ASTR 121 – Spring 2016
Lab 5 – HI Rotation Curve of the Milky Way

Important dates:

- Prelab due: Monday, 4/10
- Rough draft due: Monday, 4/17
- Final draft due: Friday, 4/21
- **Science Goals:**

At the end of this lab, you should be able to…

- Determine the tangent velocity of HI clouds using 21 cm data
- Determine the orbital speed and orbital radius of the clouds via the tangent point method
- Construct a rotation curve of the galaxy based on these measurements

MATLAB Goals:

In this lab, you will apply MATLAB knowledge to…

- Plot data with x and y error bars
- Fit experimental data and determine information from a fit

Background

The most abundant element in the universe is hydrogen, making up 75% of baryonic matter by mass. We can observe the signature of hydrogen in the spectra of objects such as stars, where a bound electron moving between energy levels in the hydrogen atom can emit or absorb radiation at characteristic wavelengths. Not all hydrogen atoms, however, are at high-enough temperatures to emit radiation consistently in the visible spectrum; neutral hydrogen atoms in the cold interstellar medium generally remain in or near the ground electronic state. Fortunately, a quantum mechanical particle property called *spin* enables us to observe hydrogen at this lowest energy.

Elementary particles possess an intrinsic form of angular momentum called spin. Though particles are not physically rotating about an axis as a planet does, experiments show that, analogously to planets, they have some form of angular momentum not accounted for by orbital angular momentum alone. Like all forms of angular momentum, spin is a vector and therefore has a direction. In the case of baryons (protons and neutrons) and electrons, there are two spin states: spin-up and spin-down. Because neutral hydrogen consists of one proton and one electron, the two particles can either have spin in the same direction (*parallel*; i.e., both spin-up or both spin-down) or in opposite directions (*anti-parallel*). The energy carried by an atom in the parallel spin configuration is greater than that of an atom in the anti-parallel configuration. Therefore, when the spin state flips from parallel to anti-parallel, energy is released in the form of a low-energy photon with a wavelength of 21 cm, in the radio part of the spectrum. The corresponding photon frequency, which we will be using in this lab, is ~1420 MHz.

Unlike optical observations of the Milky Way, 21-cm radiation is not impeded by interstellar dust, and can therefore be detected anywhere in our galaxy. For this reason, measurements of HI (neutral hydrogen) are used extensively in radio astronomy. In this lab, we will use spectra of HI clouds to create a rotation curve for the Milky Way. This is done by measuring the orbital speed of HI clouds at varying distances from the galactic center.

Theory

If one assumes that neutral hydrogen is uniformly distributed throughout our galaxy, then a 21-cm line should be detectable from all points along a given line of sight. All of these spectra, however, will have different Doppler shifts (due to the clouds having different velocities with respect to the observer), which is the basis from which we can find orbital speeds. The tangent point method allows one to determine the shift that corresponds to a distance from the galactic center. In
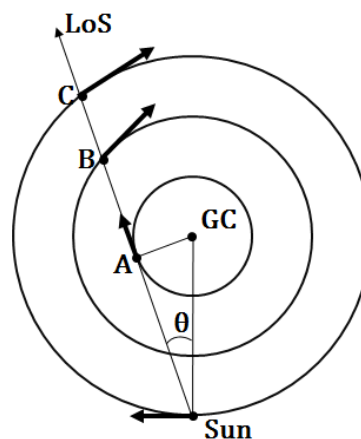

Figure 1: Tangent Point Method

Figure 1, the line of sight (LoS) from the Sun at a galactic longitude θ intersects clouds A, B, and C.

Our line of sight is tangent to the orbit of cloud A, so its orbital speed is equal to its radial velocity (the velocity along our line of sight). The orbits of clouds B and C, however, are not tangent to our line of sight, and so we can only measure a component of their orbital velocities. Thus, for this particular line of sight, cloud A will have the largest measured radial speed. Because we are looking at the first quadrant (i.e., $0° < \theta < 90°$), the galaxy is rotating away from us, thus the cloud with the largest relative speed will also have the largest redshift. Because a redshifted spectral line will have a lower frequency than its rest frequency, the speed of the tangent point cloud v_{tan} can be found by taking the lowest frequency detected and using the redshift equation:

$$v_{tan} = \frac{c \times (f_{rest} - f_{obs})}{f_{obs}}$$

where $f_{rest} = 1420.406 \, MHz$, f_{obs} is the measured (lowest) frequency, and c is the speed of light. In order to construct a rotation curve, we will measure the tangent-point speeds of a set of clouds at varying galactic longitudes, all of which would represent motion purely along our line of sight. In other words, we will only measure the radial speeds of galactic clouds like cloud A.

