Extra Credit

Task 1:

$t = 0 \to 2 \; ; \; y(0) = 1 \; ; \; y_0 = 1$

b) Eulers w/ $h = .5$; $x = t$ to match notes lol

$x_{i+1} = x_i + h$

$x_1 = x_0 + h = .5$ bc $x_0 = 0$

$\phi_0 = y_0 t^3 - 1.5 y_0 = 0 - 1.5(1) = -1.5$

$y_1 = y_0 + \phi_0 h = 1 + .5 \cdot -1.5 = .25$

t	y
0	1
.5	.25

$y_1 = .25$

Eulers w/ $h = .25$ $t_1 = t_0 + h = .25$

$\phi_0 = y_0 t_0^3 - 1.5 y_0 = 0 - 1.5 \cdot 1 = -1.5$

$y_1 = 1 - 1.5 \cdot .25 = .625$

t	y
0	1
.25	.625

c) Midpoint w/ $h = .5$ $t_1 = .5$

$y_{0 + 1/2} = y_0 + f(t_0, y_0)(h/2) = 1 + (-1.5)(.25)$
$= .625$

$\phi_{0 + 1/2} = f(t_{1/2}, y_{1/2})$

$\phi_{1/2} = .625(.25)^3 - 1.5(.625) = -.9277$

$y_1 = y_0 + \phi_{0 + 1/2} h = .536$

t	y
0	1
.5	.5361

d) RK 4th order w/ $h = .5$

$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$

$k_1 = f(t_0, y_0) = -1.5 \; ; \; k_2 = f(t_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1 h) = f(.25, .625) = -.928$

$k_3 = f(t_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2 h) = f(.25, .768) = -1.141$

$k_4 = f(t_0 + h, y_0 + k_3 h) = f(.5, .4299) = -.5912$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h = .4811$

t	y
0	1
.5	.4811

Task 2:

$\frac{d^2 y}{dx^2} = -2x^2 y + 6x^2 + 3y \; ; \; y(0) = 0 \; ; \; y'(0) = 1 \; ; \; x = 0 \to 1 \; h = \frac{1}{3}$

$y_{i+1} = y_i + \phi_i h$

$\phi_i = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1})}{2}$

$y_{i+1} = y_i + f(x_i, y_i)h$

$x_1 = 1/3 \; , \; y_1^\circ = 0$

$\phi_0 = \frac{1}{2}(f(x_0, y_0) + f(x_1, y_1^\circ)) = \frac{1}{2}\left(0 + 0 + 6\left(\frac{1}{3}\right)^2 + 0\right)$
$= 1/3$

$y_1 = y_0 + \phi_0 h = 0 + \frac{1}{3}(\frac{1}{3}) = 1/9$

$x_2 = 1/3 + 1/3 = 2/3$ wow!

$y_2^\circ = y_1 + f(x_1, y_1)h = .4362$

$\phi_1 = \frac{1}{2}(f(x_1, y_1) + f(x_2, y_2^\circ)) = 2.2814$

$y_2 = y_1 + \phi_1 h = 1/9 + 2.2814(1/3) = .872$

$y_2^* = y_1 + f(x_1, y_1)h = .4362$

$\phi_1 = \frac{1}{2}(f(x_1, y_1) + f(x_2, y_2^*)) = 2.2814$

$y_2 = y_1 + \phi_1 h = \frac{1}{9} + 2.2814(\frac{1}{3}) = .872$

$x_3 = \frac{2}{3} + \frac{1}{3} = 1$ shucker

$y_3^0 = y_2 + f(x_2, y_2)h = .8715 + [-2(\frac{2}{3})^2(.872) + 6(\frac{2}{3})^2 + 3(.872)]\cdot\frac{1}{3}$

$y_3^0 = 2.374$

$\phi_3 = \frac{1}{2}(f(x_2, y_2) + f(x_3, y_3^0)) = .5[(-2(\frac{2}{3})^2(.872) + 6(\frac{2}{3})^2 + 3(.872) + (-2)(2.374)$
$\qquad\qquad\qquad\qquad\qquad + 6 + 3(2 .374)]$

$\phi_3 = 6.44$

$y_3 = y_2 + \phi_3 h = 3.02$

X	Y
0	0
$\frac{1}{3}$	$\frac{1}{9}$
$\frac{2}{3}$	.872
1	3.02