Testing the Arbitrage Pricing Theory Versus the Capital Asset Pricing Model Across Investment Categories

Zach Johnson ECON610 Term Paper

Introduction

The motivation for a method to accurately predict the future returns of capital markets is obvious. If a trader could, with consistency, select a portfolio which returns more than the average market return, then they would be rewarded generously by clients as a financier and would make large gains in their personal portfolio.

In this paper, I aim to test the power of The Arbitrage Pricing Theory(Ross 1976). The APT states that the returns of assets and portfolios of assets can be modeled using macroeconomic factors, versus the Capital Asset Pricing Model(Sharpe 1964; Litner 1965; Black 1972), CAPM, which posits that market-asset covariance is the driver of returns. Emulating the model build by Chen, Nai-Fu, Roll, and Ross (1986), it is analyzed if the CAPM of APT better describes the stock returns of various investment sectors. The Identification of sector-specific responses would allow an investor to focus investment there while maintaining the benefits of portfolio diversification.

It is found find that the Industrial sector follows a CAPM model. Information Technology, Communications, Health Care, Consumer Staples, and Utilities weakly follow the APT model. Consumer Discretionary, Financial, Energy, and Materials semi-strongly follow the APT model. Real Estate is the sector that is best modeled with the APT. The identification of sectors that have differing pricing responses allows an investor to maintain benefits of diversification, when compared to more volatile single-stock investments.

Literature Review

Modern Portfolio Theory and The Capital Asset Pricing Model

Humans are inherently risk averse(Werner 2008). If given a chance to flip a coin for the same monetary loss or gain, most would choose not to flip the coin; an equal loss hurts more than an equal gain. Using this presupposition, Harry Markowitz (1959) pioneered Modern Portfolio Theory (MPT). His suggestion was that a portfolio with lower variation of returns was better than one with higher variation, given the two portfolios had equal returns overall or equal predicted returns. This discounted returns for a portfolio by how much risk the investor was taking on. Then, the theory posits, the investor must be compensated for taking on increased risk through increased returns. The idea that risk is a driving factor in portfolio selection was the seminal contribution to MPT and has been the groundwork for many risk-based theories developed since.

Since 1959, many have iterated on MPT and attempted to create more sophisticated models of risk. The most well-known example would be the Capital Asset Pricing Model, which was developed in the 1960s by William Sharpe (1964), John Litner (1965), and Fischer Black (1972).

The focus of each of their works was to determine which parts of a securities risk could be eliminated through diversification. In theory, an investor would only have to analyze the aggregate risk changes brought by adding a security to a portfolio. A single addition, in a well-diversified portfolio, would increase risk infinitesimally. This is the concept of diversification in a portfolio. The risk that was unavoidable was called systematic risk. The CAPM assigns each security in a portfolio a Beta, β, that represents its variance with respect to the market, the "systematic risk". A security with a β above 1 represents a security that is more volatile than the market. Mathematically:

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)} \qquad (1)$$

For the i^{th} asset in a portfolio, where r_m is market returns and r_i is the return of the i^{th} asset.

The CAPM then predicts returns of the i^{th} asset to be modelled as:

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f] \qquad (2)$$

The risk-free rate of return(i.e., government bond) is denoted r_f. The market risk premium, $[E(r_m) - r_f]$, allows a portfolio/securities' returns to be adjusted for risk. The driver of returns is the assets relative volatility, β.

Arbitrage Pricing Theory

The Arbitrage Pricing Theory was first introduced by Stephen Ross (1976). It extends the CAPM to incorporate the systematic effects into n factors to be chosen by the modeler. The model is represented mathematically as:

$$E(r_i) = r_f + \beta_{i1}\delta_1 + \beta_{i2}\delta_2 + \cdots + \beta_{in}\delta_n + \epsilon_i \qquad (3)$$

$E(r_i)$ is the expected return for the i^{th} asset, r_f is the risk-free return rate, δ_n is the n^{th} factor's risk premium, β_{in} is the factor loading specific to the n^{th} factor and i^{th} asset, and ϵ_i is the error term. Each asset i receives its own set of n β's specific to that asset and to the k^{th}factor δ.

The model is intuitive when considering certain factors. Stock prices seem to respond to the news. When inflation goes up, investors grow wary of the future and pull investment, dropping stock prices. When the Federal Reserve announces an interest rate hike, markets may respond negatively.

Choice of Factors

An analysis of the CAPM is relatively simple as each asset only has one parameter to be calculated, a single β. The APT is a model wherein the modeler decides which factors to include is their analysis. It follows that the *correct* choice of factors has been a source of debate for the past five decades. Ross (1976) said that the factors can be grouped into firm-specific and non-firm-specific, "systematic" factors. Examples of the former would be cash flows, price-earnings ratio, industry, and company age. The latter would generally include macroeconomic indicators such as interest rates, inflation, and GDP changes. This analysis focuses exclusively on systematic factors.

