Lab 2 – Stellar Parallax

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ABSTRACT

One of the most effective method astronomers have to measure the distance to stars in our night sky is through the measuring the star's parallax. By taking a set of two pictures from the same perspective exactly six months apart, we are able to observe and measure the apparent movement of stars across the night sky. Comparing this movement to a plate scale created by a known distance in the picture (such as the angular separation of two others stars), we can then estimate the parallax angle of, and hence the distance to, the star of interest. We did this for 10 pairs of pictures, or data sets, and compared this to the official data gathered by the Hipparcos satellite. We found that our data is generally in good agreement with the Hipparcos data, with only two measured distances deviating from the official data by more than its error. When using our distance from the first data set and combining it with our colleague's measured values, we found that as a whole we were able to closely estimate the value given by Hipparcos. This shows the effectiveness of this technique for calculating distance to stars, as we were able to ascertain both accurate and precise values with a relatively crude experimental procedure.

1. Introduction

Astronomers use a wide variety of methods to determine the distances of astronomical objects. These methods are arranged in something called the cosmic distance ladder, in which methods of determining distances of close objects help determine distances of farther objects. Similar to a real ladder, the success of higher rungs of the cosmic distance ladder depend on the stability of the lower rungs. In astronomy, one of the lowest rungs, and therefore, one of the most fundamental methods, is stellar parallax.

Parallax is the apparent movement of an object due to the angle of observation. Typically, astronomers use the Earth's position at opposite points in its orbit as the two observation points, as observed in Figure 1 below.

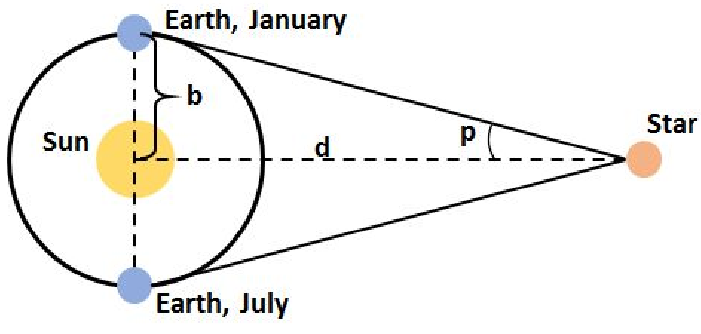


Figure 1: Stellar Parallax from Earth's Orbit

By knowing the size of the Earth's orbit and measuring the angle the star appears to move relative to 'stationary' background stars (that is, stars that do not visibly move,) one can calculate the distance to a star *d* by using simple geometry:

$$d = \frac{b}{tan(p)}$$

where *b* is the observer baseline (half the distance between observation points) and *p* is the parallax angle (half of the observed angular distance traveled), as illustrated in Figure 1. The greater the parallax angle, the closer an object is to the observer.

Though parallax can theoretically be measured from any two points in space, astronomers often use the Earth's orbit about the Sun as a baseline, which results in a particular form of the above equation:

$$d\ [pc] = \frac{1\ [au]}{p\ [arcsec]}$$

In this lab, we measured observed arcsecond parallax of stars in 10 image pairs by measuring screen space millimeter change and converting via plate scale, as derived from angular separation given by Hipparcos between constant "calibration stars" as validation.

2. Methodology

The data we used for our analysis consisted of 10 pairs of images of separate starfields, 8 of which show actual parallax and 2 in which parallax was artificially created. In each image, we identified the star that was a good candidate for a parallax measurement as the star that showed significant difference in location between images. We assumed that each set of images was taken 6 months apart to create the maximum Earth-observed parallax effect, that the two image captures were of the same stars, and that the angular separation given alongside each data set was accurate.

The first step in the analysis of each pair was to find the plate scale of the images. In order to do this, we measured the physical pixel separation in millimeters of the two "calibration stars" and derived a plate scale by comparing this measurement, with uncertainty given by the radius of scattering area of each star, to the angular separation given by Hipparcos.

After obtaining the plate scales of each image set, we measured the apparent distance the star in question moved between the two images using the same methodology as used to measure the distance between the calibration stars. The apparent movement that we measured is illustrated in Figures 2 and 3.

