Adaptive_Filters

February 28, 2020

```
In [1153]: import numpy as np
           import matplotlib.pyplot as plt

In [1154]: plt.style.use('seaborn')
           plt.rcParams['figure.figsize'] = 12,6
```

0.1 Define Constants

```
In [1404]: L = 21 # length of filter
           SeqLen = 1000 # length of test data
           SNR = 10

           lms_mu = 0.05

           nlms_mu = 1
           delta = 1e-3

           beta = .1
           reg = 1e-1
```

0.2 Generate Input Data

Run random data through a low-pass filter. LMS with same number of taps should be pretty good at guessing LPF response.

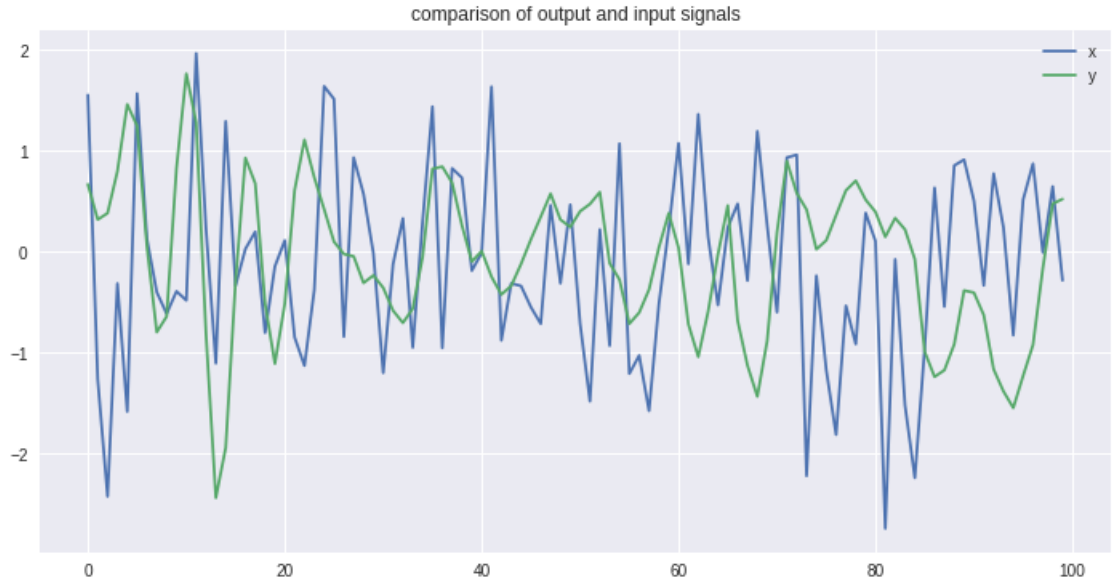
```
In [1405]: h = np.array([
                -0.02010411882885732,
                -0.05842798004352509,
                -0.061178403647821976,
                -0.010939393385338943,
                0.05125096443534972,
                0.033220867678947885,
                -0.05655276971833928,
                -0.08565500737264514,
                0.0633795996605449,
                0.310854403656636,
                0.4344309124179415,
                0.310854403656636,
```

```
        0.0633795996605449,
        -0.08565500737264514,
        -0.05655276971833928,
        0.033220867678947885,
        0.05125096443534972,
        -0.010939393385338943,
        -0.061178403647821976,
        -0.05842798004352509,
        -0.02010411882885732,
    ])
```

```
In [1406]: x = np.random.normal(0, 1, SeqLen+L-1)
           y = np.zeros(SeqLen)
           for samp in range(SeqLen):
               y[samp] = x[samp:samp+L]@h + np.random.normal(0,1/SNR)
```

```
In [1418]: plt.plot(x[L:100+L], label="x")
           plt.plot(y[:100], label="y")
           plt.title("comparison of output and input signals")
           plt.legend()
```