For guidance on choosing groups of factors, I turn to a 1986 paper in the Journal of Business by Chen, Nai-Fu, Roll, and Ross (1986). They posit that because stock prices, p, can be written as $p = \frac{E(c)}{k}$, where $E(c)$ is expected future dividend stream, and k is the discount rate, then actual returns in any period can be written as:

$$\frac{dp}{p} + \frac{c}{p} = \frac{d[E(c)]}{E(c)} - \frac{dk}{k} + \frac{c}{p} \qquad (6)$$

The factors that influence returns will be factors that change the discount factor k or expected cash flows $E(c)$. Using this logic, their paper proposes six non-market index factors that would likely influence future returns.

1. Industrial Production – Changes in industrial production are likely to change the price of an asset. If more production than had previously been expected takes place, the asset's price may rise if the information had not already been priced into the market.
2. Inflation – Changes in inflation change the discount factor and may cause investors to divest from assets as the true return has now been discounted.
3. Risk Premia – The difference between corporate bond portfolio returns and long-term government bond returns. This can be thought of as a measure of the degree of risk aversion implicit in pricing.
4. The Term Structure – Unanticipated return on long term bonds. It is measured as the difference between the long-term bond yield and all other bond yields.
5. Consumption – Changes in consumption are signals of consumer expectations that may influence investors decisions to invest.
6. Oil prices – Changes in oil prices can be a good signal as to future expenditures given the importance of energy production on business and everyday life.

In addition, there is a market index factor, the returns of a market index, that act as the control variable in the model.

7. Market Indices – Changes in market indices are a factor that this paper included because it could capture most of the systematic factors priced in by the markets that the macroeconomic variables were unable to catch. This will "absorb" the systematic impacts not caused by the other factors. Used as a control, essentially.

The analysis by Chen, Nai-Fu, Roll, and Ross (1986) ran analyses including the market index and analyses not including the market index on a set of portfolios. The objective was to test the CAPM and APT. The CAPM would imply that adding the market index to the analysis, obtaining the beta estimate, would be sufficient to explain price movements in the portfolio and the macroeconomic variables would be insignificant. The APT would suggest that the macroeconomic variables had explanatory power to explain why the prices of the portfolio changed, i.e., significant regression coefficients.

They find that the macroeconomic variables consistently have significant effects on the pricing of the portfolio when not including the market index, and that the inclusion of the market index causes a large drop in significance of the macroeconomic variables, while some still are significant. For them, in most time periods, the CAPM was the model that worked for explaining variability of portfolio returns, but market indices were not significant for predicting future returns(Chen, Nai-Fu, Roll, and Ross 1986). In this paper, I aim to perform an analysis like theirs, but only to analyze which model, APT or CAPM, fits the modelling of returns across investment sectors.

Data

Macroeconomic Data(Exogenous variables)

All the macroeconomic data were pulled from the Federal Reserve Economic Data (FRED) website using the Pandas web data reader library in Python, unless otherwise mentioned. All data are time-series, quarterly sampled, differenced data. Variables that are not rates were logged. Aliases, descriptions of data, and original data sources are listed below in table 1.

Table 1

Data Series Aliases, Descriptions, and Source

Alias	*Description*	*Source*
dIP	Logged Quarterly Changes in Industrial Production: Total Index	Board of Governors of Federal Reserve System
dEI	Quarterly Changes in Expected 1-year Inflation: 1-Year Expected Inflation	Federal Reserve Bank of Cleveland
dUPR	Quarterly Changes in Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity	Federal Reserve Bank of Atlanta Federal Reserve Bank of St. Louis
dUI	Quarterly Changes in Unexpected Inflation: Inflation(t) – Expected Inflation(t-12)	Federal Reserve Bank of Atlanta
dUTS	Quarterly Changes in Long-term bond returns in period t - total bond returns in period t-1	Organization for Economic Cooperation and Development, Board of Governors of the Federal Reserve System
dPC	Logged Quarterly Changes in Personal Consumption Expenditures	U.S. Bureau of Economic Analysis
dOP	Logged Quarterly Changes in Crude Oil Prices	U.S. Energy Information Administration
dSP500	Logged Quarterly Changes in S&P500 Price	Yahoo Finance

After review, it was decided that changes in personal consumption would be omitted from the model. It has a high correlation with industrial production(0.8). This raises the issue of multicollinearity, and the two measures would likely be explaining too much in our model. Figure 1 shows the trends of each of the pre-differenced, non-derived data series of exogenous variables. Note that IP and OP's y—axis are on the right side, with the other variables y-axis being on the left. Note: All graphs were made using Python's Matplotlib and Seaborn packages.