We must remember, however, that the Sun is also orbiting around the galaxy, and, furthermore, the Earth is rotating on its axis as well as orbiting about the Sun. These motions introduce extra signal to the redshift detected from the galactic HI clouds. The motions of the Earth are dependent on the time and location of observation, so this has already been accounted for in your data. The Sun's orbital speed about the galactic center, how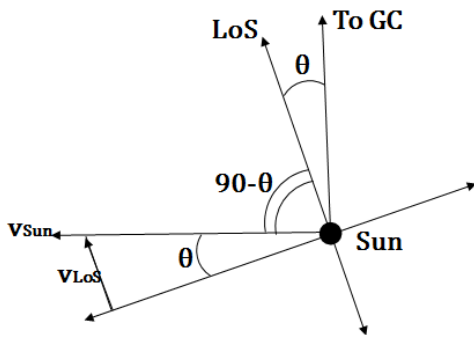ever, is not yet corrected for. The component of this motion included in a measurement of redshift depends upon the line of sight. In Figure 2, we see that the line-of-sight component v_{LoS} of the Sun's orbital speed v_{Sun} is dependent on the galactic longitude and is given by the expression:

Figure 2: Component of Sun's orbital motion along line of sight

$$v_{LoS} = v_{Sun} \times \sin(\theta).$$

Since we are moving towards the clouds we are measuring, the relative speed between us and each cloud is measured to be smaller than its true line-of-sight speed"; you must add the line of sight speed of the Sun to the measured radial speed of a cloud in order to get the full orbital speed $v_{orbital}$ of a cloud:

$$v_{orbital} = v_{tan} + v_{LoS}.$$

To construct the rotation curve of the galaxy, one more piece of information is needed: the distance of the tangent point cloud from the galactic center. Note that in Figure 1, the triangle between the Sun, cloud A, and the galactic center must be a right triangle, with a 90-degree angle at cloud A. Therefore, from simple trigonometry, cloud A's distance r from the galactic center is determined from its galactic longitude and the Sun's distance from the Galactic center r_{Sun} by the equation:

$$r = r_{Sun} \times \sin(\theta)$$

Once r and $v_{orbital}$ are obtained, you can construct a rotation curve by plotting the latter against the former.

About the Data

No external data is necessary in the first part of the lab; you will be constructing purely theoretical rotation curves. The data used in the second part of the lab consists of 13 spectra taken at different galactic longitudes. Each data file contains two columns: the first, frequency (rather than wavelength) in MHz; the second, brightness temperature (a measurement similar to flux or intensity, often used in radio astronomy) in Kelvin. The galactic longitude at which each spectrum was taken is given by the file name, e.g., 'lon17.dat' is a spectrum at a galactic longitude of 17 degrees. Assume the values of galactic longitude have an uncertainty of one degree.

Part 1: Exploring Rotation Curves

In this part of the lab, you will construct rotation curves for different mass distributions. You will use the equation for orbital speed v around an enclosed mass M_r at an orbital radius r,

$$v = \sqrt{\frac{G M_r}{r}},$$

where G is the gravitational constant.

1. Consider a system in which almost all the mass is concentrated in a point at the center, resulting in a constant value of M_r. Construct a rotation curve for this system by plotting orbital speed as a function of orbital radius. To set a reasonable scale, use a value of M_r equal to the total mass of the Milky Way and take r to vary on kiloparsec scales.

2. *Plot a similar curve for a system where enclosed mass is proportional to orbital radius, $M_r \propto r$. Use similar scales as for the previous model and show them together.*

3. Add a third plot showing a distribution where enclosed mass is proportional to volume, $M_r \propto r^3$. *What real systems can be represented by each of these rotation curves?*

4. Assume that the Milky Way can be modeled as a rigid sphere out to a radius of 2 kpc, with a total mass of 10^{12} $M_\odot$, and that there is negligible mass outside this central region. Plot a theoretical rotation curve for the galaxy with these assumptions, showing orbital speeds out to an orbital radius of 8 kpc.

