Day 23 Notes

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1 Agenda

- Review of Flow Networks
- Solving Max Flow Problsm
- Announcement: Grading too slow, expect delays
- Announcement: reading, project

2 Max Flow

- Directed graph
- Capacities - can be different in each direction
- Single pipe - pos flow one direction means negative flow in other direction
- 2 special nodes: source and sink where conservation doesn't apply
- Value leaving source is total value of flow

Solving Max Flow Problems

- Given nodes u, v, the **residual capacity**, $c_f(u, v)$ is the additional capacity left in the pipe from $u \rightarrow v$
- Critical Idea: In the residual network, if a path from source to sink exists with all positive residuals, then the flow is not optimum because it can be increased along this path.
- above question fair game on quiz...
- Given a flow network and a flow, an augmenting path is a simple path (no cycles) from s to t in the residual network. The residual capacity of an augmenting path is the minimum residual capacity of the edges on the path.
- **Ford-Fulkerson Algorithm**:

```
# Ford Fulkerson Algorithm
def FF(g):
    flow = 0
    while p = is_aug_path(source,sink):
```

```
    cfp = residual_capacity(p)
    flow += cfp
```

– How to prove output is optimal?

3 Ford-Fulkerson Proof

– **Cut**: a cut of a flow network $G(V, E)$ is a partition of V into S and $T = V - S$ such that $s \in S$ and $t \in T$, where s is source, and t is sink.

– More simply: If source is at left, Sink is at right, cut the graph with a vertical line.

– **Net Flow** across a cut (S, T) is $f(S, T)$, the sum of all flow from nodes in S to nodes in T.

– **Note:** Net flow depends on sign, flows going backwards are subtracted.

– **Cut Capacity:** $c(S, T)$ is sum of capacities of all edges from nodes in S to nodes in T.

– Cut Capacity is upper bound on net flow across the cut.