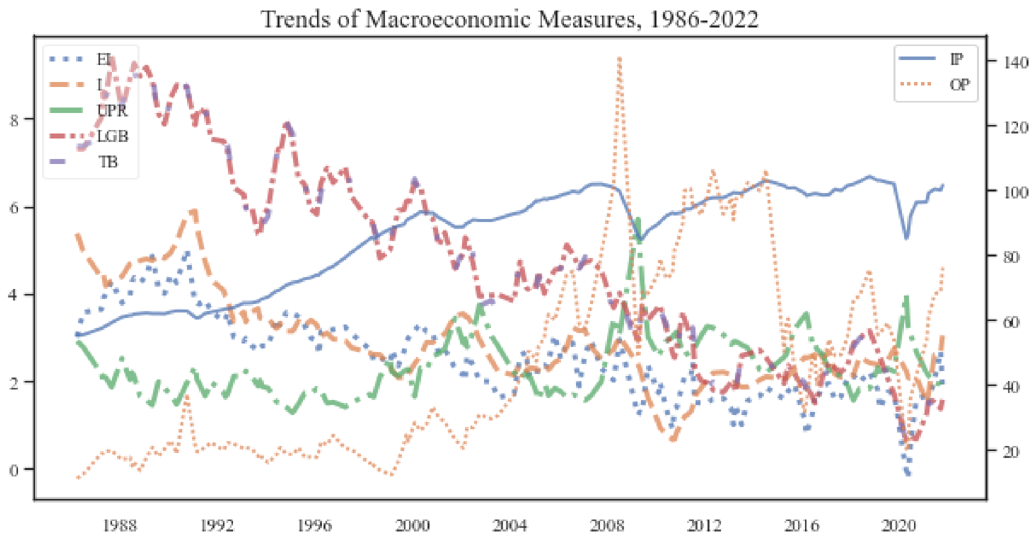

Figure 1: Macro Trends

The exogenous variables to be used in the regression models were derived from these series as described above. The correlation matrix for the differenced variables is shown below in table 2.

Table 2

Correlation Matrix for Exogenous Variables

	dIP	*dEI*	*dUPR*	*dUI*	*dUTS*	*dOP*	*dSP500*
dIP	1	0.503	-0.606	0.132	0.071	0.6	0.556
dEI	0.503	1	-0.52	0.195	0.11	0.589	0.29
dUPR	-0.606	-0.52	1	-0.034	-0.215	-0.513	-0.624
dUI	0.132	0.195	-0.034	1	0.185	0.15	-0.034
dUTS	0.071	0.11	-0.215	0.185	1	0.144	-0.111
dOP	0.6	0.589	-0.513	0.15	0.144	1	0.328
dSP500	0.556	0.29	-0.624	-0.034	-0.111	0.328	1

The correlations that do exist are mostly expected. For instance, industrial production is correlated with changes in expected inflation positively as this may increase investments in the macroeconomy, it is correlated negatively with changes in the risk premia, which may cause disinvestment, and is positively correlated with changes in the sp500 as investors invest more into production when it is increasing. None of the correlations are too high or concerning.

Table 3 shows the descriptive data for each of the exogenous variables.

Table 3

Data Descriptions of Exogenous Variables

Statistic	dIP	dEI	dUPR	dUI	dUTS	dOP	dSP500
count	148	148	148	148	148	148	148
mean	0.0041	-0.0079	-0.0072	-0.0156	0.001	0.013	0.0197
std	0.0232	0.4379	0.4199	0.2366	0.2319	0.2064	0.0785
min	-0.1806	-1.775	-1.25	-0.64	-0.88	-1.0684	-0.4601
25%	-0.0002	-0.1915	-0.16	-0.1298	-0.16	-0.062	-0.0047
50%	0.0051	0.0252	-0.04	-0.0003	-0.01	0.0331	0.0237
75%	0.0114	0.1995	0.11	0.1051	0.1325	0.1068	0.0593
max	0.0609	1.0863	2.64	0.8799	0.59	0.6814	0.2471

There were 148 months of data, as the oil data is only available starting in 1986. This was not cause for concern. The goal of this analysis is to apply the methodology to the "modern economy" – so only having data from after the paper by Chen, Nai-Fu, Roll, and Ross (1986) was a nice coincidence. The descriptive data are easy to interpret. The mean shows how much these variables have changed, on average, quarterly, since 1986. For instance, industrial production(dIP) has averaged an approximately $(100 - [e^{0.004} - 1]) = 4.01\%$ increase each quarter since 1986. Compounded to a year, this is approximately a $1.04^4 - 1$ = 16.98% increase per year. The standard deviations show the spread of the quarterly returns since 1986; industrial production varies the least – it increases at the most consistent rate and expected inflation has varied the most. The min, max, and 25%, 50%,75% percentiles show the datapoints appearing at the respective quantiles in the data. Figure 2 shows the distributions of all the macro variables except dIP, which made the other variables hard to see due to its relatively small standard deviation. The data all appear normally distributed.

