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**Assignment 1 (100 points, Appendix A)**

**Submission: Please type your answers in this WORD file and submit to Tracs.**

**1. (14) List the elements for each of the following sets:**

(1) P({a, b, c}) (Note: P refers to power set)

={{∅},{a},{b},{c},{a,b},{a,c},{b,c}}

(2) P({a, b}) - P({a, c})

={{b},{a,b}}

(3) P(∅)

={∅}

(4) {x ∈ ℕ: (x ≤ 7 ∧ x ≥ 7} (Note: ℕ is the set of nonnegative integers)

{7}

(5) {x ∈ ℕ: ∃y ∈ ℕ (y < 10 ∧ (y + 2 = x))}

={(2,3,4,5,6,7,8,9,10,11}

(6) {x ∈ ℕ: ∃y ∈ ℕ (∃z ∈ ℕ ((x = y + z) ∧ (y < 5) ∧ (z < 4)))}

={(0,1,2,3,4,5,6,7}

(7) {a, b, c} x {c, d} (Note: x refers to Cartesian product)

= {{a,c},{a,d},{b,c},{b,d},{c,c},{c,d}}

**2. (12) True or False.**

Let R = {(1, 2), (2, 3), (1, 1), (2, 2), (3, 3), (1, 3)}.

(1) R is reflexive.

True

For each a in R, (a,a) is in R

(2) R is transitive.

True

For some a,b,c in A all (a,b) in R and (b,c) in R also have (a,c) in R

In the case of R, the only two sets we have that could apply to this rule are (1,2),(2,3)

We can see from this we do also have (1,3); therefore, R is transitive.

(3) R is symmetric.

False

For all (x,y) in R (y,x) is not in R

We would also need (2,1),(3,2), and (3,1) in R.

(4) R is antisymmetric.

True

There is no example where x is not equal to y and there exists an (x,y) and (y,x) in R.

**3. (16) True or False.**

(1) Subset-of is a partial order defined on the set of all sets.

True

A partial order must be reflexive, transitive, and antisymmetric.

By definition a partial order is all of these things.

(2) Subset-of is a total order defined on the set of all sets.

False

(3) Proper-subset-of is a partial order defined on the set of all sets.

False

A proper-subset-of breaks the rules of reflexivity because x must be less than y; therefore, it is not a partial order.

(4) Proper-subset-of is a total order defined on the set of all sets.

False

We already know that a proper-subset-of is not a partial order. That is, it is not a total order.

(5) Less than or equal (<=) is a partial order defined on the set of real numbers.

True

<= follows all rules of a partial order. It is reflexive, transitive, and antisymmetric

(6) Less than or equal (<=) is a total order defined on the set of real numbers.

True

We can find a relation between all x,y on the set of real numbers.

(7) Less than (<) is a partial order defined on the set of real numbers.

False

It is not a partial order because less than breaks the rule of reflexivity.

(8) Less than (<) is a total order defined on the set of real numbers.

False

Since we know it is not a partial order, we also know it is not a total order.

**4. (12) True or False.**

(1) f (x) = 2x is onto where f: R -> R. (Note: R is the set of real numbers)

True

A function is onto iff every elements of B is the value of some elements in A.

If A is R and B is R then every elements of B will be a value in A.

(2) f (x) = 2x is one-to-one where f: R -> R.

True

A function is one-to-one iff no two elements of A map to the same elements of B.

There is no case in f(x) = 2x to give a contradiction.

(3) f(x) = x² is onto where f: R -> R.

True

It is the case that every element in B is a positive value. It is also the case that the set A contains all the positive values as well.

(4) f(x) = x² is one-to-one where f: R -> R.

False

Consider the case where both the elements 1 and -1 from A map to 1 in B.

(5) f(x) = x² is onto where f: R -> [0, ∞).

True.

It is still the case where all elements of B are of positive value. It is also the case where all the elements of A are still positive value.

(6) f(x) = x² is one-to-one where f: R -> [0, ∞).

False

It is still the case where A contains all real numbers and B is only positive values. In this case we can still consider the case where 1 and -1 are elements in A but both map to the element 1 in B.

**5. (6) Let ℕ be the set of nonnegative integers. For each of the following sentences in first-order logic, state whether the sentence is valid, is satisfiable (but not valid), or is unsatisfiable.**

(1) ∀x ∈ ℕ (∃y ∈ ℕ (y < x)).

Unsatisfiable

Contradiction: Consider the case where x = 0. Y cannot be a be less than x.

(2) ∀x ∈ ℕ (∃y ∈ ℕ (y > x)).

Valid

Consider that for any x that is a natural number then x+1 is also a natural number. Thus always true.

(3) ∀*x* ∈ ℕ (∃*y* ∈ ℕ *f*(*x*) = *y*).

Satisfiable

Consider when f(x) = x/3 and let x =2. Then f(2) = 2/3 which is not a natural number. Thus, not valid.

Now consider the case where x = 2 and f(x) = x. That is, f(2) = 2 which is true. Thus, not unsatisfiable.

Therefore, satisfiable

**6. (20) Are the following sets closed under the given operations? Answer yes or no. If the answer is no, please specify what the closure is.**

(1) The negative integers under subtraction.

No, the closure would include all reals.

(2) The odd integers under the operation of mod 3.

No, the closure would include the set of all odd integers union {0,2}

(3) The positive integers under exponentiation.

Yes, any positive integer that is exponentiated will also be a positive integer, meaning it will be in the set of positive integers.

(4) The finite sets under Cartesian product.

Yes

The cartesian product of two finite sets is also finite.

(5) The rational numbers under addition.

Yes, any two rational numbers added together would result in another rational number, which is still in the set.

**7. (20) True or False. If the answer is true, provide an example (Hint: use subsets of integers and real numbers) as a proof.**

(1) The intersection of two countably infinite sets can be finite.

True

The intersection of the set of all positive numbers and the set of all negative numbers is an empty set, which is finite.

(2) The intersection of two countably infinite sets can be countably infinite.

True

Two sets of even numbers are two countably infinite sets. The intersection of two sets of even numbers is the set of even numbers, which is a countably infinite set.

(3) The intersection of two uncountable sets can be finite.

True

Consider the sets (-∞,0] and [0, ∞). Both of these are uncountable sets. The intersection of these two sets is 0, is finite set.

(4) The intersection of two uncountable sets can be countably infinite.

True

Consider the sets A = all irrational numbers union all rationals and B = the set of infinite binary sequences union all rationals. That’s is, the intersection of set A and B is all rationals, a countably infinite set.

(5) The intersection of two uncountable sets can be uncountable.

True, the intersection of two sets where both sets are the set of all real numbers.