Sequential Time Series Forecasting Generalizations of Traditional Fama-French Style Asset Pricing Factor Models

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Abstract

Our research explores different asset pricing models and observes any improvement in performance in terms of predictive ability and explanatory power regarding excess returns. We utilize the Fama-French 3-factor model and the Capital Asset Pricing Model (CAPM) as our baseline for performance comparison and include various factors such as momentum, liquidity, sector returns, and principal component analysis (PCA). Using the Carhart momentum factor and the Pastor-Stambaugh liquidity factor, we attempt to capture investor sentiment and risk in a portfolio of assets. We then compare the performance of this new 5-factor model to our baseline models as well as the existing 4-factor Fama-French Carhart model, which includes the original three factors and momentum. Using our PCA, we attempt to find principal components (PC) that drive market returns. These PCs will either be almost identical to existing factors or be used in conjunction with the factors for asset pricing. The last model we created is based on the Global Industry Classification Standard (GICS) sector returns to analyze the risk decomposition of various portfolios. This can help identify which sectors are contributing to the portfolio's risk. The performance of these models was assessed by analyzing the t-values, incremental r^2 , and the Gibbon, Ross, and Shanken (GRS) test statistic.

1. Introduction

Asset pricing models are models used to determine a theoretically appropriate expected rate of return of an asset to make decisions about adding assets to a well-diversified portfolio. They attempt to capture the various factors that affect an asset's returns, such as market risk, firm-specific risk, and macroeconomic variables. Understanding how different factors affect the returns of a portfolio of assets is critical for making informed investment decisions and managing risks effectively. Investors seek to maximize their returns while minimizing their risk exposure, and asset pricing models help them achieve this goal by providing a framework to analyze and measure the relationship between risk and return.

Eugene Fama and Kenneth French created the Fama-French three-factor model in 1992. It builds upon the Capital Asset Pricing Model (CAPM), which only considers the systematic risk of a security or portfolio to derive an expected return. The three-factor model adds two additional factors, the size and value factors, which can capture the systematic risk of a security or portfolio. The size factor, or Small Minus Big (SMB) factor, accounts for the relative outperformance of small-cap companies to large-cap companies, while the value factor, or High Minus Low (HML) factor, accounts for the relative outperformance of high book-to-market value companies to low book-to-market value companies. In 1997, Mark Carhart proposed extending the Fama-French three-factor model to create the Fama-French four-factor or Carhart Model. This model adds a new momentum, or Up Minus Down (UMD), factor to the three-factor model, which accounts for the difference in the performance of high-momentum (positively advancing) firms to low-momentum (negatively advancing) firms.

The Fama-French three-factor model is one of the most widely used asset pricing models, but recent studies suggest that it may not fully capture all the factors that affect an asset's returns. Therefore, this research aims to investigate which factors in a portfolio will explain average excess returns most accurately and if any sets of factors will perform better than the original Fama-French three-factor model. We will be using the CAPM, Fama-French three-factor model, and Carhart model as a baseline to which we will compare the performance of models that we have created by adding additional factors, including momentum, liquidity, sector returns, and principal component analysis (PCA).

It should also be noted, despite not being used as a baseline for comparison in this paper, that in 2015, Fama and French revised their earlier 3-factor model and added two more factors to create the Fama-French 5-factor model. It includes the profitability, or Robust Minus Weak (RMW) factor, which accounts for the difference in operating profitability between firms and the investment pattern, or Conservative Minus Aggressive (CMA) factor, which accounts for the difference in returns from conservative versus aggressively invested portfolios. The formulas for these models, as mentioned above, are listed, with their variables defined below:

CAPM: $\mathbf{r_i = r_f + ß_i(r_m - r_f) + \varepsilon}$

r_i is the expected rate of return for a security
r_f is the risk-free rate of return
$ß_{MKT}$ is the systematic risk associated with the asset
r_m is the expected return of the market

Fama-French 3-factor model: $\mathbf{r_i = r_f + ß_{MKT}(r_m - r_f) + SMB * ß_{SMB} + HML * ß_{HML} + \varepsilon}$

$ß_{SMB}$ is the size factor
$ß_{HML}$ is the value factor

Fama-French Carhart model: $\mathbf{r_i = r_f + ß_{MKT}(r_m - r_f) + SMB * ß_{SMB} + HML * ß_{HML} + UMD * ß_{UMD} + \varepsilon}$

$ß_{UMD}$ is the momentum factor

Fama-French 5-factor model: $\mathbf{r_i = r_f + ß_{MKT}(r_m - r_f) + SMB * ß_{SMB} + HML * ß_{HML} + RMW * ß_{RMW} + CMA * ß_{CMA} + \varepsilon}$

$ß_{RMW}$ is the profitability factor
$ß_{CMA}$ is the investment pattern factor

2. Literature Review

2.1 Fama-French Model

There are many factors included across all of the variations of the Fama-French model, and it is essential to analyze the data set used to determine the best-fit model. Ahmed, Bu, and Tsvetanov (2019) explain that at least ten types of factor models exist. The categories tested were investment, profitability, intangibles, and trading friction. In our study, it is essential to see how much more beneficial it is to use the Fama-French 3 and 5-factor model and recognize the pros and cons of our pricing models. This article helps us be able to see what kinds of factors are important when using these models for our specific data set and what is not recommended to use.

Another vital factor to consider when choosing which model is better for us is the expected returns. Liu and Shi (2022) helped show us the implementation of the three-factor, four-factor, and five-factor models in the context of the Chinese Funds Market and gave more insight into the construction of both the general form of the model and its variations. They did this by discussing all the added factors and their purpose.

Risk and return are also significant to consider. Pettengill, Chang, and Hueng (2013) formed ten portfolios based on estimated security betas and tested the ability of the Fama-French three-factor model to predict return and return variation. This shows how to implement the Fama-French model to estimate returns and use different factors for predictions. In addition, it shows some of the possible flaws in this model. We even saw that this model worked with predicting expected returns with international data, as seen in Hanauer, Jäckel, and Kaserer (2014), which showed us how to find a reasonable proxy for expected returns and how useful this model was for capturing time variations in expected returns.