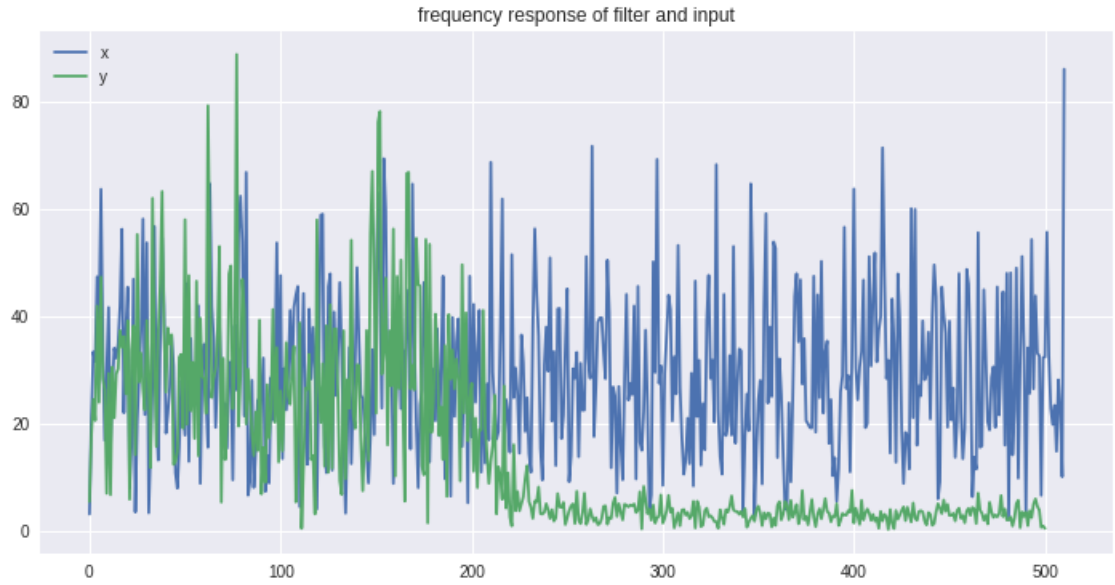
```
Out[1418]: <matplotlib.legend.Legend at 0x7fea4f80fd90>
```



Confirm that low pass filter is working

```
In [1417]: plt.plot(abs(np.fft.rfft(x)), label="x")
           plt.plot(abs(np.fft.rfft(y)), label="y")
           plt.title("frequency response of filter and input")
           plt.legend()
```

```
Out[1417]: <matplotlib.legend.Legend at 0x7fea4f87d850>
```



1 Create and use filters

```
In [1409]: class LMS:
               """ Least mean squares filter
                   Arguments:
                       L : length of filter in samples
                       mu: learning rate of filter
               """
               def __init__(self, L, mu):
                   self.w = np.zeros(L)
                   self.mu = mu

               def process(self, X, y):
                   yhat = self.w.T@X
                   e = y-yhat
                   self.w += self.mu * e * X
                   mse = e**2
                   return mse, yhat
```

```
In [1410]: class NLMS:
               """ NLMS filter class
                   Arguments:
                       L     : length of filter in samples
                       mu    : learning rate of filter
                       delta : factor to add for numerical stability
```

```
            """
            def __init__(self, L, mu, delta):
                self.w = np.zeros(L)
                self.mu = mu
                self.delta = delta

            def process(self, X, y):
                yhat = np.dot(self.w, X)
                e = y-yhat
                nu = self.mu / (self.delta + np.dot(X, X))
                self.w += nu * e * X
                mse = e**2
                return mse, yhat
```

In [1411]:
```
            class RLS:
                """ Recursive least squares filter
                    Arguments:
                        L    : length of filter in samples
                        beta : forgetting factor
                        reg  : regularization factor
                """
                def __init__(self, L, beta, reg):
                    self.w = np.zeros(L)
                    self.beta = beta
                    self.reg  = reg
                    self.P = 1/reg*np.eye(L,L)

                def process(self, X, y):
                    yhat = np.dot(self.w.T, X)
                    e = y-yhat

                    num = np.dot(np.dot(np.dot(self.P,X),X.T),self.P)
                    denom = self.beta + np.dot(np.dot(X,self.P),X.T)

                    self.P = 1/self.beta * (self.P - num/denom)
                    self.w += e*(self.P@X.T)
                    mse = e**2
                    return mse, yhat
```

