12.5-6 a) False. It's the other way around. That's why it's called a "relaxation," because it relaxes (expands) the feasible region.

b) True. If you then restrict the feasible region to integers you've only eliminated points that weren't optimal anyway. You can't do better.

c) False! See section 12.4 and its problems for examples.

12.5-7 (a) Initialization Step: Set $Z^*=+\infty$. Apply the bounding and fathoming steps and the optimality test as dexcribed below on the whole problem. If the whole problem is not fathomed, it becomes the initial subproblem for the first iteration below.

Iteration:

1. Branching: Choose the most recently created unfathomed subproblem (breaking ties by selecting the one with the smallest bound). Among the assignees not yet assigned for the current subproblem, choose the first one in the natural ordering to be the branching variable. Subproblems will correspond to each of the possible remaining assignments for the branching assignee.
   Form a subproblem for each remaining assignment by deleting the constraint that each of the unassigned assignees must perform exactly one assignment.

2. Bounding: For each new subproblem, obtain its bound by choosing the cheapest assignee for each remaining assignment and totaling the costs.

3. Fathoming: For each new subproblem, apply the two fathoming test below:
   Test 1. Its bound>= $Z^*$
   Test 2. The optimal solution for its relaxation is a feasible assignment (if this solution is better than the incumbent, it becomes the new incumbent and Test 1 is reapplied to all unfathomed subproblems with the new smaller $Z^*$).

Optimality Test: Identical to that in the text.

(cont.)

12.6-1 c) Let $x_1 = y_{11} + 2y_{12}$

$x_2 = y_{21} + 2y_{22}$

The BIP formulation is:

Maximize $Z = -3y_{11} - 6y_{12} + 5y_{21} + 10y_{22}$

subject to: $5y_{11} + 10y_{12} - 7y_{21} - 14y_{22} \geq 3$

$y_{11}, y_{12}, y_{21}, y_{22}$ binary.

d)

```
                --- X4=1,Inf.,F
               /
              /
             /
            /
           /
          /
         /
X=(1,1,0,0.86),Z=-0.4
         \
          \
           \
            \
             \
              \
               \
                --- X4=0,X=(0,1,1,0),Z=-1
```

The optimal solution is:

$(y_{11}, y_{12}, y_{21}, y_{22}) = (0,1,1,0)$

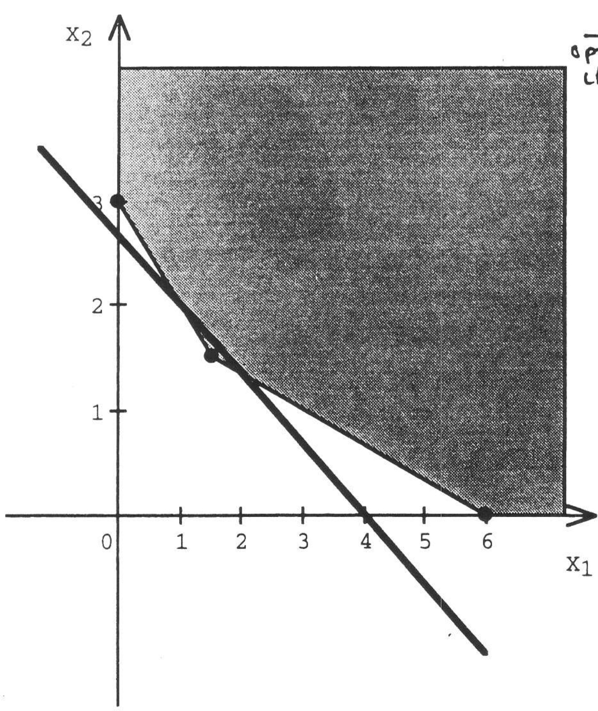
$Z = -1$

so $x_1 = 2$, $x_2 = 1$ as in part (a).

Solve Interactively by the Graphical Method:

12.6-2

(a)




| | Corner Points | | Z |
|---|---|---|---|
| optimal LP-relax. → | ( 1.5, | 1.5) | 7.5 |
| | ( 0, | 3) | 9 |
| | ( 6, | 0) | 12 |