Li and Duan (2021) and Dirkx and Peter (2020) were also helpful for our research, consisting of tests in different countries. Li and Duan showed a clear improvement in explanatory power using the Fama & French models during the Pandemic and showed that there were drawbacks to the five-factor model compared to the three-factor model. They also suggested that the five-factor model might not be a great instrument when the market is complex but becomes a great instrument when it is simplified. Dirkx and Peter exposed us to four and five-factor models and many new types of regressions. Both articles helped provide a framework for extending previously established models and new testing models.

2.2 Capital Asset Pricing Model

Blanco (2012) compares the CAPM to the Fama-French Three Factor Model and explains how the Fama-French model controls the size, leverage, E/P, Book-to-market, and beta in a single cross-sectional study and how it adds to the explanation of expected stock returns provided by market beta. In this article, they ran both models on six different types of portfolios. This gave us a framework for using the Fama-French three-factor model and showed us how the CAPM is lacking. Fama and French (2004) and Bello (2008) also contributed similar insights.

Rockafellar et al. (2006) expand on the ups and downs of various CAPM-like models over the classical CAPM; they attempt to determine conditions for optimality, assuming general deviation measures are minimized. This helped us gain further context for how we should behave with Fama-French Style models to optimize conditions for CAPM-like models.

2.3 Constructing and Testing Factor Models

Hou, Xue, and Zhang (2015) played a significant role in our inspiration for our project, as they challenge the Fama-French three-factor model and provide evidence that it may not fully capture the sources of risk and returns in the stock market. The authors argue that the three-factor model has limitations, such as its inability to fully capture the profitability of firms, which is an important determinant of stock returns. To address this issue, they propose a new factor, which they call "profitability," based on the difference in return on assets (ROA) between high and low-profitability firms. They also suggest a refinement to the value factor based on a measure of investment-to-assets, which they argue is a better indicator of value than the book-to-market ratio. Their paper introduced us to the idea of improving upon the Fama-French 3 factor model and the idea of constructing novel factor models.

Harris (1995) proposes a method for principal component analysis (PCA) of cointegrated time series. Cointegration is a statistical concept that describes a long-term relationship between two or more non-stationary time series, and PCA is a technique for reducing the dimensionality of data by identifying the most important patterns or trends in the data. Harris's method involves transforming the cointegrated time series into stationary time series using a common trend and then performing PCA on the transformed data. Harris helped us to understand the concepts behind PCA and furthered our ability to test and compare the efficiency of our models with rolling window analysis with PCA.

Gibbons, Ross, and Shanken (1989) introduced us to a powerful way to evaluate the performance of our novel factor models in the GRS test. They provided us with additional understanding of the importance of testing portfolio efficiency, the importance of improving portfolio construction and risk management, and also helped to familiarize ourselves with the GRS test, notably on learning how to perform one and the contexts in which they can and should be used. Diallo, Bagudu, and Zhang (2019), and Li, Xiao, and Teng (2022) provided us with insight for factor model construction with their papers about their novel 5 and 7 factor models, respectively.

2.4 Principal Component Analysis of Stock Returns For Robust Risk Factor Identification

The goal of Malkov (2019) in this paper was to find an effective way to identify risk factors and significant underlying factors that portray the variation in equal-weighted and capitalization-weighted stock returns. This was achieved through PCA, in which selected factors had significant t-statistics that correlated with the return covariance matrix eigenvectors. He found that the five-factor model that used market, size, book-to-market equity, momentum, and profitability factors portrayed the best variation in equally-weighted stock market returns. For finding the S&P 500 fluctuations specifically, he found that market, book-to-market-equity, profitability, investment, one-month-lagged short-term reversal, and one-month leading HMLD were significant factors.

Markov's paper was informative for our group because it gave us some understanding of where to start when constructing our models and how to find which factors would best explain the average excess return. He used tactics like the Sharpe ratio to determine the rationality of the proposed models and verified the robustness of his protocol by comparing it with the results he got for the Least Absolute Selected and Shrinkage Operator (LASSO), a regression technique that helps eliminate insignificant factors. He also used Fama-Macbeth (FM) regressions in order to calculate the factors betas, or OLS coefficients estimates, and risk premia, the expected returns associated with the risks of trading with the selected factors.

This paper showed us a starting point of how to begin our PCA by outlining their statistical analysis by measuring the statistical significance of their canonical correlations between the principal components of his return covariance matrix and how he used this to determine the eigenvalue and eigenvectors. Malkov plotted this on a scree plot to clearly show which eigenvalue explains the most variance for both the equal-weighted returns and the S&P 500. Multiple steps in our research project somewhat parallel the test that was conducted in this paper. This gave structure to how we went about creating the factor model using PCA and helped to give us an outline of what steps to perform to get proper results.

3. Data Source

Kenneth R. French's data library contains Fama-French three-factor and five-factor models from July 1926 to September 2022. Our group uses daily aggregated data from December 2016 to November 2021. The data we have gathered on the Fama-French three-factor model, our baseline, was found in U.S. research returns from Kenneth R. French's data library. To test our models, we used 5 different sets of portfolios for a total of 101 portfolios. We have tested all of our factor models against OEF, an exchange-traded fund (ETF) that tracks the S&P 100 index, a subset of the S&P 500, composed of a hundred large-cap U.S. stocks across various sectors. The other hundred portfolios consist of four sets of twenty-five portfolios each, formed based on the Size and Book-to-Market ratio, Size and Momentum, Size and Operating Profitability, and Size and Investment factors, respectively.