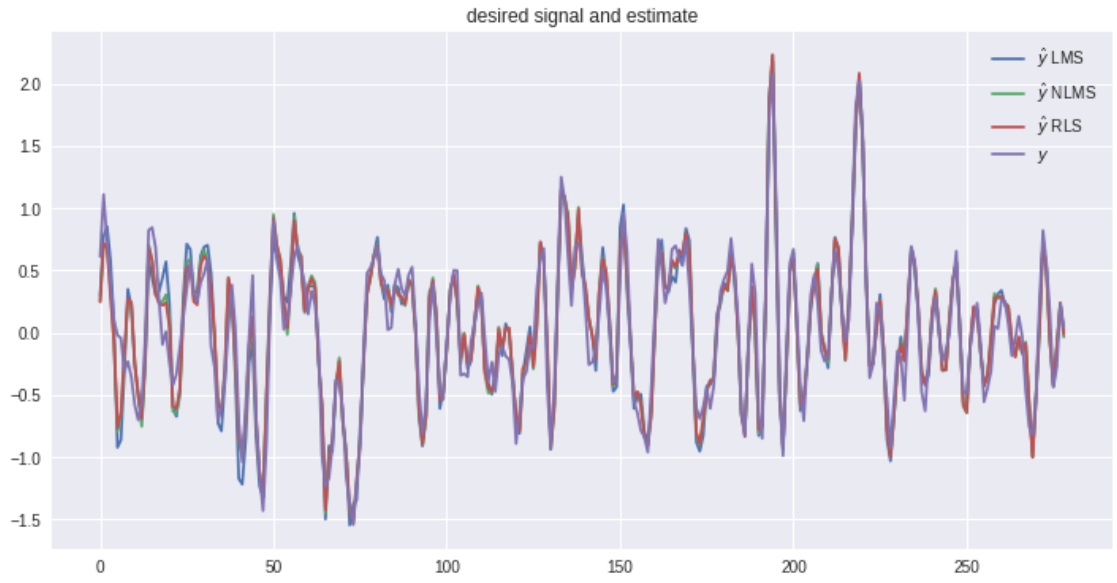
In [1412]:
```
            lms = LMS(L,lms_mu)
            nlms = NLMS(L, nlms_mu, delta)
            rls = RLS(L, beta, reg)
```

In [1413]:
```
            yhat_lms  = np.zeros(y.shape)
            yhat_nlms = np.zeros(y.shape)
            yhat_rls  = np.zeros(y.shape)
            es_lms    = np.zeros(y.shape)
```

```
    es_nlms    = np.zeros(y.shape)
    es_rls     = np.zeros(y.shape)
    for i in range(SeqLen):
        es_lms[i], yhat_lms[i] = lms.process(x[i:i+L], y[i])
        es_nlms[i], yhat_nlms[i] = nlms.process(x[i:i+L], y[i])
        es_rls[i], yhat_rls[i] = rls.process(x[i:i+L], y[i])
```

