iSpy: Detection of Signals in Noise
(EECE4688)
Spring 2019

Homework 2
(Assigned Jan. 17, 2019; due Jan. 24, 2019 in class.)

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1) In this problem, we study detection of a signal in noise (signal present/absent problem), but the signal itself is random. Specifically, under hypothesis 0, the observation Y contains only the noise Z, which is zero-mean Gaussian with variance σ_Z^2. Under hypothesis 1, the observation Y contains the signal A in noise Z. The signal is Gaussian distributed with zero mean and variance σ_A^2. (Hopefully, $\sigma_A^2 > \sigma_Z^2$.) The signal is independent of the noise.
   (i) Develop the maximum likelihood detection rule.
   (ii) If the signal to noise ratio (SNR) is $\sigma_A^2/\sigma_Z^2 = 10$ dB, determine the probability of false alarm P_{fa} and the probability of missed detection P_{md}. What is the corresponding probability of error P_e?
   (iii) Use Matlab to plot P_{fa} and P_{md} vs. the SNR.

2) In this problem, we study the usual case of detecting a deterministic signal in noise (signal present/absent problem), but the noise is non-Gaussian. Specifically, under hypothesis 0, the observation Y contains only the noise Z, which is distributed according to $f_Z(z) = \frac{\lambda}{2}e^{-\lambda|z|}$. This distribution is called Laplacian. In particular, the noise Z has zero mean and variance $\sigma_Z^2 = E\{Z^2\} = \frac{2}{\lambda^2}$. Under hypothesis 1, the observation Y contains the signal A in noise Z. The signal amplitude A is known.
   (i) Develop the maximum likelihood detection rule.
   (ii) If the signal to noise ratio (SNR) is $\sigma_A^2/\sigma_Z^2 = 10$ dB, determine the probability of false alarm P_{fa} and the probability of missed detection P_{md}. What is the corresponding probability of error P_e?
   (iii) Use Matlab to plot P_{fa} and P_{md} vs. the SNR.

3) Consider the detection problem of Homework 1, where the signal present/absent decision has to be made in zero-mean Gaussian noise. The prior probabilities are $P_0 = 0.3$, $P_1 = 0.7$, and the SNR is 3 dB. Compare P_{fa}, P_{md} and P_e of maximum likelihood (ML) detection with those of maximum a-posteriori (MAP) detection.

4) Imagine that the MAP detector of the previous problem has incorrect information about P_0, P_1. Namely, let us say that the detector is implemented using 0.1 and 0.9 instead of the actual 0.3 and 0.7 values for P_0 and P_1, respectively. What will be the probability of false alarm?

Reporting: Your report should be typed, and not exceed two single-sided pages. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include any Matlab code as an appendix. Please put your name on top of the report.