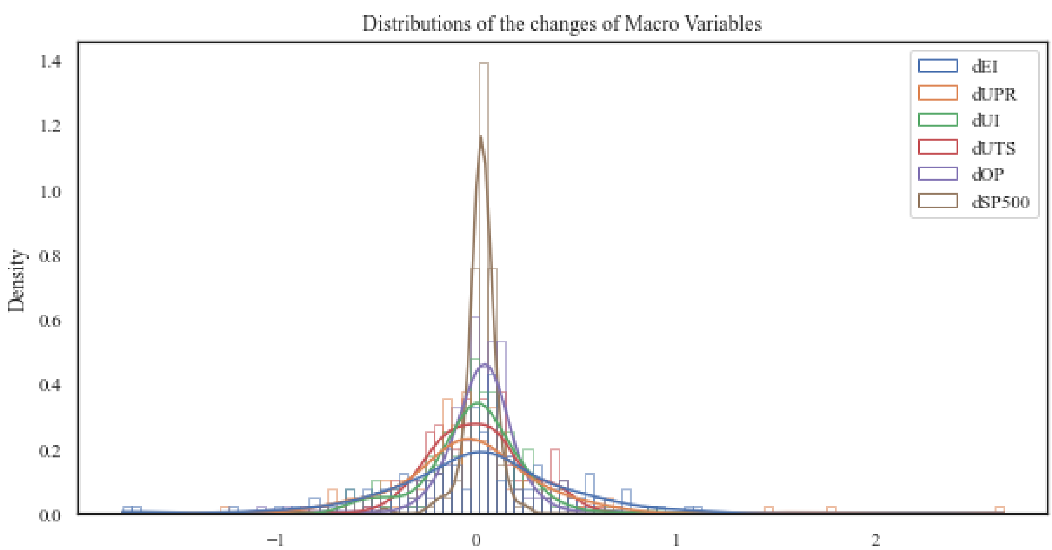

Figure 2: Distributions of Changes in Macro Trends

Stock Data (Segmented by Sector)

The complete list of S&P500 constituents was pulled from TopForeignStocks.com and was up to date as of November 15th, 2022(Top Foreign Stocks 2022). The dataset contained the name of the company, stock ticker, and sector. There are 510 stocks in the index, with 12 possible sectors: Information Technology(IT), Consumer Discretionary(CD), Communication Services(Com), Financials(Fin), Health Care(HC), Energy(E), Consumer Staples(CS), Utilities(U), Industrials(Ind), Materials(Mat), Real Estate(RE), and Unassigned. Only one stock was in the Unassigned sector, so it was not included in the analysis. The data were scraped from Yahoo Finance using Python and averaged out by sector. Below in table 4 are the number of stocks from the S&P500 in each sector.

Table 4

Counts of Each Sector in the S&P500

	IT	*CD*	*Com*	*Fin*	*HC*	*E*	*CS*	*U*	*Ind*	*Mat*	*RE*
count	76	58	26	66	65	21	33	29	71	28	31

Table 5 is the summaries of the log-relative quarterly changes for each investment sector of the S&P500 from 1986 to 2022. *Count*, in this case, is the number of months that had data that could be averaged, the same as the macro-variables. The data are quarterly to index-match the exogenous variables, many of which are delivered quarterly.

Table 5

Summaries of S&P500 Quarterly Changes by Sector, 1986-2022

	IT	*CD*	*Com*	*Fin*	*HC*	*E*	*CS*	*U*	*Ind*	*Mat*	*RE*
count	148	148	148	148	148	148	148	148	148	148	148
mean	0.037	0.033	0.036	0.028	0.033	0.024	0.029	0.023	0.029	0.026	0.029
std	0.137	0.119	0.108	0.106	0.088	0.154	0.066	0.074	0.096	0.098	0.101
min	-0.446	-0.476	-0.49	-0.634	-0.353	-0.859	-0.234	-0.368	-0.483	-0.535	-0.703
25%	-0.016	-0.01	-0.007	-0.008	-0.015	-0.024	-0.003	-0.011	-0.009	-0.014	-0.007
50%	0.036	0.035	0.041	0.026	0.033	0.041	0.028	0.029	0.03	0.029	0.036
75%	0.101	0.093	0.087	0.088	0.087	0.101	0.06	0.063	0.086	0.081	0.078
max	0.515	0.392	0.406	0.335	0.263	0.353	0.25	0.292	0.233	0.281	0.316

The mean here is interesting as, for instance, we can see that IT has done the best in terms of returns since 1986. The IT sector has averaged approximately a $100 * [e^{0.037} - 1] = 3.77\%$ average quarterly increase in stock prices. This, compounded out to an entire year is roughly a $100 * [1 - (1 + 0.0377)^4] = 15.95\%$ increase.

The histograms of each sector display obvious normality with their slight positive mean, shown in figure 3 below. This is a better representation of the data than a line graph, which shows sporadic movements of little value.

