Department of Electrical and Computer Engineering
EECE 4542: Advanced Engineering Algorithms

Homework #2

Solve the following problems.

1. Define the following terms:
   (a) the complexity class NPC, and
   (b) a reduction from problem P_1 to problem P_2.

2. Argue that each of the following problems is or is not in NP (note that this is a tricky problem). A tour is a Hamiltonian Cycle.
   (a) Given a weighted graph $G = (V, E)$ and an integer k, is there at least one tour with cost $\leq k$?
   (b) Given a weighted graph $G = (V, E)$ and an integer k, is the number of tours with cost $\leq k$ equal to one million?
   (c) Given a weighted graph $G = (V, E)$ and an integer k, is the number of tours with cost $\leq k$ greater than or equal to one million?
   (d) Given a weighted graph $G = (V, E)$ and an integer k, does the one millionth-shortest tour have cost $\leq k$? (a tour t is the *n-shortest tour* if the number of tours whose cost is less than the cost of t is $n - 1$. Assume that no two different tours have the same cost.)
   (e) Given a weighted graph $G = (V, E)$ and an integer k, does the one millionth-shortest tour have cost $> k$? Assume that no two different tours have the same cost.

3. Prove that $HC \leq TSP$.

4. Prove that $SP \leq SS$.

5. Prove that for decision problems A_1 and A_2 such that $A_1 \leq A_2$, if $A_1 \notin P$ then $A_2 \notin P$.

6. Prove that for decision problems A_1 and A_2, if $A_1 \leq A_2$, $A_1 \in NPC$, and $A_2 \in NP$, then $A_2 \in NPC$.

7. Prove that for decision problems A_1 and A_2, if $A_1, A_2 \in NPC$, then $A_1 \leq A_2$ and $A_2 \leq A_1$.

8. Prove that for decision problems A_1 and A_2, if $A_1, A_2 \in NPC$ and $A_1 \in P$ then $A_2 \in P$.

9. Why is it considered unlikely that any NP-complete problem is tractable? Use the definition of NPC to give a precise argument.

10. Prove the following or give a counter example. Given decision problems $\Pi(D, Y)$ and $\Pi'(D', Y')$ such that Π' is a subproblem of Π, if Π' is NP-complete, then Π is NP-complete. Assume $\Pi \in NP$.