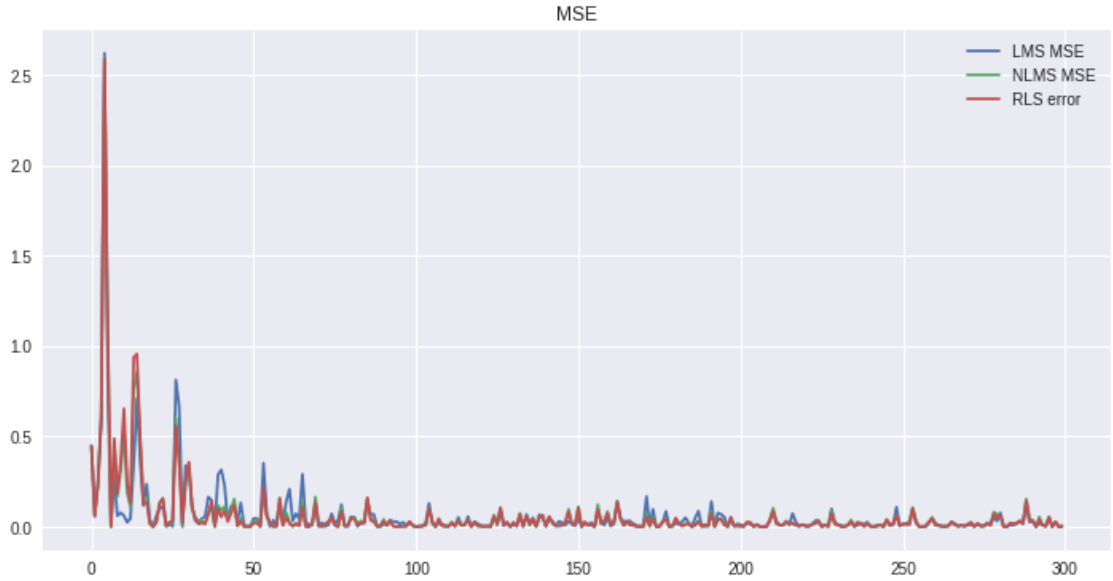
In [1414]:
```
npoints = 300
plt.plot(yhat_lms[L:npoints], label=r"$\hat{y}$ LMS")
plt.plot(yhat_nlms[L:npoints], label=r"$\hat{y}$ NLMS")
plt.plot(yhat_rls[L:npoints], label=r"$\hat{y}$ RLS")
plt.plot(y[L:npoints], label=r"$y$")
plt.title("desired signal and estimate")
plt.legend()
```

Out[1414]:
```
<matplotlib.legend.Legend at 0x7fea4f9e79d0>
```



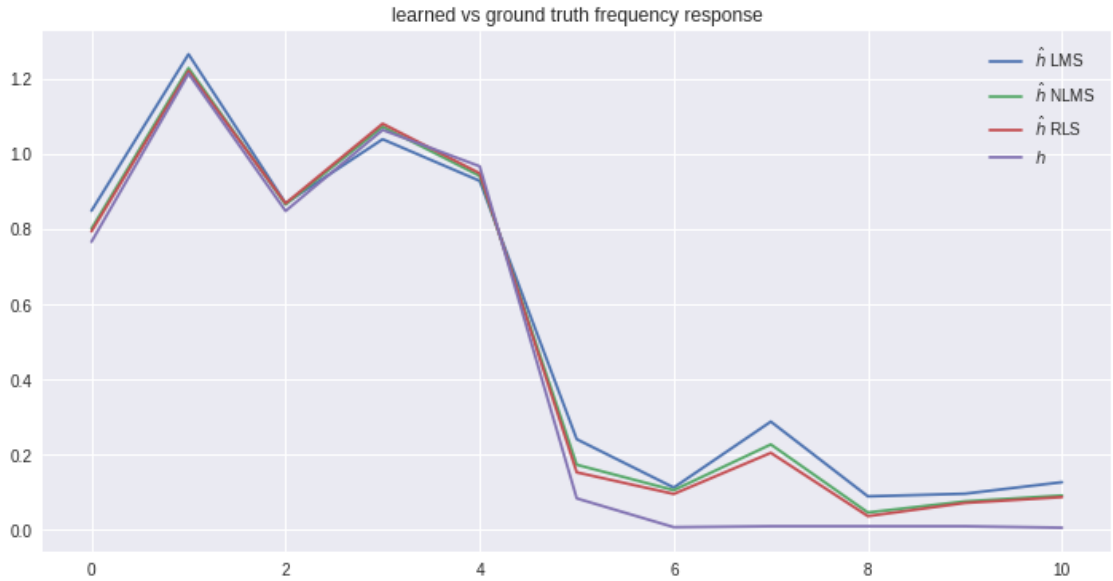
In [1415]:
```
nerrors = 300
plt.plot( es_lms[:nerrors], label="LMS MSE")
plt.plot(es_nlms[:nerrors], label="NLMS MSE")
plt.plot( es_rls[:nerrors], label="RLS error")
plt.legend()
plt.title("MSE")
```

