HOMEWORK ASSIGNMENT 8

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PROBLEM 1

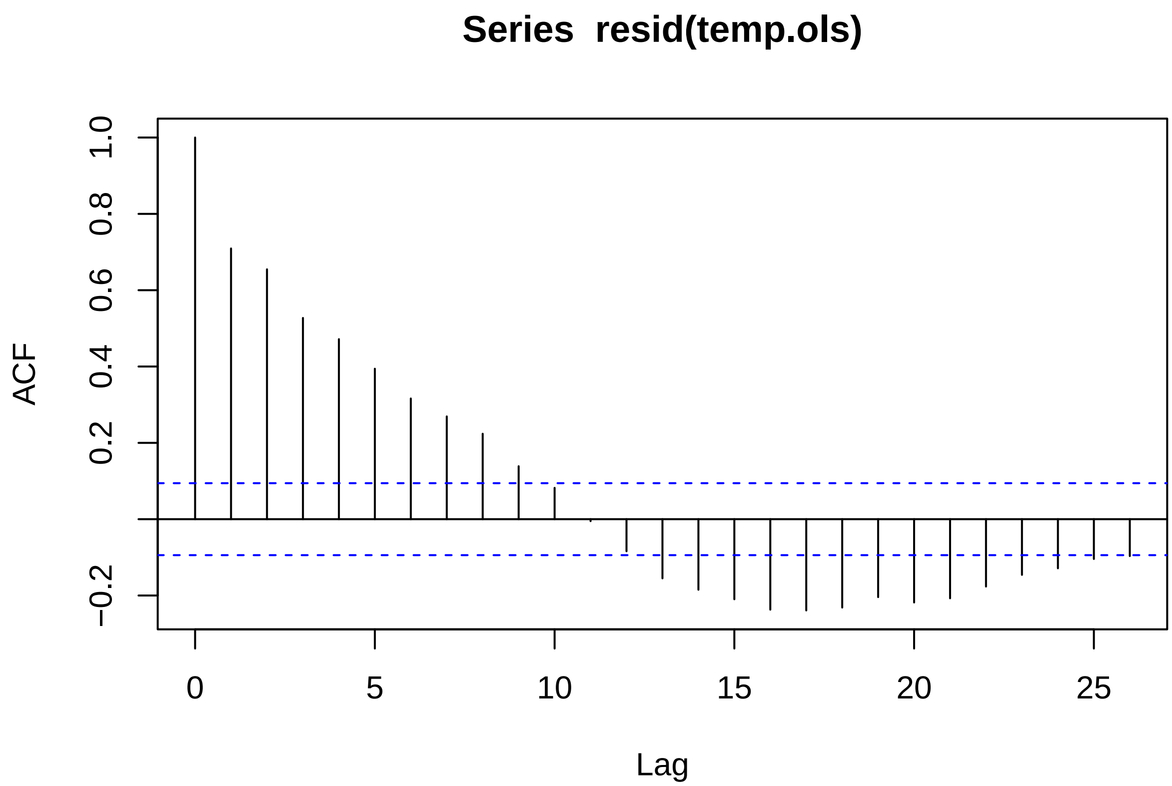
The standard errors of the parameter estimates of a fitted regression model are likely to be underestimated if there is positive serial correlation in the data. This implies that explanatory variables may appear as 'significant' when they should not. Use GLS to check the significance of the variables of the fitted model from Section 5.6.3. Use an appropriate estimate of the lag 1 autocorrelation within gls.

```
## Window the series from 1970--
temp <- window(Global.ts, start = c(1970, 1))

## Harmonic Model
sin <- cos <- matrix(nrow = length(temp), ncol = 6)
for (i in 1:6) {
  cos[, i] <- cos(2 * pi * i * time(temp))
  sin[, i] <- sin(2 * pi * i * time(temp))
}

## Standardize Time Variable
time.std <- (time(temp) - mean(time(temp)))/sd(time(temp))

## OLS Model
temp.ols <- lm(temp ~ time.std + I(time.std^2) +
                 cos[,1] + sin[,1] + cos[,2] + sin[,2] +
                 cos[,3] + sin[,3] + cos[,4] + sin[,4] +
                 cos[,5] + sin[,5] + cos[,6] + sin[,6])

## Print Coefficient Z-Scores
coef(temp.ols)/sqrt(diag(vcov(temp.ols)))
```

```
##   (Intercept)      time.std I(time.std^2)      cos[, 1]      sin[, 1]
##    18.2569117    30.2800742     1.2736899     0.7447906     2.3845512
##      cos[, 2]      sin[, 2]      cos[, 3]      sin[, 3]      cos[, 4]
##     1.2086419     1.9310853     0.6448116     0.3971873     0.5468025
```

```
##      sin[, 4]      cos[, 5]      sin[, 5]      cos[, 6]      sin[, 6]
##     0.1681753     0.3169782     0.3504607    -0.4498835    -0.6216650
```

```
acf(resid(temp.ols))
```




```
## Autocorrelation at lag 1 is approx. 0.709
temp.gls <- gls(temp ~ time.std + I(time.std^2) +
                  cos[,1] + sin[,1] + cos[,2] + sin[,2] +
                  cos[,3] + sin[,3] + cos[,4] + sin[,4] +
                  cos[,5] + sin[,5] + cos[,6] + sin[,6], cor = corAR1(0.709))

## Print Coefficient Z-Scores
coef(temp.gls)/sqrt(diag(vcov(temp.gls)))
```

```
##   (Intercept)      time.std I(time.std^2)      cos[, 1]      sin[, 1]
##     7.2656177    11.9434659     0.5001722     0.4969154     1.6129465
##      cos[, 2]      sin[, 2]      cos[, 3]      sin[, 3]      cos[, 4]
##     1.3088569     2.3769718     1.0005592     0.5746192     1.0031744
##      sin[, 4]      cos[, 5]      sin[, 5]      cos[, 6]      sin[, 6]
##     0.2626412     0.5600524     0.7743843    -1.5831847    -2.1388852
```

