Exam 1

2:00-3:15pm, Wed. Oct. 14, 2020

Upload your exam to gradescope or email it to Kathryn by 3:22pm (7 minutes after the end of the exam). Please keep your Zoom video on during the exam. The exam is "closed book" – you are not allowed to use notes, the textbook, or consult any other resources. Send Kathryn a chat message if you have any questions.

Question 1. Show that if $A \subset \mathbb{R}$ is a bounded set that is a null set for Lebesgue measure, then $A^2 := \{a^2 : a \in A\}$ is also a null set.
Hint: outer measure

Question 2. Let $(X, \mathcal{M}, \mu)$ be a measure space. Fix a measurable function $f : X \to [0, \infty)$ and define the measure ν on $\mathcal{M}$ by

$$\nu(E) = \int_E f d\mu.$$

Prove that for every measurable function $g : X \to \mathbb{R}$,

$$\int_E g \, d\nu = \int_E gf \, d\mu$$

whenever at least one side is defined.
Hint: simple functions

Question 3. Let $(X, \mathcal{M}, \mu)$ be a measure space and let $f : X \to [0, \infty)$ be integrable (i.e. $\int_X f d\mu < \infty$). Prove that for each $\epsilon > 0$, there exists a set E with $\mu(E) < \infty$ such that

$$\int_E f d\mu > \left(\int_X f d\mu \right) - \epsilon.$$