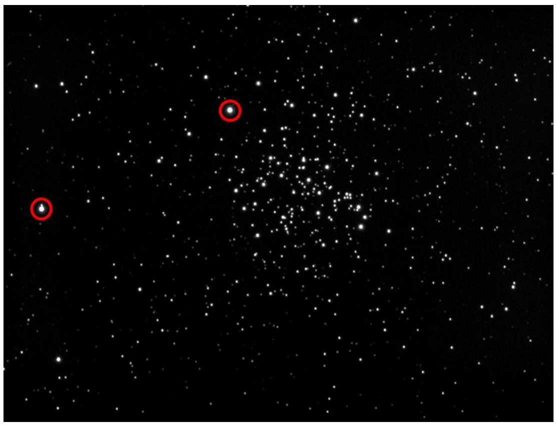


Figure 2: Starting image of data set 1. Calibration stars are circled in red.

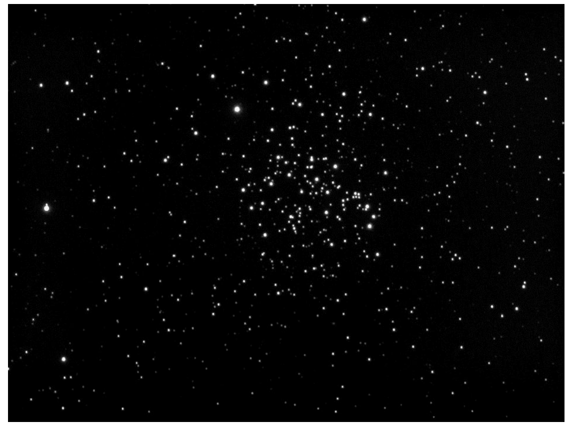


Figure 3: Ending image of data set 1 with parallax motion

With the plate scale and apparent distance traveled by each star, we could then determine the parallax angle by dividing the apparent distance traveled by the plate scale to convert to an angle. Finally, we were able to calculate the distance to the star, using the equation for Earth-orbit observed parallax defined in the introduction.

This process was repeated for all 10 data sets. Additionally, we combined the entire class's data for Star 1 by compiling each group's measurements of dataset 1.

There were issues with some of the datasets. For instance, dataset 2 had no visible calibration stars, forcing us to use the radii of the indicating circles themselves as the uncertainty. For dataset 5, the scattering area of the measured stars were large and not circular, making the uncertainty for these sets even higher than just their radii.

3. Analysis

The most significant source of uncertainty in this lab arose from the areas of scattering of each star in the images; these prevented us from obtaining the true positions of each star without introducing uncertainty equal to the full radius of the scattering area. This uncertainty changed between datasets as exposure times differed, but rarely changed within datasets. We chose to make the uncertainty in our measurement between these two "calibration" stars as the average of their radii. The stars in both data sets were of larger radius than precision of the ruler, making the error it introduces small enough to ignore.

In order to determine the uncertainty of the plate scale σ_s, we used the uncertainty of the scattering area, and propagated the error using the equation derived from the error propagation formula

$$\sigma_s = s(\sigma_c / c)$$

where s is the plate scale itself, σ_c is the error introduced by the scattering area, and c is the distance we measured between the calibration stars.

Similarly, we found the uncertainty in the calculated parallax angle σ_p by using using the same scattering area method as above to find the uncertainty of the apparent motion of the parallaxed star and adding in quadrature the relative uncertainties of it and the plate scale, then dividing by 2:

$$\sigma_p = p\sqrt{\left(\tfrac{\sigma_a}{a}\right)^2 + \left(\tfrac{\sigma_s}{s}\right)^2}$$

Where $p = as/2$ is the parallax angle, a and σ_a is the apparent separation and its uncertainty in mm, and s and σ_s is the plate scale and its uncertainty.

Finally, we propagated the uncertainty of the object distance σ_d by using using the error propagation formula on the simple equation d [pc] = 1[AU]/p[arcsec] given in the introduction*:*

$$\sigma_d = d\left|\tfrac{\sigma_p}{p}\right|$$

The values measured from each pair of images are recorded in Table 1, and the calculated values of parallax angle and distance in Table 2. The difference between our distances and those of the Hipparcos data is shown in a residual plot in Figure 4.

