0 \to \mathcal{F}_1 \to \mathcal{F}_2 \to \mathcal{F}_3 \to 0$ is exact if

(1) $\forall$ admissible opens $\mathcal{U} \subseteq X$: $0 \to \mathcal{F}_1(\mathcal{U}) \to \mathcal{F}_2(\mathcal{U}) \to \mathcal{F}_3(\mathcal{U})$ is exact;

(2) $\forall$ admissible open $\mathcal{U} \subseteq X$ and $s_3 \in \mathcal{F}_3(\mathcal{U})$ $\exists$ admissible cov. $\mathcal{U} = \bigcup \mathcal{U}_i$ s.t. $\forall i$: $s_3|_{\mathcal{U}_i} \in \text{image}(\mathcal{F}_2(\mathcal{U}_i) \to \mathcal{F}_3(\mathcal{U}_i))$

Warning: This is stronger than exactness on stalks because (2) needs admissible covers.

Čech cohom. works as usual, including getting LES from SES of sheaves.

Def: Sheaf $\mathcal{F}$ on X is loc. acyclic if $\exists$ admissible cov. $X = \bigcup_{i \in I} \mathcal{U}_i$ s.t. $\forall$ admissible open $\mathcal{U}$ contained in some $\mathcal{U}_i$ we have $\check{H}^n(\mathcal{U}, \mathcal{F}|_{\mathcal{U}}) = 0\ \forall n \geq 1$

Prop: Loc. acyclic covers realize the Čech cohom. of X.

Remark: $H = \check{H}$ for all G-topologies in the book!

In the weak top., extend $\mathcal{O}$'s definition by $\mathcal{O}(\mathbb{P}) := k$, $\mathcal{O}(\emptyset) := 0$.

Thm (Tate Acyclicity for $\mathbb{P}$): $\mathcal{U} \mapsto \mathcal{O}(\mathcal{U})$ is sheaf for weak top. Moreover, $\check{H}^n(\mathfrak{U}, \mathcal{O}) = 0\ \forall n \geq 1$ and $\mathfrak{U}$ admissible cov. of some admissible open.

We will prove this next time.