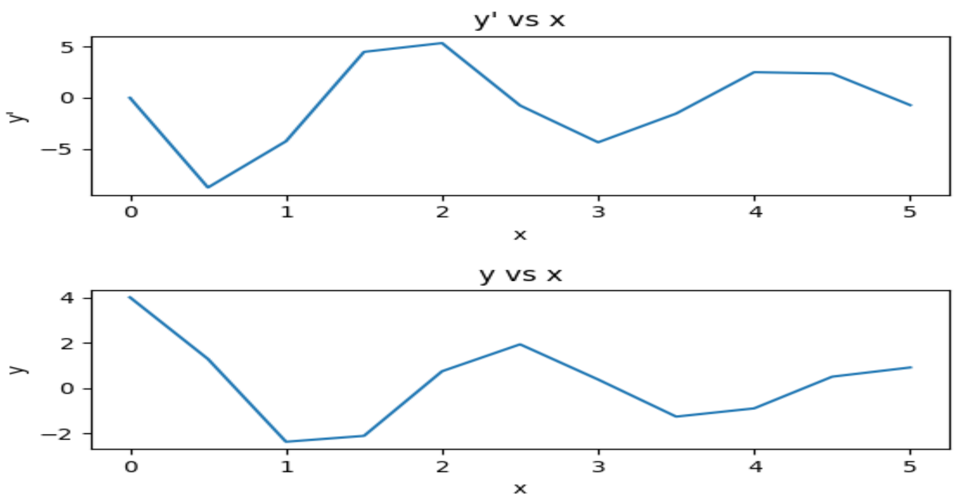

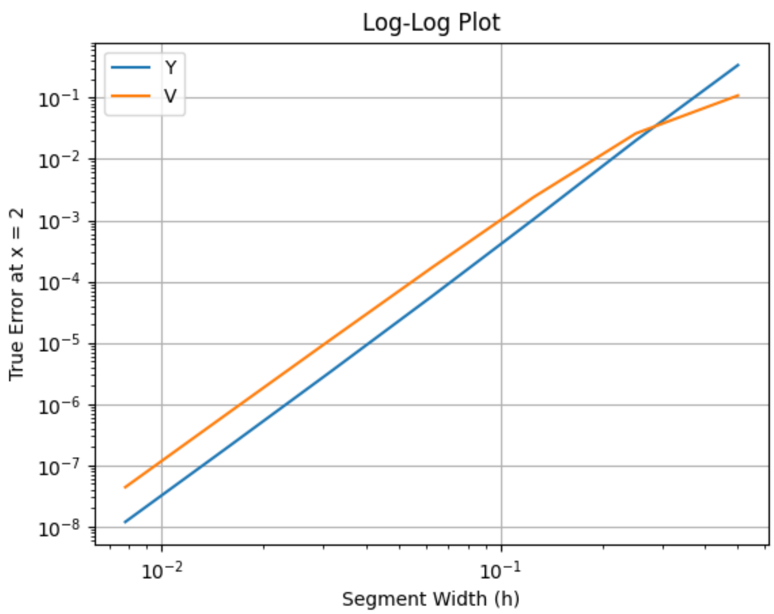

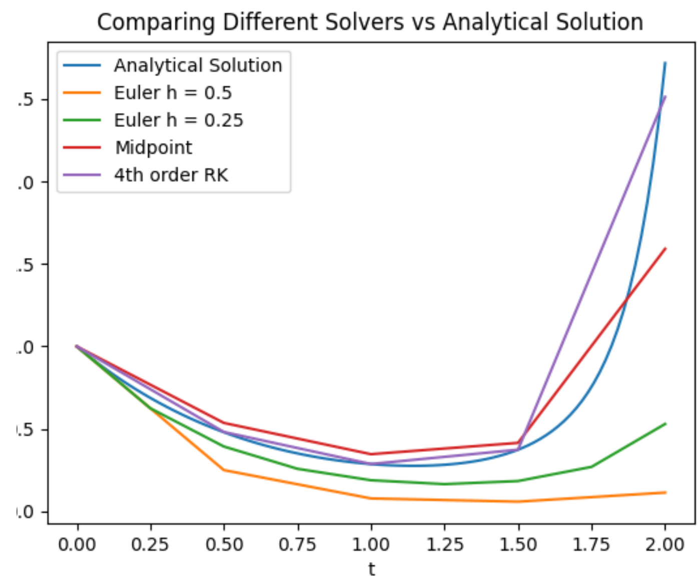

The approximate slope of Y is about 4, while the approximate slope of V is roughly the same. These both converge at about the same quadratic rate of O(h^4). This makes sense because between the two, not much changed other than the drop off past a certain extent.