For the collective class data, the distance to Star 1 was $d_{avg} = 4.6 \pm 1.04\, pc$. Class measurements for Star 1 are shown in Figure 5. For this class-wide data set, we determined the error of the average object distance for Star 1 by summing everyone's absolute error as determined by their estimation of instrumental error, and dividing by the number of entries, as shown in the equation

$$\sigma_{d_{avg}} = \frac{\sum_{i=1}^{N} \sigma_{di}}{N}$$

Table 1: Measured Values

Data Set	Plate Scale s (arcsec/mm)	Plate Scale Error σ_s (arcsec/mm)	Apparent Separation a (mm)	Apparent Separation error σ_a (mm)
1	0.03529411765	0.000830449827	13	1
2	0.00487804878	0.0005948839976	37	2
3	0.00243902439	0.00005948839976	16	4
4	0.03921568627	0.000768935025	26	2
5	0.04132231405	0.007683901373	15	8
6	0.005063291139	0.0002884153181	40	3
7	0.06315789474	0.001662049861	23	1
8	0.00218579235	0.00007166532294	37	2
9	0.06349206349	0.006550768456	13	5
10	0.003225806452	0.00005202913632	34	2

Table 2: Calculated Values

Data Set	Parallax Angle p (arcsec)	Parallax Angle Error σ_p (arcsec)	Object Distance d (pc)	Object Distance Error σ_d (pc)
1	0.2294117647	0.01845416666	4.358974359	0.3506412995
2	0.09024390244	0.01203798885	11.08108108	1.47814896
3	0.01951219512	0.004901208786	51.25	12.8733312
4	0.5098039216	0.04046965741	1.961538462	0.1557123949
5	0.3099173554	0.1750476217	3.226666667	1.822486918
6	0.1012658228	0.009537107627	9.875	0.9300170109
7	0.7263157895	0.03691285149	1.376811594	0.06997237654
8	0.04043715847	0.002556453854	24.72972973	1.563423724
9	0.4126984127	0.16434208	2.423076923	0.9649019461
10	0.05483870968	0.003344870585	18.23529412	1.112256274

Figure 4: Residuals with Hipparcos Data

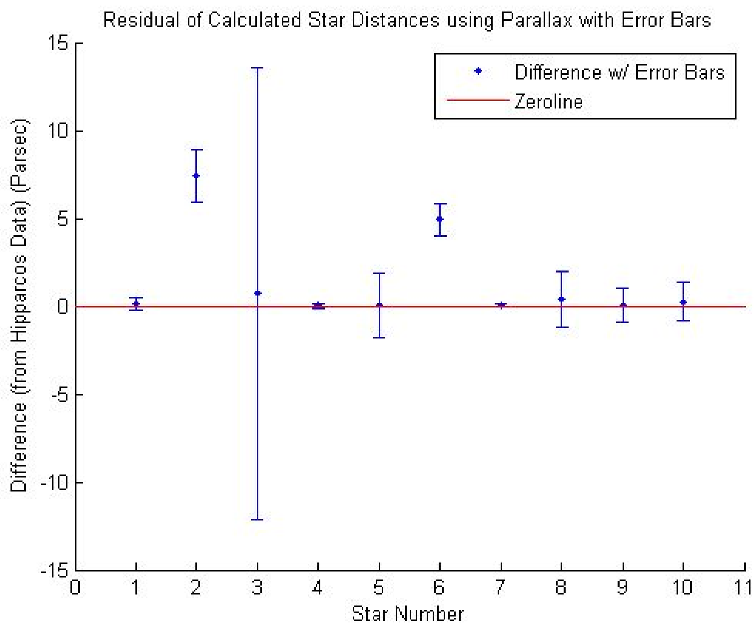
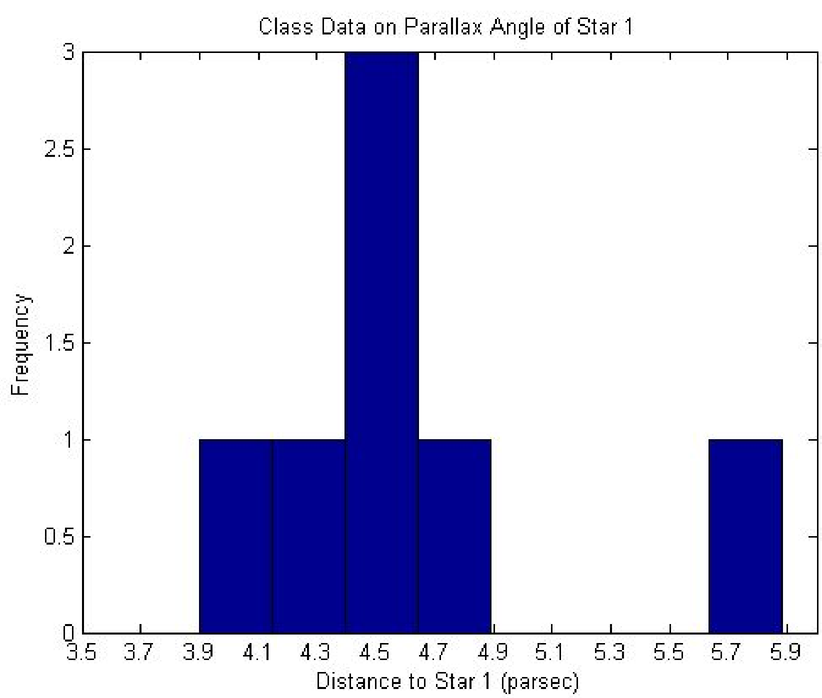