Out[1415]:
```
Text(0.5, 1.0, 'MSE')
```



In [1416]:
```
plt.plot(np.abs(np.fft.rfft(lms.w)), label=r"$\hat{h}$ LMS")
plt.plot(np.abs(np.fft.rfft(nlms.w)), label=r"$\hat{h}$ NLMS")
plt.plot(np.abs(np.fft.rfft(rls.w)), label=r"$\hat{h}$ RLS")
plt.plot(np.abs(np.fft.rfft(h)), label=r"$h$")
plt.title("learned vs ground truth frequency response")
plt.legend()
```

Out[1416]: <matplotlib.legend.Legend at 0x7fea4f8965d0>



2 Inspect Loss from C++ Implementation

This section inspects the `.csv` files exported by the c++ testbench

```
In [1367]: import pandas as pd
```

```
In [1398]: lmsf = "./hls/lms_classes/solution1/csim/build/LMS_MSEs.csv"
           nlmsf = "./hls/nlms_classes/solution1/csim/build/NLMS_MSEs.csv"
           lmsdf = pd.read_csv(lmsf)
           nlmsdf = pd.read_csv(nlmsf)
```

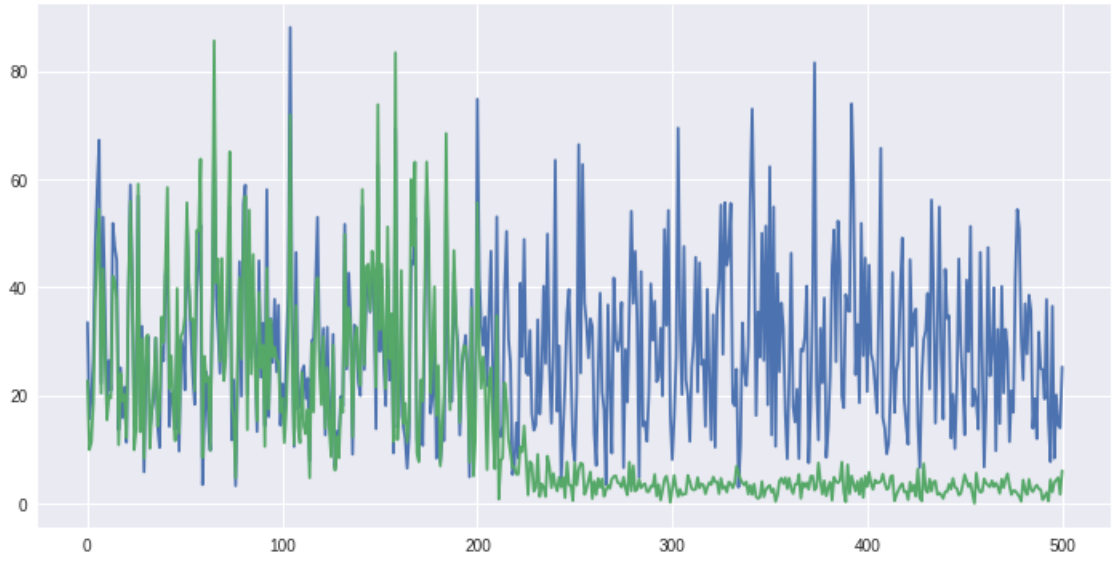
```
In [1399]: df = nlmsdf
```

```
In [1400]: nt = 200
           plt.plot(np.array(df.x[:nt]), label="x")
           plt.plot(np.array(df.y[:nt]), label="y")
           plt.legend()
```

```
Out[1400]: <matplotlib.legend.Legend at 0x7fea4fca3ad0>
```

