Day 15 Notes

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1 Agenda

- Wrap up advanced ILP techniques
- Applying branch and bound

2 Advanced LP Techniques

- Review of cutting plane example from day 14.

3 Branch and Bound Beyond ILP

- Job scheduling problem
- Given a set of jobs to process.
- Each job takes the same amount of time to process.
- Setup time depends on the current job and the previous job
- Goal: minimize total time - same as minimizing setup time
- Brute force - $O(n!)$

Table 1: Job Scheduling Example Problem

Type	1	2	3	4	5
None	4	5	8	9	4
1	0	7	12	10	9
2	6	0	10	14	11
3	10	11	0	12	10
4	7	8	15	0	7
5	12	9	8	16	0

- Branch and Bound technique
- Possible starting bounds
  - $-\infty$
  - 0

  - min value * num jobs - 20 in this case
  - $\sum$ min value in each column - 30 in this case
  - Can't easily tighten this now, but as tree progresses can tighten quickly
- Assume schedule begins with (3,1) - cost so far 8 + 10 = 18
- To compute bound, eliminate rows `none` and 3
- Take minimum from remaining problems excepting these rows
- Optimistic bound 8 + 10 + 7 + 10 + 7 = 42
- This bound is infeasible solution - jobs 2 and 4 both done directly after job 1
- This seems similar to the infeasible optimistic bounds from solving ILP using LP!
- If bound is feasible, then this path is fathomed!