4. Methodology

Our research explores different Fama-French style asset pricing models and observes any improvement in performance in terms of predictive ability and explanatory power regarding excess returns using the Fama-French three-factor model and the Capital Asset Pricing Model (CAPM) as our baseline for performance comparison. Using the Carhart momentum factor and the Pastor-Stambaugh liquidity factor, we attempt to capture investor sentiment and risk in a portfolio of assets. We then compare the performance of this new five-factor model to our baseline models as well as the existing four-factor Fama-French Carhart model, which includes the original three factors and momentum. Using our PCA, we attempt to find principal components (PC) that drive market returns. These PCs will either be almost identical to existing factors or be used in conjunction with the factors for asset pricing. The last model we created is based on the Global Industry Classification Standard (GICS) sector returns to analyze the risk decomposition of various portfolios. This can help identify which sectors are contributing to the portfolio's risk. These models' performance is assessed by performing ordinary least squares regression, rolling window analysis with PCA, and backward stepwise selection, and by analyzing our metrics: t-values, incremental r^2, and the GRS test statistic.

4.1 Liquidity and Momentum

We added the Pastor-Stambaugh liquidity and Carhart momentum factors to the original Fama-French three-factor to create a five-factor model. We had three driving reasons for why we decided to do this. The three reasons are to capture investor sentiment and certain risk types and address anomalies such as the “momentum anomaly” and “liquidity premium.”

Momentum and liquidity factors can be used as proxies for investor sentiment, which can significantly impact asset prices. As an example, optimistic investors may bid up the prices of certain assets, causing them to become overvalued. The opposite is also true, if investors are pessimistic, they may sell off certain assets, causing them to become undervalued. One can capture some of the effects of investor sentiment on asset prices by including momentum and liquidity factors.

Momentum and liquidity factors can also be used to capture various types of risk that may be present in financial markets. For instance, assets with high momentum may be more volatile and subject to greater fluctuations in price, which can be interpreted as a higher level of risk. Similarly, less liquid assets may be subject to greater price fluctuations, as fewer buyers and sellers may be willing to trade them. By including momentum and liquidity factors, one can better capture the underlying risks of different assets.

The inclusion of momentum and liquidity factors can help address certain empirical anomalies that have been observed in financial markets. For instance, momentum is often used to explain the "momentum anomaly," which is the observation that assets that have performed well in the recent past tend to continue to perform well in the near future. Similarly, liquidity is often used to explain the "liquidity premium," which is the observation that less liquid assets tend to have higher returns than more liquid assets.

4.2 Sector Factors

. By incorporating industry sector factors, the CAPM can provide a more nuanced and accurate assessment of systematic risk. Different sectors may have varying levels of sensitivity to market movements and macroeconomic factors. Including industry-specific factors allows for a closer analysis of risk within a diversified portfolio. Incorporating industry sector factors can also aid in portfolio diversification. By considering the unique risk and return characteristics of various sectors, it may help in diversifying across industries, reducing concentration risk. This diversification can help mitigate the impact of adverse events or economic downturns affecting specific sectors. The sectors we used to create this factor model are Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Communication Services, and Utilities.

4.3 Ordinary Least Squares Regression

This is a commonly used method for estimating linear regression coefficients. It is used to describe the relationship between the independent and dependent variables. By minimizing the prediction error, these coefficients are found. There is a simple linear regression which follows the form: $y = \beta_0 + \beta_1 x_1 + \varepsilon$ and multiple linear regression which follows the form: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n + \varepsilon$. This research will use multiple linear regression to assess the relationship between the factors and portfolios. The t-statistic of the intercept obtained will be used to evaluate the model. A t-value that is not statistically significant ($|t| \leq 1.96$) would provide evidence that the intercept is 0 and support that the model is a good fit.

4.4 GRS Test

The Gibbons, Ross, and Shanken (GRS) test is used to evaluate the accuracy of asset pricing models in predicting expected returns on financial assets. It compares the performance of a given asset pricing model with that of a benchmark model, such as the CAPM, that assumes that all assets have the same expected return. The test assesses whether the asset pricing model is statistically superior to the benchmark model in explaining the cross-sectional variation in asset returns. It does this by assessing the joint significance of the factors by testing the null hypothesis that the factor risk premiums are all zero.

The GRS test is based on the Fama-MacBeth procedure and employs a cross-sectional regression framework. It is a multivariate test that takes into account the correlation between asset returns and calculates test statistics to determine if the factors are jointly significant in explaining asset returns beyond what could be attributed to chance. Using a modified F-statistic, the GRS tests assess whether the additional information included in the asset pricing model improves the accuracy of expected return predictions relative to the benchmark model. If the test rejects the null hypothesis of no difference between the two models, it implies that the asset pricing model is statistically superior to the benchmark model. Overall, the GRS test is commonly used for evaluating the performance of asset pricing models and assessing whether they provide a better explanation of expected returns than simpler benchmark models. We use the test in this way to determine whether the inclusion of additional factors to the Fama-French Three Factor Model improves its explanatory and predictive power. Setting a p-value of 0.05, we analyze whether the null hypothesis should be rejected for each model we create given different sets of anomaly portfolios.

4.5 Principal Component Analysis

Principal Component Analysis (PCA) is a statistical technique used for dimensionality reduction and data analysis. It is commonly used to identify patterns and relationships within a dataset by transforming the data into a new coordinate system called the principal components. The main objective of PCA is to find a set of orthogonal axes (principal components) that capture the maximum amount of variation present in the original data. These principal components are linear combinations of the original variables, where the first component accounts for the largest possible variance, the second component accounts for the second largest variance, and so on.

We performed our PCA by standardizing our data from 2010-2021 and 2016-2021, then by calculating a covariance matrix for our data, which represents the relationships between variables in the dataset, we were able to compute eigenvectors and eigenvalues from it. We compared the explained and cumulative explained variance. After computing the eigenvectors and eigenvalues, we were able to decide how many factors we wanted to choose. The number of principal components chosen depends on the desired level of dimensionality reduction or variance explained. We use ten or fewer components in all of our results due to the decreased significance of each subsequent component.

