HOMEWORK ASSIGNMENT 5

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PROBLEM 1: Exponential Smoothing

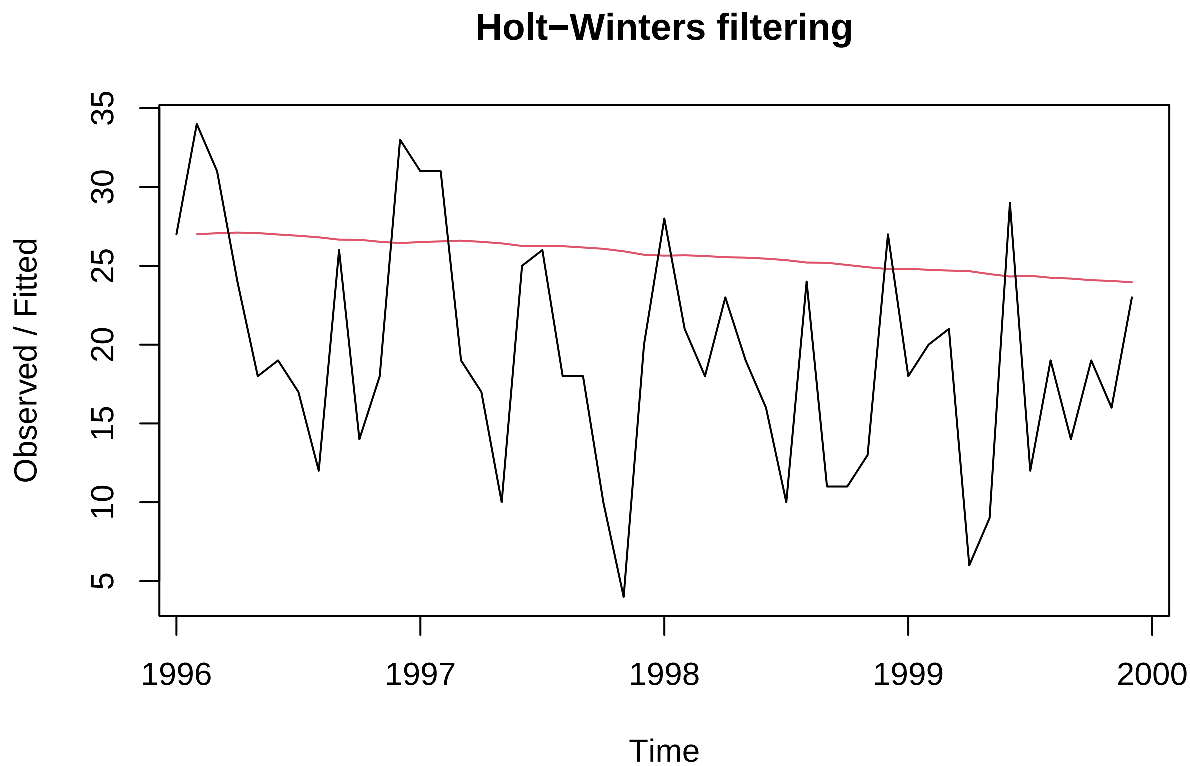
(a) Using the motor organisation complaints series, refit the exponential smoothing model with weight $\alpha = 0.01$. Extract the last residual from the fitted model and verify that the last residual satisfies Equation (3.19).

```
## Create Time Series
Comp.ts <- ts(complaints, start = c(1996, 1), freq = 12)

## Create Exp. Smooth Model with alpha = 0.01
Comp.hw1 <- HoltWinters(Comp.ts, alpha = .01, beta = FALSE, gamma = FALSE)

## Last Residual
LR <- Comp.ts[48] - Comp.hw1$fitted[47, 1]

## Calculate a_{t-1} = a_t - \alpha*x_t / (1-\alpha)
a.1 <- (Comp.hw1$coefficients - (.01)*Comp.ts[48]) / (.99)

## (e_t = x_t - a_{t-1}
FR <- Comp.ts[48] - a.1

## Verifying Equation 3.19
LR == FR
```

```
## xhat
## TRUE
```

(b) Redraw Figure 3.8 using the new value of α, and comment on the plots, explaining the main differences.

```
plot(Comp.hw1)
```



The main difference between the two plots is that the fitted smooth curve with $\alpha = 0.01$ has a slope much closer to 0 than that in Figure 3.8. Also, the fitted curve in Figure 3.8 has more curvature than that of the fitted smooth curve with $\alpha = 0.01$. The plot in Figure 3.8 follows the time series more closely.

(c) Using the motor organisation complaints series, refit the exponential smoothing model with weight $\alpha = 0.99$. Extract the last residual from the fitted model and verify that the last residual satisfies Equation (3.19).