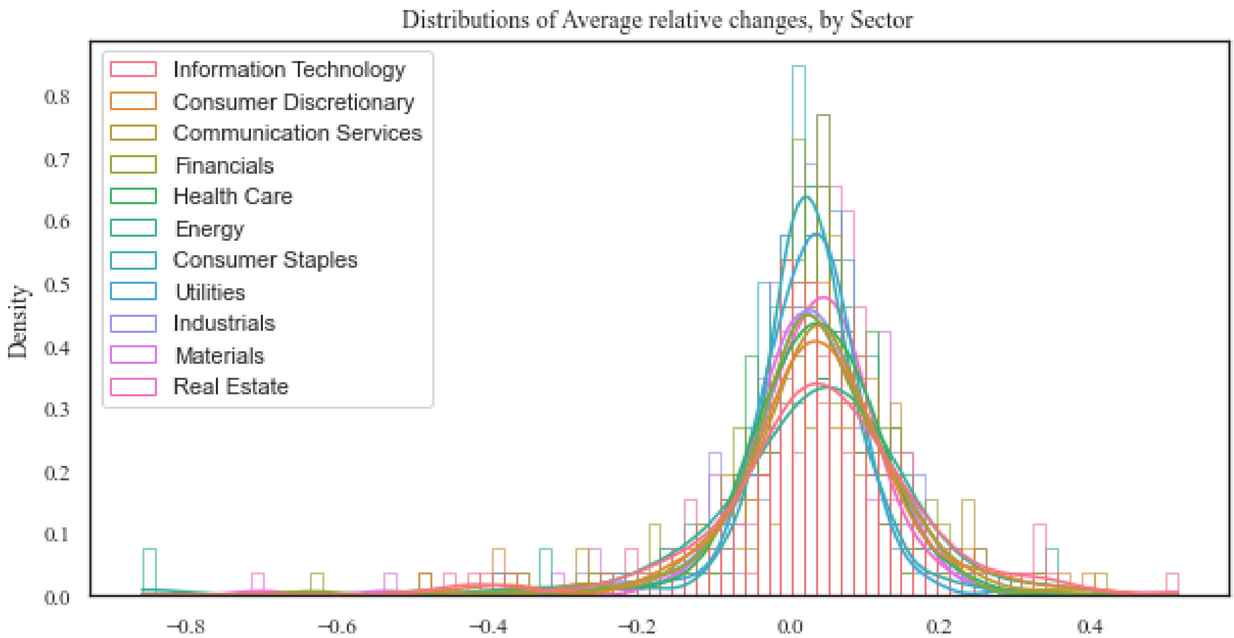

Figure 3: Distributions of quarterly log-relative changes across sector for the S&P500, 1986-2022

Figure 4 shows the non-differenced average price of stocks across each sector in a line graph. This is a crude measurement as stock price does not inform us of market cap, but this shows the obvious upward trend.

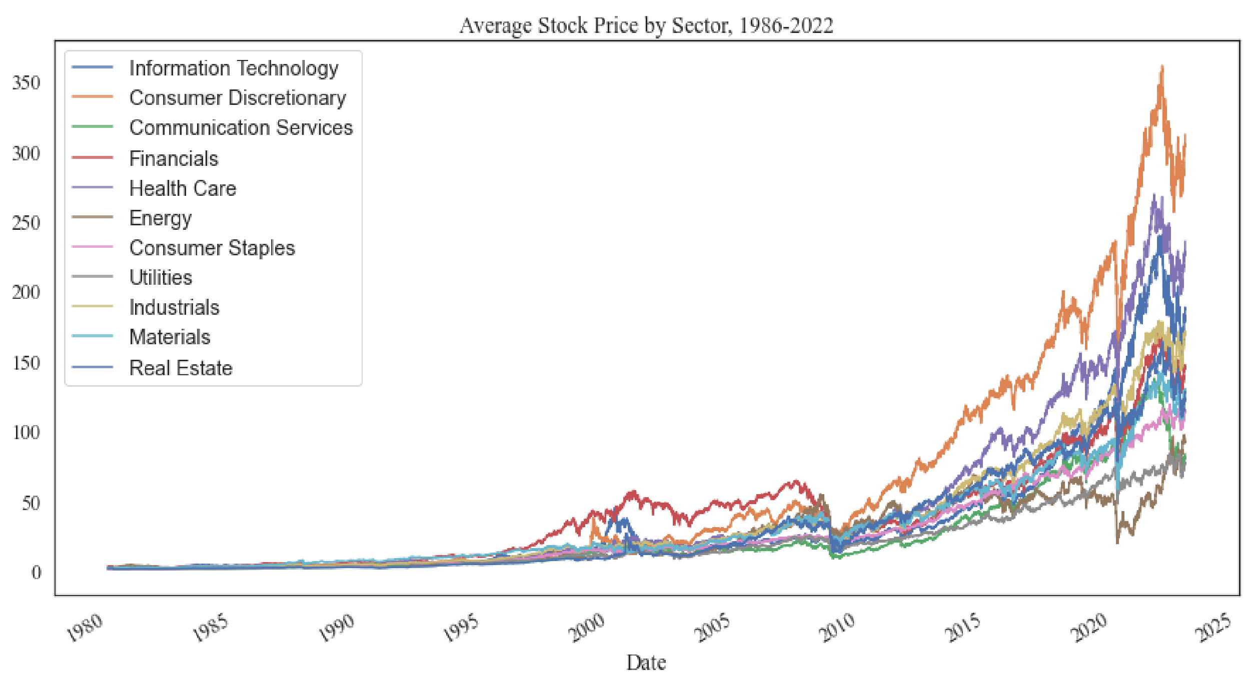

Figure 4: Trends in stock prices by sector

Results

Individual Variable Significance

Table 6 displays the parameter values obtained by regressing each sectors data on each exogenous variable. The dSP500 coefficient is the coefficient is the β parameter described earlier in the CAPM. Hence, sectors with higher dSP500 coefficients are more reactive to the market environment. Sectors that only have a significant dSP500 coefficient are better modeled by the CAPM, sectors with significant macroeconomic factor coefficients are modeled by the APT.