COLLABORATION STATEMENT:

The extent of my collaboration is as follows: No one.

Task 1:

$t = 0 \to 2$; $y(0) = 1$; $y_0 = 1$

b) Eulers w/ $h = .5$; $x = t$ to match notes lol

$x_{i+1} = x_i + h$

t	y
0	1
.5	.25

$x_1 = x_0 + h = .5$ bc $x_0 = 0$

$\phi_0 = y_0 t^3 - 1.5 y_0 = 0 - 1.5(1) = -1.5$

$y_1 = y_0 + \phi_0 h = 1 + .5 \cdot -1.5 = .25$

$y_1 = .25$

Eulers w/ $h = .25$ $t_1 = t_0 + h = .25$

$\phi_0 = y_0 t_0^3 - 1.5 y_0 = 0 - 1.5 \cdot 1 = -1.5$

$y_1 = 1 - 1.5 \cdot .25 = .625$

t	y
0	1
.25	.625

c) Midpoint w/ $h=.5$ $t_1=.5$

$y_{0+1/2} = y_0 + f(t_0,y_0)(h/2) = 1+(-1.5)(.25)$

$= .625$

$\phi_{0+1/2} = f(t_{1/2}, y_{1/2})$

$\phi_{1/2} = .625(.25)^3 - 1.5(.625) = -.9277$

$y_1 = y_0 + \phi_{0+1/2}h = .536$

t	y
0	1
.5	.5361

d) RK 4th order w/ $h=.5$

$y_{i+1} = y_i + \frac{1}{6}(k_1+2k_2+2k_3+k_4)h$

$k_1 = f(t_0,y_0) = -1.5$; $k_2 = f(t_0+\frac{1}{2}h, y_0+\frac{1}{2}k_1h) = f(.25,.625) = -.928$

$k_3 = f(t_0+\frac{1}{2}h, y_0+\frac{1}{2}k_2h) = f(.25,.768) = -1.141$

$k_4 = f(t_0+h, y_0+k_3h) = f(.5,.4299) = -.5912$

$y_1 = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)h = .4811$

t	y
0	1
.5	.4811

Task 2:

$$\frac{d^2y}{dx^2} = -2x^2y + 6x^2 + 3y \,;\; y(0)=0 \,;\; y'(0)=1 \,;\; x=0 \to 1 \;\; h=1/3$$

$y_{i+1} = y_i + \phi_i h$

$$\phi_i = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1})}{2}$$

$y_{i+1} = y_i + f(x_i, y_i)h$

$x_1 = 1/3 \,,\; y_1^\circ = 0$

$\phi_0 = \frac{1}{2}(f(x_0^\circ, y_0^\circ) + f(x_1, y_1^\circ)) = \frac{1}{2}\left(0 + 0 + 6\left(\frac{1}{3}\right)^2 + 0\right)$

$= 1/3$

$y_1 = y_0 + \phi_0 h = 0 + \frac{1}{3}(1/3) = 1/9$

$x_2 = 1/3 + 1/3 = 2/3$ wow!

$y_2^\circ = y_1 + f(x_1, y_1)h = .4362$

$\phi_1 = 1/2\,(f(x_1, y_1) + f(x_2, y_2^\circ)) = 2.2814$

$y_2 = y_1 + \phi_1 h = 1/9 + 2.2814(1/3) = .872$

$x_3 = 2/3 + 1/3 = 1$ shocker

$y_3^\circ = y_2 + f(x_2, y_2)h = .8715 + [-2(2/3)^2(.872) + 6(2/3)^2 + 3(.872)] \cdot 1/3$

$y_3^\circ = 2.374$

$\phi_3 = \frac{1}{2}(f(x_2, y_2) + f(x_3, y_3^\circ)) = .5[(-2(2/3)^2(.872) + 6(2/3)^2 + 3(.872) + (-2)(2.374)$
$+6 + 3(2.374)]$

$\phi_3 = 6.44$

$y_3 = y_2 + \phi_3 h = 3.02$

x	y
0	0
1/3	1/9
2/3	.872
1	3.02