4.6 Rolling Window

Rolling window is used to assess the stability of a time series model in two ways. One is to assess the stability of a model, and the other is to forecast the accuracy of a model. In this case, the stability of a model is being assessed, to be exact, the stability of a PCA model. A rolling window of size m is chosen to do this, and this will be the sample size. This is then incremented throughout the dataset until it is broken into *N* subsamples. The model is then run on each of the subsamples, and an estimate and its confidence interval are plotted. In this case, because it is a PCA model, the cumulative variance explained will be plotted over time.

4.7 Backward Stepwise Selection

Backward stepwise selection is a variable selection technique used in statistical modeling to determine the most relevant subset of predictor variables to include in a model. It is a stepwise regression procedure that starts with a model containing all predictor variables and iteratively removes variables that are found to be the least significant or contribute the least to the model's performance. The algorithm starts with a model containing all predictor variables, then fits the model and assesses the statistical significance or importance of each predictor variable using certain criteria; in our case, we observe the R-squared and RMSE of each model. It then removes the least significant variable and refits the model without the removed variable. This process is repeated until all remaining variables are statistically significant. Backward stepwise selection helps to eliminate non-significant or less important variables from the model, simplifying it and reducing the risk of overfitting. The procedure aims to retain the most informative and relevant predictors for explaining the response variable by iteratively removing variables.

In our study, we implement backward stepwise selection to evaluate the significance of each of the factors we add to the Fama-French 3 Factor model. Given a set of factors, we create all possible combinations of 1-, 2-, 3-, 4-, etc. factor models and analyze the average R-squared and RMSE values for each. We can then determine the scale importance of each factor by observing how many times it appeared, on average, in the most statistically significant model using this algorithm. In this way, we are able to clearly identify which factors improve upon the Fama-French three-factor model in terms of explanatory power and predictability given different sets of anomaly portfolios.

4.8 Metrics

The metrics we will use are R-squared and RMSE. These metrics will help find a better-predicting model (Pedregosa *et al.* 2011). Small MSE and MAE indicate a small error; therefore, the models' predicted values are closer to the actual values. The higher the R-squared, the better the model fits the data.

Using these algorithms and the given metrics, we will fit the data and regress them in search of a model that will give better results given the time-series data. We will also compare it to a linear regression model, which is the CAPM model, and the goal is to get a model that will be better than CAPM.

5. Results

5.1 CAPM

To start working on our research question, even though we want to find a model better than the Fama-French model, we still need to compare it to the CAPM model. Our first objective is to run regressions among all the 101 portfolios and get their t-values.

5.1.1 T-values

	Size/BM	Size/Momentum	Size/Investment	Size/Operating Profitability
1	0.135	0.005	0.370	-0.381
2	0.046	-0.394	-0.171	-0.315
3	-0.599	-0.495	-0.364	0.176
4	-0.956	-0.407	-0.181	0.523
5	0.377	0.060	-0.470	-0.723
6	0.037	-0.237	-1.098	-0.427
7	-0.165	-0.537	-0.748	-0.924
8	-0.841	-0.282	-1.006	-0.549
9	-1.299	-1.180	-0.564	-0.537
10	-0.823	0.199	-0.664	-1.083
11	-0.072	-0.345	-0.734	-0.312
12	-0.459	-0.828	-1.074	-1.696
13	-1.363	-1.393	-1.598	-0.682
14	-0.852	-0.590	-0.924	-0.145
15	-1.359	-0.150	-0.415	-0.820
16	1.376	-0.412	-0.667	-0.388
17	0.071	-0.088	-0.882	-0.477
18	-0.764	-0.286	-0.589	-0.132
19	-1.304	-0.628	-0.294	-0.222
20	-1.374	0.216	0.811	-0.029
21	3.037	-0.206	-0.478	-1.230
22	0.476	0.075	0.244	-0.609
23	-1.145	0.542	-0.607	0.649
24	-1.706	0.927	1.599	0.955
25	-0.805	-0.230	1.754	2.362

port	t_value.(Intercept)
OEF	0.203

As explained in the methodology, the t-values should be less than the absolute value of 1.96 for the intercept term to show that the model is a good fit. Here we see that 2 out of 101 portfolios are statistically significant and the CAPM model is not a good fit. To get more in-depth, we ran a GRS test.

5.1.2 GRS Test

Portfolio	GRS Stat	GRS p-value
OEF	0.04118431	0.8392154
Size/BM	2.741941	9.620067e-06
Size/Momentum	0.842191	6.886366e-01
Size/Investment	2.518616	5.549720e-05
Size/Operating Profitability	2.612646	2.662443e-05

The GRS test of the CAPM model shows us that for 75 portfolios, CAPM is not a good fit.

5.2 Fama-French 3 Factor Model

5.2.1 T-values

	Size/Mom	Size/Inv	Size/OP	Size/BM
1	1.32800234	1.26083484	-0.39071341	0.28031521
2	1.22931827	1.54051449	1.85347190	0.28581411
3	0.71871821	1.09375214	$\boxed{3.12076953}$	0.08048020
4	0.48333352	1.48733643	1.30658579	0.08819871
5	0.25074487	-0.05661002	0.28874920	1.90248596
6	1.13978342	-1.27618089	-0.92639212	-0.16802159
7	1.25853258	0.72993606	0.51967083	0.71628377
8	1.78721839	-0.04575678	1.56317098	0.09453614
9	-0.87493653	1.10389471	1.07606003	-0.57004739
10	0.52008793	-0.72122810	-0.23908715	0.83581201
11	0.84688953	0.06802211	-1.04561824	-0.51751245
12	0.45810216	-0.07222949	-1.46747916	0.35581820
13	-0.69482495	-1.20974588	1.05309336	-0.70817419
14	0.44280979	0.22895894	1.57866102	0.93938930
15	-0.31895210	-0.48544002	0.17777942	-0.37102292
16	0.65121356	0.12960457	-0.99813897	1.02571772
17	1.46374143	0.24774815	0.74854768	0.85596903
18	1.33515888	0.79400701	1.24113296	0.42039180
19	0.07974585	0.74530692	0.84258897	-0.24662081
20	-0.18095809	0.40267458	0.87961104	-0.28875225
21	0.74549553	-0.43749587	-1.68495610	$\boxed{3.74588418}$
22	1.36946382	1.37472987	-0.06068263	0.26388776
23	1.73411374	0.35761984	1.24712553	-0.38089977
24	1.04094097	1.28738430	0.95194922	-1.11089567
25	-1.26041950	1.04040034	$\boxed{2.22700009}$	0.95662018

```
port  t_value.(Intercept)
 OEF                -0.352
```

Here, 3 out of 101 portfolios are statistically significant, where the Fama-French 3 Factor Model is not a good fit.