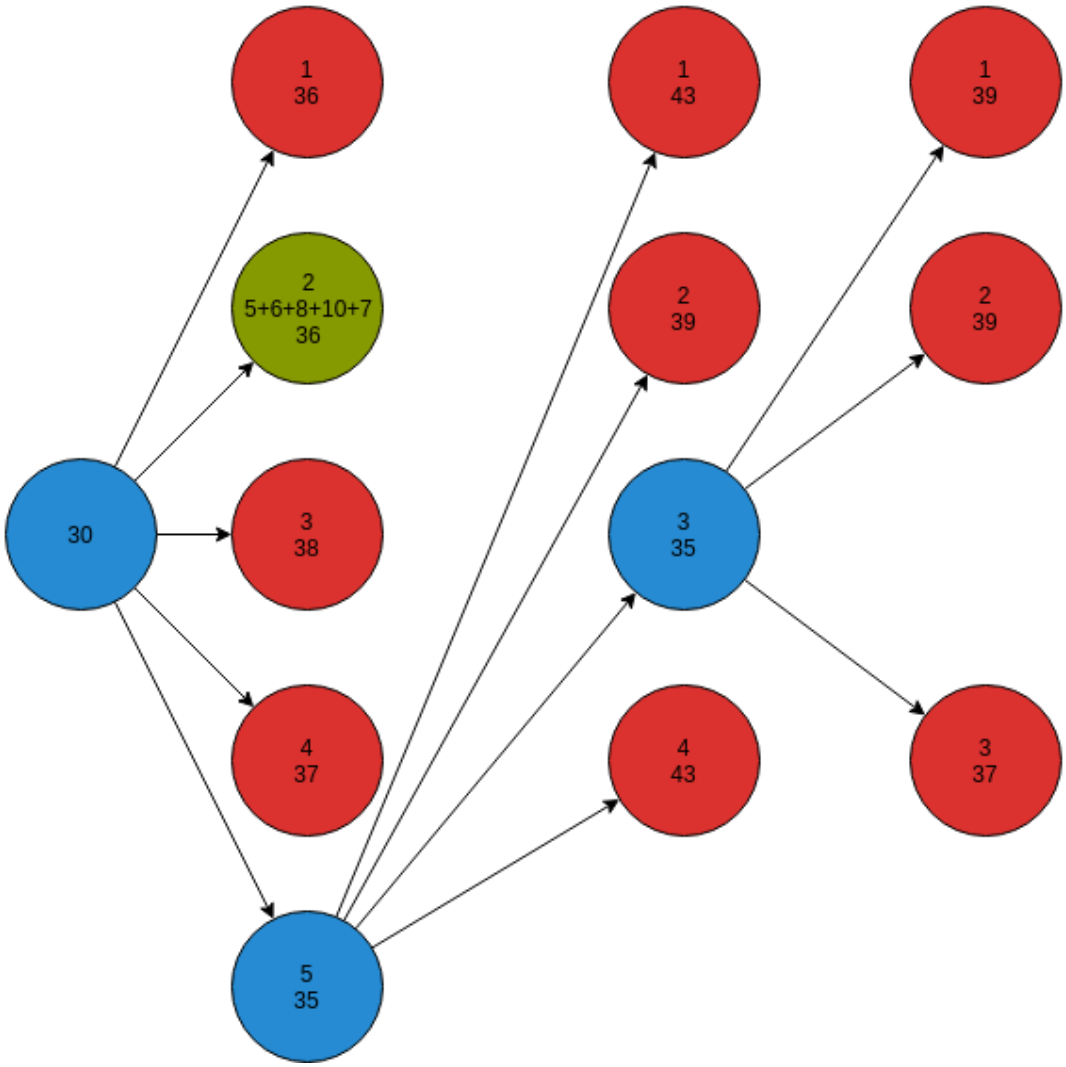

Figure 1: Job Scheduling Search Tree

Knapsack w/ Branch & Bound

Table 2: Knapsack Instance, cost limit = 100

	1	2	3	4	5	6
v_i	10	35	40	18	2	4
w_i	10	50	100	45	5	20
ratio	1	0.7	0.4	0.4	0.2	

- Sort by value/weight ratio
- Start tree at highest ratio item
- Optimistic bound:

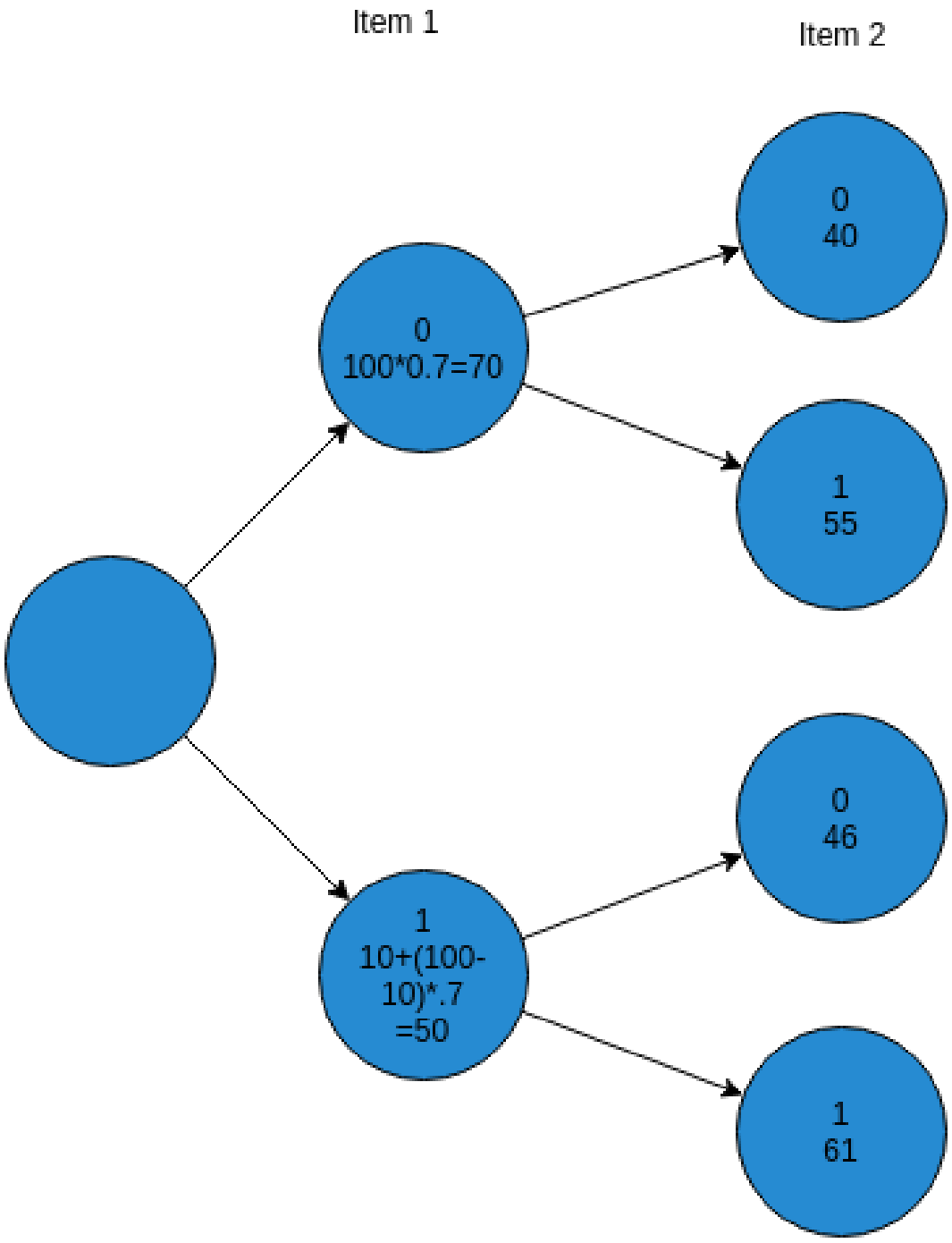

Figure 2: Knapsack Example