Lecture 5: Adaptive Filtering

Zach Neveu

October 3, 2019

LMS Algorithm

- Initialize filter $\theta_{-1} = 0$ (for all l filters)
- Choose step size μ
- For each time step
- Note: x_n means `x[n-l:]`
  - $e_n = y_n - \theta_n^T x_n$
  - $\theta_n = \theta_{n-1} + \mu e_n x_n$
- Parameters to choose: filter length, step size μ

Normalized LMS

- Idea: change step size based on how much energy is in signal
- Same initialization
- Select $0 < \mu < 2$
- For each time step:
  - same error
  - $\theta_n = \theta_{n-1} + \frac{\mu}{x_n^T x_n + \delta} e_n x_n$

1 Convergence

- LMS: small step size, slow convergence, but good optimal value
- LMS: large step size, fast convergence, bad optimal value
- LMS converges slower for colored noise than white noise
- NLMS: similar convergence rate to LMS but better final MSE
- NLMS generally preferred over LMLS
- APA: see ML ch 5.6 - similar type of algorithm
- LMS: Optimal step size: $0 < \mu < \frac{2}{\lambda_{max}}$ where λ is the eigenvalues of Σ_x
- LMS: fastest convergence step size: $\frac{2}{\lambda_{max} - \lambda_{min}}$

- Bias-Variance tradeoff: smaller μ varies less, but stays off by more on convergence.

2 Derivation of LMS

- Goal: develop an iterative scheme where the cost function $J(\theta_{n+1}) < J(\theta_n)$
- Require that cost function is differentiable
- Basics: LMS just SGD with a single example for the gradient.