5.2.2 GRS Test

Portfolio	GRS Stat	GRS p-value
OEF	0.1238094	0.7249982
Size/BM	2.746369	9.193292e-06
Size/Momentum	0.867588	6.529428e-01
Size/Investment	2.474858	7.782568e-05
Size/Operating Profitability	2.721912	1.118611e-05

The GRS test of the 3-factor model shows us that for 75 portfolios, that it is not a good fit.

5.3 4 Factor Model

5.3.1 T-values

	Size/Mom	Size/BM	Size/Op	Size/Inv
1	0.90120023	0.28029973	-0.312660115	1.45100733
2	0.92453291	0.32184110	1.788804186	1.50872982
3	0.73189382	0.03318270	3.085609319	1.05188588
4	0.75545877	0.08294738	1.358919334	1.47947403
5	0.78607141	2.00168249	0.081611687	-0.09374818
6	0.64039406	-0.16209540	-0.869980071	-1.21146844
7	0.83173101	0.63732705	0.450534436	0.63646649
8	1.76906418	0.05589110	1.465059421	-0.19120592
9	-0.58735600	-0.75650062	0.922131196	0.95401583
10	1.34473933	0.79936854	-0.530768188	-0.76188632
11	0.06659733	-0.59666091	-1.004140983	0.03657104
12	-0.21837077	0.26282962	-1.516957170	-0.19139755
13	-1.04592119	-0.77774143	0.873469670	-1.28506114
14	0.90405604	0.84170207	1.442587205	0.05297628
15	0.42209330	-0.48853619	-0.007366144	-0.53497048
16	-0.17534666	1.07275735	-0.935172564	0.15102122
17	1.06701883	0.78216333	0.774594845	0.13457432
18	1.12033333	0.38666839	1.237852273	0.67506038
19	0.49679861	-0.49789013	0.711827299	0.68445414
20	0.64297682	-0.40203120	0.756805687	0.49882062
21	0.05330021	3.78914767	-1.604541276	-0.38112093
22	0.97915665	0.06802576	0.072499968	1.35684306
23	1.44211663	-0.58988384	1.254130928	0.19476747
24	1.35922919	-1.24045266	0.793596014	1.42383914
25	-0.88065198	1.00830624	2.159540163	1.00316768

	t_value. (Intercept)
1	-0.3828071

<< OEF

Here, 4 out of the 101 portfolios are statistically significant, where the Fama-French 4 Factor Carhart Model is not a good fit.

5.3.2 GRS Test

Portfolio	GRS Stat	GRS p-value
OEF	0.1465420	0.7019271
Size/BM	2.7144937	1.187402e-05
Size/Momentum	0.8808771	0.6339811
Size/Investment	2.4782129	7.586251e-05
Size/Operating Profitability	2.6700097	1.692809e-05

The GRS test of the 4-factor model shows us that for 75 portfolios, that it is not a good fit.

5.4 5 Factor Model

5.4.1 T-values

	Size/Mom	Size/BM	Size/Op	Size/Inv
1	0.71722144	0.26418101	-0.29152138	1.36699351
2	1.07400874	0.41851511	1.88635680	1.46627397
3	0.86154823	0.18386747	3.15765978	1.23531607
4	0.87872804	0.16040679	1.33883387	1.68505482
5	0.90407831	1.96180119	0.14435995	-0.03785511
6	0.51835968	0.02328335	-0.78848745	-1.06782670
7	0.90438446	0.79052813	0.63574457	0.59829716
8	1.97178681	0.25924654	1.54811931	-0.06903058
9	-0.49032378	-0.70177792	0.99636462	1.08174998
10	1.37760031	0.76451205	-0.51280709	-0.68477713
11	-0.02871920	-0.71570580	-1.06616078	-0.08884841
12	-0.09886871	0.29318310	-1.42507580	-0.20570764
13	-1.00329204	-0.63138788	0.90984453	-1.16360169
14	0.99884487	0.76167176	1.36623603	0.12733147
15	0.42678894	-0.47497241	-0.05662321	-0.59297205
16	-0.25573015	1.03726921	-0.95367088	0.24290288
17	1.00681424	0.72349574	0.79052421	0.23178296
18	1.12398741	0.51702745	1.22683064	0.69347232
19	0.64976979	-0.61353853	0.65873854	0.64671098
20	0.60339106	-0.43267113	0.72601987	0.34085781
21	0.17284732	3.72683517	-1.54209052	-0.34923038
22	0.99686528	0.07815668	-0.06002958	1.30305516
23	1.57243845	-0.51119024	1.26112201	0.28502610
24	1.50638610	-1.03868618	0.86401317	1.40249462
25	-0.94180788	0.98505983	2.16882214	0.98932235

	t_value.(Intercept)
1	-0.3733112

<< OEF

Here, 5 out of the 101 portfolios are statistically significant, where the 5-factor model (3-factor model plus momentum and liquidity) is not a good fit.

5.4.2 GRS Test

Portfolio	GRS Stat	GRS p-value
OEF	0.07372529	0.7860328
Size/BM	2.66269577	1.794562e-05
Size/Momentum	0.91322100	0.5873717
Size/Investment	2.39148672	1.470013e-04
Size/Operating Profitability	2.64110645	2.129301e-05

The GRS test of the 5-factor model shows us that for 75 portfolios, it is not a good fit.

