Day 10 Notes

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1 Agenda

- Quiz
- LP Standard Form
- More LP examples
- Solving LP Problems

2 LP Examples

Personnel Scheduling Problem

- Goal: find an assignment of people to shifts such that enough people are working in each slot and total cost is minimized.
- Objective function: $z = 170x_1 + 160x_2 + 175x_3 + 180x_4 + 195x_5$
- Constraint example: $x_1 + x_2 \geq 79$
- Constraint form: $\sum$ active shifts during period $\geq$ required personnel during period.
- Following constraints for all time periods create many redundant constraints
- **redundant constraint:** Constraint can be deleted without changing the solution to the LP
- Solution to this problem: x=(48,31,39,43,15), z=30,610

Table 1: Personnel Schedules

Shift	Time Range	Hourly Wage $
1	6a-2p	170
2	8a-4p	160
3	12p-8p	175
4	4p-12a	180
5	10p-6a	195

3 LP Standard Form

- Maximize $\sum_{j=1}^{n} c_j x_j$ subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1:m, x_j \geq 0$

Table 2: People Required

Shift	People Needed
6-8	48
8-10	79
10-12	65
12-2	87
2-4	64
4-6	73
6-8	82
8-10	43
10-12	52
12-6a	15

- Always maximizing
- Constraints always use $\leq$
- All variables ≥ 0
- Any LP problem can be rephrased into a standard form equivalent
- **Equivalent:** Two LP formulations are equivalent if they are both maximizing and for every feasible solution in L there is a corresponding feasible solution in L' with the same objective value and visa versa.
- If L is minimizing and L' is maximizing, multiply objective function by -1
- If not all variables are constrained by ≥ 0 : create 2 positive variables and take their difference to get the unbounded value (e.g. $x_2 \to (x_2' - x_2'')$)
- If equality used instead of inequality in constraint: Split into $\leq$ and $\geq$ constraints, and flip $\geq$ one.