Optimal value of Z: 8

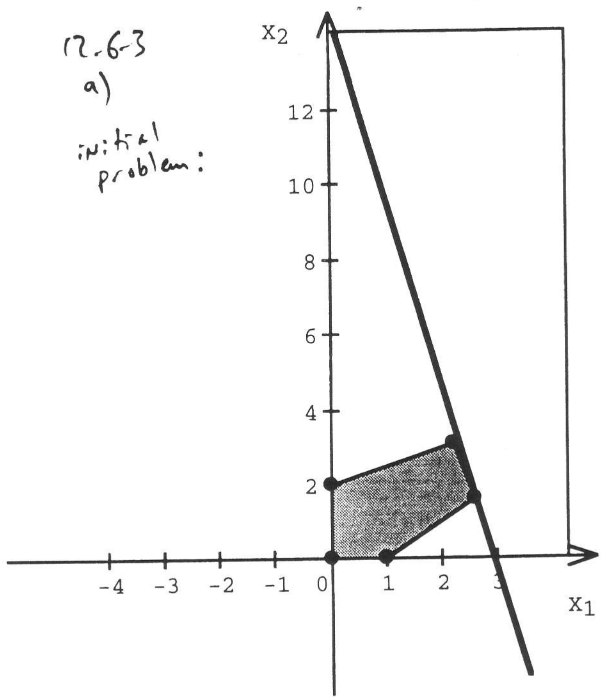
Optimal integer solution: (1,2)

(cont.)

12.6-3

a)

initial problem:



| Corner Points | Z |
|---|---|
| (2.2222,3.1111) | 14.222 |
| ( 0, 2) | 2 |
| ( 2.6, 1.6) | 14.6* |
| ( 1, 0) | 5 |
| ( 0, 0) | 0 |

Optimal value of Z: 14.6

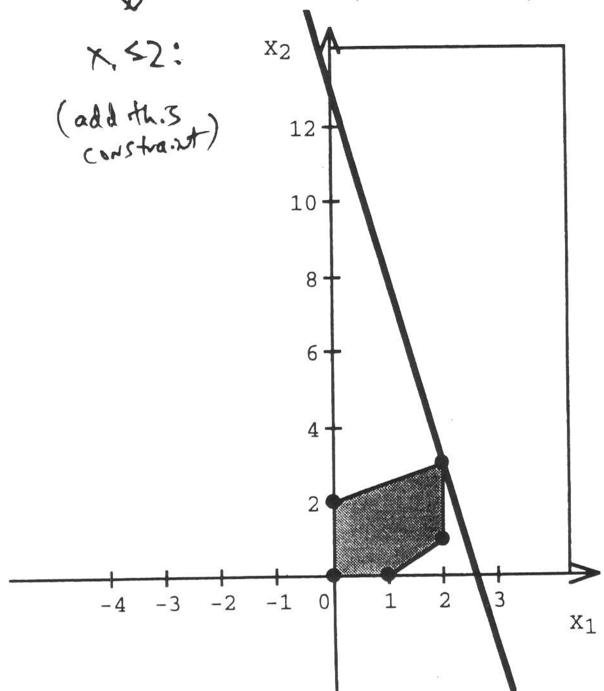
Optimal solution: ( 2.6, 1.6)

Branch: $x_1 \geq 3$ infeasible (empty feasible region)

$\downarrow$

$x_1 \leq 2$:

(add this constraint)



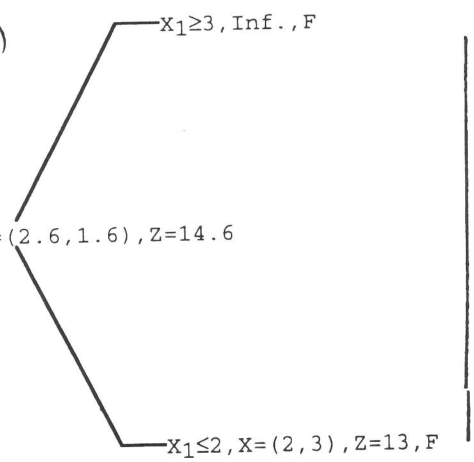
| Corner Points | Z |
|---|---|
| ( 2, 3) | 13* |
| ( 0, 2) | 2 |
| ( 2, 1) | 11 |
| ( 1, 0) | 5 |
| ( 0, 0) | 0 |

Optimal value of Z: 13

Optimal solution: ( 2, 3)

This is integer, so we're done.

b)




Optimal Solution:
**X** = ( 2, 3)
Z*= 13

c) Solution:
$X_1 = 2$
$X_2 = 3$

$Z = 13$

12.6-8

```
            --- X1>=3,X=(3,0,0,0),Z=15,F
           /
          /
         /
        /
       /
      /
     /
X=(2.5,2.5,0,0),Z=22.5
     \
      \
       \
        \
         \
          \
           \
            --- X1<=2,X=(2,2,1,0),Z=22
```

The optimal solution is

$(x_1, x_2, x_3, x_4) = (2, 2, 1, 0)$

with $z = 22$.