Table 6

Regression Coefficients for Each Exogenous Variable, by Sector

Variable	***IT***	***CD***	***Com***	***Fin***	***HC***	***E***	***CS***	***U***	***Ind***	***Mat***	***RE***
const	0.011	0.009	0.013	0.005	0.016	0.001	0.015	0.011	0.009	0.009	0.01
	(0.007)	(0.007)	(0.005)	(0.005)	(0.004)	(0.007)	(0.004)	(0.005)	(0.004)	(0.005)	(0.005)
dIP	-0.905*	1.333	0.293	0.907**	-0.747**	0.663	0.219	0.831*	0.275	-0.137	1.315**
	(0.378)	(0.793)	(0.364)	(0.312)	(0.231)	(0.744)	(0.336)	(0.35)	(0.262)	(0.313)	(0.335)
dEI	0.00	-.031*	-0.033*	-0.026*	0.007	-0.051*	-0.003	-0.014	-0.012	-0.025	-0.04**
	(0.016)	(0.014)	(0.013)	(0.013)	(0.012)	(0.024)	(0.011)	(0.015)	(0.012)	(0.013)	(0.015)
dUPR	-0.035	-0.036	-0.017	-0.031	0.008	-0.063	0.019	-0.015	-0.035	-0.054**	-0.06*
	(0.032)	(0.027)	(0.022)	(0.017)	(0.014)	(0.035)	(0.016)	(0.02)	(0.018)	(0.02)	(0.023)
dUI	-0.014	-0.036	-0.007	0.028	0.007	0.041	0.026	0.03	0.015	0.006	0.033
	(0.03)	(0.029)	(0.018)	(0.023)	(0.018)	(0.027)	(0.018)	(0.021)	(0.017)	(0.022)	(0.022)
dUTS	-0.01	-0.018	-0.046	0.004	0.007	-0.016	-0.011	-0.031	-0.009	-0.019	-0.005
	(0.032)	(0.033)	(0.024)	(0.021)	(0.019)	(0.034)	(0.017)	(0.023)	(0.02)	(0.023)	(0.024)
dOP	-0.044	-.151**	0.006	-0.047	-0.014	0.371**	-0.063*	-0.024	-0.031	0.087*	-0.034
	(0.042)	(0.047)	(0.032)	(0.033)	(0.039)	(0.051)	(0.027)	(0.033)	(0.026)	(0.034)	(0.043)
dSP500	1.498**	1.01**	1.087**	0.992**	1.072**	0.765**	0.7**	0.441**	0.956**	0.835**	0.664**
	(0.142)	(0.151)	(0.107)	(0.081)	(0.063)	(0.167)	(0.093)	(0.126)	(0.072)	(0.095)	(0.104)

Note: each column represents one independent model, which includes all sector variables listed regressed on the sector

*Note: ** denotes p< 0.05, * denotes p<0.01, Robust Standard Errors are reported in parentheses*

The coefficients can be interpreted as relative changes. For instance, the real estate coefficients would suggest that a 1 percent change in expected inflation would result in a 1.315 percent change in the returns of real estate stocks, and a 1 percent increase in unexpected inflation would result in a 0.6% decrease in the returns of real estate stocks, and none of the other factors are significant. Summary data for each sector's model are shown in table 7.

Table 7

Selected Statistics from Regressions of Macroeconomic Variables on Each Investment Sector

Statistic	***IT***	***CD***	***Com***	***Fin***	***HC***	***E***	***CS***	***U***	***Ind***	***Mat***	***RE***
R^2	0.702	0.679	0.734	0.77	0.708	0.733	0.594	0.44	0.79	0.735	0.678
Ad R^2	0.687	0.663	0.721	0.759	0.693	0.719	0.573	0.412	0.78	0.721	0.662
F-Value	32.319	20.885	62.121	58.088	66.57	30.471	18.869	14.159	74.552	50.025	15.965
P(F)	0	0	0	0	0	0	0	0	0	0	0
Durbin-Watson	1.903	1.784	2.061	1.695	1.898	2.059	1.78	2.121	1.812	1.768	1.819
Sample Size	148	148	148	148	148	148	148	148	148	148	148

These statistics provide valuable information about each model. The R-squared value shows how much of the variance in a sectors average quarterly returns were explained by the exogenous variables in the model. Most sectors have a value of around 0.7, which is quite good for financial data. The notably lower R-squared is from the Utilities sector. This may suggest that Utility stocks follow different pricing mechanisms than other sectors. The F-values are all very high, suggesting that the model's exogenous variables are jointly significant at explaining changes in quarterly sector average price changes. The Durbin-Watson test statistic is meant to detect autocorrelation, but its results can often be ambiguous and a better method of detecting autocorrelation is employed in the next section.

