Day 9 Notes

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1 Agenda

- Quizzes back
- No simple mapping from # to grade - based on "expected number"
- Intro to LP
- Examples of LP

2 HW #2

- Definition of NP: **yes** example has certificate which can be verified in polynomial time.
- Definition is not symmetrical!
- Many problems in NP have inverse outside of NP

3 Linear Programming

- Example: Political Election categories (see day 8)
- Ex. Input: $20k on roads, $0 on guns, $4k on farms, $9k on gas
- Minimize $x_1 + x_2 + x_3 + x_4$ (total cost)

$$20(-2)+0(5)+4(0)+9(10) = 50{,}000$$
$$20(5)+0(2)+4(0)+9(0) = 100{,}000$$
$$20(3)+0(-5)+4(0)+9(-2) = 200{,}000$$

General Linear Program

Given a set of constants $a_1, \ldots, a_n$ and a set of variables $x_1, \ldots, x_n$, a **linear function** f on the variables is:

$$f(x_1, \ldots, x_n) = a_1(x_1) + \ldots + a_n(x_n) = \sum_{\alpha=1}^{n} a_\alpha x_\alpha$$

Given a constant b and a linear function f,
$f() \leq b$ and $f() \geq b$ are linear inequalities
$f() = b$ is a linear equality

< and > are not linear!

LP Problem: The problem of maximizing or minimizing a linear function subject to a finite set of linear constraints.
Example:
Maximize: $x_1 + x_2$
Subject to:
$4x_1 - x_2 \leq 8$
$2x_1 + x_2 \leq 10$
$5x_1 - 2x_2 \geq -2$
$x_1, x_2 \geq 0$

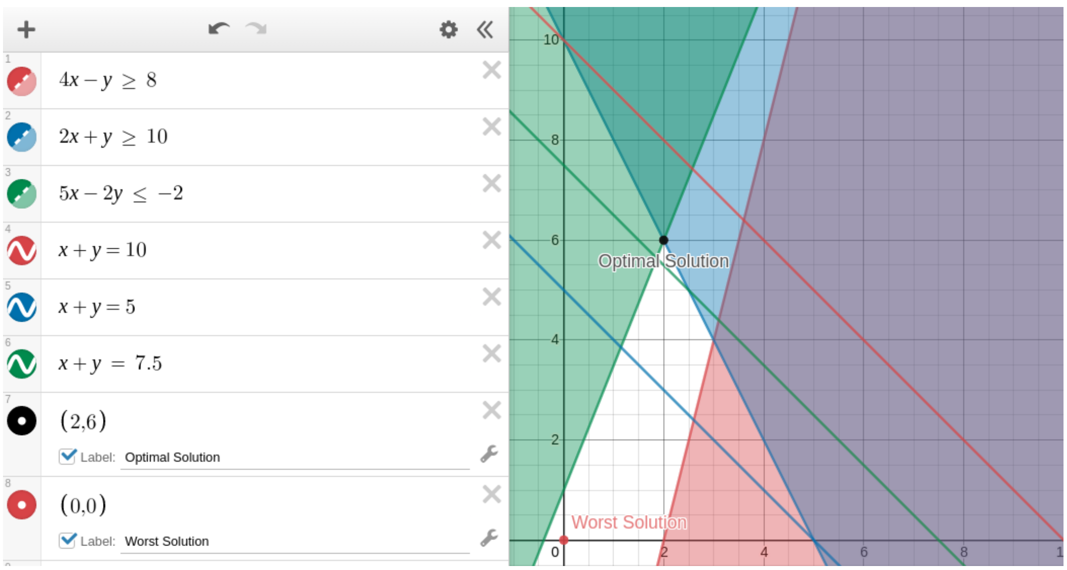

Figure 1: Graph of Solution Space: where all regions overlap is valid (marked in white). $x = y$ lines show value -> further from origin is better.

Example: Reclaiming Solid Waste

Inputs are 3 types of materials: 1,2&3

Table 1: Material Availability

Material	Available Pounds/Week
1	100
2	200
3	300

Problem:
Maximize: $5Y_A + 10Y_B$

Table 2: Grades

Grade	Spec	profit/pound
A	$M_1 \leq 30\%, M_2 \leq 40\%$	5
B	$M_2 = 50\%, M_2 \leq 20\%$	10

Let: Z_{MN} = the proportion of grade M that is material N
Subject to:
$Z_{A_1} \leq 0.3$
$Z_{A_2} \leq 0.4$
$Z_{B_2} = 0.5$
$Z_{B_3} \leq 0.2$
$Y_A * Z_{A_1} + Y_B * Z_{B_1} \leq 100$
$\vdots$
UH OH! This multiplies variables which isn't linear...
Fix:
Let: X_{MN} = <u>amount</u> of Material N in grade M
New Constraints:
$X_{A_1} + X_{B_1} \leq 100$
$X_{A_2} + X_{B_2} \leq 200$
$X_{A_3} + X_{B_3} \leq 300$
New Objective Function:
Maximize: $5(X_{A_1} + X_{A_2} + X_{A_3}) + 10(X_{B_1} + X_{B_2} + X_{B_3})$

Takeaways

- Variable choices matter! Effects speed and solve-ability
- Seemingly non-linear variables can be re-written as linear variables

Standard Form

Given constants $c_1, \ldots, c_n$, $b_1, \ldots, b_m$ and $m * n$ values a_g for $i = 1 \to m$ and $j = 1 \to n$, find $x_1, \ldots, x_n$ such that:
Maximize: $\sum_{j=1}^{n} c_j x_j$
Subject to: $\sum_{i=1}^{n} a_{ij} x_j \leq b_i$ for $i = 1 \to m$
$x_j \geq 0$ for $j = 1 \to n$
n = # variables m = # constraints

- In standard form, all variables $x \geq 0$
- Only maximization as operation
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Concise Standard Form

- $\boldsymbol{A} = (a_{ij})$
- $\boldsymbol{b} = (b_i)$
- $\boldsymbol{c} = (c_j)$
- $\boldsymbol{x} = (x_j)$

An LP formulation in standard form is: maximize $C^T x$ subject to $Ax \leq b$, $x \geq 0$.