Department of Electrical and Computer Engineering
ECE 4542: Advanced Engineering Algorithms

Homework #1

Solve the following problems.

1. Consider the following implementation of a bubble sort. Given an array a[] containing n values, the algorithm repeatedly scans the list from top to bottom. In each pass, adjacent entries are compared and swapped if they are in the wrong order. Assume that each instruction takes a constant amount of time.

```
for (i=0; i<n-1; i++)
  for (j=0; j<n-1-i; j++)
    if (a[j+1] < a[j])    /* compare the two neighbors */
    {
      tmp = a[j];         /* swap a[j] and a[j+1] */
      a[j] = a[j+1];
      a[j+1] = tmp;
    }
```

   (a) What is the best-case running time of this algorithm? Give the tightest asymptotic bound on the best-case running time using O notation.
   (b) What is the worst-base running time of this algorithm? Give the tightest asymptotic bound on the worst-case running time using O notation.

2. Consider a sequential search algorithm. Given a value, the algorithm scans a linear array until the value is found. If the value is not found, that algorithm returns a fail code

   Answer the same questions as above.

3. Prove the following, or give a counter example:

   (a) $f(n) = O(g(n))$ and $g(n) = O(h(n))$ implies $f(n) = O(h(n))$.
   (b) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.

4. Give optimization and decision versions of the following problems:

   (a) Minimum spanning tree in a weighted graph
   (b) Maximum matching in a graph
   (c) Shortest path between two vertices in a weighted graph

5. Give precise definitions for the following terms:

   (a) a heuristic,
   (b) a polynomial-time algorithm,
   (c) an intractable problem,
   (d) the complexity class P.