Day 25 Notes

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June 18, 2019

1 Agenda

- Min-Cost Flow Networks

2 Min-Cost Flow Networks

- Extend the idea of a flow network by adding a non-negative cost $d(u,v)$ to each edge
- For each node v, $b(v)$ is the net flow generated at v.
- If $b(v) > 0$, then v is a source, if $b(v) < 0$ it's a sink, otherwise it's neither.
- Flow definition:
  1. $f(u,v) \leq c(u,v)$
  2. $f(u,v) = -f(v,u)$
  3. For all $u \in V$, $\sum_{v \in V} f(u,v) - \sum_{v \in V} f(v,u) = b(u)$
- Goal: minimize $\sum_{u,v \in V} f(u,v) * d(u,v)$
- A flow is given, goal is to find cheapest way to make this happen
- Necessary condition for a flow: $\sum_{v \in V} b(v) = 0$
- This condition does NOT mean that a flow exists that is legal
- To find a legal flow, we must solve the decision version of the max flow problem
- To solve this, add single s, t with capacities = b() values
- Solve the max flow problem
- Max-flow is a special case of min-cost flow. To solve max-flow using min-cost-flow solver:
- Set source to large positive b, sink to very negative b
- Add extra path from source->sink with huge cost

Big Idea 2

- Given an expensive, but legal flow, how to make cheaper?
- Augmenting flow around a cycle does not change the net flow anywhere.

- Goal: find a where the sum of the costs is negative
- Add one to flow around this cycle as long as capacities allow
- Keep finding these cycles. When no more cycles exist, min val found.