5.5 Sector Factor Model

5.5.1 T-values

	Size/BM	Size/Momentum	Size/Investment	Size/Operating Profitability
1	0.415	1.381	1.340	-0.264
2	0.390	1.175	1.615	1.719
3	0.151	0.786	1.090	3.135
4	-0.049	0.600	1.580	1.186
5	1.893	0.445	-0.117	0.353
6	-0.182	1.147	-1.071	-0.968
7	0.673	1.185	0.641	0.290
8	0.088	1.808	-0.102	1.476
9	-0.772	-0.772	0.933	1.126
10	0.941	0.575	-0.793	-0.242
11	-0.517	0.845	0.267	-0.947
12	0.342	0.412	0.032	-1.801
13	-0.734	-0.734	-1.271	1.018
14	1.004	0.522	0.154	1.822
15	-0.464	-0.270	-0.520	0.254
16	1.513	0.708	0.601	-0.734
17	1.311	1.638	0.549	1.043
18	0.918	1.675	1.223	1.648
19	-0.092	0.429	1.149	1.280
20	-0.173	-0.083	0.778	1.433
21	3.750	0.890	-0.353	-1.286
22	0.683	1.497	2.166	0.198
23	0.210	1.636	0.644	1.088
24	-0.922	0.890	1.291	1.352
25	0.895	-1.684	0.591	1.890

OEF T-value

OEF
-1.009

Here, 3 out of the 101 portfolios are statistically significant, showing where the Sector Factor Model would not be a good fit.

5.5.2 GRS Test

Portfolio	GRS Stat	GRS p-value
OEF	0.9723242	0.324293
Size/BM	2.853058	3.889719e-06
Size/Momentum	1.103858	3.292086e-01
Size/Investment	2.595332	3.060827e-05
Size/Operating Profitability	2.977475	1.399746e-06

The GRS test of the sector factor model shows us that for 75 portfolios, it is not a good fit.

5.6 Principal Component Analysis

Here, we compared the principal components' explained variance and cumulative explained variance for both the 2010-2021 and 2016-2021 datasets. We see that the 2016-2021 years have better explained variance on both a cumulative and component-by-component basis. The 9th principal component there explains a cumulative 58% of variance as opposed to the 52% of the earlier years.

Explained Variance

	2010	2016
0	0.371444	0.395641
1	0.040169	0.060645
2	0.028497	0.035582
3	0.019596	0.024049
4	0.018050	0.019641
5	0.011628	0.013314
6	0.011365	0.012560
7	0.010273	0.011273
8	0.009290	0.010304

Cum. Explained Variance

	2010	2016
0	0.371444	0.395641
1	0.411612	0.456286
2	0.440109	0.491868
3	0.459705	0.515917
4	0.477755	0.535557
5	0.489383	0.548871
6	0.500748	0.561431
7	0.511021	0.572704
8	0.520311	0.583008

5.7 Important Factors

These tables show the important factors found in the PCA (left), 5 factor model (middle), and sector factor model (bottom). The number in each cell represents the percentage of time that each factor from each model was included in the optimal model.

	Size/Mom	Size/Inv	Size/OP	Size/BM	OEF
Market	87.32600998	91.385124	90.022565	89.238975	100.0000000
SMB	23.13031216	23.318857	23.745553	24.290010	0.4077964
HML	19.06847147	13.587983	16.456438	19.076476	0.7429843
Momentum	16.46557035	5.267477	6.709257	7.720445	0.8775584
Traded Liq	0.02814256	0.000000	0.000000	0.000000	0.0000000
PC1	97.49867374	96.612534	95.993775	96.897289	81.9214804
PC2	6.76309385	5.277926	6.216578	6.779266	13.5687366
PC3	5.03303740	5.970621	6.087655	6.140019	5.8522500
PC4	4.51073273	4.077917	4.266752	4.726805	1.5589512
PC5	7.07630790	6.520243	6.985291	7.007592	4.8009620

	Size/Mom	Size/Inv	Size/OP	Size/BM	OEF
Market	100.00000	100.000000	99.532841	100.000000	100.0000000
SMB	26.80715	26.202524	27.198189	27.446798	0.4077964
HML	23.55196	15.582424	19.474082	23.286815	0.7429843
Momentum	20.52405	6.015544	7.890482	9.345374	0.8775584
Traded Liq	0.00000	0.000000	0.000000	0.000000	0.0000000

For the PCA, the most important factors were the market, SMB, and HML factors, as well as PC1.

For the 5 factor model, the most important factors were the market, SMB, and HML factors.

For the sector factor model, the most important factors were the market factor, followed by all of the sector factors.

	Size/BM	Size/Mom	Size/Inv	Size/Op	OEF
Market	91.697707	93.236566	97.472552	94.02027	100.0000000
SMB	15.488284	13.351953	14.701260	12.38949	0.0000000
HML	9.211047	8.450089	2.129369	3.08374	0.3098818
XLB	82.774221	84.819797	86.164142	84.22465	73.0628604
XLE	67.747057	73.260358	66.900601	65.92810	46.4267552
XLF	87.152234	89.536981	87.447973	86.96161	71.4550247
XLI	87.901286	91.564972	91.079979	89.86822	76.4417022
XLK	62.427146	61.690539	68.062449	63.16049	94.3327344
XLP	39.107492	37.323360	42.764838	37.72343	64.5949865
XLRE	48.042108	48.954080	50.587748	47.51679	56.8686071
XLU	27.176322	25.503693	28.994296	24.45855	42.8754387
XLV	55.228334	54.340698	59.578049	54.60135	78.9301824
XLY	78.364717	80.914022	83.690381	80.00946	86.3783991

There are a few things to note regarding the info on this page:

1. The first PC is known to closely mimic the market factor.
2. The high importance for all of the sector factors likely reflects an issue of multicollinearity with the market factor.

