

# Day 13 Notes

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## 1 Agenda

- ILP Advanced Techniques
- Project 3
- Intro to Branch & Bound
- TODO:
  - Reading
  - HW due Tomorrow
  - Project due Monday
  - Quiz next Tuesday

## 2 ILP

$$3x_1 + 2x_2 \leq 18$$

OR

$$x_1 + 4x_2 \leq 16$$

- Binary variables  $x_1, x_2$
- How can we phrase this in terms of ILP?
- Typically, both would need to be solved
- Add a large number multiplied by a new binary variable
- Negate new variable in one equation so that this variable can control
- Now at least one constraint must be satisfied, or both can be satisfied

$$3x_1 + 2x_2 \leq 18 + yM, M \rightarrow \infty$$

$$x_1 + 4x_2 \leq 16 + (1 - y)M$$

- Say we have three constraints now
- How to implement or with more than 3 inputs?
- Add 3 new variables
- Add constraint that 3 variables sum to less than 3

$$\begin{aligned}
3x_1 + 2x_2 &\leq 18 + y_1 M \\
x_1 + 4x_2 &\leq 16 + y_2 M \\
2x_1 + 3x_2 &\leq 14 + y_3 M \\
y_1 + y_2 + y_3 &\leq 2
\end{aligned}$$

### 3 Branch & Bound

- Useful for solving ILP
- Also very useful for other stuff
- Based on divide and conquer
- In 1,  $x_0 = (1.5, 2.5)$  is the solution to the LP relaxation of this problem.
- $x_0$  is non-integral, so we keep going
- Subdivide the feasible region into two non-overlapping feasible regions such that no feasible solutions are excluded.
- SP1: add constraint  $x \leq 1$
- SP2: add constraint  $x \geq 2$
- Keep subdividing on integer boundaries
- for SP1: add  $y \leq 1$  SP3, add  $y \geq 2$  SP4
- For SP2: add  $x \leq 1$  SP5,  $y \geq 2$  SP6
- SP4, SP6 infeasible, no points in those areas
- Find LP solns to SP3, SP5
- LP soln to SP3 turns out to be integral - no need to keep going
- For SP5, add  $x \leq 2$  SP7, and  $x \geq 3$  SP8
- SP8 infeasible
- SP7 has integral soln
- Compare integral solutions to SP7 and SP3
- No subproblems left to expand, choose best LP soln (SP7)

### 4 Project

- Solve knapsack and coloring using ILP
- Instances all converted already

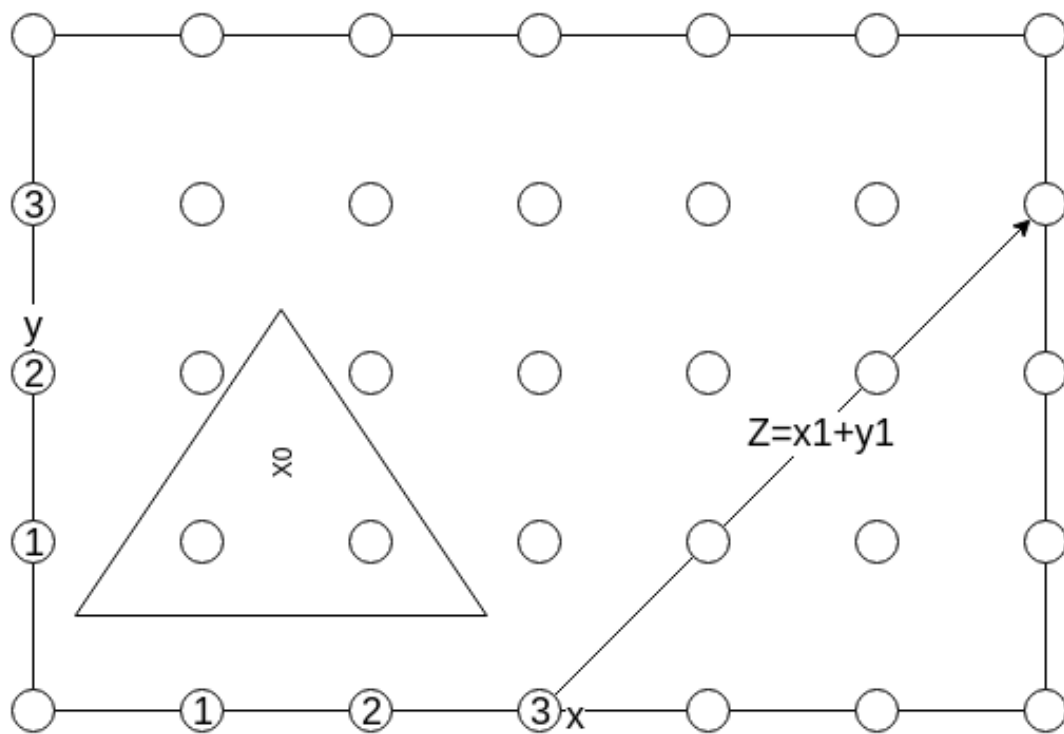


Figure 1: ILP w/ Branch and Bound Example

- Must solve formulation of problem
- Turn in model files, output files
- Limit solver runtime to 10 minutes