The ‘Intercept’, ‘Time.STD’, ‘Sin[, 2]’ are significant. The ‘Sin[, 1]’ term is very close to significant. These results are very similar to those obtained from OLS in class, but the ‘t-statistics’ are smaller for GLS than OLS. This is likely due to the fact that OLS underestimates each coefficient’s standard error, opposed to GLS.

PROBLEM 2

From the refitted harmonic model to the temperature series using `gls`,

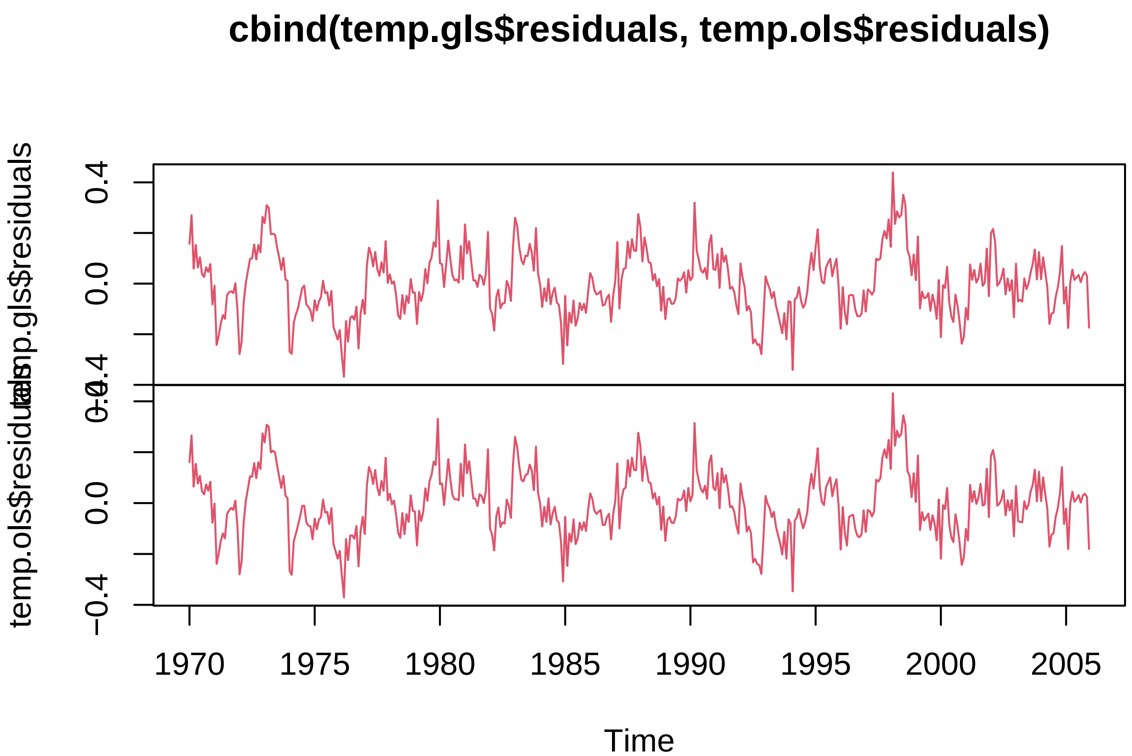
(a) Construct a 99% confidence interval for the coefficient of time.

```
confint.99 <- confint(temp.gls, level = .99)
confint.99[2, 1:2] ## 99% Conf. Int. for 'Time.STD'
```

```
##      0.5 %    99.5 %
## 0.1418164 0.2198072
```

(b) Plot the residual error series from the model fitted using GLS against the residual error series from the model fitted using OLS.

```
plot(cbind(temp.gls$residuals, temp.ols$residuals), lty = 1, col = 2)
```



The residual error series look very similar, if not identical between the GLS model and OLS model. There are no obvious deviations between the two.

(c) Fit an AR(p) model to the residuals from the fitted GLS and OLS models.

```
## GLS Model
temp.gls.ar <- ar(resid(temp.gls), method = 'mle')

## OLS Model
temp.ols.ar <- ar(resid(temp.ols), method = 'mle')
```

(d) How different are the fitted models?

```r
## Compare Order
temp.gls.ar$order
```

```
## [1] 12
```

```r
temp.ols.ar$order
```

```
## [1] 12
```

```r
## Compare Coefficients
temp.gls.ar$ar
```

```
##  [1]  0.48345640  0.31146425 -0.06350000  0.07515868  0.03601876 -0.08416256
##  [7]  0.05118148  0.08442445 -0.06142872  0.02946672 -0.02404747 -0.13111059
```

```r
temp.ols.ar$ar
```

```
##  [1]  0.48283330  0.31301732 -0.06427645  0.07548848  0.03248904 -0.07898203
##  [7]  0.04777619  0.08794487 -0.06798546  0.03073885 -0.02462791 -0.12686749
```

Both models are essentially identical. The estimated coefficients are roughly equivalent, but since the estimated order for the residual series is 12, which is not equal to 1, for both models, the residuals of both full models are not well-fitting to an AR(1) process.