Figure 5: Class Measurements of Star 1 Distance



4. Discussion

In general, our calculated distances were close to the Hipparcos data. The most accurate measurement was of Star 4, with a distance of 1.96 parsecs and an uncertainty of 0.2 parsecs. The most precise measurement was of Star 7, with a distance of 1.37 parsecs and an uncertainty of 0.07 parsecs. The least accurate and least precise measurements were Star 2 and Star 3, respectively, with distances of 11.1 +/- 1.5 parsecs, and 51.25 +/- 13 parsecs. These measurements seem plausible for the actual distances.

Our class measurement of Star 1's distance was $d_{avg} = 4.6 \pm 1.04\ pc$. This agrees with the given value for this star, $d_{hipp} = 4.514pc$. To improve the uncertainty of this measurement, we could regulate the method by which the class collects their data. This way everyone will be able to use the method that generates the smallest amount of uncertainty, and it will in turn reduce the uncertainty in the average measurement..

The biggest contribution to the discrepancy between our data and the Hipparcos data is the method of measurement. To minimize this error in the future, we could use pixel tools instead of a ruler.

We encountered two notable instances of strange data, namely in Dataset 2 where there were no visible calibration stars and in Dataset 3 were the uncertainty severely outclassed the measurement.

Our results appear to support the data from the Hipparcos satellite, assuming the calibration stars are accurately placed. We found that, on average, the datasets and our measurements correlated positively with the data given by Hipparcos. From looking at the residual, only 2 of the 10 datasets disagree with the Hipparcos data, and while the measured uncertainty was quite large, the class was able to synthesis their data to get an effective measurement for the distance to the star in data set 1.

5. Appendix

The following function, calc_parallax was used in matlab to perform calculations on the data in this lab. We input an array of our data for each parameter in the function

<u>Inputs:</u>

```
a = [13, 37, 16, 26, 15, 40, 23, 37, 13, 34];

sigma_a = [1, 2, 4, 2, 8, 3, 1, 2, 5, 2];

s = [0.03529, 0.004878, 0.002439, 0.03922, 0.04132, 0.00506, 0.06316, 0.002186, 0.06352, 0.003225];

sigma_s = [0.00083, 0.00059, 0.00006, 0.00077, 0.0077, 0.00029, 0.00166, 0.00007, 0.00656, 0.00005];
```

<u>calc_parallax</u>

```
function [ p, sigma_p, d, sigma_d ] = calc_parallax(a, sigma_a, s, sigma_s )

%This function takes an array of apparent distances, the attributed plate

%scale s, and their uncertainties, and calculates the parallax angle

   p = (1/2).*(a.*s);

   sigma_p = p.*sqrt((sigma_a./a).^2+(sigma_s./s).^2);

   d = 1./p;

   sigma_d = d.*sqrt((sigma_p./p).^2);

end
```