Assumptions, Limitations, and Tests

The time series OLS model requires a few conditions to be a legitimate tool for inference and estimation. The assumptions, and their implications for the models described in this paper, are as follows:

1. Linear in Parameters, exogenous vars are stationary and weakly dependent.
   The Dickey-fuller test was run on each model to test for stationarity and trend. The null hypothesis of the test is that there is a unit root, and the alternative is that there is not. An augmented Dickey-Fuller test was run on each of our exogenous variables, and the null hypothesis was rejected for each, implying that the exogenous variables in this analysis are stationary, satisfying the first requirement. Results are shown below in table 8.

Table 8

Augmented Dickey-Fuller Test for Unit Root Results

	dIP	***dEI***	***dUPR***	***dUI***	***dUTS***	***dOP***	***dSP500***
ADF Stat	-10.4545	-7.3252	-7.5669	-9.3685	-6.9869	-10.5661	-7.4152
P(ADF)	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001

Note: Null Hypothesis is that there is a unit root; low p-value implies absence of unit root

2. No perfect collinearity – This requirement is satisfied for our data, shown in table 2.
3. Zero conditional mean – errors are exogenous
   All the exogenous variables, in every model, have a correlation with the residuals of < 0.0001. Additionally, the residuals display no skew in their distribution and have a mean of 0, shown in figure 5.

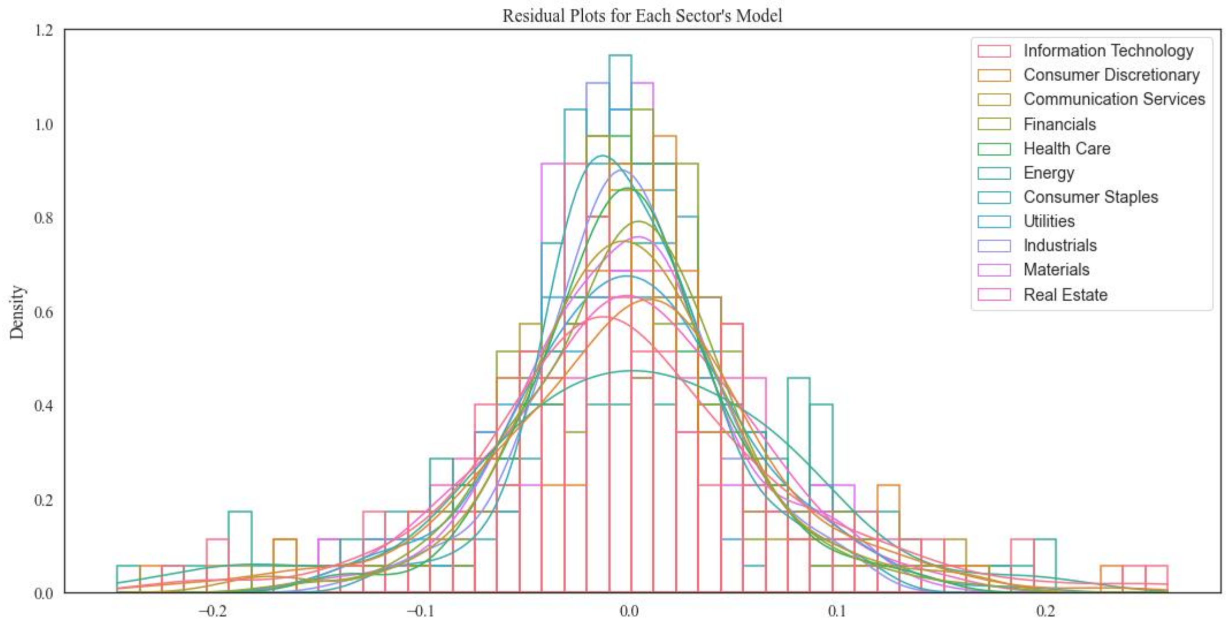

Figure 5: residual plots for each sector's model

4. The error terms are contemporaneously homoscedastic
   A Breusch-Pagan test was performed on each sector's residuals, and most of the sector's models shows results that are insignificant at the 5% level, implying that the condition holds. The sectors of consumer discretionary and consumer staples had significant results. These are shown in table 9.

Table 9

Results of Breusch-Pagan Test on Errors of Regression for Each Sector

Statistic	***IT***	***CD***	***Com***	***Fin***	***HC***	***E***	***CS***	***U***	***Ind***	***Mat***	***RE***
LM	7.824	16.439	11.491	4.741	5.02	6.641	14.076	11.728	6.865	10.115	11.346
P(LM)	0.348	0.021	0.119	0.692	0.658	0.467	0.05	0.11	0.443	0.182	0.124
F	1.116	2.499	1.684	0.662	0.702	0.94	2.102	1.721	0.973	1.467	1.661
P(F)	0.356	0.019	0.118	0.704	0.67	0.478	0.047	0.109	0.454	0.184	0.124

5. No serial correlation
   Because of the two significant results in requirement 4, Durbin-Watson is not advised for testing this requirement. Instead, for each model of each sector the residuals from the model were regressed on all the exogenous variables from the model and the lagged residuals from the model. The p-values of the parameter values obtained from these regressions are shown in table 10. P-values of <0.05 would suggest that the model suffers from autocorrelation. However, all p-values here with respect to the lagged residuals are > 0.05, so this requirement is satisfied.