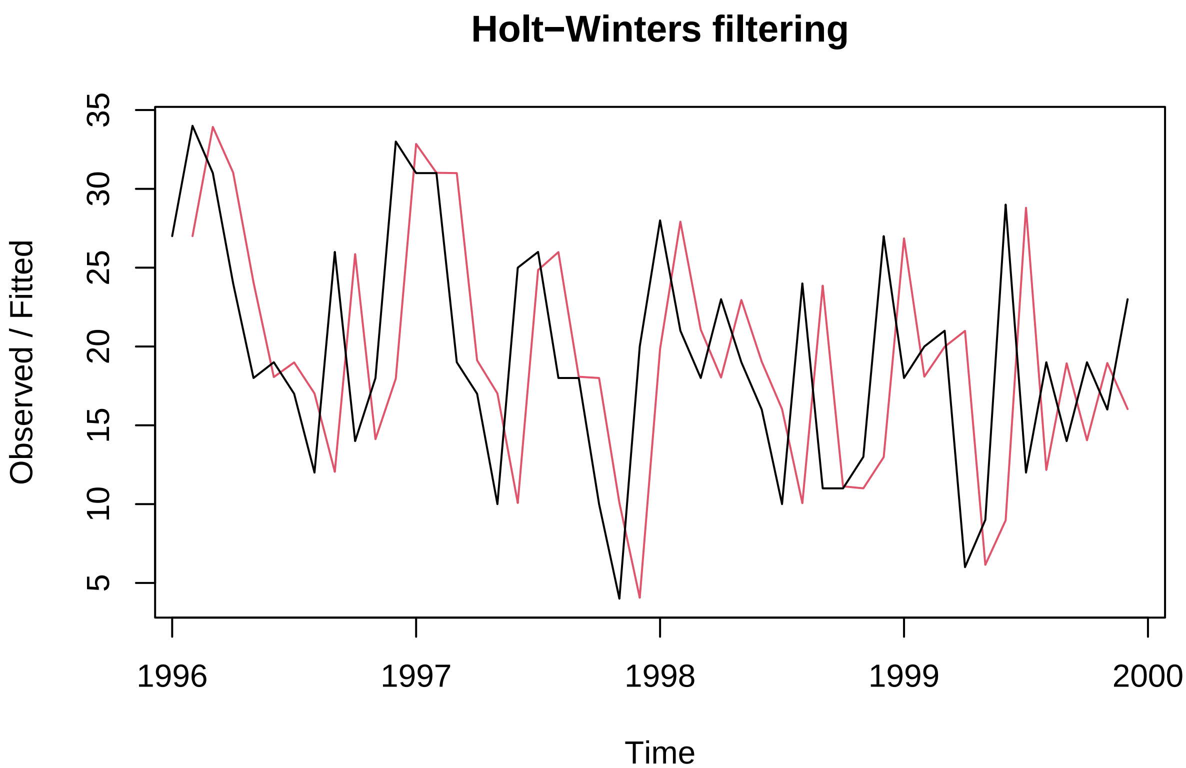
```
## Create Time Series
Comp.ts <- ts(complaints, start = c(1996, 1), freq = 12)

## Create Exp. Smooth Model with alpha = 0.99
Comp.hw2 <- HoltWinters(Comp.ts, alpha = .99, beta = FALSE, gamma = FALSE)

## Last Residual
LR <- Comp.ts[48] - Comp.hw2$fitted[47, 1]

## Calculate a_{t-1} = a_t - \alpha*x_t / (1-\alpha)
a.1 <- (Comp.hw2$coefficients - (.99)*Comp.ts[48]) / (.01)
```

```r
## (e_t = x_t - a_{t-1}
FR <- Comp.ts[48] - a.1

## Verifying Equation 3.19
LR == FR
```

```
##  xhat
## FALSE
```

(d) Redraw Figure 3.8 using the new value of α, and comment on the plots, explaining the main differences.

```r
plot(Comp.hw2)
```



This exponential smoothing curve does little to smooth the curve at all. The curve with $\alpha = 0.99$ follows the original time series too closely. This would likely be useless for forecasting. The plot in Figure 3.8 shows a good balancing at of smoothing rather than the two extremes shown with $\alpha = 0.01$ and $\alpha = 0.99$.

PROBLEM 2: Holt-Winters and sweet wine

Refer to the sweet white wine sales (Section 3.4.2)

```
Wine <- read.table("wine.dat", header = T)
attach(Wine)
```

(a) Use the `HoltWinters` procedure with α, β and γ set to 0.2 and compare the SS1PE with the minimum obtained with `R`.

```
## Create Time Series
sweet.ts <- ts(sweetw, start = c(1980, 1), freq = 12)

## Build Holt Winters Model with \alpha = \beta = \gamma = 0.2
sweet.hw1 <- HoltWinters(sweet.ts, seasonal = "multiplicative",
                         alpha = .2, beta = .2, gamma = .2)

## Build Holt Winters Model calculated by R
sweet.hw2 <- HoltWinters(sweet.ts, seasonal = "multiplicative")

sweet.hw1$SSE > sweet.hw2$SSE
```

```
## [1] TRUE
```

The Holt Winters model with parameters calculated by R has a smaller SSE than that of the Holt Winters model with parameters $\alpha = \beta = \gamma = 0.2$.

(b) Use the `HoltWinters` procedure on the logarithms of `sweetw` and compare SS1PE with that obtained using `sweetw`.

```
## Take log of Sales and Create a Time Series of log(Sales)
logsweet <- log(sweetw)
logsweet.ts <- ts(logsweet, start = c(1980, 1), freq = 12)

## Create Holt Winters Model with log(Sales)
logsweet.hw <- HoltWinters(logsweet.ts, seasonal = "multiplicative")

## Compare Holt Winters Sales to Holt Winters log(Sales)
sweet.hw2$SSE > logsweet.hw$SSE
```

```
## [1] TRUE
```

The Holt Winters model using the logarithm of sales has a smaller SSE than that of regular sales. This makes sense due to the fact the taking a log(x) normalizes all x, reducing its variance.

(c) What is the SS1PE if you predict next month's sales will equal this month's sales?

```
## SSIPE = \Sigma_{i=2}^{n+1} e_i^2 = \Sigma_{i=2}^{n} e_i^2 + (x_{n+1} - a_n)^2
sweet.hw2$SSE + (sweetw[length(sweetw)] - sweet.hw2$coefficients[1])^2
```

```
##        a
## 477830.5
```