– **Minimum Cut**: cut whose capacity is the minimum of all cuts in the network

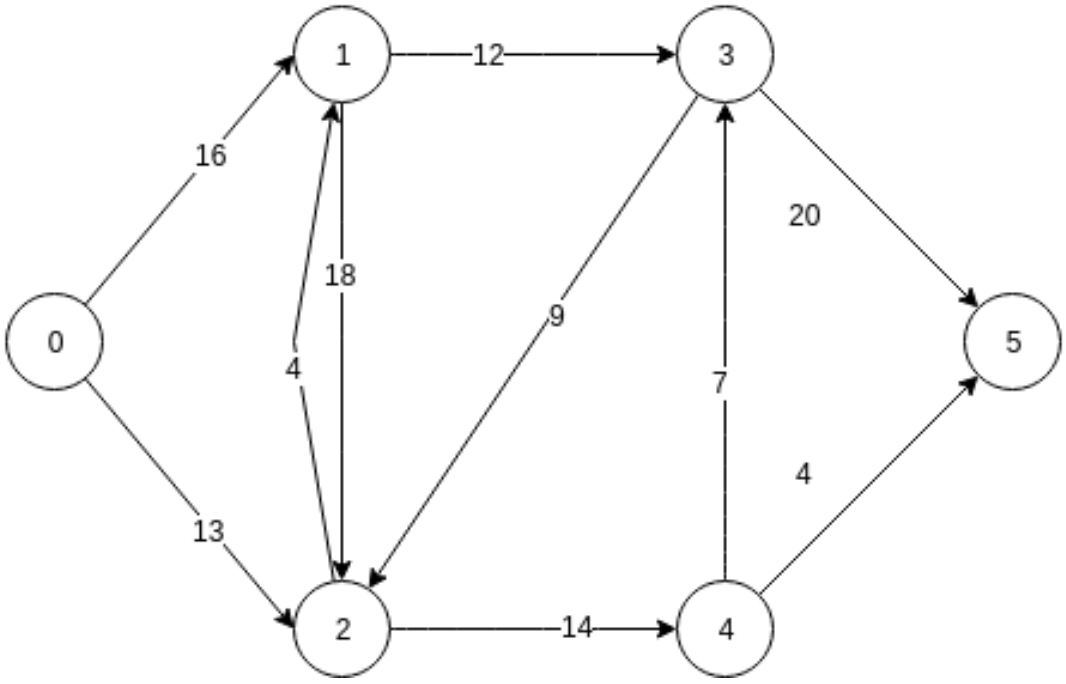

Figure 1: Max Flow Capacities

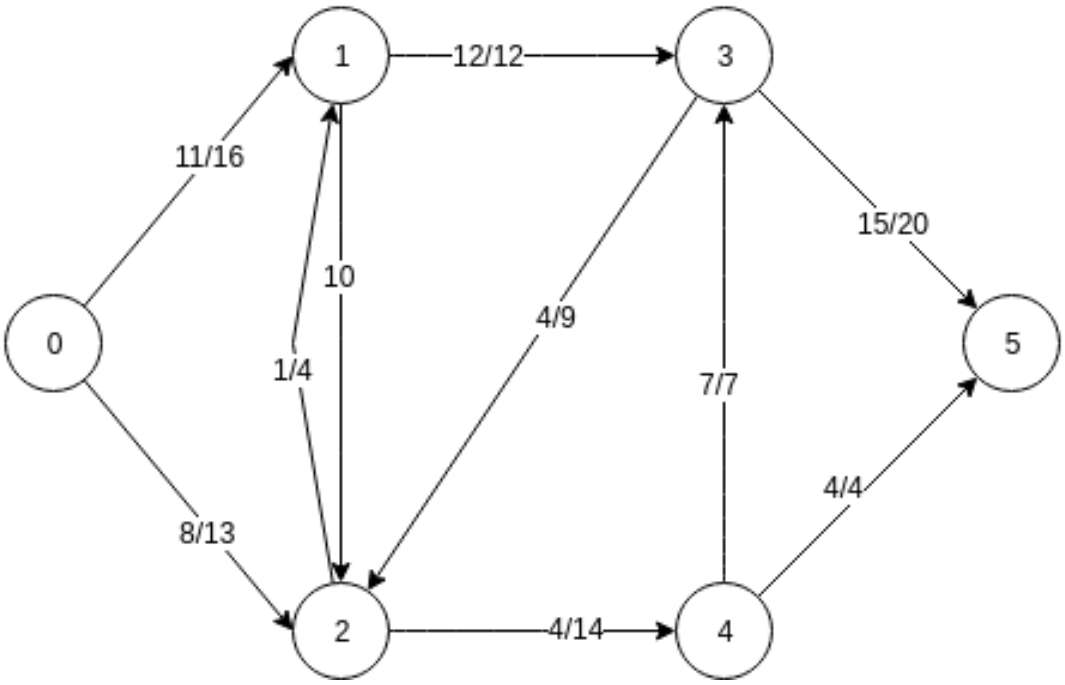

Figure 2: Example Flow

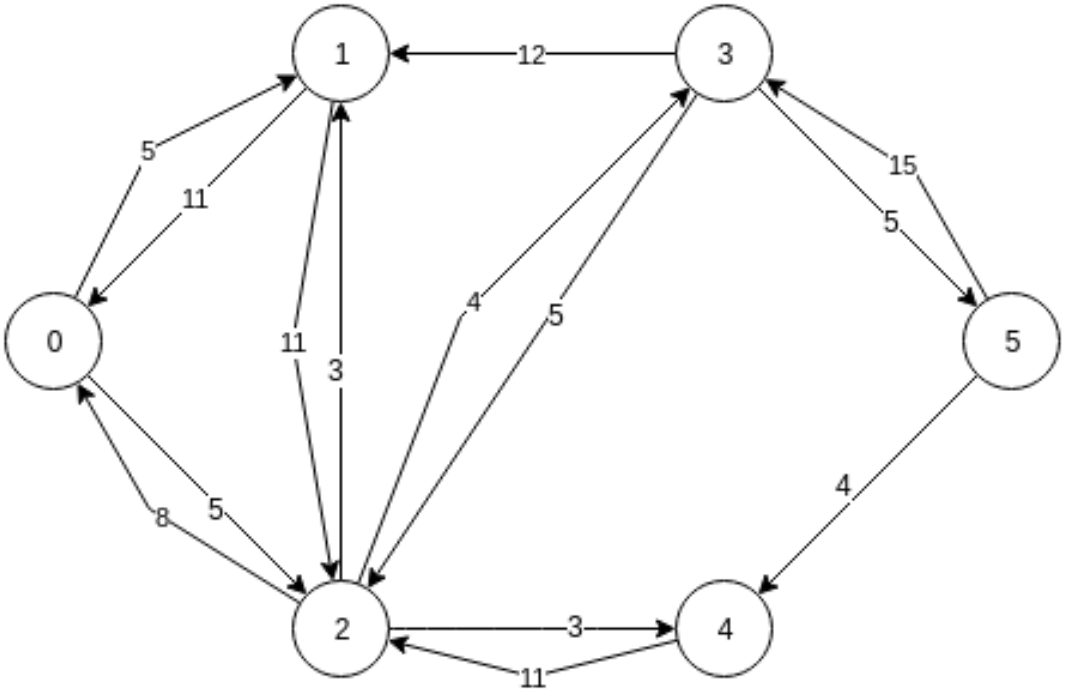

Figure 3: Residual Flow Diagram from Flow in Fig. 2