Table 10

Autocorrelation Test: P-values from Regression of Exogenous Variables and lagged Residuals on Residuals for Each Model

	IT	***CD***	***Com***	***Fin***	***HC***	***E***	***CS***	***U***	***Ind***	***Mat***	***RE***
const	0.9932	0.9581	0.9907	0.9466	0.9926	0.9864	0.9924	0.9812	0.9503	0.9694	0.9964
dIP	0.9646	0.83	0.9871	0.9368	0.9792	0.9156	0.9741	0.9496	0.994	0.9668	0.9569
dEI	0.953	0.8211	0.9526	0.9346	0.9927	0.9999	0.9921	0.925	0.8834	0.951	0.9432
dUPR	0.9974	0.9626	0.9724	0.9313	0.9846	0.9941	0.969	0.9925	0.8687	0.9711	0.9759
dUI	0.9793	0.9976	0.9978	0.9218	0.961	0.9923	0.9459	0.9291	0.9439	0.9829	0.9614
dUTS	0.9698	0.884	0.9631	0.997	0.9745	0.9939	0.9245	0.931	0.8685	0.8939	0.9804
dOP	0.929	0.7577	0.963	0.9907	0.9996	0.9642	0.9652	0.974	0.9038	0.865	0.9342
dSP500	0.9429	0.9174	0.9999	0.8073	0.9963	0.9849	0.9768	0.9091	0.8192	0.9027	0.9789
u(t-1)	0.6321	0.1816	0.6599	0.0669	0.5558	0.6853	0.2998	0.3805	0.2716	0.1796	0.3424

Note: u(t-1) is the lagged residuals and p-values < 0.05 would suggest the model suffers from autocorrelation

Discussion and Conclusion

The results shown in table 6 demonstrate that each sector of the S&P500 is significantly influenced by the returns of the whole S&P500. The CAPM would suggest that the regression coefficient obtained for the S&P500, β, would be the *only* significant one as no other variables would add any more information. However, there is only one sector where this was the case: Industrials; its particular β determines average returns. Using the significant coefficients in table 6, its returns can be modeled to take the form:

$$r_{Ind}(t) = 0.009 + 0.956dSP5009(t) + u_t \qquad (7)$$

The interpretation equation 7, and all other equations that could be constructed (but weren't because they would use too much paper space), is that a 1% change in the price of the S&P500 in period t will result in a 0.956% average change in the average price of stocks in the Industrial Sector. The 10 remaining sectors each had at least one significant factor other than their market β that significantly estimated its average returns.

The degree to which the sector followed an APT model can be described by how many significant factors its model produced. Sectors with one significant variable are weakly APT modeled sectors, ones with two significant factors are semi-strongly APT modeled sectors, and ones with three or more significant factors are strongly APT modeled sectors.

Information Technology, Communications, Health Care, Consumer Staples, and Utilities are weakly modeled APT sectors. These sectors do not have many unique influences that effect their pricing relative to the market. Consumer Discretionary, Financial, Energy, and Materials are the semi-strongly APT modeled sectors, with multiple non-market influences that change their pricing. Real Estate is the only sector that is a strongly modeled APT sector. Home buyers and sellers may be extra sensitive to news about macroeconomic conditions and thus this market is susceptible to swings based on macroeconomic factors.

The individual factors are interesting to look at as well. Unexpected Inflation and The Term Structure were not significant in any model. Perhaps these are variables that would be significant lagged, but lagging variables was something I chose not to do in this model as it was not done in the paper by Chen, Nai-Fu, Roll, and Ross (1986) I am emulating and due to paper length constraints. This is something that future papers could model.

Expected Inflation and Industrial Production were the variables that were significant in the most models, 5 each. These are important pricing factors as they are signals about the future earnings potential. Expected inflation had a negative coefficient in each model it was significant, which implies that investors respond negatively to increases in expected inflation. The coefficient for Industrial Production is positive for all sectors except HC and IT, which may be an artifact of the COVID-19 pandemic where technology and healthcare companies saw massive gains, and industrial production slowed. Increases in industrial production could also be a sign for a disinvestment in technology in a zero-sum investment economy.

The single most significant factor for any sector was oil prices for energy, with a strong positive coefficient. This connection is obvious, as increased energy prices allow for higher returns from energy companies, as happened in Summer 2022. Oil price was significant for consumer discretionary and consumer staples, both with negative coefficients. This implies that consumers tend to decrease consumption as oil prices change, and those sectors returns suffer. Materials returns was positive with oil prices as they likely benefit from increased energy prices.

Unexpected Risk Premia had significant effects on real estate and materials, which may be sectors that suffer from an increase in risk aversion.

These results suggest that different sectors of the economy respond differently to macroeconomic conditions, with varying levels of significance. Some are modeled better by the CAPM, and some fit an APT model. This could be a tool used by investors to focus their investments or disinvestments in sectors that respond to certain macroeconomic factors, while still maintaining the benefits of portfolio diversification.

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