Solution Tree:

$X_3 \geq 1$, X=(2,3,1,0,0),Z=14,F

$X_1 \geq 2$, X=(2,0,0.33,0,0),Z=10.3

$X_3 \leq 0$, X=(2,2,0,0,0),Z=12

X=(1.42,0.08,0.42,0,0),Z=7.58

$X_3 \geq 1$, X=(1,0,1,0,2),Z=12

$X_1 \leq 1$, X=(1,0,0.5,0,2),Z=11.5

$X_3 \leq 0$, X=(1,0,0,3,2),Z=17,F

Optimal Solution:
**X** = ( 1, 0, 1, 0, 2) , (2,2,0,0,0)
Z*= 12

12.6-11

a) (1) Use the quadratic programming (QP) relaxation:

$$\max.\ Z = 2x_1 - x_1^2 + 3x_2 - 3x_2^2$$

s.t. $x_1 + x_2 \le 3/4$ | same

$\frac{1}{5} \le x_1 \le \frac{1}{2}$ , $\frac{1}{5} \le x_2 \le \frac{1}{2}$ | relaxed

(2) The fathoming tests are the same as those stated in the chapter. A subproblem is fathomed if it's infeasible, has value $\le Z^*$ for the current incumbent, or has a qualifying solution, where to qualify here we need $x_1, x_2 \in \{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}\}$.

(3) To branch from a value for $x_j^*$ where $(m+1)^{-1} < x_j^* < m^{-1}$ for some $m = 2,3,4$, we use the two alternative constraints

$x_j \le (m+1)^{-1}$ or $x_j \ge m^{-1}$

b) The branch & bound tree will look like this:

relaxed sol'n.
(.4375, .3125) $Z = 1.328$

① branch

- $x_1 \le \frac{1}{3}$
  - $(\frac{1}{3}, .4167)$ $Z = 1.29$
  - ② branch
    - $x_2 \le \frac{1}{3}$: $(\frac{1}{3}, \frac{1}{3})$ $Z = 1.22$ F
    - $x_2 \ge \frac{1}{2}$: $(\frac{1}{4}, \frac{1}{2})$ $Z = 1.188$ F
- $x_1 \ge \frac{1}{2}$ ($x_1 = \frac{1}{2}$ in this problem)
  - $(\frac{1}{2}, \frac{1}{4})$ $Z = 1.3125$ Incumbent
  - optimal!

So $x_1 = \frac{1}{2}$, $x_2 = \frac{1}{4}$, $Z = 1.3125$ is optimal, agreeing with 12.3-5.

12.7-1

a) $x_1 = 0$, $x_3 = 0$

b) $x_1 = 0$

c) $x_1 = 1$, $x_3 = 1$

12.7-2

a) $x_1 = 0$

b) $x_1 = 1$, $x_2 = 0$

c) $x_1 = 0$, $x_2 = 1$

12.7-3. From eqn. 1, $x_3 = 0$; this makes eqn. 1 redundant.
From eqn. 3, $x_5 = 0$, $x_6 = 1$; now eqn. 3 is redundant
Since $x_6 = 1$, from eqn. 2 $x_2 = x_4 = 0$, and this is redundant.
Finally eqn. 4 reduces to $x_1 = 0$, which leaves all equations redundant. We have $x_1 = x_2 = x_3 = x_4 = x_5 = 0$, $x_6 = 1$, $x_7$ ?

12.7-4

a) redundant, because even if all variables are 1, $2+1+2 \le 5$

b) not redundant, as $(1,0,1)$ violates this $(8 \nleq 5)$

c) not redundant, as $(0,0,0)$ (for example) violates

d) redundant, because $(0,1,1) \rightarrow 0-1-2 \ge -4$ still; this is the worst case, because we let variables with positive coefficients $=0$ and variables with negative coefficients $=1$ to try to violate the $\ge -4$ condition, and we can't do it.

12.7-5

$$3x_1 - 2x_2 + x_3 \le 3$$

$$S = 4 < 3 + |a_1| = 6$$

$$\bar{a}_1 = S - b = 1 \qquad \bar{b} = S - a_1 = 1$$

$$\rightarrow x_1 - 2x_2 + x_3 \le 1$$

$$S = 2 < 1 + |a_2|$$

$$a_2 := b - S = -1$$

$$\rightarrow \boxed{x_1 - x_2 + x_3 \le 1}, \text{ done.}$$

Check for yourself that the same binary $(x_1, x_2, x_3)$ vectors satisfy both the original and the tightened constraint.

12.7-6

$$x_1 - x_2 + 3x_3 + 4x_4 \ge 1$$

$$-x_1 + x_2 - 3x_3 - 4x_4 \le -1$$

$$S = 1 < -1 + |a_3| = 2$$

$$\rightarrow -x_1 + x_2 - 2x_3 - 4x_4 \le -1$$

$$S = 1 < -1 + |a_4| = 3$$

$$\rightarrow \boxed{-x_1 + x_2 - 2x_3 - 2x_4 \le -1}, \text{ done.}$$

(these steps could have been done at the same time, since they involve $a_j < 0$)