Day 5 Notes

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1 Agenda

- Review of NPC
- Proving problems NP Complete
- Examples
- Subproblems

2 Announcements

- Quiz on Wednesday - through most recent homework (NOT NPC)
  - Know big ideas
  - Know important terms
  - Know practical applications
  - Re-Solve problems we've seen for practice
  - Make sure we've done the reading
  - No code on quiz
- Finish up reading about NPC stuff

3 NP Completeness Review

To be in NPC, a problem, L, must:

- $L \in NP$
- For every problem $L' \in NP$, $L' \leq L$

How to Prove

- Prove that $SAT \leq L$
- Given a problem $\pi \in NP$ whos complexity is unknown, how to find?

- Define special case π' containing a subset of the instances of π
- Prove that π' is NPC
- $\pi' \leq \pi$ because every special case is already a regular case as well
- π is NPC, since its simpler subset, π' is NPC
- QUIZ: Explain why the last bullet is true

4 Examples

Partition: given a set **A** and size $s(\boldsymbol{A})$ for all $a \in \boldsymbol{A}$ is there a subset $\boldsymbol{A'} \in \boldsymbol{A}$ such that $\sum s(a) = \sum s(!a)$ where $!a$ is the set of elements not in s. Basically, divide A into two sets with equal size.

Knapsack: given a set, U, a size $s(u)$ and a value $v(u)$ for all $u \in U$, and size constraint B, and a value goal K, is there a subset $u' \in U$ such that $\sum s(u') \leq B$ and $\sum v(u') \geq K$?

- Claim: Partition $\leq$ Knapsack
- Prove: Given an instance of Partition, show that we can produce an instance of knapsack with the same answer.
- Answer: Set $K = B = \frac{1}{2} \sum s(u)$.
- Idea: sandwich K & B such that knapsack will find same answer as partition
- If Knapsack is yes instance, Partition will be yes instance
- If Partition is yes instance, Knapsack is yes instance
- If partition is NPC, Knapsack is NPC

5 Problem as Tuple

- Consider a problem $\pi = (D, Y)$ where D is all instances, Y is all yes instances
- Sub-problem $\pi' = (D', Y')$ reduces to π

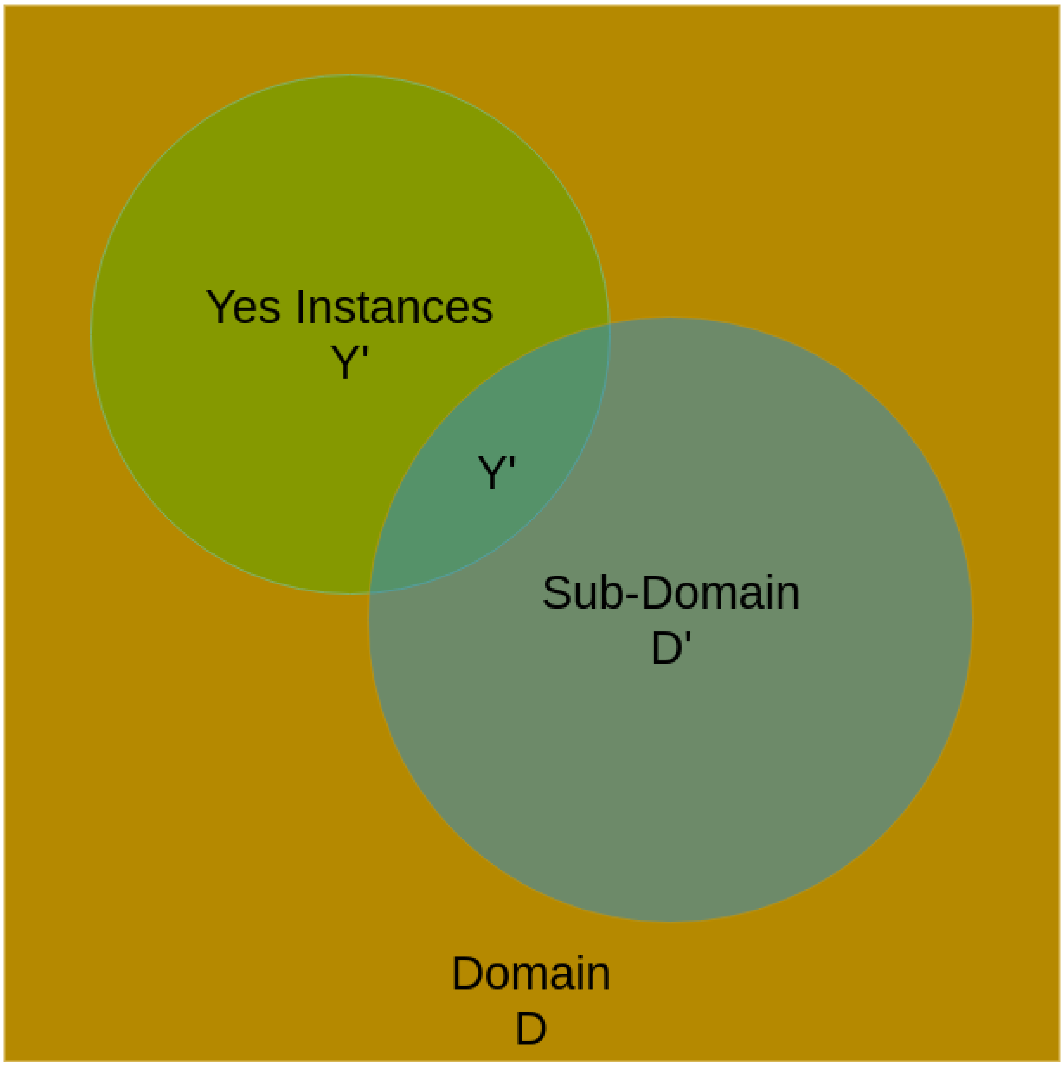

Figure 1: Relation of D, D', Y, Y'