iSpy: Detection of Signals in Noise
(EECE4688)
Spring 2019

Homework 5
(Assigned Feb.14, 2019; due Feb.20, 2019 in class.)

Objective: The objective of this exercise is to experiment with binary hypothesis testing using multiple observations.

Task: The Matlab file `hwk5.mat` contains $N = 1000$ realizations of a random vector $\mathbf{Y}$. Each realization (each vector $\mathbf{y}$) has $M = 16$ elements corresponding to multiple observations. Your task is to design the detector, and make $N = 1000$ decisions, one for each realization $\mathbf{y}$. A decision is either 1, corresponding to "signal present," or 0, corresponding to "signal absent."

The detection problem is specified as follows:

$H_0 : \mathbf{Y} = \mathbf{Z}$
$H_1 : \mathbf{Y} = A\mathbf{s} + \mathbf{Z}$

The noise is circularly symmetric complex Gaussian, $\mathbf{Z} \sim \mathcal{CN}(\mathbf{0}, \sigma_{Z_1}^2 \mathbf{I})$. In other words, each element of $\mathbf{Z}$ consists of independent zero-mean Gaussian real/imaginary parts of variance $\sigma_{Z_1}^2/2$. The elements (different noise observations) are uncorrelated, as evident from the diagonal structure of the covariance matrix $\mathbf{C_Z} = E\{\mathbf{Z}\mathbf{Z}'\} = \sigma_{Z_1}^2 \mathbf{I}$. The amplitude A is real-valued and constant, and the vector $\mathbf{s}$ is defined as through a phase ϕ as

$$\mathbf{s} = \begin{bmatrix} 1 \\ e^{-j\phi} \\ e^{-j2\phi} \\ \vdots \\ e^{-j(M-1)\phi} \end{bmatrix} \tag{1}$$

This type of problem is found in radar/sonar, or any type of sounding or probing, when an array of M receivers is used to detect presence/absence of a signal. If the array receivers are separated by a distance d, and the signal of wavelength λ (frequency $f = c/\lambda$) arrives to the array from direction θ, the phase ϕ is given by

$$\phi = 2\pi \frac{d}{\lambda} \sin\theta \tag{2}$$

In the present problem, the direction of arrival is $\theta = 20°$, and $d = \frac{\lambda}{2}$. The variance of the noise is $\sigma_{Z_1}^2 = 1$.

1) Specify the relevant probability density functions $f_0(\mathbf{y})$ and $f_1(\mathbf{y})$.
2) State the likelihood ratio test (LRT).
3) Show that the LRT reduces to $\frac{1}{M} Re\{\mathbf{s}'\mathbf{y}\} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$. (Prime here denotes conjugate transpose.)
4) Determine the threshold γ so as to ensure probability of false alarm equal to $P_{fa}^* = 10\%$.
5) Apply the resulting Neuman-Pearson detection rule to the data stored in `hwk5.mat`. How many "signal present" decisions have you made?
6) Imagine that a genie comes to you after you've done all this work, and tells you that the signal was actually present in 8 out of 1000 realizations; specifically, in realizations 148 607 624 626 645 711 727 and 780. Did you have any missed detections? If so, in which realizations did they occur?

Reporting: Your report should be typed, and not exceed two single-sided pages. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include any Matlab code as an appendix. Please put your name on top of the report.