6. Additional Results

Our results from the GRS test show that every factor model, including CAPM, Fama-French 3-factor model, Carhart model, 5-factor model, and sector factor model, were not a good fit for 75 out of the 101 portfolios. Something interesting to note, however, is that all of the models, including the benchmarks, yielded the same overall significance results for all portfolios. What we mean by this is that every model had a significant GRS test statistic for each of the Size/Book-to-Market, Size/Operating Profitability, and Size/Investment portfolios, and all had insignificant scores for both OEF and Size/Momentum. This could mean that portfolio selection likely played a much larger part in our research and results than we realized it would have. It can even be found in our Important Factors section, where, for the 5-factor model and PCA, momentum is only shown to be somewhat important when in the context of the Size/Momentum portfolio. In future attempts, we would emphasize the usability of portfolios chosen to create and test models from/on.

In our important factors section, multicollinearity can make it challenging to interpret the individual effects of the market factor and industry sector factors. The high correlation between the factors can make it difficult to disentangle their specific contributions toward explaining the expected rate of return for a security. This hinders the ability to assess each factor's unique risk and return characteristics. The high number of factors so closely related to the market factor may also have reduced the relative importance of the SMB and HML factors. Multicollinearity may lead to unreliable estimates of the factor model coefficients, when it becomes difficult for the model to determine the individual impact of each factor on the output, it can result in coefficients with large standard errors and imprecise estimates.

7. Robustness Checks

OEF

	rmse	rsq
1	0.002082219	0.9657253
2	0.001485969	0.9833592
3	0.001271264	0.9874735
4	0.001127453	0.9900871
5	0.001086252	0.9907882
6	0.001076469	0.9909411
7	0.001079798	0.9908978
8	0.001076216	0.9909992
9	0.001078444	0.9909698
10	0.001077694	0.9909730

Size and Investment

	rmse	rsq
1	0.6491944	0.7903470
2	0.4466136	0.9007259
3	0.3636799	0.9328807
4	0.3520890	0.9370085
5	0.3461317	0.9389222
6	0.3435359	0.9397260
7	0.3419250	0.9403538
8	0.3416412	0.9404922
9	0.3415126	0.9405082
10	0.3417447	0.9404374

Size and Momentum

	rmse	rsq
1	0.8209507	0.7253052
2	0.5538400	0.8759888
3	0.4202146	0.9279008
4	0.4027205	0.9332714
5	0.3949278	0.9351611
6	0.3919429	0.9360217
7	0.3898499	0.9365480
8	0.3890044	0.9367641
9	0.3888301	0.9368504
10	0.3887685	0.9368581

Size and Operating Profitability

	rmse	rsq
1	0.7049105	0.7786744
2	0.5349834	0.8717659
3	0.4301994	0.9153275
4	0.4205662	0.9187471
5	0.4155495	0.9204591
6	0.4139914	0.9210511
7	0.4118231	0.9217155
8	0.4104807	0.9220754
9	0.4104432	0.9221088
10	0.4105053	0.9220871

Size and Book-to-Market

	rmse	rsq
1	0.7381311	0.7588906
2	0.4663488	0.9034566
3	0.3762964	0.9352740
4	0.3652470	0.9388206
5	0.3598100	0.9404798
6	0.3557114	0.9416241
7	0.3550684	0.9418024
8	0.3549140	0.9418416
9	0.3547113	0.9419254
10	0.3545149	0.9420280

OEF

	RMSE	R^2
1 factor	0.2201723	0.9656995
2 factor	0.1624266	0.9815586
3 factor	0.1495417	0.9841637
4 factor	0.1486355	0.9841082
5 factor	0.1458773	0.9846507
6 factor	0.1423850	0.9852003
7 factor	0.1415646	0.9852312
8 factor	0.1400816	0.9855353
9 factor	0.1400474	0.9854719
10 factor	0.1385901	0.9857440
11 factor	0.1390526	0.9856947
12 factor	0.1391895	0.9856986
13 factor	0.1386704	0.9857878

Size/BM

	RMSE	R^2
1 factor	0.7747291	0.7393174
2 factor	0.5023372	0.8881772
3 factor	0.3797008	0.9351689
4 factor	0.3655232	0.9399434
5 factor	0.3590837	0.9417552
6 factor	0.3573805	0.9422728
7 factor	0.3553565	0.9428754
8 factor	0.3548249	0.9429773
9 factor	0.3546412	0.9430935
10 factor	0.3542602	0.9431558
11 factor	0.3538147	0.9433104
12 factor	0.3537385	0.9433592
13 factor	0.3537466	0.9433552

Size/Mom

	RMSE	R^2
1 factor	0.8539746	0.7114749
2 factor	0.6390531	0.8388555
3 factor	0.5414041	0.8811345
4 factor	0.5296886	0.8862992
5 factor	0.5252430	0.8881869
6 factor	0.5244857	0.8884204
7 factor	0.5224860	0.8891754
8 factor	0.5218410	0.8894403
9 factor	0.5209462	0.8898598
10 factor	0.5207810	0.8899777
11 factor	0.5206110	0.8900332
12 factor	0.5206725	0.8900186
13 factor	0.5207274	0.8899705

Size/Inv

	RMSE	R^2
1 factor	0.6986549	0.7657744
2 factor	0.4693347	0.8924709
3 factor	0.3730960	0.9301068
4 factor	0.3610796	0.9345263
5 factor	0.3556436	0.9364647
6 factor	0.3536324	0.9370724
7 factor	0.3519619	0.9376253
8 factor	0.3508147	0.9379742
9 factor	0.3506914	0.9379764
10 factor	0.3504697	0.9380462
11 factor	0.3502892	0.9381065
12 factor	0.3502203	0.9381251
13 factor	0.3501719	0.9381464

Size/Op

	RMSE	R^2
1 factor	0.7962146	0.7271737
2 factor	0.6149439	0.8369514
3 factor	0.4448643	0.9108786
4 factor	0.4313254	0.9156456
5 factor	0.4225687	0.9188354
6 factor	0.4189732	0.9202138
7 factor	0.4179084	0.9205500
8 factor	0.4162258	0.9211727
9 factor	0.4148979	0.9217503
10 factor	0.4147621	0.9217376
11 factor	0.4143259	0.9219151
12 factor	0.4139774	0.9220674
13 factor	0.4139924	0.9220308

The three tables above show the RMSE and r^2 values for PCA factors (top), the sector factor model factors (middle), and the 5-factor model factors (bottom) obtained from backward selection across each set of portfolios. This data is used to reach the results in the Important Factors section.