Part 2: Constructing a Rotation Curve of the Milky Way

Now we will look at real data: given spectra of HI lines at varying galactic longitudes, you will create a rotation curve of the Milky Way.

1. Load and plot the first spectrum at a galactic longitude of $17°$. Considering the axes, the range of frequencies plotted, and features present, *how is this spectrum different from those we have looked at in previous labs?* Include this figure in your report.

2. The frequency you will record for each galactic longitude will be the lowest frequency for which there is a signal. For this spectrum, determine where the beginning of the signal is, and assign an uncertainty to this frequency based on how precisely you can determine the beginning. *Why are we using the lowest frequency to represent this spectral line, even though the line is present over a range of frequencies?*

3. Record the frequency of the beginning of the signal (hint: you may want to make use of the 'ginput' command). Repeat this for all 13 galactic longitudes, making sure to keep track of which lowest frequency corresponds to which galactic longitude. *Is it important that you keep track of brightness temperature as well? Why or why not?*

4. The frequencies you've measured are observed frequencies that have been Doppler shifted. From these frequencies, calculate the corresponding tangent-point speeds. Then, for each speed and its corresponding galactic longitude, apply the correction for the motion of the Sun to calculate the orbital speed. (Note: MATLAB assumes angles in radians).

5. Using the uncertainty you attributed to the minimum signal frequency, calculate the uncertainty of the tangent-point speeds and orbital speeds using error propagation. Include a table showing tangent-point and orbital speeds for each galactic longitude, as well as the uncertainties of all values.

6. For each of the 13 spectra, calculate the distance of the tangent-point cloud from the galactic center, as well as the uncertainty in the value. Add these to your table.

7. Add a fourteenth data point to your set representing the Sun's orbital speed and radius.

8. You now have all of the necessary data to construct a rotation curve for the Milky Way. In a new figure, plot this curve, including both x and y error bars. Include this figure in your report. (Note: Though MATLAB doesn't have an x error bar function, you can try searching for and downloading user-created functions online. Besides writing your own functions, using user-created functions allows you to expand MATLAB's functionality.)

9. To compare your data to central mass model Milky Way, overlay your model for a central-mass-dominated Milky Way from Part 1 onto your data. *How do the two curves compare? What does the shape of your data imply?*

10. Fit your data and y uncertainties to a line and use this fit to determine the enclosed mass at a radius of 4 kpc. Plot both the fit line and this point on your rotation curve, making sure to distinguish them from the measured data. Determine the uncertainty of the enclosed mass value.

Report

Your report should contain the following:

1. *Cover page*: Follow the model and guidelines in the rubric.

2. *Abstract:* In a paragraph or two, summarize the scientific purpose of the lab, what was done, the results, and your conclusions.

3. *Introduction:* In several paragraphs, discuss the background information regarding the purpose of the lab and its goals.

4. *Methodology:* Describe the data you used in this lab. Explain how you created your models for Part 1. Include rotation curves for the three different mass distributions, as well as a separate figure showing the two component model. For Part 2, describe the measurements you made in order to construct the rotation curve.

5. *Analysis:* Explain the calculations used on your measured data to find the values and uncertainties needed in your rotation curve. Include a table with values of galactic longitudes, tangent-point speeds, orbital speeds, orbital radii, and uncertainties for all values. Additionally, include a figure showing your data, best-fit line, interpolated point, and comparison curve.

6. *Discussion:* Compare your expectation to your results. Discuss the difference between the two-component model made for the Milky Way in Part 1 and what your data shows. What are the implications of the rotation curve from the data? To what extent can we interpolate data (as you did in Part 2, question 10) from this data? Make sure to also answer any questions in this handout (in italics).

7. *Appendix:* Include all scripts, functions, and data that aren't already represented in your report.

Remember to upload any MATLAB code as .m files on ELMS, along with a PDF of your lab report. When uploading a .m file that will run on its own, replace any interactive commands (e.g., 'ginput') with variables containing the values that you got from the command. If you have a lot of variables from interactive commands, put their values directly in the script. If you have only a few variables, you might investigate the 'save' and 'load' commands.

Credits: Ideas and background material for Lab 5 are based on a project devised by the MIT Haystack Observatory and available at:
http://www.haystack.mit.edu/edu/undergrad/srt/SRT%20Projects/index.html