Section 5 Math 2202

Parameterizations, Level Curves and Level Surfaces

1. **Parameterizing Ellipses, Parabolas and Hyperbolas** Find a parameterization for the following curves. Specify the domain of your parameter. In other words, what values must your parameter go through in order to trace out the entire curve.

   - The curve $x^2 + 4y^2 = 9$ in the xy-plane

   - The curve $x - z^2 = 4$ in the xz-plane

   - The curve $x^2 - y^2 = 1$ in the xy-plane
     Hint: Think about the trigonometric identity $\sec^2 t = 1 + \tan^2 t$.

2. **Parameterizing Level Curves**

   (a) Parameterize the level curves of $f(x, y) = x^2 + 4y^2$ with $k = \pm 9, \pm 10$

   (b) Find a parameterization of the part of the curve $x^{2/3}y^{1/3} = 3$ for which $x, y \geq 0$. (Hint: can you rewrite this as a function of one variable?)
   (This is a level curve of a Cobb-Douglas function. These are used in economics[1].)

3. Find a parameterization of the curve of intersection of the surfaces $x^2 + (y+2)^2 + (z-5)^2 = 4$ and $-3x + 4z = 20$.

   First ask yourself: what are these surfaces? what do you expect their intersection to look like?

[1] A Cobb-Douglas function is of the form $P(L, K) = bL^aK^{1-a}$ and gives the total production (the monetary value of goods produced in a year) we get when the labor and capital invested is L and K. Labor, L, is measured in person-hours (the work done by an average worker in one hour) per year and K, the capital invested, is the value of machinery, equipment and buildings. Here a and b are positive constants (representing elasticities and a productivity factor). In this example x is L and y is K, and the constants are $b = 1$ and $a = 2/3$. What would $x^{2/3}y^{1/3} = 3$ mean in this context?

4. **Level Surfaces, or Visualizing Functions Whose Graph is 4 Dimensional**

   To visualize something like $f(x, y, z) = x^2 + y^2 + z^2$, we would need 4 dimensions to plot this in. What we do instead is use level surfaces - we look at $f(x, y, z) = k$ for values of k and plot those in 3 space. This is analogous to using level curves to understand a function of 2 variables. Try it out for the following:

   (a) $f(x, y, z) = x^2 + y^2 + z^2$

   (b) $h(x, y, z) = x + 2y + z$