Formal Models: Section 4 Exercises*

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Exercise 1

There are two friends, Zach and Christian (Z and C), who are visiting Chicago and agreed to meet up at their favorite restaurant, Cafecito. Unfortunately, this restaurant has two locations: South Loop (SL), and Gold Coast (GC). Furthermore, both friends have already boarded the CTA on separate trains but, sharing one brain cell, have let their cell phones run out of battery and die. As a result, both friends must independently decide which location they should head to, hoping that the other friend will head there. As part of the structure, we assume that both Zach and Christian both equally prefer ending up at the same location to *not* ending up at the same location, but neither has a preference for South Loop over Gold Coast.

(a) Write the strategy/action profile for each player; the set of outcomes; and their utilities.

*Any errant mistakes are mine alone.

(b) Based on the above, write the game in matrix form.

(c) In this game, are there any pure strategy Nash Equilibria? If so, identify them.

Exercise Two: Battle of the Cohort II

With the Oscar season fully underway, I decide to see a movie with my friend Tom. As we're both busy graduate students, we pick a date in advance and agree to meet at the theater at a pre-arranged time. There are two movies in the running for Best Picture that we are interested in watching, with both movies showing at the same theater and with the same showtime: *Marty Supreme* and *The Secret Agent*.

On the big day, I get caught grading 23 problem sets and have to stay late at the office, so I tell Tom I'll just meet him at the theater. When I arrive, I text Tom, but while I can see that his location is in the theater the text doesn't deliver and his phone appears to be dead.[1] Furthermore, I'm quite late, so I need to pick a movie and go in ASAP.

Fortunately, I remember that earlier that week Tom and I discussed our preferences. As a proud bisexual stereotype, I am a fan of Tyler the Creator, Timothee Chalamet, and A24 so thus I prefer to watch *Marty Supreme* in order to maintain my street cred, but I wouldn't mind *The Secret Agent* given that another friend highly recommended it. Tom, who carries a better sense of taste, national pride, and an interest in political thrillers, prefers to see *The Secret Agent*, but also wouldn't mind seeing *Marty Supreme*. Both of us, however, very strongly prefer to coordinate and see a movie together so that we can discuss afterwards.

To make my decision, I draw out the game as follows:

	Marty Supreme$_{Tom}$	The Secret Agent$_{Tom}$
Marty Supreme$_{Zach}$	(3,1)	(0,0)
The Secret Agent$_{Zach}$	(0,0)	(1,3)

(a) In this game, are there any pure strategy Nash Equilibria? If so, identify them.

[1] You'll notice these examples have a lot of dead phones, which is necessary to impose non-coordination constraints on these games while keeping them fun and externally valid. Sorry!

I know that Tom, as a fellow student of formal theory, is unlikely to have expected us to coordinate on a pure strategy Nash equilibrium. After all, it doesn't make much sense for either of us to "concede" by choosing the other person's preferred movie. We have an underlying disagreement where we both prefer an outcome where we get 3 jollies rather than 1, and without a method of communication there's no apparent way to coordinate or resolve the situation. For a moment, I think we're stuck.

But, standing in front of the box office, my formal theory training comes back to me. I need to form a belief about what Tom chose. Of course, earlier that night Tom, reasoning symmetrically, was forming a belief about *me*. To solve this, I'll need to form a strategy that is robust to Tom's reasoning *about my reasoning*. The only way to do this is to find a strategy that I can play where Tom has *no incentive to adjust his behavior*; he will stick to the plan regardless of what he believes I am doing. In other words, I need to make him indifferent between his own choice of *Marty Supreme* and *The Secret Agent*.

So, I realize that we are in a world of **mixed strategies**. Tom picks *Marty Supreme* with some probability q and *The Secret Agent* with probability $1-q$; similarly, I should pick *Marty Supreme* with some probability p and *The Secret Agent* with probability $1-p$. We can now represent the mixed strategies on the game like so:

		q	$1-q$
		Marty Supreme$_{Tom}$	The Secret Agent$_{Tom}$
p	Marty Supreme$_{Zach}$	(3,1)	(0,0)
$1-p$	The Secret Agent$_{Zach}$	(0,0)	(1,3)

(b) What does it mean for Tom to be indifferent between picking *Marty Supreme* and *The Secret Agent*? Can you solve that expression to recover one of the probabilities?

(c) What, then, is the mixed-strategy Nash Equilibrium (if any exist)?

(d) Given this, what is my best response function? What is Tom's?