iSpy: Detection of Signals in Noise
(EECE4688)
Spring 2019

Homework 6
(Assigned Mar. 27, 2019; due Apr. 3, 2019 in class.)

Objective: The objective of this exercise is to review various types of estimation.

Task: The vector $\mathbf{Y}$ of random variables $Y_m,\ m = 1, \dots M$ is know to depend on some quantity θ as $\mathbf{Y} = \mathbf{g}\theta + \mathbf{Z}$, where $\mathbf{g}$ is a known deterministic vector of size $M \times 1$ and $\mathbf{Z}$ is zero-mean noise. Given an observation $\mathbf{y}$ of Y, our goal is to estimate θ.

1) Specify the least squares estimate $\hat{\theta}_{LS}$.
2) If the noise covariance $\mathbf{C_Z}$ is known, specify the minimum variance unbiased estimate $\hat{\theta}_{MVUB}$.
3) If θ is treated as random with $E\{|\theta|^2\} = R_\theta$, specify the minimum mean squared error linear estimate $\hat{\theta}_{MMSE,lin}$.
4) If $\mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \mathbf{C_Z})$, specify the maximum likelihood estimate $\hat{\theta}_{ML}$.
5) If $\mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \mathbf{C_Z})$, and θ is known to be either θ_0 or θ_1, specify the ML rule that can be used to decide between the two possibilities. Can you interpret this decision rule as consisting of two steps, one in which an estimate of θ is formed, and another in which this estimate is compared to a threshold?
6) If $\mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \mathbf{C_Z})$ and $\theta \sim \mathcal{CN}(0, \sigma_\theta^2)$, show that $\theta|Y \sim \mathcal{CN}(\frac{\sigma_\theta^2}{1+\sigma_\theta^2 \mathbf{g}'\mathbf{C_Z}^{-1}\mathbf{g}}\mathbf{g}'\mathbf{C_Z}^{-1}\mathbf{y}, \frac{\sigma_\theta^2}{1+\sigma_\theta^2 \mathbf{g}'\mathbf{C_Z}^{-1}\mathbf{g}})$, and specify the following estimates:
   a) Minimum mean squared error estimate $\hat{\theta}_{MMSE}$
   b) Minimum absolute squared error estimate $\hat{\theta}_{ABS}$
   c) Maximum a-posteriori probability estimate $\hat{\theta}_{MAP}$

Reporting: Your report should be typed, and not exceed two single-sided pages. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include any Matlab code as an appendix. Please put your name on top of the report.