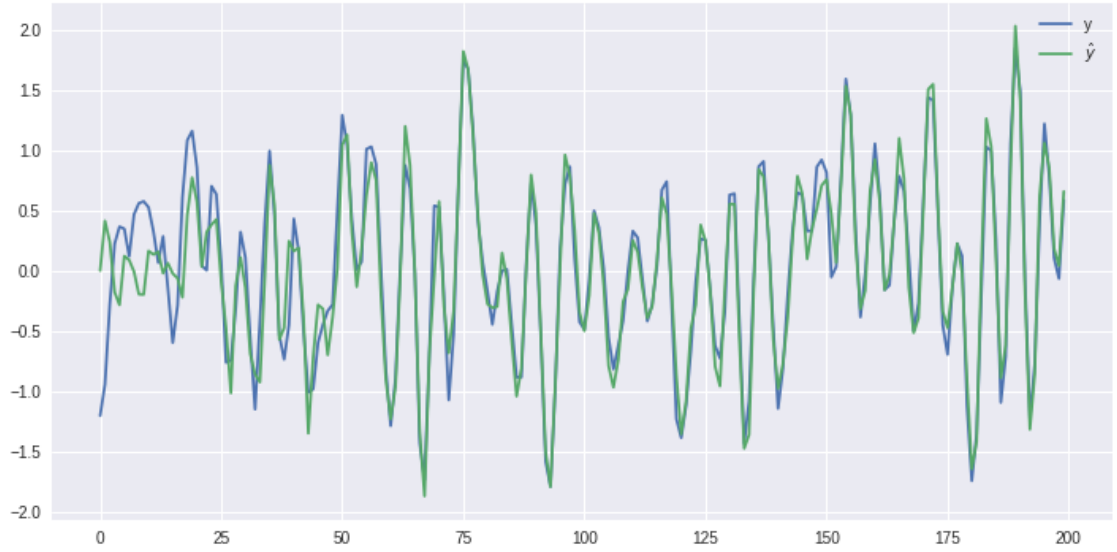

```
In [1401]: plt.plot(abs(np.fft.rfft(df.x)))
           plt.plot(abs(np.fft.rfft(df.y)))
```

```
Out[1401]: [<matplotlib.lines.Line2D at 0x7fea4fc5a910>]
```



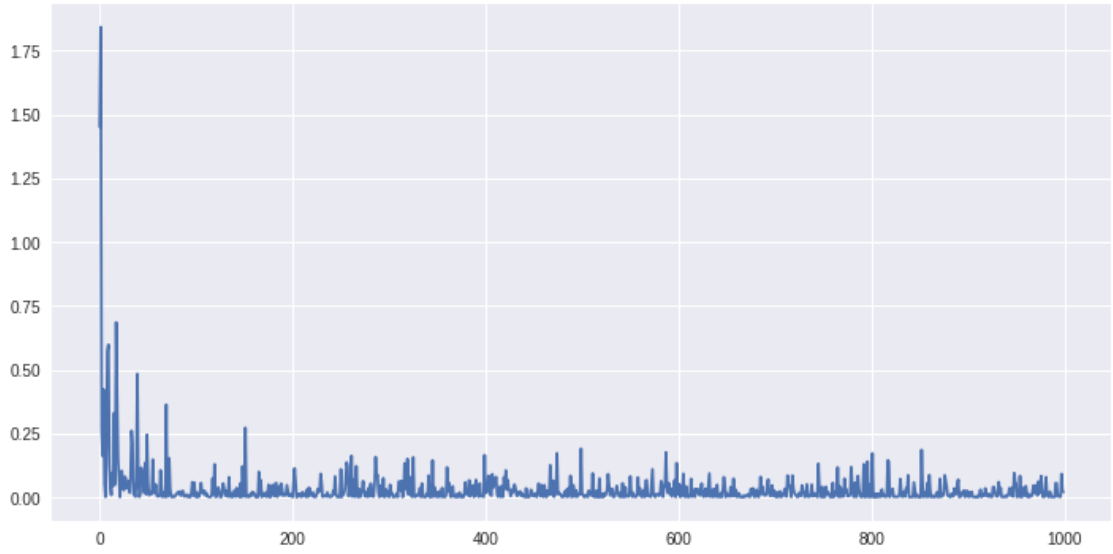
In [1402]:
```
plt.plot(df.y[:nt], label='y')
plt.plot(df.yhat[:nt], label=r'$\hat{y}$')
plt.legend()
```

Out[1402]: <matplotlib.legend.Legend at 0x7fea4fbe9510>



In [1403]:
```
plt.plot(df.MSE)
```

Out[1403]: [<matplotlib.lines.Line2D at 0x7fea4fb1cf90>]



In [ ]: