

# Day 5 Notes

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## 1 Agenda

- Review of NPC
- Proving problems NP Complete
- Examples
- Subproblems

## 2 Announcements

- Quiz on Wednesday - through most recent homework (NOT NPC)
  - Know big ideas
  - Know important terms
  - Know practical applications
  - Re-Solve problems we've seen for practice
  - Make sure we've done the reading
  - No code on quiz
- Finish up reading about NPC stuff

## 3 NP Completeness Review

**To be in NP a problem,  $L$ , must:**

- $L \in NP$
- For every problem  $L' \in NP$ ,  $L' \leq L$

**How to Prove**

- Prove that  $SAT \leq L$
- Given a problem  $\pi \in NP$  whos complexity is unknown, how to find?

- Define special case  $\pi'$  containing a subset of the instances of  $\pi$
- Prove that  $\pi'$  is NPC
- $\pi' \leq \pi$  because every special case is already a regular case as well
- $\pi$  is NPC, since its simpler subset,  $\pi'$  is NPC
- QUIZ: Explain why the last bullet is true

## 4 Examples

Partition: given a set  $A$  and size  $s(a)$  for all  $a \in A$  is there a subset  $A' \in A$  such that  $\sum s(a) = \sum s(!a)$  where  $!a$  is the set of elements not in  $s$ . Basically, divide  $A$  into two sets with equal size.

Knapsack: given a set,  $U$ , a size  $s(u)$  and a value  $v(u)$  for all  $u \in U$ , and size constraint  $B$ , and a value goal  $K$ , is there a subset  $u' \in U$  such that  $\sum s(u') \leq B$  and  $\sum v(u') \geq K$ ?

- Claim: Partition  $\leq$  Knapsack
- Prove: Given an instance of Partition, show that we can produce an instance of knapsack with the same answer.
- Answer: Set  $K = B = \frac{1}{2} \sum s(u)$ .
- Idea: sandwich  $K$   $B$  such that knapsack will find same answer as partition
- If Knapsack is yes instance, Partition will be yes instance
- If Partition is yes instance, Knapsack is yes instance
- If partition is NPC, Knapsack is NPC

## 5 Problem as Tuple

- Consider a problem  $\pi = (D, Y)$  where  $D$  is all instances,  $Y$  is all yes instances
- Sub-problem  $\pi' = (D', Y')$  reduces to  $\pi$

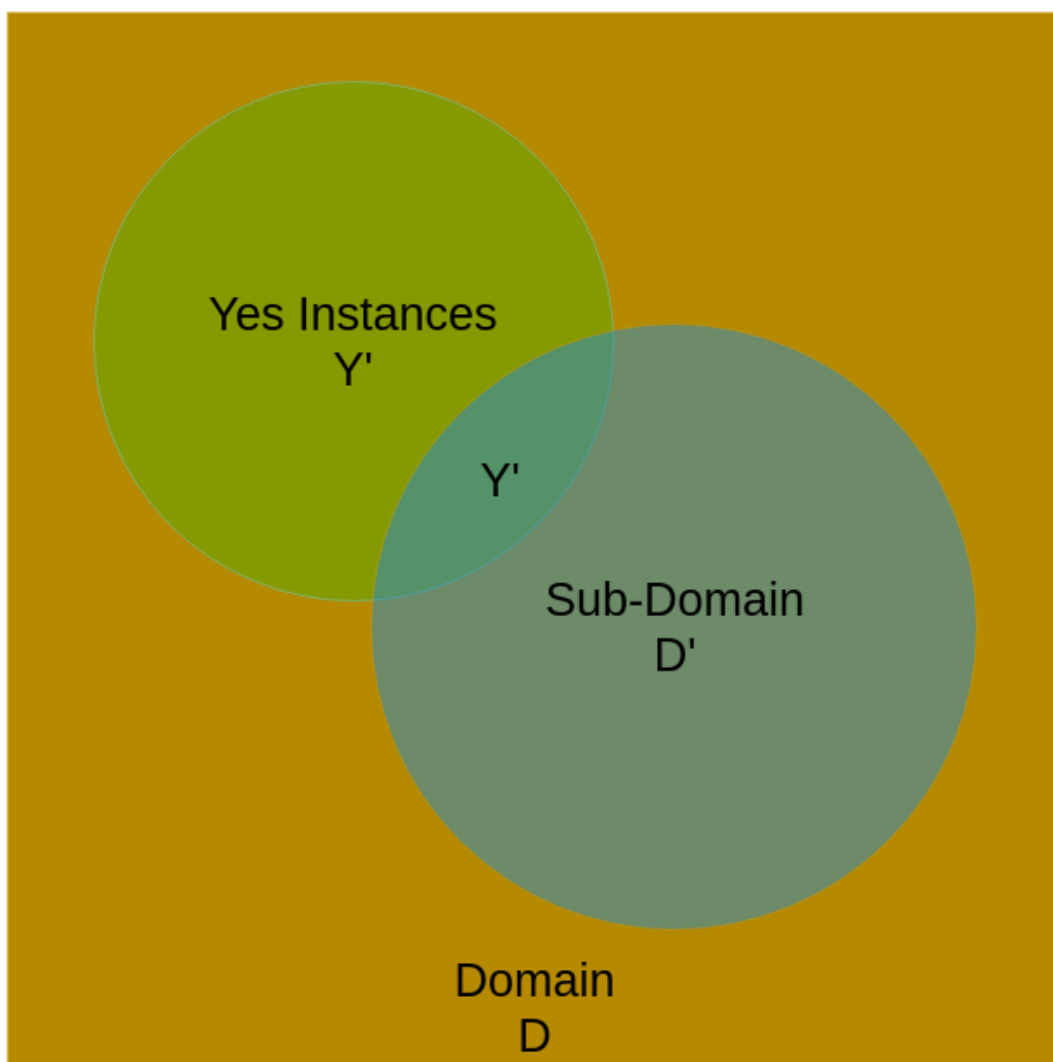


Figure 1: Relation of  $D$ ,  $D'$ ,  $Y$ ,  $Y'$