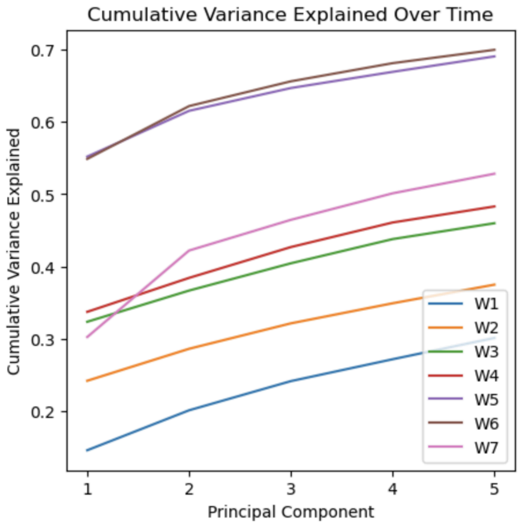
S&P 100	RMSE	R2
1 factor	0.002098095	0.9663548
2 factor	0.001517203	0.9837387
3 factor	0.001473289	0.9845740
4 factor	0.001477888	0.9844789
5 factor	0.001476104	0.9845020

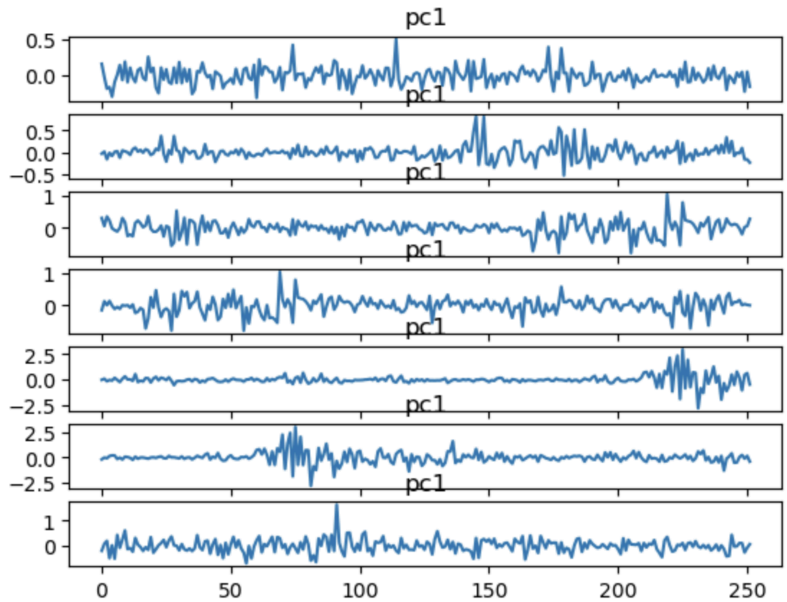
Size/Mom	RMSE	R2
1 factor	0.8716885	0.6989304
2 factor	0.5715838	0.8706022
3 factor	0.4414107	0.9210097
4 factor	0.4256391	0.9258692
5 factor	0.4253979	0.9259596

Size/BM	RMSE	R2
1 factor	0.7992380	0.7264313
2 factor	0.4898287	0.8946894
3 factor	0.3892207	0.9311893
4 factor	0.3881089	0.9317157
5 factor	0.3880884	0.9317254

Size/Inv	RMSE	R2
1 factor	0.7737928	0.7433782
2 factor	0.5835372	0.8533406
3 factor	0.4466360	0.9099857
4 factor	0.4442818	0.9110077
5 factor	0.4442641	0.9110417

Size/Op	RMSE	R2
1 factor	0.6990634	0.7650014
2 factor	0.4694018	0.8910462
3 factor	0.3765903	0.9281578
4 factor	0.3755496	0.9285087
5 factor	0.3756565	0.9284812





This rolling window analysis uses a window size of 252 days with a shift size of 150 days on data from December 2016 to November 2021. The graph with the PC1 labels shows the explained variance of the first principal component over time (denoted as 0 in the chart). The chart shows a numerical value for the cumulative explained variance for each principal component at the 150-day shift. The other graph visualizes this. We can see that the more recent data has a greater cumulative explained variance, apart from the 7th window. This change in particular, based on the timing, would approximately line up with the Covid-19 market crash as a probable reason for why it is less accurate than prior shifts.

We performed an ordinary least squares regression on each model and there were between 2-5 portfolios out of 101 for each model which had a T-value that was statistically significant. This is within expectations, given the 95% confidence interval used. This may help to support what was said earlier in Additional Results about the importance of portfolio section, as we had initially expected the overall positive OLSR results to be paralleled in the GRS tests.

8. Conclusion

Out of all the factors we have researched, the market, SMB, and HML factors explain excess returns most accurately. The momentum factor was not wholly unfruitful, as it did experience some importance in certain scenarios and in portfolios built for it, but it was ultimately less significant than the SMB and HML factors even when it performed best. The liquidity factor ended up almost never being included in optimal portfolios, which suggests that it may not be as important as we had thought. The sector factors performed well and were all more significant than SMB and HML factors in their backward selection test, but with concerns about multicollinearity, we could not veritably say that it would perform in real-world applications as well as it suggests. The first principal component from PCA does have more explanatory power than the market, SMB, and HML factors, but it is significantly limited in how recent the data is, is closely linked with the market factor, and, even then, does not provide any *significant* performance improvement. Therefore, we have found no sets of factors that outperform the original Fama French three-factor model, and found that portfolio construction plays an important role in the performance of each individual model .

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